2
Literature review and preliminary concepts

2.1
Literature review

Oil well drill string dynamics has been the subject of many academic and industrial studies. In the literature, several works were published in order to understand the involved phenomena and propose methods to mitigate undesired drill strings behaviors.

Gnirk *et al.* [11], 1968, described the rotary drilling system by idealized conditions of bit-rock interactions. Dimensionless relationships between weight on bit, rotary speed, bit diameter, among others, were developed.

Johancsik [20],1984, developed a computational model of directional wells modeling the friction torque influence and rubbing borehole/casing of the drill string. In this article, if a friction coefficient is given then the torque and drag can be obtained. Similarly, if the torque and drag are given then the friction coefficient can be calculated.

As faster computers were developed, some finite elements models were created to better describe the drilling system. Brakel et al. [6], 1989, used this method to design trajectories of drilling considering bit/rock interactions and Bottom Hole Assembly (BHA). Brakel et al. presented a 3D model simulating the dynamic behavior of the BHA during drilling with two types of bit: tricone and PDC (Polycrystalline Diamond Compact).

Jogi *et el.* [19], 1992, presented a new model to identify the lithology and drilling conditions using MWD (Measurement While Drilling) data. The model was proposed to obtain information about lithological changes, formation porosity, pore pressure, tooth/ cutter and bearing conditions. The model provided ”drilling alerts” that identified inefficient drilling conditions. This model, called Drilling Response, was tested during drilling and with post-drilling data.

Pavone *et al.* [29], 1994, approached the stick-slip phenomena from experimental data and developed a friction model to describe the resistive torque. Also, they analyzed the stability around 34 RPM and 161 kN, as well as 125 RPM and 50 kN. In the end, two stick-slip prevention methods were observed: PID control and Anti Stick-Slip Tool. PID control avoided stick-slip but the system continued suffering high vibrations. The second method avoids stick-slip with a positive borehole friction slope and provided stability to the process.

Focusing on torsional dynamics of drill strings, Jansen *et al.* [17], 1995, presented a model considering the drill string as a torsional pendulum with two degrees- of-freedom (DOF). The inertia representing the rotary table rotating with constant velocity was driven by a DC motor. Jansen described his system in terms of relative displacements to avoid singularity. The friction torque on bit is modeled as a constant dynamic friction torque if bit velocity was greater than zero and is modeled as static friction torque if bit velocity turns equal to zero.

In 1995, Jansen *et al.* [18], proposed to eliminate stick-slip of a drilling system using active damping (top-drive). A control system was implemented to control the torque from surface aiming to adjust the velocity of the system. The control system was based on a hydraulic system. The purpose of this active damping was to operate as a dynamic absorber.

Yigit and Christoforou [41], 1998, present a coupled torsional and bending model of the drill string. Using lumped parameters they obtained the equations of motion in polar coordinates. The contact was modeled using a linear, or Hertzian stiffness. The rotary table speed was a function of time, i.e., it was not a prescribed excitation. The Torque on Bit (TOB) and Weight on Bit (WOB) are modeled as sinusoid functions of the bit speed. The author concluded that torsional vibrations affect the lateral vibrations and when the bit speed reaches three times the rotary table speed, the amplitudes of transverse vibrations are higher.

Abbassian *et al.* [1], 1998, described the application of a stability approach to three models of drill string. These models are: torsional model, lateral model and coupled torsional-lateral model. The first model was a two degree-of-freedom (DOF) without lateral motion. It underwent a reactive torque which varies with instantaneous bit velocity. To the steady-state solution in torsional case, the authors used the standard technique of the linear stability theory. The second model without torsional motion described a system in permanent contact with the wellbore around which it was assumed to spin on its own axis at a constant speed. The steady-state solution in the lateral model was represented by a forward synchronous motion of the bit. The coupled model was a combination of the two previous models. The solution for the coupled model was similar to the lateral model.

Aiming to improve the driller operations, Heisig et al. [15], 1998, described a real time tool in downhole which permited the driller to make adjustments to control the drilling process based on information from downhole. They concluded that the process feedback allows the driller not only to avoid dynamic problems downhole but also to optimize the drilling process for a higher Rate of Penetration (ROP).

Robnett *et al.* [32], 1999, described the use of magnetometer signals to identify stick-slip behavior downhole. They concluded that negative RPM indicates backward rotation of BHA and bit. Comparison of the differences between maximum and minimum RPM, the average RPM, and the time of backward rotation were implemented as measures of severity of torsional oscillations. The authors illustrated with examples of stick-slip with a PDC bit, stick-slip with a tricone bit, stick-slip after start of rotation, and torsional oscillations after top-drive stall. Also, some field data measurements are illustrated.

