

**Bruno Cesar Cayres Andrade**

**Numerical and experimental  
analysis of nonlinear torsional  
dynamics of a drilling system**

**DISSERTAÇÃO DE MESTRADO**

**DEPARTAMENTO DE ENGENHARIA MECÂNICA**

**Programa de Pós-Graduação em Engenharia  
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**Dissertação de Mestrado**

Dissertation presented to the Postgraduate Program in Mechanical Engineering of the Departamento de Engenharia Mecânica do Centro Técnico Científico da PUC–Rio, as partial fulfillment of the requirements for the degree of Mestre em Engenharia Mecânica.

Advisor : Prof. Hans Ingo Weber  
Co-Advisor: Dr. Romulo Reis Aguiar

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To my parents and my sister, my family, and Bárbara Lavôr,  
with love.

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## Abstract

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A successful oil and gas prospecting requires many efforts to overcome the encountered challenges, some of these challenges include drill string axial, lateral and torsional vibrations. These phenomena may cause premature component failures of the drilling system, dysfunction of measurement equipments, and increase time and costs of the prospecting process. Torsional vibrations are present in most drilling processes and may reach a severe state: stick-slip. An improved understanding about the stick-slip phenomenon provides tools to avoid the increase of prospecting time and costs, assuring the investment and success of the drilling process. Firstly, a numerical analysis of the drill string is performed with different friction models. These models are proposed in order to get familiar with the drill string dynamics. Also, it is described the experimental procedure with a nonlinear friction aiming to induce stick-slip and is performed a simple analytical modeling of the problem. The friction model is based on dry friction imposed by a break device. The nonlinear behavior of the experimental apparatus is analyzed and the numerical model is validated comparing experimental and numerical bifurcation diagrams.

## Keywords

Oil well drilling; torsional vibration; dynamic drill string; stick-slip phenomenon; nonlinear dynamics.

## Resumo

Andrade, Bruno Cesar Cayres; Weber, Hans Ingo; Aguiar, Romulo Reis. **Análise Numérica e experimental da dinâmica não linear torsional de um sistema de perfuração.** Rio de Janeiro, 2013. 88p. Dissertação de Mestrado — Departamento de Engenharia Mecânica, Pontifícia Universidade Católica do Rio de Janeiro.

Uma prospecção bem sucedida de petróleo e gás requer muitos esforços para se sobrepor os desafios encontrados, tais como vibrações axiais, laterais e torcionais. Estes fenômenos podem causar a falha prematura de componentes do sistema de perfuração, disfunção nos equipamentos de medição e aumento no tempo e custo no processo de perfuração. Em particular, vibrações torcionais estão presentes em grande parte dos processos de perfuração e podem alcançar um estado crítico: *stick-slip*. Um melhor entendimento sobre este fenômeno proporciona ferramentas para evitar o aumento do tempo e do custo da prospecção, assegurando o investimento e sucesso do processo de perfuração. Neste trabalho, é descrito um procedimento experimental com um atrito não linear objetivando induzir *stick-slip* e é feita uma modelagem analítica simples do problema. O modelo de atrito é baseado em um atrito seco imposto por um dispositivo de freio desenvolvido. O comportamento não linear da bancada experimental é analisada e o modelo numérico é validado comparando diagramas de bifurcações numérica e experimentais.

## Palavras-chave

Poços de perfuração; dinâmica de coluna de perfuração; vibração torcional; fenômeno de stick-slip; dinâmica não linear .

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## Nomenclature

$\delta$	Logarithm decrement
$\mu$	Friction coefficient
$\nu$	Poisson ratio
$\omega_d$	Damped frequency
$\omega_n$	Natural frequency
$\Omega$	Vector of velocities
$\rho_{BHA}$	Density of the Bottom Hole Assembly
$\rho_{DP}$	Density of the Drill Pipe
$\varphi$	Vector of displacements
$\xi$	Damping factor
<i>BHA</i>	Bottom hole assembly
<i>C</i>	Matrix of damping
<i>DOF</i>	Degree of freedom
<i>DP</i>	Drill Pipe
<i>DP</i>	Drill pipe
<i>Dr</i>	Damping factor of the mud per length unit
<i>DRPM</i>	Downhole rotation per minute
<i>E</i>	Young modulus
<i>G</i>	Shear modulus
<i>I</i>	Area moment of inertia
<i>IC</i>	Initial conditions
<i>ID</i>	Inner diameter
<i>J</i>	Matrix of inertia
<i>K</i>	Stiffness coefficient or Stiffness matrix
<i>K</i>	Stiffness of test rig
$K_{BHA}$	Stiffness of Bottom Hole Assembly
$L_{BHA}$	Length of Bottom Hole Assembly
$L_{DP}$	Length of Drill Pipe

<i>MCF</i>	Modified Coulomb Friction
<i>NDOF</i>	Number of Degree of Freedom
<i>OD</i>	Outer diameter
<i>P<sub>f</sub></i>	Proportional factor
<i>PDF</i>	Probability density function
<i>RPM</i>	Rotation per minute
<i>SRPM</i>	Surface rotation per minute
<i>SSS</i>	Severity criteria
<i>STOR</i>	Surface Torque
<i>T</i>	Period
<i>T<sub>1</sub></i>	Torque at bottom end
<i>T<sub>2</sub></i>	Torque at surface end
<i>TOB</i>	Torque on bit
<i>WOB</i>	Weight on bit

*Try not to become a man of success but rather  
to become a man of value.*

**Albert Einstein, .**