Pontifícia Universidade Católica
do Rio de Janeiro

Bruno Fanzeres dos Santos

Hedging Renewable Energy Sales in the Brazilian Contract Market via Robust Optimization

DISSERTAÇÃO DE MESTRADO

Dissertation presented to the Programa de Pós-Graduação em Engenharia Elétrica of the Departamento de Engenharia Elétrica, PUC–Rio as partial fulfillment of the requirements for the degree of Mestre em Engenharia Elétrica.

Advisor: Prof. Alexandre Street

Rio de Janeiro
March 2014

Pontifícia Universidade Católica do Rio de Janeiro

Bruno Fanzeres dos Santos

Hedging Renewable Energy Sales in the Brazilian Contract Market via Robust Optimization

Dissertation presented to the Programa de Pós-Graduação em Engenharia Elétrica of the Departamento de Engenharia Elétrica do Centro Técnico Científico da PUC–Rio, as partial fulfillment of the requirements for the degree of Mestre.

Prof. Alexandre Street
Advisor
Departamento de Engenharia Elétrica — PUC–Rio

Prof. Luiz Augusto Barroso
PSR

Prof. Eduardo Thomaz Faria
ENEVA

Prof. Davi Michel Valladão
Department of Industrial Engineering — PUC–Rio

Prof. José Eugenio Leal
Coordinator of the Science and Engineering Center — PUC–Rio

Rio de Janeiro, March 27, 2014

Bruno Fanzeres dos Santos

Bruno Fânzeres dos Santos graduated in 2011 in Electrical and Industrial Engineering in Pontifical Catholic University, Rio de Janeiro - Brazil. In 2012, he started its master's program in electrical engineering in the same university. During this period, he joined the research group in power system economics of the Laboratory of Applied Mathematical Programming and Statistics (LAMPS) where he worked as a researcher in the area of financial and operational evaluation of power plants, renewable and conventional energy commercialization, risk management modeling, statistical modeling for renewable production and optimization applied to power system.

Bibliographic data

Fanzeres, Bruno

Hedging Renewable Energy Sales in the Brazilian Contract Market via Robust Optimization / Bruno Fanzeres dos Santos ; advisor: Alexandre Street — 2014.
100 f. : il. ; 30 cm

Dissertação (Mestrado em Engenharia Elétrica)-Pontifícia Universidade Católica do Rio de Janeiro, Departamento de Engenharia Elétrica, Rio de Janeiro, 2014.
Inclui bibliografia

1. Engenharia Elétrica – Teses. 2. Otimização Robusta e Estocástica. 3. Energia Renovável. 4. Conditional Value-at-Risk (CVaR). 5. Programação Não Linear. 6. Risco de Preço-Quantidade. 7. Mercado Livre de Contratos. 8. Opção Térmica de Compra de Energia. 9. Contratos por Capacidade. 10. Contrato Futuro de Energia Elétrica. I. Street, Alexandre. II. Pontifícia Universidade Católica do Rio de Janeiro. Departamento de Engenharia Elétrica. III. Título.

CDD: 621.3

Acknowledgments

First and foremost, I would like to thank my academic advisor, Alexandre Street, for his phenomenal support and guidance over the course of my Master's work at PUC-Rio. Apart from academic research, Street played a remarkable role in my personal development throughout my stay at PUC-Rio, and I would be ever grateful to him for that.

I would like to thank my greatest friend Alexandre Moreira, who helped me greatly in improving this work. His constant encouragement and friendship was crucial for me to carry on during my Master's degree pursuing.

I would also like to thank every LAMPS member for the daily exchanges and their insightful considerations, and also their involvement that makes LAMPS such a unique environment for work. A warm hug in Lucas Freire, Joaquim Garcia, Mario Souto, Henrique Helfer, Aderson Campos, Moises Lima and Ana Luiza Lopes whom I have worked with on LAMPS both academically as well to have fun. I would like to thank Luiz Augusto Barroso for his acceptance to join my dissertation committee, and also for the friendship and the constant contributions to the development of my academic research.

I would like to thank PUC-Rio, CAPES, CNPq and the UTE Parnaíba Gerção de Energia S/A for the financial support during my Master's work. I would also like to thank my distinguished dissertation committee members, Dr. Eduardo Thomaz Faria and Prof. Dr. Davi Michel Valladão, for the keen intuition and interesting feedback that helped refine the work of this dissertation

I would like also to thank Luciana Moreira for the love and friendship during my Master's course. Thank you my dear for taking on this lifelong journey with me and making life easier.

Most importantly, I would like to thank my family. My parents, Sandra and Ricardo, have always supported me in each of my decisions and provided nothing but unconditional love. I would like to thank my brother Rafael for his affection and constant support. My deepest gratitude for my family support, love and encouragement.

Finally, this dissertation is dedicated to my grandmother Maria Regina who were, is and will be by my side all the time in every sense and to Marly and Luiza Oliveira, who were there in any time and situation for everything I needed. They were responsible for this project to happen. I am eternally grateful to them. Thank you.

Abstract

Fanzeres, Bruno; Street, Alexandre (Advisor). **Hedging Renewable Energy Sales in the Brazilian Contract Market via Robust Optimization**. Rio de Janeiro, 2014. 100p. MSc Dissertation — Departamento de Engenharia Elétrica, Pontifícia Universidade Católica do Rio de Janeiro.

Energy spot price is characterized by its high volatility and difficult prediction, representing a major risk for energy companies, especially those that rely on renewable generation. The typical approach employed by such companies to address their mid- and long-term optimal contracting strategy is to simulate a large set of paths for the uncertainty factors to characterize the probability distribution of the future income and, then, optimize the company's portfolio to maximize its certainty equivalent. In practice, however, spot price modeling and simulation is a big challenge for agents due to its high dependence on parameters that are difficult to predict, e.g., GDP growth, demand variation, entrance of new market players, regulatory changes, just to name a few. In this sense, in this dissertation, we make use of robust optimization to treat the uncertainty on spot price distribution while renewable production remains accounted for by exogenously simulated scenarios, as is customary in stochastic programming. We show that this approach can be interpreted from two different point of views: stress test and aversion to ambiguity. Regarding the latter, we provide a link between robust optimization and ambiguity theory, which was an open gap in decision theory. Moreover, we include into the optimal portfolio model, the possibility to consider an energy call option contract to hedge the agent's portfolio against price spikes. A case study with realistic data from the Brazilian system is shown to illustrate the applicability of the proposed methodology.

Keywords

Stochastic and Robust Optimization; Renewable Energy; Conditional Value-at-Risk (CVaR); Nonlinear Programming; Price-Quantity Risk; Contract Market; Energy Call Options; Capacity Contracts; Forward Contracts.

Resumo

Fanzeres, Bruno; Street, Alexandre (Orientador). **Modelo de Contratação para Fontes Renováveis com Rubustez ao Preço de Curto-Prazo**. Rio de Janeiro, 2014. 100p. Dissertação de Mestrado — Departamento de Engenharia Elétrica, Pontifícia Universidade Católica do Rio de Janeiro.

O preço da energia no mecado de curto-prazo é caracterizado pela sua alta volatilidade e dificuldade de previsão, repesentando um alto risco para agentes produtores de energia, especialmente para geradores por fontes renováveis. A abordagem típica empregada por tais empresas para obter a estratégia de contratação ótima de médio e longo prazos é simular um conjunto de caminhos para os fatores de incerteza a fim de caracterizar a distribuição de probabilidade da receita futura e, então, otimizar o portfólio da empresa, maximizando o seu equivalente certo. Contudo, na prática, a modelagem e simulação do preço de curto prazo da energia é um grande desafio para os agentes do setor elétrico devido a sua alta dependência a parâmetros que são difíceis de prever no médio e longo, como o crescimento do PIB, variação da demanda, entrada de novos agentes no mercado, alterações regulatórias, entre outras.

Neste sentido, nesta dissertação, utilizamos otimização robusta para tratar a incerteza presente na distribuição do preço de curto-prazo da energia, enquanto a produção de energia renovável é tratada com cenários simulados exógenos, como é comum em programação estocástica. Mostramos, também, que esta abordagem pode ser interpretada a partir de dois pontos de vista: teste de estresse e aversão à ambiguidade. Com relação ao último, apresentamos um link entre otimização robusta e teoria de ambiguidade. Além disso, incluímos no modelo de formação de portfólio ótimo a possibilidade de considerar um contrato de opção térmica de compra para o hedge do portfólio do agente contra a iregularidade do preço de curto-prazo. Por fim, é apresentado um estudo de caso com dados realistas do sistema elétrico brasileiro para ilustrar a aplicabilidade da metodologia proposta.

Palavras–chave

Otimização Robusta e Estocástica; Energia Renovável; Conditional Value-at-Risk (CVaR); Programação Não Linear; Risco de Preço-Quantidade; Mercado Livre de Contratos; Opção Térmica de Compra de Energia; Contratos por Capacidade;Contrato Futuro de Energia Elétrica.

– What is jazz, Mr. Armstrong?
– My dear lady, as long as you have to ask that question, you will never know it.

Louis Armstrong, *Musician.*

Contents

List of Figures

List of Tables

1
Introduction

Over the last decades, energy markets worldwide have undergone a major transition. In the past, the energy delivery process by power Generation Companies (Gencos) to end-users (residential, commercial and industrial consumers) was done by regulated facilities, typically owned by national, regional or local governments, using self-owned transmission and distribution lines. Because of their monopoly status, regulators used to periodically set the tariff in which the companies earn a "fair" rate of return over investments and recover operational expenses. Therefore, these firms used to maximize their profits subject to many regulatory constraints. But because utilities were allowed to pass cost on to consumers through regulated tariffs, there has been little incentive to reduce costs or to make necessary investments on the grid [3][4].

After the deregulation process has started, this structure gradually changed to market mechanisms [5], with unbundled generation, transmission and distribution companies. In addition, several countries stated a high degree of competition among agents, especially in the generation sector, aiming to reach an economically efficient solution and eliminate the market power, among other reasons. Within this liberalized market, two environments emerged as the most common places for energy trading: (i) the short-term market (day-ahead market) where generators and demands daily bid, in a hourly basis, a set of quantities at certain price and a central operator clears the market, setting the energy spot prices for the day ahead settlement [6][7][8]; and (ii) the forward market, which comprise mid- and/or long-term contracts negotiations (typically financial instruments only) and are usually used by Gencos to hedge its cash flow against the volatility of the spot price [9][10][11]. Both environments are present in almost all deregulated electricity markets and play an important role in the worldwide power sector reform.

Particularly, the Brazilian power sector reform started in 1996 [12]. Similarly to other countries, the new rules were designed to encourage competition in generation and retailing, leaving the distribution and transmission sectors still regulated activities with provision for open access [13][14]. Following the main guidelines, the current power sector model relies on a combination of competition and planning to guarantee supply adequacy and provide a "safe" environment to attract new investors. The principal driving force is the hydro predominance in the country, with huge reservoirs that control multiple river systems distributed over a vast area. Due to this particular characteristic, the system has a centralized coordination which takes advantage of the regulation capability of the reservoirs to manage them as a portfolio and obtain the long-term minimum overall operating cost [15]. In practice, the system's scheduling is carried out by the Brazilian Independent System Operator (ONS) [1], which uses a multi-stage stochastic optimization model that takes into account the plants' operating characteristics and inflow uncertainty [16]. This least-cost dispatch does not consider any commercial arrangements and determines the dispatch of every plant in the system and also the short-run marginal cost, which is used as the clearing price in the short-term energy market.

A collateral effect of such centralized dispatch is the high volatility pattern of the short-run marginal cost (which is, ultimately, the energy spot price) and its strong correlation with the system's inflow [17]. Since the system is designed to withstand under very harsh conditions (by means of an overcapacity in power and energy), in most of the time it has an excess of recourses to meet the demand. As a consequence, the short-run marginal cost stays at lower values in most of the time, reflecting an expected "normal" inflow conditions, reaching extremely high values when the system future reliability is expected to be in danger. Moreover, the dispatch model takes into account several *ex ante* hypothesis on market uncertainties, such as fuel prices, plant availability, supply expansion scenario, hydrology (just to name a few), in which any deviation on these hypotheses distort the observed probability distribution of the prices (*ex post*) with respect to the simulated (planned) one.

Therefore, from the point of view of energy commercialization, the irregular pattern of the spot price poses a major risk for trading companies which sell electricity contracts backed on renewable production in the Brazilian contract market. The main reasons are the intermittent energy production pattern of renewable sources [18] and the modality of contracts typically traded in the contract market, the standard forward contract [3][4]. On the one hand, although renewable plants are known for the low emission of greenhouse gases, they are also known for their energy uncontrollability and difficulty to predict. On the other hand, energy standard forward contracts are bilateral agreements in which the seller counterpart (Gencos) has the obligation to delivery a fixed amount of electricity to the buyer (end-users) against a fixed payment. Therefore, when a standard forward contract is supplied by renewable energy, the uncertain profile typical to this kind of sources exposes the generation company to the so-called price-quantity risk [10][19][20][21][22], which occurs whenever the Genco must purchase in the short-term market the amount of energy sold but not produced, at high prices. In a nutshell, risk-mitigation mechanisms are of utmost importance when renewable energy is traded in the Brazilian contract market.

In technical literature, several works dealt with such issue: (i) in [23], a strategic bidding model that takes into account the agent's risk profile, by means of piecewise linear utility functions, and the main uncertainties factors that affect the long-run Genco's revenue are considered to assess the Willingness-to-Supply (WtS) curve of a renewable generator in a multi-item auction-based environment; (ii) [20] addresses the hedging problem of a load serving entity. By exploiting the correlation between consumption volume and energy spot price, an optimal zero-cost hedging function characterized by a set of payoffs is derived. It is also illustrated how such hedging strategy can be implemented through a portfolio of standard forward contracts and call and put options; (iii) in [10], a two-stage stochastic optimization model that defines the optimal composition of a portfolio based on complementary renewable sources is proposed. The model aims to maximize the revenue of an Energy Trading Company (ETC) selling a standard forward contract in the Brazilian contract market. An interesting feature of the model proposed in [10] is that the ETC bears all the trading risk, leaving the renewable plants and the end-user free of such risk; (iv) in [21], a renewable energy hedge pool composed with the three main renewable sources of the Brazilian power matrix (in [22], the model is extended to a finite number of renewable plants) is proposed to jointly sell a single standard forward contract in the Brazilian contract market. The price-quantity risk is mitigated since the renewable plants have a complementary production profile and thus a less intermittent joint generation and exposure to the short-term market; (v) [24] proposes a model for a Wind Power Producer (WPP) based on a joint-selling strategy with a Small Hydro (SH) run-of-river Genco. In such model, the hydro Genco receives a surplus payment in comparison to the amount it would receive in the market, thus being always a good business for the SH, and the WPP the remainder of the income. It is shown that such commercial model is able to mitigate the exposure to the short-term market due to the complementary production profile of such sources and promote a safer entrance of wind power generation into the Brazilian contract market.