Hansen [14], 2002, approached mechanical systems with friction in control analysis. He mentioned some advantages and drawbacks of compensation techniques of friction. He also explained about tools of analysis for friction induced stick-slip phenomena summarizing the tools most frequently used, as well as friction models. Also, he presented an experimental Linear Motor Motion System (LiMMS) and analyzed the friction induced limit cycles. He used a PID controller to validate and compare the model predictions with the experimental apparatus.

Spanos *et al.* [36], 2004, described a drilling process and discussed about axial, torsional, lateral oscillations, as well as buckling and fatigue. Also, emerging techniques of analysis are remarked as a new form to approach the subject such as stochastic analysis and optimization analysis.

Franca [10], 2004, described a model to increase the rate of penetration (ROP) in hard rocks by means of percussive-rotary self-excited drilling. In this thesis, the influence of the torsional and axial vibrations on ROP and the study of drilling in resonance of the BHA were analyzed. He concluded that is possible to increase the ROP using drilling in resonance of the BHA but the rotary speed has to be well controlled to create a cyclic variation of the contact forces. An experimental apparatus to validate the numerical model was proposed.

López and Suárez [28], 2004, modeled the torsional dynamics of the drilling system as a simple torsional pendulum driven by an electric motor. The dynamics of the motor system were not considered. The resistive friction torque on bit was modeled by a dry friction model. In this article, an analysis of different types of friction torque models is presented and a discussion about the influence of model parameters in stick-slip was performed.

Mihajlović *et al.* [26], 2004, developed an experimental drill string with DC motor, power amplifier, two discs, a string and a brake device. This brake device was responsible for generating stick-slip. The angular positions of the upper and lower discs were measured using encoders and the velocities were calculated by numerical differentiation of the angular positions. The parameters (moments of inertia of the lower and upper discs, stiffness, as well as parameters of the friction torque model) were estimated by the response of the system when constant and white-noise input voltages are applied.

The following year, Mihajlović [24] presented his Ph.D. thesis about torsional and lateral vibrations in systems with friction. The author approached nonlinear aspects from the adopted friction model. Several methods to encounter stable and unstable branches of the bifurcation diagrams and the steady-state of the test rig are presented. Torsional and lateral vibrations are analyzed separately and when both appear in the system. The friction model is described as a discontinuous function. This work is strongly referenced throughout the this dissertation.

Alamo [3], 2006, developed a dynamic model of the drilling system by means of the Cosserat theory considering axial, lateral and torsional dynamics. A curved beam into the uniform cavity was designed as an experimental apparatus in order to study the dynamic behavior of a simplified curved drill string. Alamo concluded that Cosserat theory could be an efficient method to solve dynamic problems of slender structures using fewer nodes.

In 2007, Khulief *et al.* [21], 2007, developed a model consisting of coupled torsional-bending-axial vibrations by means of finite element method (FEM). Using 24 finite shaft elements, they described 1000 m of drill pipe and 200 m of drill collar. A full order model and a reduced order model were compared. They observed the response of the system when a lateral excitation was applied. Also, the behavior of the system in stick-slip was illustrated demonstrating a limit-cycling behavior.

Ritto [31], 2010, described a model of drilling system by means of FEM taking into account the uncertainties of system parameters (such as inertia, damping and stiffness) and bit- rock interaction parameters (parameters of the friction model used). He analyzed stochastic ROP, stochastic angular velocity on bit of the system in time and frequency domains, with an envelope of 95% of the mean value.

2.2 Preliminary concepts

Some preliminary concepts are necessary to understand all the procedures and analysis adopted in this dissertation. The Parameter estimation method and nonlinear dynamic definitions and tools are presented in this section.

Several methods of parameter estimation have been created over the years. Kalman filter, extended Kalman filter, linear and nonlinear least-square techniques, Gaussian estimation, among others are examples of techniques to estimate parameters. The estimators can be divided in non-recursive and recursive methods. Non-recursive estimators are those that collect all the data before of the estimation process. By contrast, recursive estimators provide an estimate of the parameters continually as the data are updated. Herein, the parameter estimation is achieved by means of a non-recursive nonlinear least-square technique which is presented in section 2.2.1. Inverse problem solutions and estimation techniques are more thoroughly described in references [34, 38, 39].

Nonlinear behavior is present in several mechanical systems. Aiming for a best understanding about these behaviors, many analytical and numerical tools are widely used. Several works approach the nonlinear aspects of the drilling systems in order to analyze its stability, prediction and estimation of steady-state in oil field operation. Nonlinear dynamic conceptions are richly found in references [13, 35, 37, 40]. Also, nonlinear dynamics of drilling systems are presented in the references of this dissertation.

