

Bruno Cesar Cayres Andrade

**Numerical and experimental
analysis of nonlinear torsional
dynamics of a drilling system**

DISSERTAÇÃO DE MESTRADO

DEPARTAMENTO DE ENGENHARIA MECÂNICA

**Programa de Pós-Graduação em Engenharia
Mecânica**

Rio de Janeiro
August 2013

Bruno Cesar Cayres Andrade

**Numerical and experimental analysis of
nonlinear torsional dynamics of a drilling system**

Dissertação de Mestrado

Dissertation presented to the Postgraduate Program in Mechanical Engineering of the Departamento de Engenharia Mecânica do Centro Técnico Científico da PUC–Rio, as partial fulfillment of the requirements for the degree of Mestre em Engenharia Mecânica.

Advisor : Prof. Hans Ingo Weber
Co–Advisor: Dr. Romulo Reis Aguiar

Rio de Janeiro
August 2013

Bruno Cesar Cayres Andrade

**Numerical and experimental analysis of
nonlinear torsional dynamics of a drilling system**

Dissertation presented to the Postgraduate Program in Mechanical Engineering of the Departamento de Engenharia Mecânica do Centro Técnico Científico da PUC–Rio, as partial fulfillment of the requirements for the degree of Mestre em Engenharia Mecânica. Approved by the following commission:

Prof. Hans Ingo Weber

Advisor

Pontifícia Universidade Católica do Rio de Janeiro

Prof. Romulo Reis Aguiar

Co–Advisor

Brazil Research & Geoengineering Center - Schlumberger Ltd

Prof. Kátia Lucchesi Cavalca Dedini

Universidade Estadual de Campinas

Prof. Arthur Martins Barbosa Braga

Pontifícia Universidade Católica do Rio de Janeiro

Prof. Thiago Gamboa Ritto

Universidade Federal do Rio de Janeiro

Prof. José Eugenio Leal

Coordinator of the Centro Técnico Científico
Pontifícia Universidade Católica do Rio de Janeiro

Rio de Janeiro — August 29, 2013

All rights reserved. It is forbidden partial or complete reproduction without previous authorization of the university, the author and the advisor.

Bruno Cesar Cayres Andrade

Studied Mechanical Engineering at the Universidade Federal do Pará and at the Institut National Polytechnique de Grenoble.

Bibliographic data

Andrade, Bruno Cesar Cayres

Numerical and experimental analysis of nonlinear torsional dynamics of a drilling system / Bruno Cesar Cayres Andrade; advisor: Hans Ingo Weber; co–advisor: Romulo Reis Aguiar . — 2013.

88 f. : il. (color.) ; 30 cm

Dissertação (mestrado) – Pontifícia Universidade Católica do Rio de Janeiro, Departamento de Engenharia Mecânica, 2013.

Inclui bibliografia

1. Engenharia Mecânica – Teses. 2. Poços de perfuração. 3. Dinâmica de coluna de perfuração. 4. Vibração torcional. 5. Fenômeno de stick-slip. 6. Dinâmica não linear. I. Weber, Hans Ingo. II. Aguiar, Romulo Reis. III. Pontifícia Universidade Católica do Rio de Janeiro. Departamento de Engenharia Mecânica. IV. Título.

CDD: 621

To my parents and my sister, my family, and Bárbara Lavôr,
with love.

Acknowledgments

I would like to thank God for blessing me and giving me strength to keep my way.

My parents and my sister always have had huge importance over my whole life. I would like to thank them for making me the person that I am nowadays and always were by my side. My girlfriend Bárbara Lavôr has a great importance in this dissertation, always giving me support, love and friendship during this 3 years and 7 months that we are together. For these important people, I dedicate this dissertation.

To my big and beautiful family that has continually offered kind words of motivation. Especially to Everton and Juliana Caires for their friendship, love and care, and for giving me a gorgeous niece, Evellin.

I would like to thank my advisor Hans I. Weber and my co-advisor Romulo Aguiar, for their patience, friendship, advice and knowledge. With them, I learned what research means.

Special acknowledgment goes to my long-time friends Amanda Pinheiro, Hamilton Cavalcante, and Romulo Pimentel, for their friendship since our childhood. I cannot forget my godson, Iuri, a surprise gift in my life that has brought me happiness and peace.