In this dissertation, the business structure presented in [10] is extended to consider contracts with wind power plants. A mixed stochastic-robust optimization model is proposed to define the risk-adjusted optimal contracting strategy for a trading company backed by a renewable portfolio. This approach is different from the aforementioned works in the sense that the uncertainty in the spot price realization is treated by means of robust optimization [25][26]. In addition, we add the possibility to consider in the portfolio an energy call option with a thermal power plant in order to give robustness and protection to the ETC's portfolio under a worst-case analysis. The complete model is converted as two-stage stochastic programming suitable for off-the-shelf solvers. Finally, we show that the proposed contracting model can be interpreted from two different point of views: the stress test [27] and ambiguity averseness [28][29][30].

The latter relation (ambiguity and robust optimization) has been widely studied in recent technical literature. For instance, [31] provides conditions that guarantee the satisfaction of probability constraints subjected to ambiguity in a linear programming framework; [32] studies the properties of a ambiguous chance-constrained problem; [33] introduces an approach for constructing uncertainty sets for robust optimization using deviation measures. Such measures capture the distributional asymmetry and lead to better approximations of chance constraints; [34] provides a polynomial-time algorithm for sample-driven robust stochastic programs with uncertainty in the mean and covariance; in [35], a framework for robust optimization that relaxes the standard notion of robustness by allowing the decision maker to vary the protection level in a smooth way across the uncertainty set is proposed. This approach is applied to the problem of maximizing the expected value of a payoff function when the underlying distribution is ambiguous. In this dissertation, we provide a formal relation between robust optimization and ambiguity by means of a re-parametrization of the problem. We show that solving the robust-stochastic contracting model presented in Section 5 (also presented in [36]) is equivalent to solve a contracting model with ambiguity aversion. In addition, the applications involving robust optimization and ambiguity are, typically, in financial markets. Here, we present an application to the optimal renewable contracting strategy. In the next section, the objective and contributions of this dissertation are summarized.

1.1 Objective and Contributions

The commercialization of renewable energy in Brazil is typically made in the regulated market [23]. In this environment, long-term energy contracts with special clauses that transfer the production risk to consumers are auctioned, reducing thus the renewable unit's exposure to the spot price volatility. However, as a consequence of these less risky contracts, the number of new renewable power plants grew rapidly in the past few years. This movement, though, created a pressure in the auctioned prices of the regulated market due to the high competition among agents, pushing out risk averse investors. In the view of these considerations, the contract market became a way out to keep the growing pace of renewable energy in the Brazilian power system. However, the contracts typically traded in the contract market are standard forward contracts. The main characteristic of these contracts is the obligation by the seller (Gencos) to delivery a fixed amount of energy to the buyer (consumer) against a fixed payment, i.e., unlike the regulated one, they have no clauses that mitigate the unit exposure to the short-term market. Thus, the combination of intermittent generation profiles with volatile spot prices creates a highly undesirable uncertainty on the power generation company's cash flow when they are long in standard forward contract backed on renewable energy. This undesirable uncertainty is known as price-quantity risk.

In addition to price-quantity risk, electricity traders also face the difficulty to model the behavior of the energy spot price due to the complex formation of this variable. It is well known that spot prices are driven by complex interactions between participants in the market and largely depend on unpredictable market conditions. Therefore, it becomes extremely difficult to capture the true underlying stochastic process of the spot price and accurately simulate a set of mid- and long-term scenarios needed to feed the stochastic model usually used to obtain an optimal contracting strategy in the Brazilian contract market.

Therefore, the objective of this dissertation is to propose a different approach to represent the uncertainty in the problem of electricity portfolio allocation involving renewable energy. Since the energy spot price modeling is a very difficult task, we argue that the actual models to simulate it are inaccurate and only represent an approximation of the true underlying probability distribution. Therefore, we make use of robust optimization to characterize the spot price uncertainty in the ETC's portfolio allocation problem, while renewable generation are treated as usual, via scenarios generated by Monte Carlo methods. In this sense, for each scenario of renewable production and a (trial) portfolio allocation, the proposed methodology finds the series of spot price that creates the worst payoff for the ETC inside a given credible (uncertainty) set. At last, a certainty-equivalent maximization problem is derived in order to find the "best" portfolio of renewable sources taking into account the worst-case realization of the energy spot price. We show that this approach has two different interpretations widely used in practical applications, the stress test and ambiguity aversion.

Our major contributions to the literature are fourfold:

- Present a new methodology to support an Energy Trading Company (ETC) to devise contracting strategies under an optimal risk-averse renewable portfolio problem in which the uncertainty in the generation of renewable energy sources is accounted for by exogenously simulated scenarios, as is customary in stochastic programming, and the uncertainty on spot price realization by means of robust optimization;
- Show that the proposed model can be interpreted from two different points of view: stress test and ambiguity aversion;
- Develop a business structure which considers the most traded contracts in the Brazilian contract market (standard forward contracts, capacity payment contracts and energy call options) within a robust optimization framework;
- Provide a link between robust optimization and ambiguity aversion models with applications to the electricity portfolio allocation problems;

The main tools used in those commercial models are: two-stage stochastic optimization, robust optimization, ambiguity and bilevel programming.

1.2 Organization

This dissertation is organized in the following chapters: Chapter 2 presents an overview of the Brazilian power system in which we discuss the difference between the contracting environments stated in Brazil during the power sector reform, the process to obtain the energy spot price, the different types of energy procurements used in this dissertation and the main renewable sources of the Brazilian power matrix. Chapter 3 discusses the problem of making decisions under uncertainty. We present a widely used risk measure, the Conditional Value-at-Risk (CVaR), and an induced Certainty Equivalent (CE) as well as description of the main tools used in decision-making problems, such as stochastic optimization and robust optimization. Chapter 4 describes the modeling approach proposed for the representation of the uncertainties in the contracting model. Chapter 5 develops the revenue of each modality of contract considered in the proposed business and also present the electricity portfolio allocation model. Chapter 6 discusses the two different point of views of the proposed model with their interpretation. A link between robust optimization and ambiguity theory is also presented in this Chapter. Chapter 7 illustrates the applicability of the proposed model in which the ETC has only the renewable sources to back up a forward contract. Two case studies are presented, one for mid-term contracts and one for a long-term business. In Chapter 8, we present two more studies for the complete model, which includes a thermal call option in the ETC's portfolio. Finally, Chapter 9 concludes this dissertation and discusses extensions and future research.

1.3 Publications Related to this Dissertation

During the MSc degree pursing, several publications were produced related to the theme of this dissertation. The following list presents the most important ones:

I) Journal Publications

- B. Fanzeres with A. Street, and L. A. Barroso, "Contracting Strategies for Renewable Generators: a Hybrid Stochastic and Robust Optimization Approach," accepted to *IEEE Trans. Power Syst.*

II) Conference Proceedings

- B. Fanzeres with A. Passos, A. Street, and S. Bruno, "A Novel Framework to Define the Premium for Investment in Complementary Renewable Projects," in *Proc. XVIII Power System Computation Conference (XVIII PSCC) 2014*, Wroclaw, Poland, pp. 1-7, Aug 2014.
- B. Fanzeres with A. Street, and L. Barroso, "Contracting Strategies for Generation Companies with Ambiguity Aversion on Spot Price Distribution," in

Proc. XVIII Power System Computation Conference (XVIII PSCC) 2014, Wroclaw, Poland, pp.1-8, Aug 2014.

- B. Fanzeres with A. Street, D. Lima, J. Garcia, L. Freire, and R. Rajagopal, "Mecanismo de Realocação de Energia Renovável: Uma Nova Proposta para Fontes Alternativas," in *Proc. XXII Seminário Nacional de Produção e Transmissão de Energia Elétrica (XXII SNPTEE) 2013*, pp. 1-9, Brasília, Distrito Federal, Brazil, Oct. 2013.

- B. Fanzeres with A. Street, A. Veiga, D. Lima, A. Moreira, J. Garcia, and L. Freire, "Simulação da Geração de Usinas Renováveis Coerentes com os Cenários de Operação do Sistema Elétrico Brasileiro," in *Proc. XXII Seminário Nacional de Produção e Transmissão de Energia Elétrica (XXII SNPTEE) 2013*, pp. 1-8, Brasília, Distrito Federal, Brazil, Oct. 2013.

- B. Fanzeres with A. Street, A. Veiga, D. Lima, L. Freire, and B. Amaral, "Fostering Wind Power Penetration into the Brazilian Forward-Contract Market," in *Proc. IEEE PES General Meeting 2012*, pp. 1-8, San Diego, California, USA, Jul. 2012.

- B. Fanzeres with A. Street, A. Veiga, D. Lima, L. Freire, and B. Amaral, "Comercialização de Energia Eólica no Ambiente Livre: Desafios e Soluções Inovadoras," in *Proc. XII Symposium of Specialists in Electric Operational and Expansion Planning (XII SEPOPE) 2012*, pp. 1-10, Rio de Janeiro, Rio de Janeiro, Brazil, May. 2012.

- B. Fanzeres with A. Street, "Cálculo da Curva de Disposição a Contratar de Geradores Hidroelétricos: Uma Abordagem Robusta ao Preço de Curto-Prazo," in *Proc. XXI Seminário Nacional de Produção e Transmissão de Energia Elétrica (XXI SNPTEE) 2011*, pp. 1-8, Florianópolis, Santa Catarina, Brazil, Oct. 2011.

2
Brazilian Power Sector

The Brazilian power system is the largest in Latin America with 125 GW installed [37]. Almost 98.3% of the country is connected at the bulk power level by a 116,000 km meshed high-voltage transmission network, with voltages ranging from 230 kV to 750 kV ac, plus two 600 kV dc links connecting the binational Itaipu power plant (14,000 MW) to the main grid. The main direct international interconnections are the back-to-back links with Argentina (2,200 MW), plus some smaller interconnections with Uruguay (70 MW) and Venezuela (200 MW). The remainder 1.7% of the system comprises small isolated systems located mainly in the Amazon region. Fig. 2.1 shows the Brazilian transmission grid in 2013 [1].

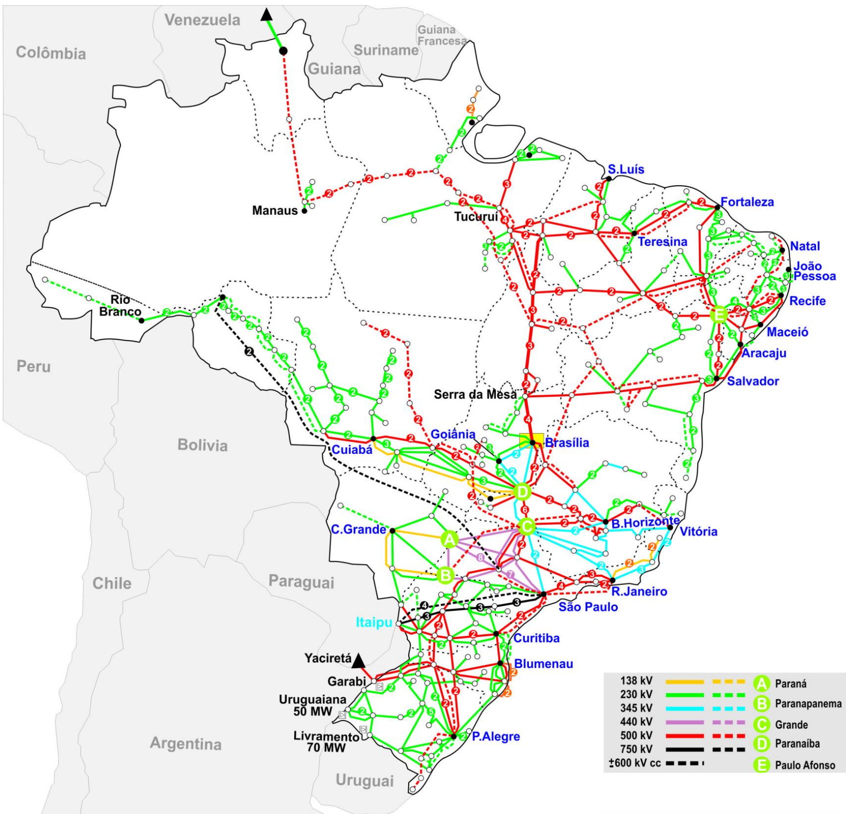

Fig. 2.1: Brazilian transmission system in 2013 [1].

The main driving force of the system is hydroelectricity. Almost 70% of total installed capacity is hydropower, with more than 75% of the total system load met by this source. The remaining generation sources mix includes the renewable plants: wind and solar power, and the thermal generation: natural gas, nuclear, oil, bioelectricity (co-generation from ethanol production, using sugarcane bagasse as a fuel) and coal (see Fig. 2.2 for data of capacity installed and energy generation of each source in Sep/2013). In particular, wind power is emerging as a competitive new source. For instance, according to the Brazilian expansion plan for 2020, wind power will represent 6.1% of the total capacity installed in the country, becoming the second largest source in capacity, only behind hydroelectricity [38].

Source	Capacity Installed (Sep/2013)		Energy Produced (Sep/2013)	
	MW	**% Total**	**GWh**	**% Total**
Hydro	83,437	69.75%	33,604	77.52%
Wind	1,747	1.46%	682	1.57%
Solar	3	>0.01%	0.12	>0.01%
Natural Gas	13,382	11.19%	4,011	9.25%
Nuclear	2,007	1.68%	1,251	2.89%
Oil	7,338	6.13%	530	1.22%
Bioelectricity	9,757	8.16%	2,178	5.02%
Coal	1,944	1.63%	1,090	2.51%
Total	**119,615**	**100.00%**	**43,346.12**	**100.00%**

Fig. 2.2: Capacity installed and energy produced in the Brazilian interconnected system in September 2013 [1].

