

**Bruno Cesar Cayres Andrade**

**Nonlinear dynamic analysis of dry  
friction-induced torsional vibration  
in a drill-string experimental set-up**

**TESE DE DOUTORADO**

**DEPARTAMENTO DE ENGENHARIA MECÂNICA**  
**Programa de Pós-graduação em Engenharia**  
**Mecânica**

Rio de Janeiro  
August 2018



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### Bruno Cesar Cayres Andrade

The author was born in June 14th, 1988 in Belém - Pa, Brazil. In 2006, he started to study Mechanical Engineering at Universidade Federal do Pará (UFPA), Brazil, and graduated in 2011. For his performance, the author won the Kawaguchi Awards - which is an awards to the best student. Meanwhile, in 2009-2010, he partially studied at *Institut Polytechnique de Grenoble* (INP-G) - France. His master's dissertation was entitled "Numerical and experimental analysis of nonlinear torsional dynamics of a drilling system" and was conducted under supervision of Prof. Dr.-Ing. Hans I. Weber and Dr. Romulo R. Aguiar at PUC-Rio. In the same university, in September 2013, he started his doctoral research in Applied Mechanics under supervision of Prof. Dr.-Ing. Hans I. Weber. The results and contributions are presented in this thesis which focused in nonlinear dynamics of the drill-string experimental set-up. Since 2013, the author is lecturer at CEFET/RJ.

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## Abstract

Andrade, Bruno Cesar Cayres; Weber, Hans Ingo (Advisor). **Non-linear dynamic analysis of dry friction-induced torsional vibration in a drill-string experimental set-up**. Rio de Janeiro, 2018. 101p. Tese de Doutorado - Departamento de Engenharia Mecânica, Pontifícia Universidade Católica do Rio de Janeiro.

The latter round bids of the pre-salt for exploration and production of oil and natural gas in Brazil indicate the drilling operations will become more intense in coming years. The rotational drilling process is largely used to reach the oil reservoirs and because of diameter-to-length ratio of the drilling system, torsional vibration mode is present in most all drilling processes and may reach an undesired severe stage: the stick-slip phenomenon. In order to address this problem, the torsional vibration mode is isolated and the stick-slip is observed in a fully instrumented drill-string experimental set-up in this work. During this phenomenon, another torque may be applied on an intermediate position of the test bench. The lumped parameter mathematical model is obtained and it is compared to experimental data to validate whether the mathematical model represents the experimental apparatus. A stability analysis is performed using the validated mathematical model in order to identify stable solutions of the system. Therewith, one observed that there is a range of the bifurcation parameter in which stable equilibrium and periodic solutions may coexist. For a given stick-slip situation in bi-stability range, two mitigation strategies of torsional vibration were considered which consisted of imposing perturbations in the system via torques on the intermediate position of the test bench: (i) torques applied only against the direction of motion of the system, and (ii) torques applied in both directions. The strategies were tested numerically and presented efficiency so that the stick-slip was completely mitigated: the energies of the system and the work created by the intermediate torque were compared in order evaluate the feasibility and reasonableness of the strategy. Experimentally, the system continued to oscillate, however it presented a significant reduction of stick phase even with limitations of torque applications.

## Keywords

Drill-string dynamics; Stick-slip phenomenon; Nonlinear dynamics; Stability analysis; Mitigation strategy.



## Resumo

Andrade, Bruno Cesar Cayres; Weber, Hans Ingo (Orientador). **Análise da dinâmica não linear de uma bancada experimental de uma coluna de perfuração com vibração torcional induzida por atrito**. Rio de Janeiro, 2018. 101p. Tese de Doutorado - Departamento de Engenharia Mecânica, Pontifícia Universidade Católica do Rio de Janeiro.