To Leonardo Pereira, Michelle Azulay, Hadrien Zarah, Mateus Carniatto, Jordana Colman, and Felipe Alfaia, for those countless conversations about our academic careers, funny moments and friendship.

Also, I would remiss if I did not acknowledge my new friends Americo Cunha Jr., Roberta Lima, and Jonathan Clay, for their partnership, support and lighthearted moments in this endeavor. Cesar Fosenca, Marcelo Pereira, Mario Sandoval, Guilherme Rodrigues, and Wagner da Cruz were extremely important for the successful closure of this dissertation. Sincerely, thank you very much. Friends for life.

Finally, I would like to take this opportunity to acknowledge the Department of Mechanical Engineering of PUC-Rio and Schlumberger Ltd. for the partnership that provided the financial support for this research. Special acknowledgments to the Drilling Optimization Engineering team from the Brazil Research & Geoengineering Center (Schlumberger) and the Laboratory of Dynamics and Vibrations (PUC-Rio), as well as to Coordenação de Aperfeiçoamento de Pessoal de Ensino Superior (CAPES) for the financial support during two years.

Abstract

Andrade, Bruno Cesar Cayres; Weber, Hans Ingo (Advisor); Aguiar, Romulo Reis (Co-advisor). **Numerical and experimental analysis of nonlinear torsional dynamics of a drilling system**. Rio de Janeiro, 2013. 88p. MSc. Dissertation — Departamento de Engenharia Mecânica, Pontifícia Universidade Católica do Rio de Janeiro.

A successful oil and gas prospecting requires many efforts to overcome the encountered challenges, some of these challenges include drill string axial, lateral and torsional vibrations. These phenomena may cause premature component failures of the drilling system, dysfunction of measurement equipments, and increase time and costs of the prospecting process. Torsional vibrations are present in most drilling processes and may reach a severe state: stick-slip. An improved understanding about the stick-slip phenomenon provides tools to avoid the increase of prospecting time and costs, assuring the investment and success of the drilling process. Firstly, a numerical analysis of the drill string is performed with different friction models. These models are proposed in order to get familiar with the drill string dynamics. Also, it is described the experimental procedure with a nonlinear friction aiming to induce stick-slip and is performed a simple analytical modeling of the problem. The friction model is based on dry friction imposed by a break device. The nonlinear behavior of the experimental apparatus is analyzed and the numerical model is validated comparing experimental and numerical bifurcation diagrams.

Keywords

Oil well drilling; torsional vibration; dynamic drill string; stick-slip phenomenon; nonlinear dynamics.

Resumo

Andrade, Bruno Cesar Cayres; Weber, Hans Ingo; Aguiar, Romulo Reis. **Análise Numérica e experimental da dinâmica não linear torsional de um sistema de perfuração**. Rio de Janeiro, 2013. 88p. Dissertação de Mestrado — Departamento de Engenharia Mecânica, Pontifícia Universidade Católica do Rio de Janeiro.

Uma prospecção bem sucedida de petróleo e gás requer muitos esforços para se sobrepor os desafios encontrados, tais como vibrações axiais, laterais e torcionais. Estes fenômenos podem causar a falha prematura de componentes do sistema de perfuração, disfunção nos equipamentos de medição e aumento no tempo e custo no processo de perfuração. Em particular, vibrações torcionais estão presentes em grande parte dos processos de perfuração e podem alcançar um estado crítico: *stick-slip*. Um melhor entendimento sobre este fenômeno proporciona ferramentas para evitar o aumento do tempo e do custo da prospecção, assegurando o investimento e sucesso do processo de perfuração. Neste trabalho, é descrito um procedimento experimental com um atrito não linear objetivando induzir *stick-slip* e é feita uma modelagem analítica simples do problema. O modelo de atrito é baseado em um atrito seco imposto por um dispositivo de freio desenvolvido. O comportamento não linear da bancada experimental é analisada e o modelo numérico é validado comparando diagramas de bifurcações numérica e experimentais.

Palavras-chave

Poços de perfuração; dinâmica de coluna de perfuração; vibração torcional; fenômeno de stick-slip; dinâmica não linear .