On the other side of the equation, the Southeastern area of Brazil has the largest energy consumption among the five geographic regions, with 52% of total consumption (39,440 GWh), followed by the Northeastern and the South area (both with 17%) and the Middle-East and North area (both with 7%). Moreover, in Sep/2013, Brazilian total consumption was divided into the following classes: 27% residential users, 40% industrial users, 18% commercial users and 15% others users (rural areas, public light, public service and losses).

2.1 General Aspects of the Brazilian Power Sector Reform

The Brazilian power sector reform started in 1996 [12]. Before the restructuring process, the system was bundled and government owned. The central power was the main agent which financially supported the system's expansion and also coordinate of the system's operation. As occurred in other countries with the same structure, the power plants construction cost overran over time and the load forecast was constantly biased downwards leaving the system with (apparently) over capacity. Therefore, the tariffs for the consumers were not adequate and not reflected the system's true cost, being mostly an instrument to control the exploding inflation. This situation led to a disastrous financial situation in the Brazilian power sector with several delays in construction plants and almost no capital for new investments to meet the growing demand.

In this sense and following the main guidelines of the reform occurred worldwide, the Brazilian power sector replaced the regulated procedures used in the decision-making process by market mechanisms. The expansion planning is now driven by market forces, i.e., neither are mandatory expansion plans for the generation capacity nor when/how to meet a demand projection. Agents decide on their own when and where to build a new facility and its characteristics, such as the plant installed capacity, the energy source, the machinery used, among others. The main objective of the reform was to induce a reliable and efficient energy supply with adequate tariffs for end-users. In order to give more reliability to the system, was established the creation of an Independent System Operator (ONS) [1] to, independently, operate the system; a market administrator (CCEE) [39] which coordinates all the bilateral contracts, the short-term market and the electricity auctions; and an agency that regulates and supervises the power system agents (ANEEL) [37]. Sector reform brought about 85% of total regulated load privatization and the transmission expansion had been carried out with strong participation of private agents. However, due to political opposition, the privatization of power generation companies resulted in only 15% of total generation capacity, being appointed to be one of the main reasons for the 2001-2002 supply crisis that affected all consumers around the country [17].

Although competition became a keyword in the Brazilian power sector, the system's operation remain centralized coordinated and cost-based. The basic reason for not moving to a bidding scheme was the vast number of interconnected basins spread out through a complex cascade topology in which plants with different ownerships lie in the same cascade (Fig. 2.3). This central scheme leads to an efficient use of the reservoirs since the operator can take advantage of their regulation capability to manage then as a portfolio and obtain the long-term minimum overall operating cost [15]. Because the system dispatch is cost-based, the spot price that clears the short-term market in Brazil is constructed to represent the expected value of the water opportunity in a long-term operation setting and is obtained by means of the dual variables associated with the power constraints (marginal operative costs). Thus, the Brazilian spot prices are not based on the equilibrium between supply and demand as many electrical systems around the globe are.

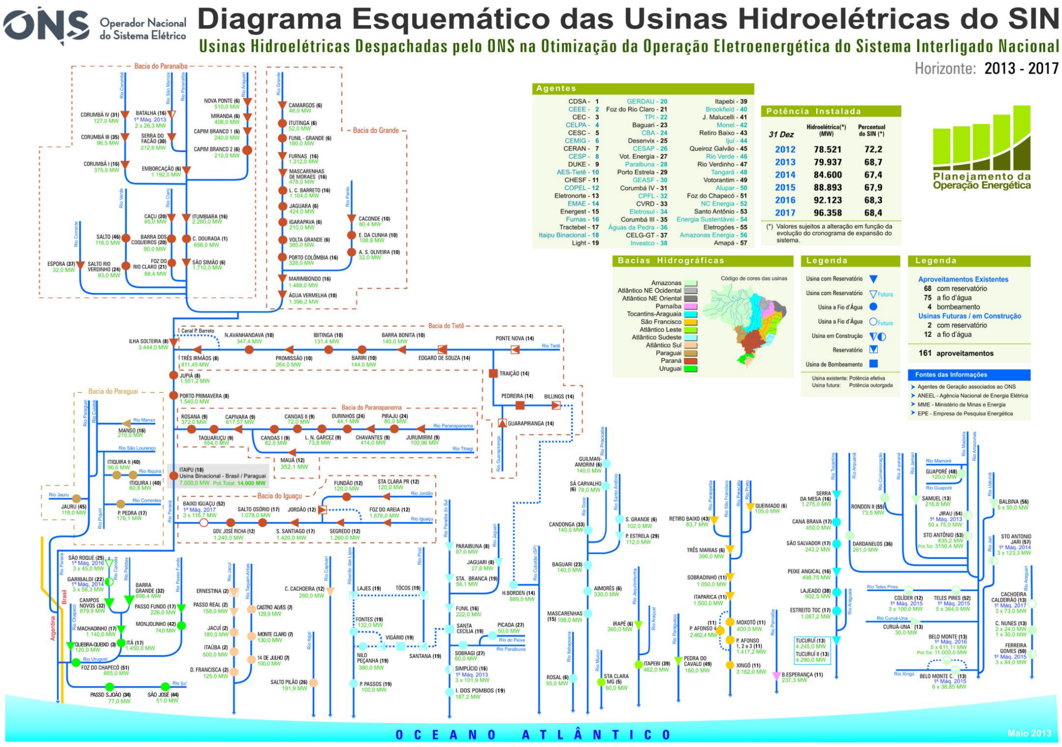

Fig. 2.3: Brazilian interconnected system hydro plants schematic diagram [1].

A collateral effect of such centralized dispatch is the high volatility of the spot price and its strong correlation with the system's inflow [17]. The spot price typically stays at lower values in most of the time, reflecting a expected "normal" inflow conditions, reaching extremely high values when the system's future reliability is expected to be in danger. Moreover, the dispatch model takes into account some *ex ante* hypothesis on market uncertainties, such as fuel prices, plant availability, supply expansion scenario, hydrology (just to name a few), in which any deviation on this hypothesis distort the observed probability distribution of the prices (*ex post*) with respect to the simulated (planned) one (see Fig. 2.4).

A direct consequence of this structure is that the spot price in Brazil does not provide a clear economical signal for the entrance of new generation as economic theory suggests [14]. In this sense, a strict rule was established to create incentives

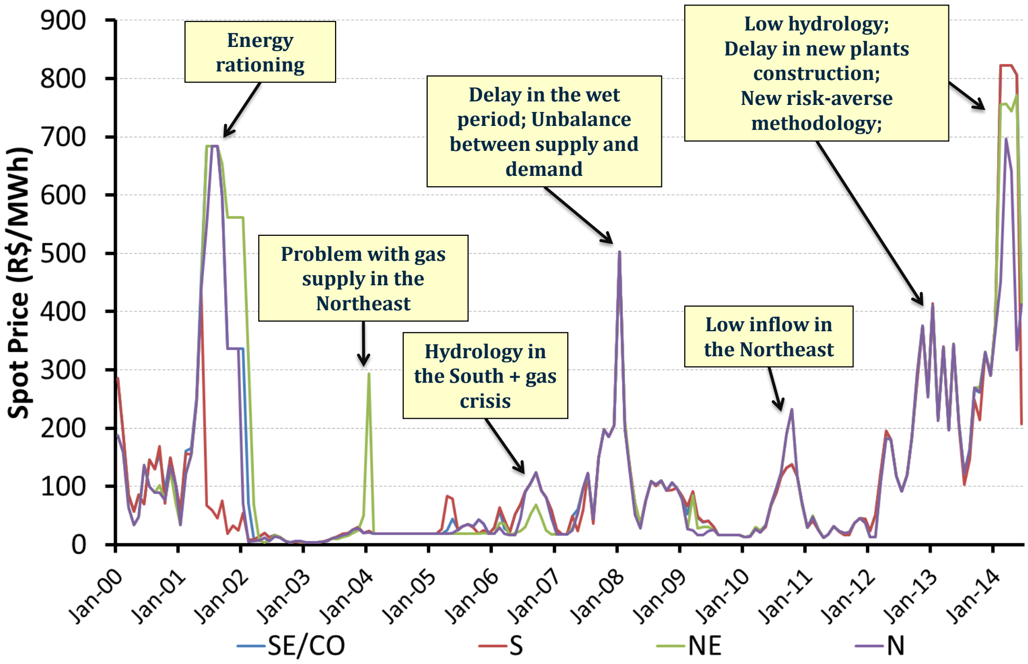

Fig. 2.4: Brazilian interconnected system hydro plants schematic diagram [1].

for the entrance of new generation: all consumers, both regulated and free, should have contracts that back up 100% of their loads. The contract coverage is verified *ex post*, comparing the cumulative MWh consumed in the previous year with the cumulative MWh contracted. If the contracted energy is smaller than the consumed energy, the user pays a severe penalty. Although the contracts are financial instruments only, they must be covered by Firm Energy Certificates (FECs), measured in avgMW[1], issued by the system regulator (ANEEL) for each power unit of the power plant [40]. For hydro units, the FEC is evaluated as the maximum demand that can be supplied within the worst hydrological condition of the historical record of inflow, being thus a probabilistic variant of the well-known concept of "firm energy" [41]. In the case of thermal plants, the FEC is assessed by means of a simulation procedure using the official dispatch model and is dependent of the operating cost given by the unit [14]. With regard to renewable units, the FEC represents a quantile of the historical production, typically being the 10%, 25% or 50% quantile of the renewable unit production [24]. This energy certificate tries to estimate the maximum amount of electricity that the generator can deliver on a sustainable basis and it is commonly used to measure the reliability of supply. In addition, the amount of

[1] An average MW (avgMW) is equivalent to the continuous production/consumption of one MW during the relevant period. One avgMW during one year corresponds to 8760 MWh.

energy that the generator is allowed to sell in contracts is constrained by this certificate in order to not put the reliability of the system's supply in danger. Therefore, since each consumer has a contract that covers 100% of its energy consumption and each contract is backed up by a firm energy certificate, a link between physical generation expansion and load growth has been made, creating thus incentives from the entrance of new generation.

In the next section, we discuss the two environments created by the restructuring process for contract trading backed on electricity in Brazil.

2.2 Contracting Environments

Aiming to establish a formal way for energy trading in Brazil, the national regulatory agency [37] created in 2004 two different trading environments: the Regulated Market and the Contract Market [42].

In the regulated market, Distribution Companies (Discos) purchase mid- and long-term bilateral contracts through public open auctions. The contracts negotiated have standardized rules and are designed to allocate risk between generators, distributors and consumers, and also to promote efficient energy purchase. The future delivery allows investor in generation plants to build the projects and the long-term contract creates conditions for project financing. These auctions are divided into two main groups: (i) *New Energy* auction, carried out three and five years in advance and aim to auction off new capacity to cover the foreseen load growth; and (ii) *Existing Energy* auction, which is designed to cover the existing load, acting as a contract renewal, and the unexpected load growth. Operationally, the auctions are carried out for the total load of all Discos, which means that the winners must sign bilateral contracts with each distribution company in proportion to their energy needs. This environment correspond to almost 70% of total contracts negotiated.

Some important auctions were made under the revised regulatory model in the Brazilian's regulated market. The first energy procurement were carried out in 2004 and three types of long-term energy contracts were offered to generators. Each product was an eight-year financial energy supply contract with start dates in 2005, 2006 and 2007. The final result were about 18.5 avgGW contracted [23]. In 2005, an auction for 15-years contracts for delivery in 2008, 2009 and 2010 were carried out aiming to contract new capacity. The novelty of this auction, with respect to the one made in 2004, is that the contracts auctioned were energy call options in which the thermal generator needed to bid both its strike price and the option premium [43]. The final result were 3.2 avgGW contracted [14]. Another important auction carried out in 2009 were a single product were auctioned for a specific technology, the wind power. The motivation of this auction was to take advantage of the 2008-2009 world financial crisis that has lowered equipment cost as well as to foster competition among the interested investors, thus starting the development of this technology in the country in a larger scale. The product offered was a 20-year energy contract for delivery in 2012 and had very specific characteristics that mitigate the exposition of the producer to the short-term market. The final result were 1.8 avgGW contracted [44]

On the other hand, in the contract market, Gencos and consumers with loads higher than 3 MW freely negotiate short, medium and long term bilateral contracts. This environment has gain substantial attention, especially regarding to renewable energy, since the downwards movement occurred in the regulated market prices. Moreover, consumers that purchase energy in the contract market from renewable plants (wind power, small hydros run-of-river, biomass and solar farms) with less than 30 MW of capacity installed have an discount of more than 50% in the transmission fee [45]. However, the contracts typically negotiated in this environment are standard forward contracts, in which the seller counterpart (Gencos) has the obligation to delivery a fixed amount of energy to the buyer (consumers) against a fixed payment. This obligatory delivery associated with the inherent intermittency of energy production of renewable sources exposes the generation company to the so-called price-quantity risk [10][19][20][21][22], which occurs whenever the Genco must purchase in the short-term market, at high prices, the amount of energy sold but not produced in order to fulfill the contract. Thus, risk-mitigation mechanisms are of utmost importance when trading renewable energy in the Brazilian contract market.

The proposed model of this dissertation aims to mitigate the price-quantity risk of an Energy Trading Company (ETC) when selling standard forward contracts backed on renewable energy by treating the main risk factor of the business, the irregularity of the spot price, by means of robust optimization. In the next section, we briefly explain the formation of the short-term market price with its interpretation and also motivate the use of robust optimization to model its uncertainty instead of using the classical stochastic approach.