2.2.1 Least-square technique

Usually, in theoretical environment, the variables are encountered when the parameters are known. By contrast, the experimentalists use the inverse problem solution, i.e the parameters can be estimated by given (or measured) variables. Least-square technique is one of several methods of solution for inverse problems. Bayesian inference provides the theory to relate observed results with theoretical predictions [38]. In the context of this dissertation, Eq. 2-1 shows the posterior probability density function (PDF) of parameters $\{x\}$ given measured variables $\{y\}$ from the experimental apparatus,

$$p(\{x\}|\{y\}) \approx \aleph\, p(\{y\}|\{x\}), \tag{2-1}$$

where $\aleph = p\{x\}/p\{y\}$ is the normalization constant (for constants $p\{x\}$ and $p\{y\}$) and $p(\{y\}|\{x\})$ is the forward PDF. Now, $\{y\} = \{\hat{y}_{(\{x\})}\} + \{n\}$ is defined where $\{\hat{y}_{(\{x\})}\}$ is the data from the theoretical model and $\{n\}$ is a gaussian noise.

Thus, the likelihood function is defined as

$$\begin{aligned} P(\{y\}|\{x\}) &= P(\{n\} = \{y\} - \{\hat{y}_{(\{x\})}\}) \\ &= \frac{1}{(2\pi)^{N/2}\sqrt{det(\Gamma)}} e^{-\frac{1}{2}(\{y\}-\{\hat{y}_{(\{x\})}\})^T\Gamma^{-1}(\{y\}-\{\hat{y}_{(\{x\})}\})}, \end{aligned} \quad (2\text{-}2)$$

where Γ corresponds to the covariance matrix and N is the number of samples. The superscript T denotes transposition.

The least-square technique assumes that measuring errors have Gaussian distribution and the trials are independent identically distributed (i.i.d.). This assumption means that the covariance matrix Γ is diagonal and $\Gamma_{ii} = \sigma^2$. Thus, $\Gamma = \sigma^2 \underline{\mathbf{I}}$ e $\Gamma^{-1} = \frac{1}{\sigma^2} \underline{\mathbf{I}}$, where σ^2 is the variance. Herein, $\underline{\mathbf{I}}$ denotes identity matrix.

The functional shape of the Gaussian distribution is an unimodal function (one peak), thus the maximum of the likelihood function drives to the minimum of its exponent, called misfit function $\varepsilon(\{x\};\{y\})$ (Eq. 2-3).

$$\varepsilon(\{x\};\{y\}) = \frac{1}{\sigma^2}(\{y\} - \{\hat{y}\}_{(\{x\})})^T(\{y\} - \{\hat{y}_{(\{x\})}\}). \quad (2\text{-}3)$$

Summarizing, least-squares technique is equivalent to the maximum likelihood function with independent identically distributed Gaussian noise. The Bayesian theory and parameter estimation applied to mechanical system are thoroughly described in reference [34].

2.2.2
Nonlinear dynamics concepts

In a broader context of dynamics, there are two types of systems: differential equations and iterated maps. The first one describes the evolution of the system in continuous time, whereas in the second one the time is discrete. For differential equation representations, a nonlinear dynamical system can be expressed as

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}), \quad (2\text{-}4)$$

where $\mathbf{x} \in \Re^n$ contains the state with initial condition $\mathbf{x}_0$, $t \in \Re$ and $\mathbf{f}: \Re^n \to \Re^n$ is the vector field. The overdot denotes differentiation with respect t.

To find the solution of Eq. 2-4, the Lagrangian coordinates are adopted. These coordinates describe the trajectory of an imaginary particle that places in x_0 position at t_0, i.e $\{\mathbf{x}(\mathbf{x_0}, t)| - \infty < t < \infty\}$ is the trajectory and represents the solution of the differential equation starting x_0 at time t_0. This solution is called *flow*. Thereby, the dependence of $\mathbf{x}$ with respect to its initial conditions was evidenced to achieved the solution. By contrast, the Eulerian coordinates propose a description of motion in terms of the space coordinates. This description observes what is occurring at a fixed point in space over time. This type of approach is typically applied in studies of fluid mechanics where the properties of greatest interest is the rate of changes of the kinematics of the motion than the shape of the body of fluid in a given time.

