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**Rui Francisco Pereira Moital Loureiro da Cruz**

**An XFEM element to model intersections between  
hydraulic and natural fractures in porous rocks**

**TESE DE DOUTORADO**

Thesis presented to the Programa de Pós-graduação em Engenharia Civil of PUC-Rio in partial fulfillment of the requirements for the degree of Doutor em Engenharia Civil

Advisor: Prof<sup>a</sup>. Deane Roehl  
Co-advisor: Prof. Eurípedes Vargas

Rio de Janeiro  
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## Abstract

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A large number of hydrocarbon reservoirs are naturally fractured. When subjected to hydraulic fracturing treatments, the natural fractures may influence the propagation of the hydraulic fracture, which can grow in a complicated manner creating complex fracture networks in the reservoir. In order to better understand and simulate such phenomena an element based on the eXtended Finite Element Method is proposed. The element formulation comprises fracture intersection and crossing, fracture frictional behaviour, fully coupled behaviour between displacements, pore and fracture fluid pressure, leak-off from the fracture to the surrounding medium and the eventual loss of pressure due to filter cake. The theoretical background and implementation aspects are presented. A set of analyses is performed in order to validate different features of the implemented element. Finally, the results of four practical applications are analysed and discussed: two laboratory hydraulic fracture tests, hydraulic fracture propagation in a multi-fractured synthetic model and percolation through a dam fractured foundation. It is concluded that the implemented code provides very good predictions of the coupled fluid-rock fracture behaviour and is capable of correctly simulating the interaction between hydraulic and natural fractures. Moreover, it is shown that the hydraulic behaviour of the models and the intersection between fractures are very sensible to parameters such as differential in-situ stresses, angle between fractures, initial hydraulic aperture and fracture face transversal conductivity.

## Keywords

Finite Element Method; eXtended Finite Element Method; Hydraulic Fracturing; Intersection between hydraulic and natural fractures

## Resumo

Cruz, Rui Francisco Pereira Moital Loureiro da Cruz; Roehl, Deane; Vargas, Eurípedes. **Um elemento XFEM para modelar intersecções entre fraturas hidráulicas e naturais em rochas porosas**. Rio de Janeiro, 2018. 225p. Tese de Doutorado - Departamento de Engenharia Civil, Pontifícia Universidade Católica do Rio de Janeiro.

Um elevado número de reservatórios de hidrocarbonetos é naturalmente fraturado. Quando sujeitos a estimulação hidráulica, as fraturas naturais podem influenciar a propagação da fratura hidráulica, que pode tomar uma forma geométrica complexa, criando redes de fraturas no reservatório. De forma a melhor entender e simular tais fenômenos, um elemento baseado no Método dos Elementos Finitos Estendidos (XFEM) é proposto. A formulação do elemento inclui interseção e cruzamento entre fraturas, atrito entre as faces das fraturas, comportamento acoplado entre deslocamentos, poro-pressões e pressões do fluido da fratura, absorção de fluido da fratura para o meio poroso (leak-off) e a eventual perda de pressão nas faces da fratura (filter cake). Os fundamentos teóricos e os aspectos relevantes da implementação são apresentados. Um conjunto de análises é realizado de forma a validar em separado as diferentes funcionalidades do elemento implementado. Finalmente, os resultados de quatro aplicações práticas são analisados e discutidos: dois conjuntos de ensaios de laboratório de interseção de fratura, propagação de fratura hidráulica num modelo sintético multi-fraturado e percolação na fundação fraturada de uma barragem. Conclui-se que o código implementado fornece previsões muito boas do comportamento acoplado do meio fraturado e tem capacidade de simular corretamente a interação entre fraturas hidráulicas e naturais. Pode também verificar-se que o comportamento hidráulico dos modelos e a propagação e interseção de fraturas são muito influenciados por parâmetros tais como o diferencial de tensões in-situ, ângulo entre fraturas, a abertura hidráulica das fraturas e a condutividade transversal das faces da fratura.

## Palavras Chave

Método dos Elementos Finitos; Método dos Elementos Finitos Estendidos; Fraturamento Hidráulico; Interseção entre fraturas hidráulicas e naturais



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