2.3 Energy Spot Price Formation

In most electricity power markets worldwide, the price that clears the short-term market is obtained via an optimization problem. Operationally, an independent system operator receives energy offers from generation companies and energy bids from consumers, and determines, for every hour, the market-clearing price, as well as the individual power production of each generator and the consumption levels of each demand [46][47]. The procedure used by the operator is typically formulated as the following economic dispatch problem:

$$(\boldsymbol{\pi}, \boldsymbol{g}, \boldsymbol{d}) \in \arg\left\{ \min_{\substack{g_{i,t}, d_{j,t},\\ \delta_{t,n}}} \sum_{t \in \mathcal{T}} \left(\sum_{i \in \mathcal{I}} P^G_{i,t} g_{i,t} - \sum_{j \in \mathcal{J}} P^D_{j,t} d_{j,t} \right) \right. \quad (2\text{-}1)$$

subject to

$$\left(\sum_{i \in \Psi(n)} g_{i,t} - \sum_{j \in \Psi(n)} d_{j,t} \right) = \sum_{m \in \Theta(n)} B_{n,m}(\delta_{t,n} - \delta_{t,m}) : \pi_{t,n}, \quad \forall t \in \mathcal{T}, n \in \mathcal{N}; \quad (2\text{-}2)$$

$$0 \leq g_{i,t} \leq Q^G_{i,t}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}; \quad (2\text{-}3)$$

$$0 \leq d_{j,t} \leq Q^D_{j,t}, \quad \forall j \in \mathcal{J}, t \in \mathcal{T}; \quad (2\text{-}4)$$

$$f^{\min}_{n,m} \leq B_{n,m}(\delta_{t,n} - \delta_{t,m}) \leq f^{\max}_{n,m}, \quad \forall t \in \mathcal{T}, n \in \mathcal{N}, m \in \Theta(n); \quad (2\text{-}5)$$

$$\delta^{\min}_{t,n} \leq \delta_{t,n} \leq \delta^{\max}_{t,n}, \quad \forall t \in \mathcal{T}, n \in \mathcal{N} \backslash \{1\}; \quad (2\text{-}6)$$

$$\left. \delta_{t,n} = 0, \quad \forall t \in \mathcal{T} \text{ and } n = 1. \right\} \quad (2\text{-}7)$$

The decision variables in (2-1)-(2-7) are: (i) the energy production ($g_{i,t}$) of each generator i in the set of generators $\mathcal{I}$, during the considered dispatch period $t \in \mathcal{T}$ (usually t represents one hour and $\mathcal{T} = \{1, ..., 24\}$); (ii) the consumption level ($d_{j,t}$) of each demand j in the set of end-users $\mathcal{J}$, during the same period; and (iii) the voltage angle ($\delta_{n,t}$) of bus $n \in \mathcal{N}$ in time period $t \in \mathcal{T}$. Within the set of parameters, $B_{n,m}$ is the susceptance of the line that connects the buses n and m, and $f^{\min}_{n,m}$ and $f^{\max}_{n,m}$ are the transmission capacity of the respective line; $\delta^{\min}_{t,n}$ and $\delta^{\max}_{t,n}$ are the bounds of each angle bus $n \in \mathcal{N}$ during the considered dispatch period t and are typically set to be the mathematical constant $-\pi$ and π, respectively; $\Psi(n)$ identifies the unit (generation or demand) located at bus n; and $\Theta(n)$ identifies the buses connected to the bus n.

In this framework, equation (2-1) maximizes the social welfare by obtaining the minimum difference among the cost of dispatch and the cost of consumption. Note that $P^G_{i,t}$ and $P^D_{j,t}$ are the offer and bid prices by the generators and end-users, respectively. A dc linear approximation of the grid, equation (2-2), is used to represent the power balance at each node. The price which clears the short-term market is assessed by the dual variable of this constraint and, for a given level of consumption, can be interpreted as cost to meet an addition MW of demand. Equations (2-3) and (2-4) are, respectively the bounds for the generation dispatched and the level of demand consumed, where $Q^G_{i,t}$ and $Q^D_{i,t}$ are the offer and bid quantities by the generators and end-users, respectively. Finally, (2-5) and (2-6) enforce the bounds of each transmission line and angle buses in the grid, respectively, and (2-7) imposes $n = 1$ as the slack bus. A complete representation of the dispatch problem with no-load offers, ramping rates, minimum generation limits, and minimum up and down times can be incorporated to (2-1)-(2-7) and has been widely studied in the literature [48][49][50][51].

The unit commitment problem presented in (2-1)-(2-7) is typically applied to systems with thermal predominance. However, the hydro dominance of the Brazilian power system in addition to different plant owners in the same cascade, as discussed in Section 2.1, poses additional challenges to the system's operator with respect to a simple thermal dispatch [15]. Aiming to take advantage of the large reservoirs that are capable of multi-year regulation (up to five years), the system is centrally operated and a long-term cost-based stochastic optimization model is applied to define the dispatch of the generators (demand is considered fixed) and the price that clears the short-term market [16]. The main decision that should be made is to assess the optimal use of the water stored in the reservoirs, i.e., the operator should decide if is optimal to use the water today (in the short-term) or to store it and use in the future (in the long-term). For instance, assume a two-period decision process with two possible realizations for the future inflow (high or low inflows). If the operator decides to use the reservoirs in the short-term and a high inflow occurs, then the decision was made "correctly" since the demand was met at the minimum cost and the reservoirs are full again. On the other hand, if a low inflow occurs, the system enters in deficit, thus possible needing an expensive source (thermal units) to meet the future demand, configuring the "wrong" decision. Now assume that the operator decides to not use the reservoirs in the short-term. A thermal unit is used to meet the demand in the short-term and water is stored in the reservoirs. If a high inflow occurs, the reservoir spills, losing energy stored and, as a consequence, the system was not met at minimum cost. On the other side, if a low inflow occurs, then no energy is lost and the decision made was made “correctly”. In Fig. 2.5, the decision process for this simple example is presented.

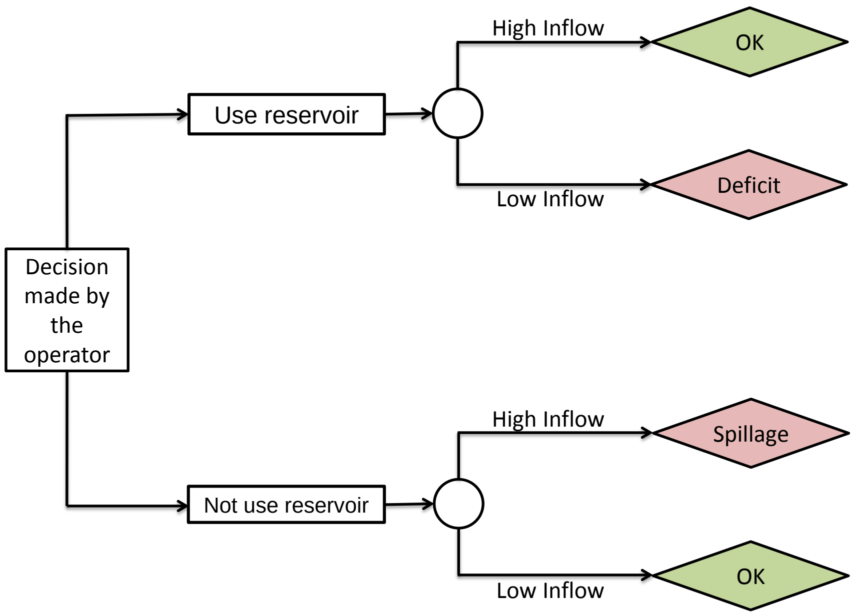

Fig. 2.5: Decision process in hydrothermal systems.

As a result, the decision process must take into account a composition between immediate cost and the future cost in a environment of uncertainty (the level of inflow in the future). In this sense, the problem in the Brazilian dispatch do not rely on which power level is needed to meet the demand that is feasible with respect to ramp and reserve constraints and/or maximum and minimum up and down times. The problem is “energetic”, i.e., relies on the optimal operation of the reservoirs under inflow uncertainty, since ramp and reserves are addressed by the hydro plants. To construct an adequate operation model for hydrothermal systems, note that Immediate Cost Function (ICF) is related to the cost of using thermal generation in the short-term and storage water for the use in the future. On the other hand, the Future Cost Function (FCF) is related to the usage of thermal generation in the future. Therefore, both functions are inversely related: (not) use the water today, (increases) decreases the level of the reservoirs in the future and (decreases) increases the expected cost of thermal unit. Thus, the optimal use of the water correspond to the point that minimizes the sum of both functions, i.e., the point that the derivative of both functions are equal in absolute value. In a stochastic framework, the optimal use of the water must be assessed for different states of the nature (scenarios), representing possible future realizations of the reservoirs levels. Hence, the optimization model (main constraints) to find the optimal long-term operating level of a hydrothermal system in a stage (period) $t \in \mathcal{T}$ can be formulated as:

$$z_t = \min_{\substack{g_{j,t}, u_{i,t},\\ v_{i,t+1}, s_{i,j}}} \sum_{j \in \mathcal{J}} c_j g_{j,t} + \alpha_{t+1}(\boldsymbol{v}_{t+1}) \tag{2-8}$$

subject to

$$\sum_{i \in \mathcal{I}} \rho_i u_{i,t} + \sum_{j \in \mathcal{J}} g_{j,t} = d_t \qquad : \pi_t; \tag{2-9}$$

$$v_{i,t+1} = v_{i,t} - u_{i,t} - s_{i,t} + a_{i,t} + \sum_{k \in \mathcal{M}(i)} (u_{k,t} + s_{k,t}), \ \forall\, i \in \mathcal{I}; \tag{2-10}$$

$$0 \leq v_{i,t+1} \leq \bar{v}_i, \qquad \forall\, i \in \mathcal{I}; \tag{2-11}$$

$$0 \leq u_{i,t} \leq \bar{u}_i, \qquad \forall\, i \in \mathcal{I}; \tag{2-12}$$

$$0 \leq g_{j,t} \leq \bar{g}_j, \qquad \forall\, j \in \mathcal{J}. \tag{2-13}$$

In (2-8)-(2-13), the decision variables are: (i) thermal production ($g_{j,t}$) of the unit j in the set of plants $\mathcal{J}$ at stage t; (ii) volume discharged ($u_{i,t}$) by the hydro plant i in the set of hydro units $\mathcal{I}$ at stage t; (iii) total volume at the hydro plant reservoir ($v_{i,t+1}$) in the next stage $t+1$; and (iv) spillage discharge $s_{i,t}$ of the hydro plant i at stage t. Within the set of parameters, $\bar{u}_i$, $\bar{v}_i$ and $\bar{g}_j$ are, respectively, the maximum value for the volume discharged, the volume stored and thermal production. Moreover, ρ_i represents the production coefficient of the hydro unit i, $u_{i,t}$ is the actual level of the reservoir i at stage t, $a_{i,t}$ the inflow to the hydro unit i at stage t and d_t the foreseen demand for the stage t.

As discussed earlier, the objective function (2-8) of the hydrothermal dispatch model aim to minimize the sum of the ICF, represented by $\sum_{j \in \mathcal{J}} c_j g_{j,t}$, and the FCF, $\alpha_{t+1}(\boldsymbol{v}_{t+1})$. Note that the FCF of a stage t ($\alpha_t(\cdot)$) is parametrized by the total volume stored in the system $\boldsymbol{v_t} = \{v_{1,t}, ..., v_{|\mathcal{I}|,t}\}$ at the same stage, representing the future status of the system. Equation (2-9) is the load balance in stage t. Again, its dual variable defines the price that clears the short-term market and has the same interpretation of (2-2). In (2-10), the hydraulic continuity is modeled for each hydro plant $i \in \mathcal{I}$. In this model, it is assumed that the discharges from all upstream reservoirs, represented by the set $\mathcal{M}(\cdot)$, flow directly into the succeeding downstream plant with no time lag. Finally, the set of equations (2-11)-(2-13) bounds the volume discharged, the volume stored and thermal production, respectively. In real time operation, several particular constraints are considered to better represent the system, but the equations that form the backbone of the hydrothermal dispatch model is presented in (2-8)-(2-13). The main question that arises is how to compute the value of the FCF, $\alpha_{t+1}(\boldsymbol{v}_{t+1})$. [16] presents a recursive procedure, called Stochastic Dual Dynamic Programming (SDDP), to evaluate this function and efficiently solve the hydrothermal dispatch.

From the point of view of energy trading in Brazil, agents typically assess their optimal contracting strategy by simulating the system for the contract period using the official dispatch tool based on (2-8)-(2-13) and obtaining a series of synthetic scenarios for prices that clears the short-term market $(\boldsymbol{\pi})$. Then, a two-stage linear optimization problem is formulated to assess the optimal portfolio [10][19]. However, note that, since the operation evolves a long-term reservoir management, the operator needs to assume a series of hypothesis for the system for the long-run operation, such as hydrology, demand level, no delay on plants construction, out of merit order dispatches, among others. In the non-occurrence (*ex-post*) of some of this hypothesis in operation lead to a totally different solution for (2-8)-(2-13) and thus a totally different spot price. In this context, the optimal portfolio decided *ex-ante* can be sub-optimal or even infeasible for implementation, being in some cases (not so rare), more risky than planned. In addition, the complex structure of the spot price formation in Brazil makes a statistical model for price simulation unappropriated. Therefore, the need for a methodology that capture this systemic risk is of utmost importance in the problem of devising optimal strategies for trading companies. In this work, we propose the use of robust optimization to protect the portfolio against the worst-case realization of the spot price within a feasible set. We show, by means of a realistic case for the Brazilian system, that this approach outperforms the pure stochastic one when the realized spot price is influenced by some changes on the system structure that were not contemplated during the scenarios obtainment process.

2.4 Renewable Energy in Brazil

In Brazil, the renewable sources that are receiving special attention in the past years are the Wind Power (WP) and the Small Hydro (SH) run-of-river. According to the Brazilian expansion plan for 2020, these two sources will represent 10% of the total installed capacity in the system (see Fig. 2.6 for the individual share) [38]. The great interest in these sources comes from the appeal to reduce greenhouse emission and reach a more sustainable power system. However, on the other hand, these sources are known to have a high intermittent energy production. Therefore, from the point of view of trading WP and SH plants in the Brazilian contract market, the uncertain renewable production associated with the irregular pattern of the spot price, discussed in Section 2.3, poses a major risk to generation companies which sell forward contracts backed up on these sources.

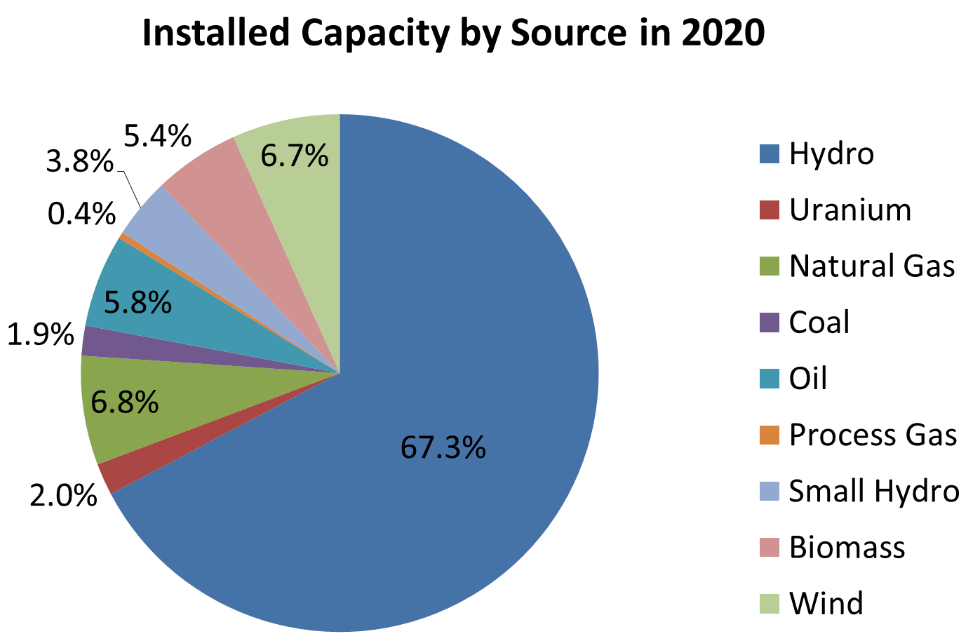

Fig. 2.6: Installed capacity in 2020 (forecast) segregated by type of source.