In order to have an easy understanding, for solutions of dynamical systems as described by Eq. 2-4, a fixed point is an equilibrium point when

$$\mathbf{f}(\mathbf{x}^*) = 0, \tag{2-5}$$

and in cases of periodic solutions, $\mathbf{x}_p$ is a solution if

$$\mathbf{x}_p(\mathbf{x}_0, t) = \mathbf{x}_p(\mathbf{x}_0, t + T), \tag{2-6}$$

where T represents the period of the solution.

A periodic solution is isolated if other periodic solution does not exist in its neighborhood. An isolated periodic solution, in autonomous systems, is called *limit cycle* [24]. Further, it is possible to find a steady-state solution that is a combination of periods and the relation between these periods (T_1/T_2, for instance) is an irrational number. These type of motions are called quasi-periodic solutions.

Addressing steady-state behavior, the stability of the solutions of dynamical systems are often investigated. There are several types of stability of differential equations, however, stability around an equilibrium solution is one of the most important type. Stable solutions are approached in theory of Lyapunov.

In general words, if $\mathbf{x}_{eq}$ is a steady-state solution and all solutions of the dynamical system that start close by $\mathbf{x}_{eq}$, remain close by $\mathbf{x}_{eq}$ in $t \to \infty$, then $\mathbf{x}_{eq}$ is Lyapunov stable (or stable in the sense of Lyapunov). If a solution is Lyapunov stable but not attracting, it is called neutrally stable. Furthermore, if this solution $\mathbf{x}_{eq}$ is asymptotically stable then all solutions that start around it converge to $\mathbf{x}_{eq}$ when $t \to \infty$. Otherwise, the solution is unstable.

Mathematically, the equilibrium solution $\mathbf{x}_S(x_{S0}, t)$ is Lyapunov stable with initial condition $\mathbf{x}_{S0}$ if, for every $\epsilon > 0$ there is a $\delta = \delta(\epsilon)$ such as

$$||\mathbf{x}_0 - \mathbf{x}_{S0}|| < \delta(\epsilon) \longrightarrow ||\mathbf{x}(\mathbf{x}_0, t) - \mathbf{x}_S(\mathbf{x}_{S0}, t)|| < \epsilon, \tag{2-7}$$

for every $t \geq t_0$ and asymptotically stable if,

$$||\mathbf{x}_0 - \mathbf{x}_{S0}|| < \delta(\epsilon) \longrightarrow \lim_{t \to \infty} ||\mathbf{x}(\mathbf{x}_0, t) - \mathbf{x}_S(\mathbf{x}_{S0}, t)|| = 0. \tag{2-8}$$

Nonlinear dynamical systems may hold more than one solution (equilibrium points and/or periodic solutions). Thereby, the solution $\mathbf{x}(\mathbf{x}_0, t)$ converges to one of those stable solutions, according the initial condition $\mathbf{x}_0$. Hence, those solutions are *locally stable*. The suite of related all initial conditions $\mathbf{x}_0$ which are solution of $\mathbf{x}(\mathbf{x}_0, t)$ of 2-4 converge to the equilibrium point (or periodic solution) over time is called as *basin of attraction* of the equilibrium point (or periodic solution). By contrast, if for all initial condition, the solution converge to one stable behavior over time, then this solution is *globally stable* [13, 24, 37].

However, in the oil industry, these stability concepts are different. For instance, if a drilling system presents vibrations (a periodic solution according to the theory of Lyapunov) then the system is unstable. Likewise, if the system does not present vibrations (an equilibrium solution according to the theory of Lyapunov).

The qualitative structure of the flow can change when certain parameter varies. Stable solutions can be created or destroyed, or become unstable. These parameters are called *control parameters* μ and these qualitative changes in the dynamics are known as *bifurcations*. The values in which these changes occur are called *bifurcation points*. The representation of the dynamical system presents a variant as following

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mu). \tag{2-9}$$

Graphically, the curves of the flow $\mathbf{x}_S(\mathbf{x}_{S0}, t)$ as function of μ are called *branches* and represent an equilibrium point or a periodic solution. These branches will be called *equilibrium branch* and *periodic branch*, respectively, as adopted in [24].