Os últimos leilões do pré-sal para exploração e produção de petróleo e gás no Brasil indicam que as operações de perfuração se tornarão mais intensas nos próximos anos. O processo de perfuração rotativo é amplamente utilizado para alcançar os reservatórios de petróleo e devido à relação diâmetro/comprimento do sistema de perfuração, o modo de vibração torcional está presente em quase todos os processos de perfuração, podendo chegar a um estado crítico indesejável: o fenômeno de *stick-slip*. Com o intuito de abordar este problema, o modo torcional é isolado e o *stick-slip* é observado em uma coluna de perfuração em escala reduzida completamente instrumentada. Durante o *stick-slip*, outro torque pode ser aplicado em uma posição intermediária da bancada de teste. O modelo matemático de parâmetros concentrados é obtido e o modelo é comparado com dados experimentais com o propósito de verificar se o modelo matemático representa o aparato experimental. Uma análise de estabilidade é feita usando o modelo validado com o objetivo de identificar soluções estáveis do sistema. Com isso, observou-se que existe uma faixa do parâmetro de bifurcação na qual soluções de equilíbrio e periódicas estáveis coexistem. Para uma dada situação de *stick-slip* na faixa de biestabilidade, duas estratégias de mitigação de vibração torcional foram consideradas e consistiram em impor perturbações no sistema por meio do torque na posição intermediária da bancada de teste: (i) torques aplicados apenas contra a direção de movimento do sistema, e (ii) torques aplicados em ambas as direções. As estratégias foram testadas numericamente e apresentaram eficiência de tal modo que o *stick-slip* foi completamente mitigado: as energias do sistema e o trabalho gerado pelo torque intermediário aplicado foram comparados com o propósito de avaliar a factibilidade e razoabilidade da estratégia. Experimentalmente, o sistema continuou a oscilar, porém apresentou uma significativa redução na fase de *stick* mesmo com limitações de aplicações de torque.

## Palavras-chave

Dinâmica de coluna de perfuração; Fenômeno de stick-slip; Dinâmica não linear; Análise de estabilidade; Estratégia de mitigação.

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## Nomenclature

### List of abbreviations

APL	amplitude pulse level
BHA	bottom hole assembly
SRPM	surface rotation per minute
WOB	weight on bit

### List of symbols

$\delta_{12}$	angular phase between $R_1$ - $R_2$
$\delta_{23}$	angular phase between $R_2$ - $R_3$
$\dot{\theta}_1$	angular velocity of $R_1$
$\dot{\theta}_2$	angular speed of $R_2$
$\dot{\theta}_3$	angular speed of $R_3$
$\dot{\theta}_m$	angular velocity of the $J_m$
$\eta$	transmission factor
$\mathbb{H}^*$	Hurwitz matrix
$\mathbb{J}^*$	Jacobian matrix
$\mu_k$	kinetic coefficient of friction
$\mu_s$	static coefficient of friction
$\omega_{ref}$	reference angular velocity
$\tau_m$	motor output torque
$\tau_s$	torque provided by the DC-motor
$\theta_1$	angular displacement of $R_1$
$\theta_2$	angular displacement of $R_2$
$\theta_3$	angular displacement of $R_3$
$\xi_i$	damping ratio (1,2)
$C_m$	speed regulation constant of the DC-motor
$D_1$	diameter of Disc 1
$D_2$	diameter of Disc 2
$d_1$	damping between $R_1$ - $R_2$



$d_2$	damping between $R_2$ - $R_3$
$E_{k_i}$	relative kinetic energy of Disc i (1,2)
$E_{p_i}$	relative kinetic energy of spring $k_i$ (1, 2)
$g$	gravitational acceleration
$i$	electric current of the DC-motor
$J_1$	inertia of the Disc 1
$J_2$	inertia of the Disc 2
$J_3$	inertia of the motor including the transmission factor
$J_m$	inertia of the motor
$K_E$	voltage constant of the DC-motor
$k_i$	integral constant of the DC-motor
$k_p$	proportional constant of the DC-motor
$K_T$	torque constant of the DC-motor
$k_1$	stiffness of the shaft between $R_1$ - $R_2$
$k_2$	stiffness of the shaft between $R_2$ - $R_3$
$L$	armature inductance of the DC-motor
$m_1$	mass of Disc 1
$m_2$	mass of Disc 2
$N_1$	normal force at Disc 1
$N_2$	normal force at Disc 2
$P_C$	Pearson correlation coefficient
$R$	armature resistance of the DC-motor
$R_1$	Disc 1
$r_1$	distance between the contact point of the pin and the geometric center of rotation of disc $R_1$
$R_2$	Disc 2
$r_2$	distance between the contact point of the pin and the geometric center of rotation of disc $R_2$
$R_3$	Disc 3 (DC-motor)
$T_f$	internal friction torque of the DC-motor
$T_{r_1}$	torque on Disc 1
$T_{r_2}$	torque on Disc 2

*A vaidade cega a sabedoria.*

**Matias Aires.**