In almost all countries, wind power has been the most developed renewable source. In 2012, its installed capacity around the world exceeded 280,000 MW with a growth rate of 15.8% over 2011, more than ever before. China appeared as the largest total wind power capacity with more than 75,000 MW installed, followed by USA (59,000 MW), Germany (31,000 MW), Spain (22,000 MW) and India (18,300 MW), summing 73% of the worldwide wind capacity (see Fig. 2.7 for the evolution of the worldwide total installed capacity of wind power) [2].

In Brazil, however, this growth movement started only in 2009, led, mainly, by the world financial crisis and regulatory/government incentives. During the 2008-2009 world financial crises, a severe cutoff in energy investment took place in Eu-

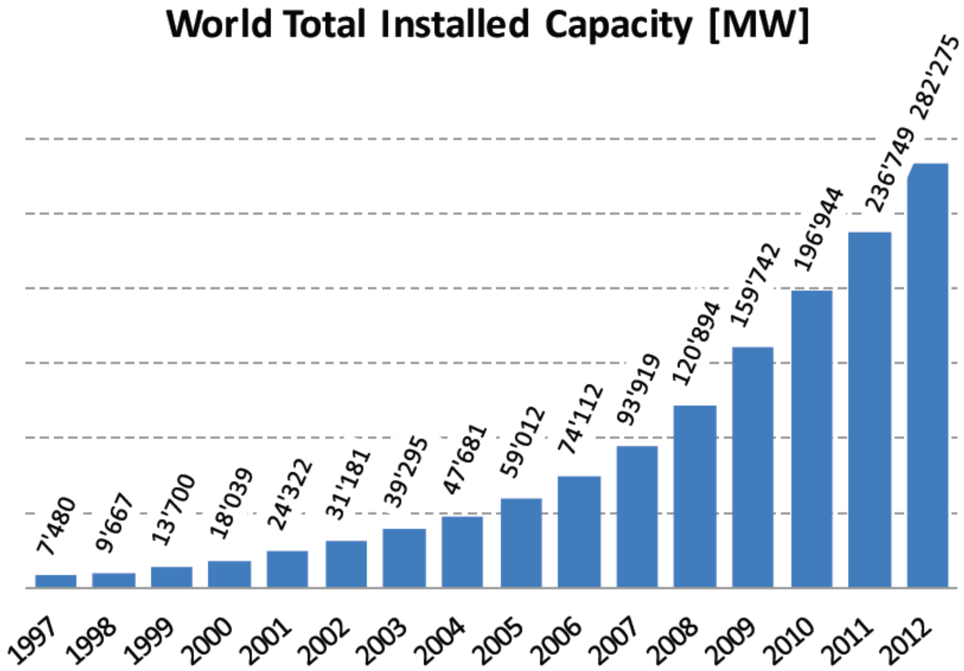

Fig. 2.7: Wind power world total installed capacity (source: WWER 2012 [2])

rope forcing the European and Asian manufactures to lower their equipment cost and find alternative markets for their products, aiming still growing economies, which is the case of Brazil. On the regulatory environment, in 2009, a specific auction to contract wind power as reserve energy to the system was carried out. This auction included clauses that alleviate the producers risk in order to foster this source into the Brazilian power system [44]. In subsequent auctions, the pace of wind energy contracted grew exponentially at the cost of a severe reduction on the auctioned price, pushing out risk averse investors. However, Brazil still has an enormous wind power potential untapped. According to [52], almost 300 GW of wind energy remain unexplored, mainly in the Northeastern area of country. In this sense, the Brazilian contract market became a way out to keep the growing pace of wind power in the Brazilian power system. Nevertheless, as discussed earlier, selling a forward contract backed up on wind power production can be very risk for the generation company, and risk-mitigation mechanisms (as the one proposed in this dissertation) should be studied in order to make a "safe" transition between the regulated market to the contract market.

Analyzing the energy production profile of a typical wind farm in the Northeastern area of Brazil (Fig. 2.8), despite the intermittency, we can observe a seasonal pattern which can be explored to reduce the agent's exposition to the short-term market via a joint commercialization with a complementary source, such as the SHs in the Southeastern area. Moreover, it is also possible to adequately generate scenarios of energy production for long periods (more than a year) by means of a periodical stochastic process without violating its dynamics [24].

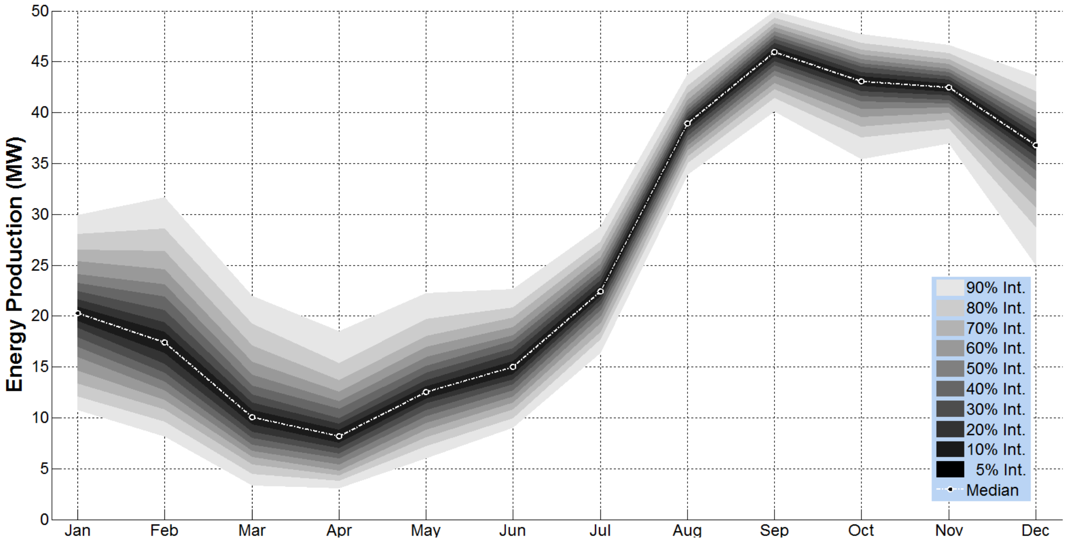

Fig. 2.8: Quantiles of energy production of a typical wind farm in the Northeastern area of Brazil

The other renewable source with great interest in the Brazilian system is the small hydro run-of-river. According to [53], a SH is a hydroelectric plant in which the capacity installed in higher than 1 MW and lower than 30 MW. Moreover, the total area of the plant reservoir must be higher than 3 km^2 and lower than 13 km^2, thus having a regularization capacity of less than a month (typically, a single day). Therefore, the energy production is a direct function of the influent inflow. In terms of energy commercialization, the price-quantity risk that affect the trading of renewable sources in the contract market is worsened for the case of the small-hydros due to the high (negative) correlation of the spot price with the system's inflow [17].

In order to mitigate this important risk, a hedging mechanism, known as Energy Reallocation Mechanism (MRE), was proposed for hydro plants, in which each hydroelectric (small hydros and "big" hydros) receives an *energy credit* proportional to the total hydro production (sum of the production of all hydros in the system). As a consequence, within the MRE, the production used to clear the contract is less intermittent and seasonal, being thus less expose to the short-term market. However, a several critics have been made to the inclusion of SHs into the MRE with the consequence that some plants have been excluded from the mechanism. Moreover, the entering of new participants (SHs) in the pool became more difficult[2]. Thus, SHs still need to study risk-mitigation mechanisms in order to make the contract market a less risk environment, especially for those outside the MRE. In Fig. 2.9, the energy production profile of a typical small hydro plant in the Southeastern area of Brazil is presented. Note that the uncertainty on the SH production is higher than the WP (Fig. 2.8), but it also presents a seasonal pattern which can be explored in the same way discussed earlier.

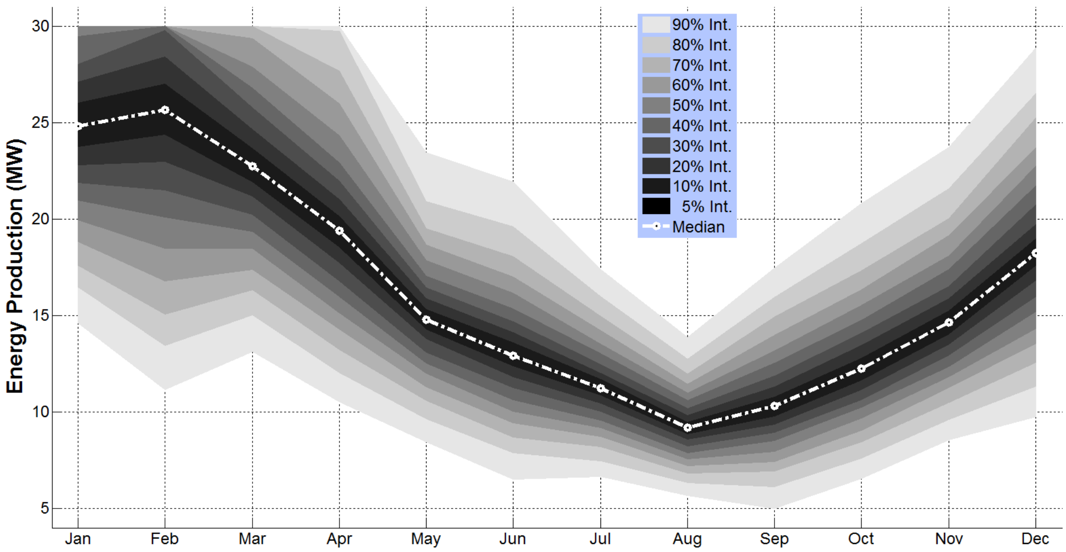

Fig. 2.9: Quantiles of energy production of a typical small hydro plant in the Southeastern area of Brazil

Finally, in order to illustrate the complementarity between the WP and the SH energy production, in Fig. 2.10 is presented the monthly average and 5%-95% quantiles for each renewable generation, in percentage of their FEC. Several works in recent literature, such as [10][21][22][24], make use of this well-known characteristic and shown positive results in mitigating the price-quantity risk when a joint-strategy is performed to sell a forward contract backed up by renewable sources in the contract market. It is important to mention that the energy production profile of a biomass (generation from sugar-cane waste) also presents a complementary profile with a SH, being suitable for joint-trading strategies. In the business structure proposed in this work, the complementarity characteristic of renewable sources to create a less intermittent joint-generation profile with higher market value will be

[2] For "big" hydros, the entry in the mechanism is compulsory.

explored too. We make use of the model presented in [24] to simulate correlated scenarios of different renewable plants and consider then in a two-stage stochastic programming model within the robust optimization framework applied in the spot price realization. The details over the formulation of the problem as well as the risk-averse measure used in the contracting model will be presented in the next chapters.

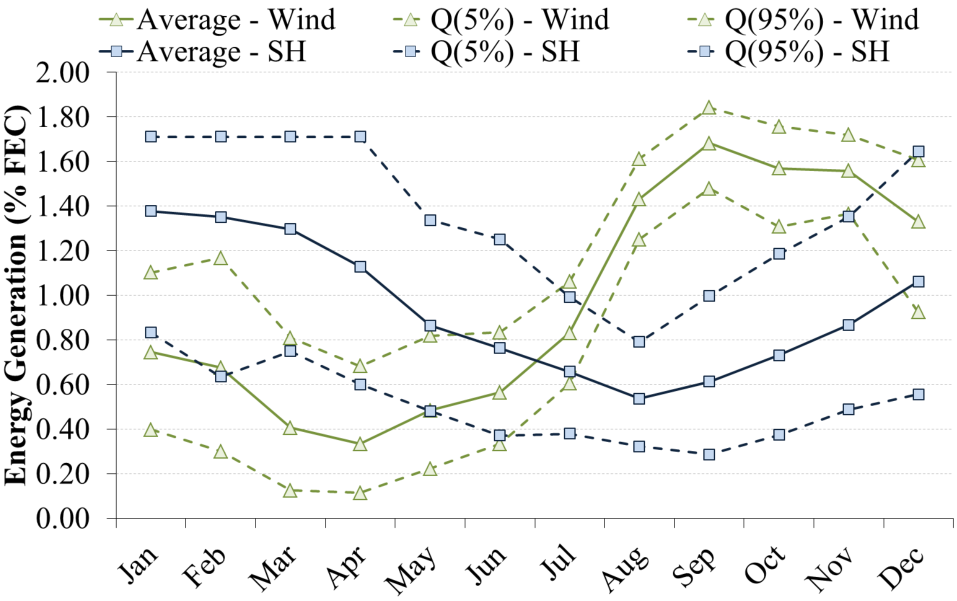

Fig. 2.10: Monthly average and 5%-95% quantiles for SH and WP generation profile in percentage of their FEC

3
Decision under Uncertainty

The problem of decision under uncertainty can be defined as a problem in which an agent must define a future guiding policy with uncertainty in some of the problem parameters [54]. It contains a set of decision variables in which their definition must be determined before the realization of the uncertainty, known as *first stage* variables. In addition, some problems have also the possibility of a "corrective action" after the realization of the uncertainty. If a single action can be made, the problem is known as a two-stage problem and the actions are called *second stage* variables. For instance, in typical electricity trading, the contracting decision, first stage decision, is made before the observation of the uncertainty (demand, spot price, energy availability, etc.) and the second stage decision is the short-term market settlement. On the other hand, if a series of actions over time is allowed as the information is revealed, the problem is known as a multi-stage problem and the actions are called *multi-stage* variables [55]. The classical approach to deal with such problems is to treat the "uncertain parameters" as random variables with an associated joint probability distribution function.