The Hopf bifurcation diagrams are addressed. This type of diagram appears when the Jacobian eigenvalues λ of the system around the equilibrium (linearization) has two complex conjugate roots which cross the imaginary axis from the left to right half-plane or vice versa (see Figure 2.1). It means there exists a system that settles down an equilibrium while μ changes for critical value μ_c. Thus, equilibrium state loses stability and a periodic solution appears around the former steady-state - *supercritical Hopf bifurcation*. Also, for a nonlinear system that holds an unstable equilibrium solution and, when $\mu = \mu_c$, it becomes stable appearing unstable oscillations is named *subcritical Hopf bifurcation*. Figure 2.2 illustrates these bifurcations. In engineering applications, this last one is potentially dangerous because it occurs drastically and may jump to a distant attractor, which may be a fixed point, another limit cycle, infinity or a chaotic attractor. *Degenerate Hopf bifurcation* is for a system

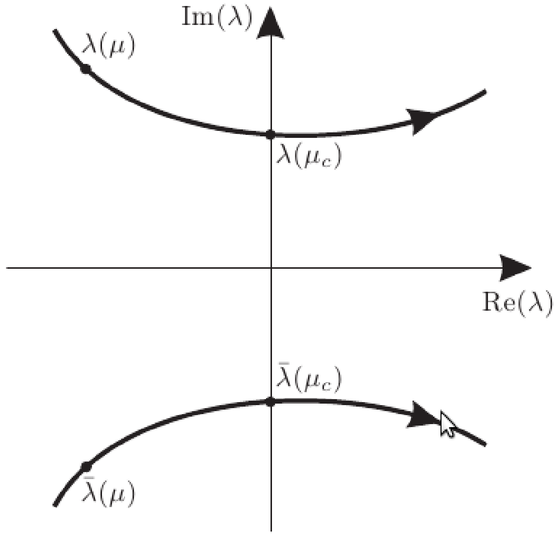

Figure 2.1: Eigenvalues of a Hopf bifurcation point. Source: Mihajlović [24].

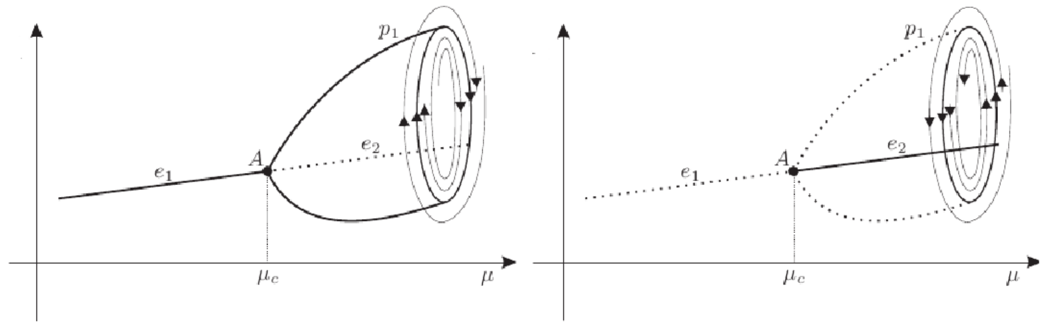

Figure 2.2: (a) Supercritical Hopf bifurcation and (b) Subcritical Hopf bifurcation. Source: Mihajlović [24].

that transits suddenly from nonconservative to conservative system.

In literature, there are several bifurcation diagrams. Each one holds its characteristics which can be found in references [13, 35, 37].

Finally, the Poincaré map is used to prove the existence of periodic orbits. Suppose an n-dimensional system as in Eq. 2-4 and $\mathbf{S}$ as an $n-1$ dimensional surface of section. All the trajectories pass trough $\mathbf{S}$, i.e. $\mathbf{S}$ is transverse to the flow. Mathematically, if $\mathbf{x}_k \in \mathbf{S}$ denotes the kth intersection of the flow (see 2.3), then the Poincaré maps P can be defined by

$$x_{k+1} = P(x_k). \tag{2-10}$$

It is noticed that if $\mathbf{x}^*$ is a fixed point in P ($P(\mathbf{x}^*) = \mathbf{x}^*$), then a trajectory starting in $\mathbf{x}^*$ returns to $\mathbf{x}^*$ after period T. In general words, this technique converts problems with stable close orbit into problems with fixed point of a mapping [37].

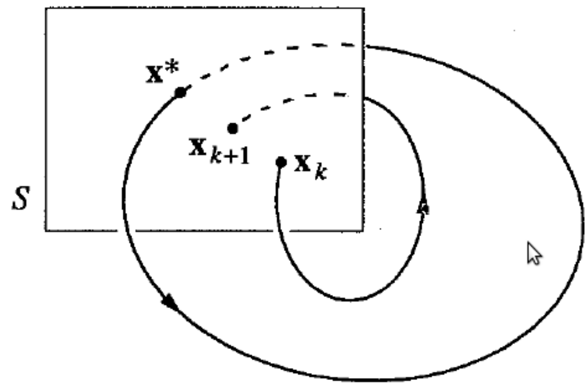

Figure 2.3: Poincaré section. Source: Strogatz [37].