Contents

1	General introduction	17
1.1	Oil well drilling system	17
1.2	Overview and objectives	20
1.3	Organization of the master dissertation	21
2	Literature review and preliminary concepts	22
2.1	Literature review	22
2.2	Preliminary concepts	26
	<i>Least-square technique</i>	26
	<i>Nonlinear dynamics concepts</i>	27
3	Mathematical modeling of the torsional dynamics of a drill string	32
3.1	Introduction	32
3.2	Torsional model	33
	<i>First modeling approach: two degrees of freedom</i>	33
	<i>Second modeling approach: multiple degrees of freedom</i>	34
	<i>Severity criteria</i>	36
3.3	Sensitivity analysis of the friction torque models	38
3.4	Convergence test	45
3.5	Results of the full scale models	47
	<i>First model: two degrees of freedom</i>	47
	<i>Second model: multi degrees of freedom</i>	53
3.6	Conclusion	55
4	Experimental study of the drill string torsional vibrations	58
4.1	Introduction	58
4.2	Description of the test rig	58
4.3	Parameter estimations	61
	<i>First estimation</i>	62
	<i>Second estimation</i>	65
4.4	Calibration of the force sensors	66
4.5	Mathematical modeling of the test rig	68
4.6	Results of the test rig model	70
4.7	Conclusion	76
5	General conclusions, future works and publication	78
5.1	General conclusions	78
5.2	Future works	80
5.3	Publication	81

Bibliography	82
A ODE23t solver	86
B Block diagrams and algorithm	87
B.1 Block diagrams	87
B.2 Arduino algorithm	88

List of Figures

1.1	Drilling system.	18
1.2	Torque transmitting element called drill string. Source: Khulief <i>et al</i> [21].	19
1.3	Types of vibration on drill string. Source: López [27].	19
1.4	Types of failures: (A) ductile; (B) fragile; (C) stress corrosion cracking and (D) fatigue. Source: Macdonald <i>et al.</i> [22].	20
2.1	Eigenvalues of a Hopf bifurcation point. Source: Mihajlović [24].	30
2.2	(a) Supercritical Hopf bifurcation and (b) Subcritical Hopf bifurcation. Source: Mihajlović [24].	30
2.3	Poincaré section. Source: Strogatz [37].	31
3.1	Torsional model of two degrees of freedom.	33
3.2	Torsional model of multiple degrees of freedom. Source: López [27].	35
3.3	Downhole speed under torsional vibrations and the limit line (dashed red line) for a set of 60 RPM and 110 kN.	37
3.4	Linear interpolation to create the friction models adopted.	39
3.5	Applied friction models. (a) Model 1, (b) Model 2, (c) Model 3, and (d) Model 4.	39
3.6	Static and dynamic points.	40
3.7	Torsional vibration map for the different friction torques.	40
3.8	Set-points of (a) 40 RPM and 100kN on vibration and (b) 140 RPM and 100 kN without vibrations.	40
3.9	Torsional vibration map of Model 2 with different friction static peaks.	42
3.10	Dynamic set-point influence on the torsional vibration map.	42
3.11	Pavone friction model.	43
3.12	Severity curve of the system using Pavone friction model.	43
3.13	3D stick-slip severity map.	44
3.14	2D stick-slip severity map.	44
3.15	Influence of the length of (a) drill pipe and (b) BHA on torsional vibration map.	45
3.16	Convergence test: (a) second and (b) third natural frequencies.	46
3.17	Frequencies relative error.	46
3.18	Torsional vibration map for the 15 DOF system.	47
3.19	Bifurcation diagram with SRPM as control parameter and constant WOB = 80 kN.	47
3.20	Time-domain response with a constant WOB = 80 kN and (a)40 RPM and (b)100 RPM.	48
3.21	Bifurcation diagram with SRPM as control parameter and constant WOB = 130 kN.	48
3.22	Time-domain response with a constant WOB = 130 kN and (a)40 RPM and (b)100 RPM.	48
3.23	Bifurcation diagram with WOB as control parameter and constant SRPM = 40 RPM.	49