A naive approach to solve these problems, i.e., to assign a value to the decision variables, is to substitute the uncertain parameters by their forecast or best prediction (typically the expected value obtained from the joint distribution). Frequently, this procedure reaches a very simple and intuitive solution. However, in those cases in which the decision variables are highly sensitive with respect to the uncertain parameters and these parameters have a significant level of variability, it is important that the model used to obtain the future guiding policy (the set of decision variables) takes into account the risk associated to this variability. Moreover, each individual (agent) has different preferences or risk profile, i.e., a different value is assigned for the same "result". In this sense, a quantitative treatment of the uncertainty and risk preference are important to avoid undesirable solutions.

In this chapter, a widely used risk measure, both in practical and academical applications, named the Conditional Value-at-Risk (CVaR), is presented and a Certainty-Equivalent (CE) induced by the CVaR is discussed. Furthermore, some important approaches to model decision-making problems with uncertainty - stochastic programming, ambiguity theory and robust optimization - are analyzed.

3.1 Conditional Value-At-Risk

An important issue in decision theory arises when the decision maker needs to define its attitudes towards uncertainty and a risk management policy. In the past years, the Conditional Value-at-Risk (CVaR) has been playing the role of the main risk measure and paving the way for an enormous number of applications [24][56][57]. The main reason is its very intuitive form and important coherence properties [58]. In a net revenue or financial profit context, for which agents or decisions makers generally express their preferences, the CVaR_α can be defined as the conditioned expectation of the revenue left-side continuous distribution below a given $(1-\alpha)$ quantile, typically 1-10% (or α from 0.99 to 0.90). To mathematically model the CVaR, we assume a probability space $(\Omega, \Sigma, \mathbb{P})$ and a stochastic continuous revenue defined as a Σ-measurable function $\tilde{R} : \Omega \rightarrow \mathcal{Q}$, which maps elements from the set of all possible "states of nature" (Ω) to the compact set of all possible revenue outcomes $\mathcal{Q} \subset \mathbb{R}$. Within these definitions, an induced cumulative probability distribution function $(F_{\tilde{R}} : \mathcal{Q} \rightarrow [0,1])$ can be defined as:

$$F_{\tilde{R}}(r) = \mathbb{P}\big\{\omega \in \Omega \mid R(\omega) \leq r\big\}, \qquad \forall\, r \in \mathcal{Q}. \tag{3-1}$$

In addition, we define $F_{\tilde{R}}^{-1}(\alpha) = \inf\big\{r \in \mathcal{Q} \,|\, F_{\tilde{R}}(r) \geq \alpha\big\}$ as the left continuous inverse of $F_{\tilde{R}}$. Therefore, for a fixed level α, the Value-at-Risk (VaR_α) is the $(1-\alpha)$-quantile of $F_{\tilde{R}}$, i.e., $\text{VaR}_\alpha(\tilde{R}) = F_{\tilde{R}}^{-1}(1-\alpha)$ [59]. Then, by definition, the Conditional Value-at-Risk is the left-tail conditional expectation of revenue values up to the $(1-\alpha)$ quantile (or $\text{VaR}_\alpha(\tilde{R})$) and have the following mathematical form:

$$\begin{aligned} \text{CVaR}_\alpha(\tilde{R}) &:= \mathbb{E}\big[\tilde{R} \,|\, \tilde{R} \leq \text{VaR}_\alpha(\tilde{R})\big] \\ &:= \int_{\{\omega \in \Omega \;|\; R(\omega) \leq \text{VaR}_\alpha(\tilde{R})\}} R(\omega) \cdot \frac{d\mathbb{P}(\omega)}{F_{\tilde{R}}\big(\text{VaR}_\alpha(\tilde{R})\big)}. \end{aligned} \tag{3-2}$$

In Fig. 3.1, the CVaR_α and VaR_α values for a smooth revenue cumulative probability function is presented. In real applications, where the agent's revenue is a function of multidimensional random variables that model exogenous risk factors, expression (3-2) can be hard to compute since it is required to assess the multidimensional integrals value. In this sense, [60] propose an equivalent approach for (3-2) based on the solution of the optimization problem presented in (3-3):

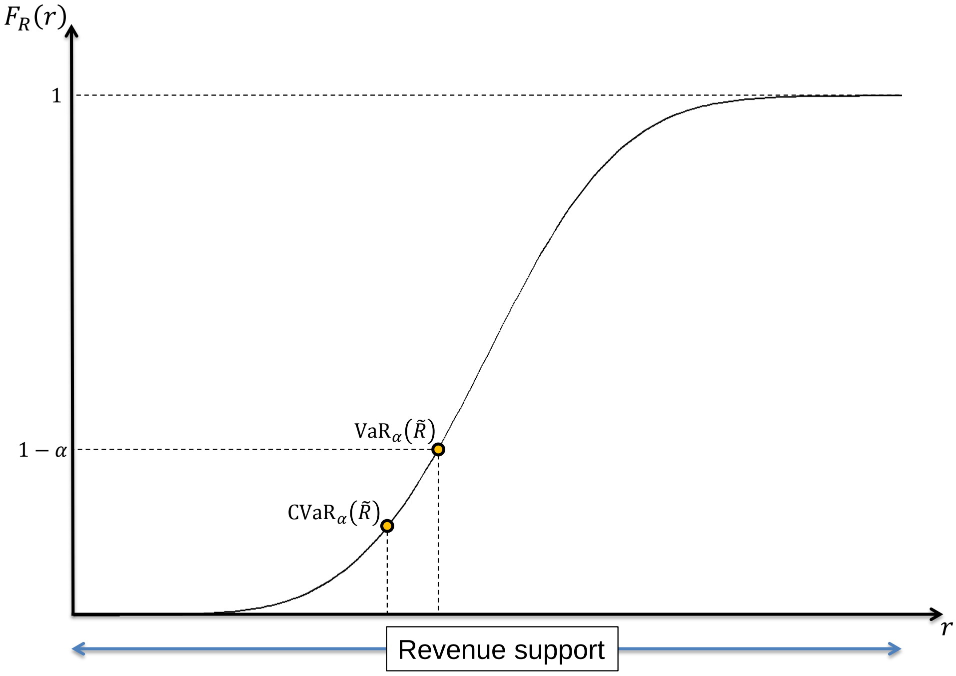

Fig. 3.1: CVaR$_\alpha$ and VaR$_\alpha$ values for a smooth revenue cumulative probability function.

$$\text{CVaR}_\alpha(\tilde{R}) := \sup_z \left\{ z - \frac{\mathbb{E}[(z-\tilde{R})^+]}{(1-\alpha)} \,\middle|\, z \in \mathbb{R} \right\}, \tag{3-3}$$

where $(x)^+ = \max\{x, 0\}$. The optimality proof as well as several properties of (3-3) can be found in [60].

Note that this approach exchanges the multidimensional integration in (3-2) by the supremum calculation over a convex family of unconditional expectation. Also, the optimization problem can take advantage of some convergence results that are provided for finite sampled scenarios, such as the convergence of the expectation operator $\left(\lim_{n\to\infty}\left(n^{-1}\sum_{i=1}^{n} x_i\right) \to \mathbb{E}[\tilde{X}]\right.$, where $\{x_i\}_{i=1}^{n}$ is a sample sequence of the random variable $\left.\tilde{X}\right)$ [61]. Therefore, (3-3) can be "solved" by sampling the exogenous variables, the so-called *Sampled Average Approximation* (SAA) [62]. Moreover, note that (3-3) is a convex maximization problem, which can be easily coupled into a decision-making problem typically modeled as a convex problem as well. In this sense, for a set $\mathcal{S}$ of sampled scenarios R_s of the revenue with probability of occurrence p_s, i.e, the pair $\{(R_s, p_s)\}_{s\in\mathcal{S}}$, the CVaR of a continuous random variable $\tilde{R}$ can be approximated by the following linear programming:

$$\text{CVaR}_\alpha(\tilde{R}) \approx \max_{z,\delta_s} \; z - \sum_{s\in\mathcal{S}} \frac{p_s\delta_s}{(1-\alpha)} \tag{3-4}$$

subject to:

$$\delta_s \geq z - R_s, \qquad \forall\, s \in \mathcal{S}; \tag{3-5}$$

$$\delta_s \geq 0, \qquad \forall\, s \in \mathcal{S}. \tag{3-6}$$

With respect to its properties, CVaR is considered a coherent risk measure [58][63]. Finally, it is shown in [64] that the CVaR_α preference index is the Certainty Equivalent (CE) of the assessed random variable $\tilde{R}$. In the next section, we discuss the concepts of CE and its importance in decision theory.

3.2 Certainty Equivalent

In decision theory, the concept of Certainty Equivalent (CE) is defined as the minimum (deterministic) value in which an agent becomes indifferent to a stochastic outcome [65]. In [64], it is shown that "*the CVaR_α preference index of a given random variable is its induced certainty equivalent* (*Remark 4*)". Moreover, in order to avoid (irrational) solutions with the same value of CVaR_α, i.e., solutions with the same risk, but lower expected values (lower return), a convex combination between the CVaR_α and the unconditioned expectation of the agent's revenue, called Extended CVaR Preference (ECP), is also presented. In this sense, a risk averse agent which follows an ECP has its certainty equivalent $(\Phi_{\alpha,\lambda})$ defined as:

$$\Phi_{\alpha,\lambda}(\tilde{R}) = \lambda\, \text{CVaR}_\alpha(\tilde{R}) + (1-\lambda)\, \mathbb{E}[\tilde{R}], \qquad \text{with } \lambda \in (0,1). \tag{3-7}$$

Note that, in (3-7), the convex combination parameter λ acts as a risk aversion parameter. For high values of λ, the weight of the CVaR_α in the agent's CE grows, representing a risk-averse agent. On the other hand, low values of λ induce a risk-neutral agent since the weight of the unconditioned expectation grows. Finally, note that (3-7) is a particularization of the widely used utility function $U(\tilde{R}) = \mathbb{E}[\tilde{R}] - \lambda\, \text{Risk}(\tilde{R})$, in which $\text{Risk}(\tilde{R})$ measures the agent's risk premium. By making the risk premium be the difference between the unconditioned expectation and the CVaR_α of the agent's revenue (see Fig. 3.2 for a graphical interpretation), (3-7) is reached:

$$\begin{aligned} U(\tilde{R}) &= \mathbb{E}[\tilde{R}] - \lambda\, \text{Risk}(\tilde{R}) \\ &= \mathbb{E}[\tilde{R}] - \lambda\Big(\mathbb{E}[\tilde{R}] - \text{CVaR}_\alpha(\tilde{R})\Big) \end{aligned}$$

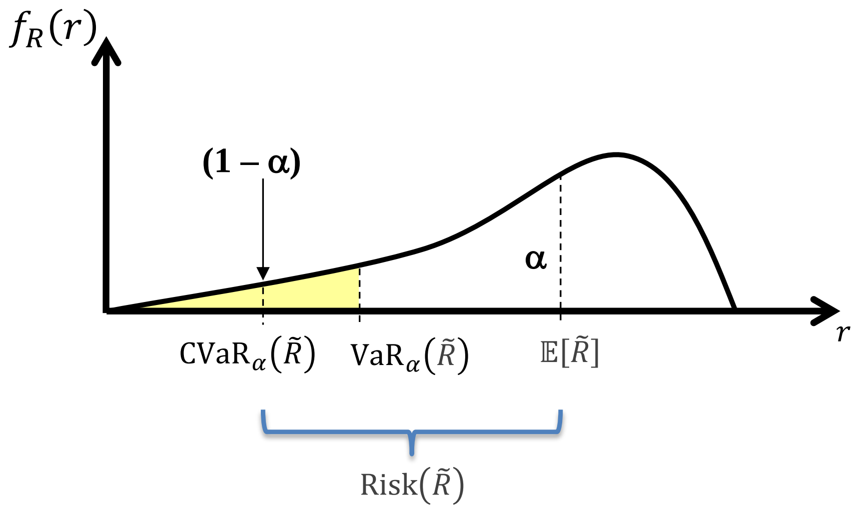

Fig. 3.2: Probability distribution function of the agent's revenue and Risk interpretation.

$$= \lambda\, \mathrm{CVaR}_\alpha(\tilde{R}) + (1-\lambda)\, \mathbb{E}[\tilde{R}]. \tag{3-8}$$

In (3-8), CE is obtained by taking the unconditioned expectation on both sides. In recent literature, several works used (3-7) as a metric for decision making [10][19][21][22][24][65]. In the business proposed by this dissertation, we use (3-7) as the agent certainty equivalent measure as well.

3.3 Mathematical Programming in Decision-Making Problems

Decision theory is the knowledge field that studies actions made by decision-making. It also evolves the identification of the uncertainties present on the business and the agent's rationality, aiming to support the decision-making process in an environment of uncertainty [54]. In this sense, several approaches have been developed, in past years, in order to find the "optimal decision" in this framework. In this section, we briefly present some of them as an introduction to contextualize the contributions and the developments of this dissertation.

