A
Método dos Multiplicadores de Lagrange

O método dos Multiplicadores de Lagrange converte problemas de minimização com restrição em problemas sem restrição, através da inserção de um novo parâmetro: o multiplicador de Lagrange - λ. As equações seguem a mesma formulação apresentada por Haykin em [45].

Um problema de otimização com restrição pode ser resumido da seguinte forma:

minimize uma função custo (real), submetida a uma função limitadora

Considere a função $f(\mathbf{w})$ a ser minimizada e submetida à restrição:

$$\mathbf{w}^H\mathbf{s} = g \tag{A-1}$$

onde g é uma constante complexa, w um vetor e s um vetor dado. Introduzindo uma nova função $c(w)$ pode-se reescrever a equação restritiva:

$$\begin{aligned} c(\mathbf{w}) &= \mathbf{w}^H\mathbf{s} - g \\ &= 0 + j0 \end{aligned} \tag{A-2}$$

Para transformar esse problema em uma minimização sem restrição, uma nova função custo é formulada:

$$h(\mathbf{w}) = f(\mathbf{w}) + \lambda_1 Re[c(\mathbf{w})] + \lambda_2 Im[c(\mathbf{w})] \tag{A-3}$$

onde λ_1 e λ_2 são multiplicadores reais de Lagrange e a função $c(w)$ é dada por:

$$c(\mathbf{w}) = Re[c(\mathbf{w})] + jIm[c(\mathbf{w})] \tag{A-4}$$

O multiplicador complexo de Lagrange pode ser definido em função de λ_1 e λ_2:

$$\lambda = \lambda_1 + j\lambda_2 \tag{A-5}$$

É possível, então, reescrever a equação A-3:

$$h(\mathbf{w}) = f(\mathbf{w}) + Re[\lambda^* c(\mathbf{w})] \tag{A-6}$$

O passo final é minimizar a função $h(\mathbf{w})$ em relação ao vetor w, fazendo $\partial h/\partial \mathbf{w}^* = 0$.

$$\frac{\partial f}{\partial \mathbf{w}^*} + \frac{\partial (Re[\lambda^* c(\mathbf{w})]}{\partial \mathbf{w}^*} = \mathbf{0} \tag{A-7}$$

A solução das equações acima e da equação A-2 define o vetor ótimo w e o multiplicador λ.

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