3.24	Time-response with a constant SRPM = 40 RPM and (a)40 kN and (b)190 kN.	49
3.25	Bifurcation diagram with WOB as control parameter and constant SRPM = 80 RPM.	50
3.26	Time-response with a constant SRPM = 80 RPM and (a)40 kN and (b)190 kN.	50
3.27	Limit cycle of dimension (a)zero and (b)one with initial conditions of 0 rad and 0 rad/s, and (c) and (d) are the time-response of the system. Set-point for (a) and (c) is $WOB = 110$ kN and $SRPM = 100$ RPM, and for (b) and (d) is $WOB = 110$ kN and $SRPM = 60$ RPM.	51
3.28	Limit cycle of dimension (a)zero and (b)one with initial conditions at surface of 100 rad and 100 rad/s, and (c) and (d) are the time-response of the system. Set-point for (a) and (c) is $WOB = 110$ kN and $SRPM = 100$ RPM, and for (b) and (d) is $WOB = 110$ kN and $SRPM = 60$ RPM.	51
3.29	Nonlinear jump in function of SRPM with (a)WOB = 80 kN and (b)130 kN.	52
3.30	Nonlinear jump in function of WOB with (a)SRPM = 40 RPM and (b)80 RPM.	52
3.31	Poincaré map with $WOB = 110$ kN and different SRPM.	52
3.32	Phase plane of the different SRPM and 100 kN. (a)40 RPM, (b)50 RPM, (c)60 RPM, and (d)70 RPM.	53
3.33	Bifurcation with (a) WOB = 80 kN and (b) WOB = 130 kN.	54
3.34	Bifurcation with (a) SRPM = 40 RPM and (b) SRPM = 80 RPM.	54
3.35	Intermediate vibration amplitudes.	54
3.36	Limit cycle of (a) zero ($WOB = 110$ kN and $SRPM = 100$ RPM) and (b) one dimension ($WOB = 110$ kN and $SRPM = 60$ RPM) with initial conditions of 0 rad and 0 rad/s. (c) and (d) are the time-domain response of (a) and (b), respectively.	55
3.37	Limit cycle of (a) zero ($WOB = 110$ kN and $SRPM = 100$ RPM) and (b) one dimension ($WOB = 110$ kN and $SRPM = 60$ RPM) with initial conditions of 100 rad and 100 rad/s. (c) and (d) are the time response of (a) and (b), respectively.	56
3.38	Nonlinear jump as function of SRPM with (a) WOB = 80 kN and (b) WOB = 130 kN.	56
3.39	Nonlinear jump as function of WOB with (a) SRPM = 40 RPM and (b) SRPM = 80 RPM.	56
4.1	Test rig set-up.	59
4.2	DC-motor of the test rig.	60
4.3	Test rig schema of measurements and positions.	60
4.4	Brake device.	61
4.5	Arduino board.	61
4.6	Measurement devices: (a) rotary encoder, (b) force sensor, (c) acquisition board, and (d) force sensor.	62
4.7	Schema to measure the applied reactive torque. (a) Front view and (b) lateral view.	62
4.8	Time-domain response rotor 1 with input voltage of 8 V.	63