3.3.1 Stochastic Programming

Due to the uncertain nature of real-life problems, the classical approach to model decision-making problems is the stochastic programming [55]. In this framework, the data uncertainty is represented as random variables and an accurate probability description of this random variables is assumed available, under the form of probability distributions, densities or, more generally, probability measures. In general, the uncertain data is represented as a vector of random variables $\tilde{\boldsymbol{\xi}} : \Omega \to \Xi$ which maps the set of all possible "states of nature" into a compact support set $\Xi \subset \mathbb{R}^n$ (the smallest closed subset in $\mathbb{R}^n$ such that $\mathbb{P}\{\tilde{\boldsymbol{\xi}} \in \Xi\} = 1$). Again, a probability space $(\Omega, \Sigma, \mathbb{P})$ is considered. Therefore, the general decision-making problem, within the framework of stochastic programming, can be formulated as the following certainty-equivalent maximization:

$$\underset{\boldsymbol{x} \in \mathcal{X}}{\text{Maximize}} \;\; \Phi_{\alpha,\lambda}\Big(R(\boldsymbol{x}, \tilde{\boldsymbol{\xi}})\Big), \tag{3-9}$$

where $\Phi_{\alpha,\lambda}$ is the agent's certainty equivalent defined in (3-7) (for convenience, we define the stochastic problem with the CE defined in Section 3.2. However, a general certainty equivalent functional can be used in (3-9)). Furthermore, $\boldsymbol{x}$ is the vector of decision variables and $\mathcal{X}$ represents the set of feasible decisions. Note that the agent revenue is redefined as a map $R : \mathcal{X} \times \Xi \to \mathcal{Q} \subset \mathbb{R}$ to contemplate the decision variables that affect the income. Usually, the maximization problem (3-9) can not be solved in its continuous form. Therefore, to numerically solve this problem, the vector of continuous random variables is approximated by a vector of discrete random variables within a discrete and finite sample space. Thus, using the CVaR reformulation presented in Section 3.1, the certainty equivalent maximization (3-9) can be redefined as:

$$\underset{\boldsymbol{x}, z, \delta(\omega)}{\text{Maximize}} \;\; \sum_{\omega \in \Omega} p(\omega) \left[(1-\lambda) R\big(\boldsymbol{x}, \boldsymbol{\xi}(\omega)\big) + \lambda \left(z - \frac{\delta(\omega)}{1-\alpha} \right) \right] \tag{3-10}$$

subject to:

$$\boldsymbol{x} \in \mathcal{X}; \tag{3-11}$$

$$\delta(\omega) \geq z - R\big(\boldsymbol{x}, \boldsymbol{\xi}(\omega)\big), \qquad \forall\, \omega \in \Omega; \tag{3-12}$$

$$\delta(\omega) \geq 0, \qquad \forall\, \omega \in \Omega. \tag{3-13}$$

One of the main challenges in application of stochastic programming to real problems is the complexity on describing the probability distribution function of some random variables (see [66] for a wide discussion). A poor description of this function can lead to a poor or even meaningless decisions. Therefore, several advances have been made in stochastic programming theory applied to decision-making problems aiming to formulate models in which the optimal solution is more "reliable" in the framework of a poor specification of the probability function. One of them is called Ambiguity Theory and is presented next.

3.3.2 Ambiguity Theory

Ambiguity was first introduced by [28] and is defined as the uncertainty over modeling of uncertain parameters, i.e., uncertainty over the joint probability distribution function of the random variables. Thus, the usual approach to obtain ambiguity-averse solutions is to take into account a set of probability distributions into the decision-making model. Hence, consider a probability space $(\Omega, \Sigma, \mathbb{P})$ and let $\tilde{\boldsymbol{\xi}} : \Omega \rightarrow \Xi \subset \mathbb{R}^n$ be a vector of uncertain parameters modeled as random variables and $F_{\tilde{\boldsymbol{\xi}}} : \Xi \rightarrow [0,1]$ the induced joint probability distribution function of these variables. Assume a decision-maker who is affected by this uncertain parameters and wants to model its problem using the classical stochastic programming approach presented in (3-9). However, with the available information, the agent was not able to precisely specify the probabilistic behavior of such variables. In this context, the certainty-equivalent maximization problem averse to ambiguity can be defined as:

$$\underset{\boldsymbol{x} \in \mathcal{X}}{\text{Maximize}} \left\{ \min_{F_{\tilde{\boldsymbol{\xi}}} \in \mathcal{F}} \Phi_{\alpha,\lambda}\Big(R(\boldsymbol{x}, \tilde{\boldsymbol{\xi}})\Big) \right\}. \tag{3-14}$$

According to [60], the CE $\left(\Phi_{\alpha,\lambda}\right)$, presented in (3-7), can be rewritten as:

$$\Phi_{\alpha,\lambda}\Big(R(\boldsymbol{x}, \tilde{\boldsymbol{\xi}})\Big) = (1-\lambda) \int_{\boldsymbol{\xi} \in \Xi} R(\boldsymbol{x}, \boldsymbol{\xi}) dF_{\tilde{\boldsymbol{\xi}}}(\boldsymbol{\xi}) \; + \\ \lambda \sup_{z} \left\{ z - (1-\alpha)^{-1} \int_{\boldsymbol{\xi} \in \Xi} \Big(z - R(\boldsymbol{x}, \boldsymbol{\xi}) \Big)^{+} dF_{\tilde{\boldsymbol{\xi}}}(\boldsymbol{\xi}) \right\}. \tag{3-15}$$

In (3-14), the inner optimization problem (minimization problem) is performed over a set $\mathcal{F}$ of credible distributions given by the agent. In this sense, considering an appropriate $\mathcal{F}$, the solution obtained by (3-14) can be interpreted as the best solution (maximization problem) for the worst distribution function that could be chosen within $\mathcal{F}$ (minimization problem). Therefore, under ambiguity, agents not only express their preference toward risk by means of their CE functionals, but also their preference toward the ambiguity level contained in each choice. Under the framework of (3-14), the main challenges in real application lies in the definition of the credible set of probability distributions $\mathcal{F}$ and how to solve the nonlinear optimization problem. We refer to [30][29][67][68] for further details about ambiguity theory and applications.

3.3.3 Robust Optimization

Robust optimization is a class of optimization problems based on worst-case analysis and was first discussed in [69]. It emerged as an alternative to stochastic programming and seeks to find the optimal solution against the worst-case possible realization of uncertainty. Using the same notation defined in the previous sections, the decision-making problem with robust optimization can be stated as:

$$\underset{\boldsymbol{x}\in\mathcal{X}}{\text{Maximize}} \left\{ \min_{\boldsymbol{\xi}\in\mathcal{U}_{\boldsymbol{\xi}}} R(\boldsymbol{x},\boldsymbol{\xi}) \right\}. \tag{3-16}$$

In (3-16), the inner optimization problem minimizes the agent's revenue over the worst possible realization of the vector of random variables within a set $\mathcal{U}_{\boldsymbol{\xi}}$, i.e., over the worst possible realization of uncertainty. In robust optimization literature, $\mathcal{U}_{\boldsymbol{\xi}}$ is known as an "uncertainty set" [26]. For instance, $\mathcal{U}_{\boldsymbol{\xi}}$ can be modeled as a polyhedral set of the form $\mathcal{U}_{\boldsymbol{\xi}} = \{\boldsymbol{\xi} \in \Xi \mid \boldsymbol{A\xi} \leq \boldsymbol{b}\}$. In Fig. 3.3, an example of a polyhedral uncertainty set is presented for a bivariate vector of uncertainties $\boldsymbol{\xi} = [\xi_1, \xi_2]^T$.

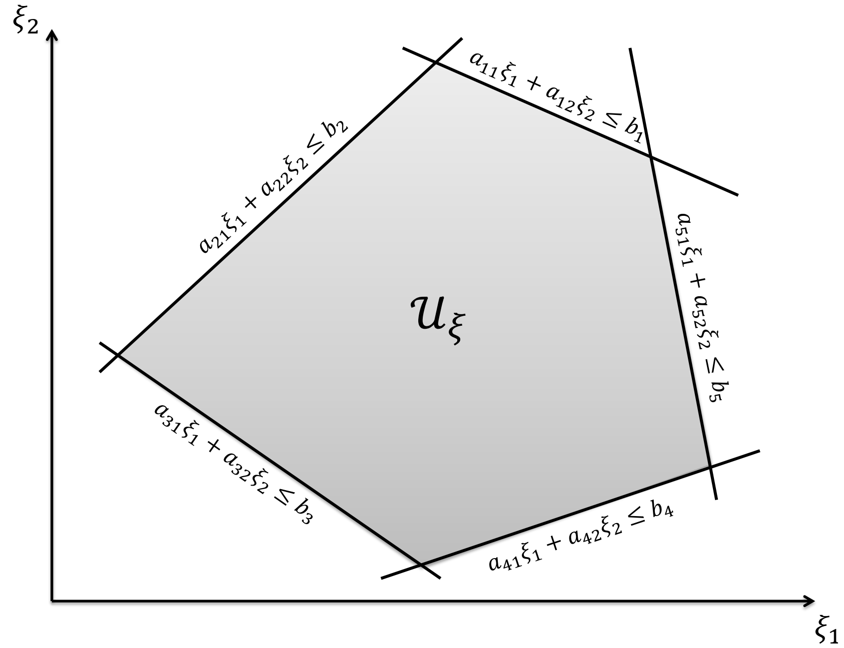

Fig. 3.3: Example of a Polyhedral Uncertainty Set.

On the one hand, the robust optimization approach has the benefit of not requiring the full specification of the joint probability distribution to represent the uncertainty factor, but only the description of an uncertainty set, which is typically written as linear equations. However, on the other hand, the main criticism to this approach in real applications is related to the fact that its solutions tends to be too conservative, which can be, sometimes, unrealistic. In this sense, a careful specification of the uncertainty set is of utmost importance to overcome high conservatism solutions.

In the literature, several works dealt with the construction of uncertainty sets. For instance, in [70], a methodology for constructing uncertainty set for linear optimization problems with uncertainty parameters that relies on decision maker risk preferences is proposed; [71] discusses a relaxed robust counterpart for general conic optimization problems that preserves the computational tractability of the nominal problem and provide a guarantee on the probability that the robust solution is feasible when the uncertainty coefficients obey independent and identically distributed Normal distributions; also [72] developed different tractable approximations to individual chance-constrained problems in robust optimization on a variety of uncertainty sets and show their interesting connections with bounds on the CVaR measure. In this dissertation, we shall be dealing with polyhedral uncertainty sets, such as the one presented in Fig. 3.3. In the next Chapter, we discuss how the uncertainty factors that affect the ETC's future cash flow were modeled in the framework of the theory presented in this Chapter.

4
Uncertainty Modeling

The main risk factors that affect an Energy Trading Company (ETC) future cash flow in the commercialization of renewable energy are: the amount of renewable production of each unit in the portfolio and the energy spot price. Using the notation introduced in Chapter 3, we can define $\tilde{\boldsymbol{\xi}}_t = \left[\tilde{\pi}_t,\, \tilde{g}^{(R)}_{1,t}, \ldots, \tilde{g}^{(R)}_{|U|,t}\right]^T$, where $\tilde{\pi}_t$ is a random variable that represents the energy spot price value in period $t \in \mathcal{H}$ and $\tilde{g}^{(R)}_{i,t}$ a random variable that accounts for the energy production of the unit $i \in U$ in period $t \in \mathcal{H}$. The "standard" approach to model optimal portfolio allocation in electricity contracts considers price-taker agents and represents uncertainty in energy production and market prices by scenarios, generated via Monte Carlo simulation. This is done by means of a production costing models (market equilibrium simulated by a fundamentalist approach) [16][46] or by statistical regressions using historical data [73][74][75]. In this dissertation, we propose a different approach to model uncertainty in energy spot price. In this Chapter, we motivate and describe how these uncertainties are treated in the proposed contracting model.

4.1 Renewable Generation

For renewable production, the modeling approach adopted in this dissertation follows the "standard" scenario-based approach. The scenarios of renewable generation is based on a well-known simulation procedure of stochastic processes, the Monte Carlo simulation procedure. We argue that physical variables, such as the wind speed and river inflows, generally exhibit a periodical and "well-behaved" pattern when they are simulated on a mid-term basis (monthly, for instance). In this sense, they are suitable for statistical modeling and can be adequately simulated for long-term periods (more than one year in a monthly basis) without violating their dynamics. Therefore, we consider that the random variables that model the renewable energy production have an objective joint probability distribution function, which means that their stochastic process can be adequately estimated from available information.

In the commercial model proposed in this dissertation, the generation of the scenarios for each renewable plant is considered as input data and thus exogenous to the model. We make use of the stochastic process proposed in [24] to simulate the renewable production used in the case study results. The model is based on a vector-autoregressive model with variance law. In this sense, the scenarios simulated contain the strong correlation structure that renewable production usually presents (correlation among the production of the power plants) and also preserve the different variance and covariance matrix for each month, reproducing thus the different variability over the months observed in this sources. It is important to mention that is outside of the scope of this dissertation to discuss or propose a methodology to simulate renewable scenarios.

4.2 Energy Spot Price

Spot price scenarios are usually obtained utilizing a production costing model (a fundamentalist approach) [16][46] or through a statistical regression on past market prices [73][74][75]. On the one hand, the fundamentalist approach takes into account *ex ante* hypothesis on market uncertainties, such as fuel prices, plant availability, supply expansion scenario, hydrology, etc. Statistical models, on the other hand, are based on the assumption that past realizations explains its future behavior. However, both approaches can be easily challenged: in the first (fundamentalist approach), any deviation on the assumptions affects the estimated probability distribution of prices, whereas the second approach (statistical approach) is not suitable for markets with technological developments, where the supply mix changes significantly over time and does not make the historical record a good proxy for the future. In this sense, an accurate description of the spot price probability distribution turns out to be a difficulty task.