4.9	Used dynamometers of (a) 3B U20034 of 5 N (0.05 N of precision) and (b) Weiheng of 40 kg (0.01 kg of precision).	64
4.10	Relation between torque and angular displacement of the rotor 1.	65
4.11	Response of the test rig (blue line) and estimation by least square (red line).	66
4.12	Convergence of the misfit function.	67
4.13	Torquimeter device with 0.5 Nm of precision.	68
4.14	Voltege response as function of the weights.	68
4.15	Modified Coulomb friction torque.	70
4.16	Numerical and experimental severity curves of the test rig.	71
4.17	Limit cycles of the numerical model of the test rig: (a) 54.5 RPM and 4.5 N, and (b) 122 RPM and 25 N.	71
4.18	Bifurcation diagrams of the (a) experimental and (b) numerical models with 7.5 N constant.	71
4.19	Experimental time-domain response with torsional vibration (a) $Frict = 7.5$ N and $MRPM = 80$ RPM, and without torsional vibration (b) $Frict = 15$ N and $MRPM = 80$ RPM	72
4.20	Bifurcation diagrams of the (a) experimental and (b) numerical models with 55 RPM constant.	72
4.21	Experimental time-domain response with torsional vibration (a) $Frict = 0.75$ N and $MRPM = 55$ RPM, and without torsional vibration (b) $Frict = 7.5$ N and $MRPM = 55$ RPM	72
4.22	Test rig behavior with 54.4 RPM and 4.5 N of friction: (a) limit cycle, and (b) time-domain response.	73
4.23	Test rig behavior with 84 RPM and 15 N of friction: (a) limit cycle, and (b) time-domain response.	73
4.24	Test rig behavior with 122 RPM and 25 N of friction: (a) limit cycle, and (b) time-domain response.	73
4.25	Time-domain response of the rotor angular velocity $RRPM$ and torque on rotor 1 $RTor$.	74
4.26	Frequency-response functions of the rotor angular velocity $RRPM$ and torque on rotor 1 $RTor$.	74
4.27	(a) Time-domain response, and (b) limit cycle of the test rig numerical model for $Frict = 25$ N and $MRPM = 122$ RPM with $\omega_p = 2.209$ rad/s.	75
4.28	(a) Time-domain response, and (b) limit cycle of the test rig numerical model for $Frict = 25$ N and $MRPM = 122$ RPM with $\omega_p = 4.510$ rad/s.	75
4.29	(a) Time-domain response, and (b) limit cycle of the test rig numerical model. $Frict = 25$ N and $MRPM = 122$ RPM with $\omega_p = 6.717$ Hz.	76
B.1	<i>LabView</i> block diagram.	87
B.2	<i>LabView</i> front panel.	87
B.3	<i>Simulink</i> block diagram.	88
B.4	Arduino algorithm.	88

List of Tables

3.1	Numerical values of the drill string system.	37
3.2	Friction model parameters.	39
3.3	Response of the drilling system under the friction models adopted.	41
3.4	Friction static peaks.	41
3.5	Different dynamic set points.	42
3.6	Length of drill pipe and stiffness values with constant BHA length (400 m).	45
3.7	Length of bottom hole assembly (BHA) and moment of inertia values with constant drill pipe length (2780 m).	45
4.1	Mechanical parameter of test rig.	61
4.2	Experimental values of the test rig system.	64
4.3	Experimental stiffness values and relative errors compared to analytical values.	65
4.4	Initial values of the parameters.	66
4.5	Estimated values of the parameters.	67
4.6	Force sensor calibration.	67
4.7	Friction coefficient and set-points values.	70
4.8	Frequency peaks ω_p .	75

Nomenclature

δ	Logarithm decrement
μ	Friction coefficient
ν	Poisson ratio
ω_d	Damped frequency
ω_n	Natural frequency
Ω	Vector of velocities
ρ_{BHA}	Density of the Bottom Hole Assembly
ρ_{DP}	Density of the Drill Pipe
φ	Vector of displacements
ξ	Damping factor
BHA	Bottom hole assembly
C	Matrix of damping
DOF	Degree of freedom
DP	Drill Pipe
DP	Drill pipe
Dr	Damping factor of the mud per length unit
$DRPM$	Downhole rotation per minute
E	Young modulus
G	Shear modulus
I	Area moment of inertia
IC	Initial conditions
ID	Inner diameter
J	Matrix of inertia
K	Stiffness coefficient or Stiffness matrix
K	Stiffness of test rig
K_{BHA}	Stiffness of Bottom Hole Assembly
L_{BHA}	Length of Bottom Hole Assembly
L_{DP}	Length of Drill Pipe

<i>MCF</i>	Modified Coulomb Friction
<i>NDOF</i>	Number of Degree of Freedom
<i>OD</i>	Outer diameter
<i>P_f</i>	Proportional factor
<i>PDF</i>	Probability density function
<i>RPM</i>	Rotation per minute
<i>SRPM</i>	Surface rotation per minute
<i>SSS</i>	Severity criteria
<i>STOR</i>	Surface Torque
<i>T</i>	Period
<i>T₁</i>	Torque at bottom end
<i>T₂</i>	Torque at surface end
<i>TOB</i>	Torque on bit
<i>WOB</i>	Weight on bit

*Try not to become a man of success but rather
to become a man of value.*

Albert Einstein, .