In the view of these considerations, our proposal considers the spot price uncertainty by means of endogenously generated scenarios following the robust optimization approach. Typically, uncertainty on the input parameters of a decision-making problem is modeled as a variation around a nominal value assumed as the expected or best prediction value. We follow this assumption and define a nominal stochastic process to represent the spot price $\{\tilde{\pi}_t^o\}_{t \in \mathcal{H}}$ (this nominal stochastic process can be obtained via fundamentalist/statistical models or given by an exogenous expert). To model the spot price uncertainty set, defined as $\Pi_\tau^o(\omega)$, a K-neighborhood around the nominal value is constructed. Moreover, to allow the ETC to express its inter-temporal risk-preference, the set of periods $\mathcal{H}$ that represents the time horizon of the business, is partitioned into subsets of sub-periods, $\{\mathcal{H}_\tau\}_{\tau \in \mathcal{T}}$, e.g., months of each year τ. Therefore, for each renewable generation scenario $g_{i,t}^{(R)}(\omega), \omega \in \Omega$, there are $|\mathcal{T}|$ sets of partial credible spot price time series, each one of them related to a given subset of periods $\mathcal{H}_\tau$. Thus, assuming a discrete sample space Ω, the uncertainty set that represents those sets are:

$$\Pi_\tau^o(\omega) = \Bigg\{ \boldsymbol{\pi}_\tau(\omega) = \Big[\pi_1(\omega), \ldots, \pi_{|\mathcal{H}_\tau|}(\omega)\Big]^T \in \mathbb{R}_+^{|\mathcal{H}_\tau|} \;\Bigg| \tag{4-1}$$

$$\pi_t(\omega) = \pi_t^o(\omega) + \Delta\pi_t^+(\omega) v_t^+(\omega) - \Delta\pi_t^-(\omega) v_t^-(\omega), \quad \forall\, t \in \mathcal{H}_\tau; \quad \mu_t(\omega) \tag{4-2}$$

$$\sum_{t \in \mathcal{H}_\tau} \big(v_t^+(\omega) + v_t^-(\omega)\big) \le K_\tau; \quad \beta_\tau(\omega) \tag{4-3}$$

$$\pi_{t+1}(\omega) \ge (1 - r_t^-)\pi_t(\omega), \quad \forall\, t \in \bar{\mathcal{H}}_\tau; \quad \gamma_t(\omega) \tag{4-4}$$

$$\pi_{t+1}(\omega) \le (1 + r_t^+)\pi_t(\omega), \quad \forall\, t \in \bar{\mathcal{H}}_\tau; \quad \theta_t(\omega) \tag{4-5}$$

$$0 \le v_t^+(\omega) \le 1, \quad \forall\, t \in \mathcal{H}_\tau; \quad \eta_t(\omega) \tag{4-6}$$

$$0 \le v_t^-(\omega) \le 1, \quad \forall\, t \in \mathcal{H}_\tau; \quad \rho_t(\omega) \Bigg\}. \tag{4-7}$$

Where, $\bar{\mathcal{H}}_\tau$ is a copy of $\mathcal{H}_\tau$ except for the last term, which is disregarded to account for (4-4)-(4-5). Lagrangian multipliers (LM) are shown after the semicolon. Expression (4-2) defines the envelope for the spot-price time series in each $\mathcal{H}_\tau$: $\pi_t^o(\omega)$ is a parameter that defines the reference or nominal scenario, $\Delta\pi_t^+(\omega)$ and $\Delta\pi_t^-(\omega)$ are parameters that define the maximum positive and negative deviations from the nominal value, respectively, and $v_t^+(\omega)$ and $v_t^-(\omega)$ express the percentage of $\Delta\pi_t^+(\omega)$ and $\Delta\pi_t^-(\omega)$, respectively, ((4-6) and (4-7)) in which the endogenous spot price deviates from the reference. In addition, the total variation within each set of periods τ is constrained by a budget represented by the conservatism parameter K_τ in constraint (4-3). Note that, roughly speaking, K_τ can be interpreted as the number of periods that the endogenous spot price can deviate from the nominal value. Thus, for a small K_τ, the uncertainty with respect to the probability distribution of the spot price considered in the robust model is also small. Finally, expressions (4-4) and (4-5) constrain the maximum and minimum returns to r_t^+ and r_t^-, respectively.

A particular variation that could be straightforwardly made in the proposed uncertainty set is allow K_τ to vary for each scenario, which induces to the model different levels of stress as a function of the renewable production and reference spot price. For instance, we might consider a modeling that, in a scenario of energy deficit, the spot price might present a stress level higher than in scenarios of energy surplus. Furthermore, for $K_\tau = 0$, the proposed model meets the traditional stochastic optimization approach in which the spot price scenarios are the nominal scenarios. In this case, the agent considers the nominal distribution as the "true" one. Therefore, increasing the value of K_τ, (4-1)-(4-7) can be understood as a polyhedral family of sets of scenarios, which defines a family of credible distributions for the spot prices. Fig. 4.1 depicts the arrangement of the polyhedral sets over time and for different renewable energy scenarios. The dots over the straight line represent the nominal values of the spot price and the shaded area is the feasible region of the endogenous price with both positive and negative deviations (expression (4-2)). Therefore, once respected the budget (4-3) and returns (4-4)-(4-5) constraints, a polyhedral family of set of scenarios for the spot price can be defined inside this shaded area, one for each period $\tau \in \mathcal{T}$ and scenario $\omega \in \Omega$.

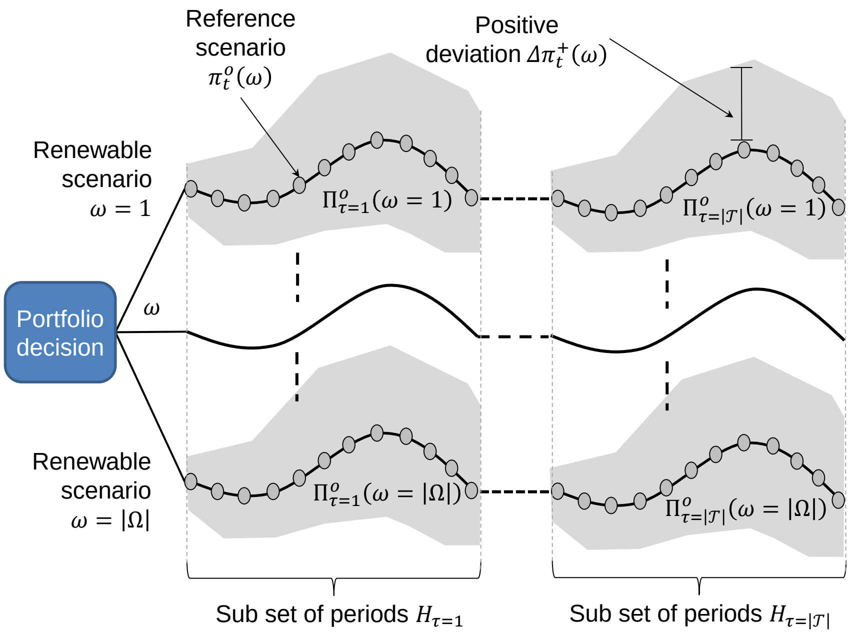

Fig. 4.1: Uncertainty characterization of the spot price random variable.

After the description of the spot price uncertainty set, we can direct apply the certainty-equivalent maximization problem discussed in Chapter 3. In the next Chapter, in order to apply the theory presented in this and in the previous Chapters, we develop the mathematical expression of the contract revenue for each modality of contract used in this dissertation and also present the electricity portfolio allocation model that takes into account the stochastic-robust framework discussed so far.

5
Modelling Electricity Contracts

As discussed in Section 2.2, the Brazilian contract market is an environment of bilateral negotiation between generator companies and consumers. In this market, the characteristics of the signed contracts are agreed between parties and, generally, rely on the definition of the modality of the contract, the price and quantity, the total maturity, the seasonalization and/or modularization, among others characteristics. Despite the full freedom in the choice of the contract, two types of arrangements are usually set in practice: standard forward contract and capacity payment contract [3]. Next, we discuss the specificities of each contract type as well as its financial payoff expression.

5.1 Standard Forward Contract

Standard forward contracts is a bilateral arrangement in which the seller counterpart has the obligation to delivery in full the agreed amount of energy against a fixed payment made by the buyer. This type of contract has been widely used in several electricity markets worldwide in different contexts (see [10][19][76][77]). A standard forward contract is defined by an initial and final supply date, an amount of energy $\left(Q_t^{(F)}\right)$ that must be delivered by the seller counterpart (typically measured in avgMW) and a price $\left(P_t^{(F)}\right)$ paid by the buyer for each MWh received. Therefore, the future (stochastic) revenue of an agent selling a standard forward contract is presented in expression (5-1).

$$R_t^{(F)}\left(\tilde{\pi}_t\right) = P_t^{(F)} h_t Q_t^{(F)} - \tilde{\pi}_t h_t Q_t^{(F)}, \qquad \forall\, t \in \mathcal{H}. \tag{5-1}$$

In (5-1), the first term represents the fixed payment received by the seller and the second term models the short-term settlement (energy bought to fulfill the contract). Typically, the agent has a second contract or a generation plant that covers the forward contract, and the risk of purchase at high price in the second term is mitigated. However, from the point of view of financial settlement of this contract in the clearing market, the energy is bought on the short-term market at the spot price $(\tilde{\pi}_t)$. The remainder parameters of (5-1) is the contract time horizon, comprised in the set of periods $\mathcal{H}$, and h_t, the number of hours of each period $t \in \mathcal{H}$.

5.2 Capacity Payment Contracts

Capacity payment contracts can be understood as a rental operation. In this framework, the generator rents a percentage of its unit (energy production and FEC) against a fixed payment [78]. The buyer, on the other hand, gains the right to trade this energy in the market (via a standard forward contract, for instance). In general, the capacity payment contract is defined by an initial and final supply date, a quantity that measures the amount of energy bought, a fixed price paid by the buyer and a Variable Operating Cost (VOC), Λ. This last term is a reimbursement made by the seller to the generation company to cover the cost of energy production.

Particularly, the characteristic and interpretation of this type of contract differs with respect to the source of generation. For renewable energy, the VOC can be considered zero ($\Lambda = 0$) since the "raw material" of the energy production is obtained from natural sources, such as wind and water inflow (for biomass plants, the sugarcane bagasse have an opportunity cost since it can be used for other purposes. Thus, for these plants, the VOC can be considered greater than zero). Therefore, the payment is related only to the amount of energy contracted. For instance, if we assume a trading in which the quantity is equal to the FEC of the plant, then the buyer has the right to receive all the plant's production, configuring a total leasing of the unit. On the other hand, for thermal units, the VOC can not be considered zero, since the "raw material" of the energy production is related to (sometimes expensive) commodities, such as coal and oil. In the Brazilian power system, in which thermal production is centrally determined, the amount of energy produced by thermal plants (consequently the energy received under this contract) is a function of the spot price. Thus, buying a capacity payment contract related to thermal units can be compared to buying an energy call option [14][43].

Next, we present the cash-flow expression of an agent buying a capacity payment contract for both types of sources: renewable and thermal plants.

5.2.1 Renewable Leasing

The net revenue expression of an ETC buying energy from renewable plants through a capacity contract is the following:

$$R_t^{(R)}\left(\tilde{g}_t^{(R)}, \tilde{\pi}_t\right) = \tilde{\pi}_t h_t \tilde{g}_t^{(R)} Q_t^{(R)} - P_t^{(R)} h_t Q_t^{(R)}, \qquad \forall\, t \in \mathcal{H}. \tag{5-2}$$

In (5-2), the first term represents the income from the settlement of the renewable energy in the short-term market, in which $\tilde{g}_t^{(R)}$ represents the energy production in period t and is modeled as a random variable, and the second-term models the fixed payment to the generator. In order to adjust the "size" of the renewable plant to the quantity of the contract, the renewable generation in expression (5-2) is accounted for per unit of FEC (% FEC). The remaining parameters h_t and $\mathcal{H}$ have the same interpretation of (5-1).

Note that (5-2) can be interpreted from the point of view of a long-term investment in a renewable plant. For instance, consider a Genco that aims to devise its optimal investment strategy in a renewable technology. Under this setting, the second term can represents the annualized investment cost and interests expenses. Thus, the decision that should be made by the Genco is to determine the optimal size of the renewable plant that it is willing-to-invest, i.e., define the optimal amount of $Q_t^{(R)}$ that maximizes the company's value by inserting this unit in the Genco's portfolio. Therefore, the term $P_t^{(R)} h_t Q_t^{(R)}$ in (5-2) could be replaced by a simpler fixed payment term, $E_t^{(R)}$, representing those expenses in each period of the cash flow time horizon.

5.2.2 Energy Call Option

The revenue of a capacity payment contract with a thermal unit is slightly different with respect to (5-2) due to the need to consider the reimbursement to the plant's owner on its declared VOC whenever the plant runs. In (5-3), the revenue expression of a agent buying this contract is presented.

$$R_t^{(T)}\big(\tilde{\pi}_t\big) = g_t^{(T)}(\tilde{\pi}_t) Q_t^{(T)} h_t \tilde{\pi}_t - P_t^{(T)} h_t Q_t^{(T)} - \Big(g_t^{(T)}(\tilde{\pi}_t) - \underline{g}^{(T)}\Big) Q_t^{(T)} \Lambda h_t, \quad \forall\, t \in \mathcal{H}. \tag{5-3}$$

The first term represents the income from the settlement of the thermal generation in the short-term market, in which $g_t^{(T)}(\tilde{\pi}_t)$ represents the energy produced by the thermal unit in period t as a function of the spot price $\tilde{\pi}_t$, the second-term models the fixed payment to the generator and the third term is the reimbursement to the plant's owner on its declared VOC (Λ) with $\underline{g}^{(T)}$ representing the plant's inflexibility (a must run constraint declared by the owner). Again, in order to adjust the "size" of the thermal plant to the quantity of the contract $Q_t^{(T)}$, the thermal generation is considered per unit of FEC (% FEC). The remaining parameters h_t and $\mathcal{H}$

have the same interpretation of (5-1) and (5-2).

As discussed in Section 2, the Brazilian system operation is cost-based. Therefore, a thermal plant is dispatched whenever the system marginal cost (spot price) is higher than plant's variable cost. In this sense, the function that models the thermal production is:

$$g_t^{(T)}(\tilde{\pi}_t) = \begin{cases} \bar{g}^{(T)}, & \tilde{\pi}_t \geq \Lambda \\ \underline{g}^{(T)}, & \tilde{\pi}_t < \Lambda \end{cases} \tag{5-4}$$

where $\bar{g}^{(T)}$ is the maximum production of the thermal unit, also declared by the plant's owner. By analyzing the revenue expression (5-3) and the thermal production rule presented in (5-4), the capacity payment contract with a thermal unit can be compared with an energy call option [3][14][43]. To better understand the comparison, assume, without loss of generality, a thermal plant 100% flexible $\left(\underline{g}^{(T)} = 0\right)$. Therefore, equation (5-3) can be rewritten as:

$$R_t^{(T)}(\tilde{\pi}_t) = \left[\left(\tilde{\pi}_t - \Lambda\right) h_t Q_t^{(T)} g_t^{(T)}(\tilde{\pi}_t)\right] - P_t^{(T)} h_t Q_t^{(T)}, \quad \forall\, t \in \mathcal{H}. \tag{5-5}$$

Note that, if the spot price $\tilde{\pi}_t$ is lower than the plant's VOC (Λ), than the thermal generation is $g_t^{(T)}(\tilde{\pi}_t) = \underline{g}^{(T)} = 0$. Therefore, the resulting income is negative, deterministic and is equal to the contract payment $\left(R_t^{(T)} = -P_t^{(T)} h_t Q_t^{(T)}\right)$. On the other hand, if the spot price $\tilde{\pi}_t$ is higher than the plant's VOC (Λ), than the thermal generation is maximum, i.e., $g_t^{(T)}(\tilde{\pi}_t) = \bar{g}^{(T)}$. As a consequence, the resulting income in stochastic (depends on the value of the spot price) and is $R_t^{(T)}(\tilde{\pi}_t) = \left[\left(\tilde{\pi}_t - \Lambda\right) h_t Q_t^{(T)} \bar{g}^{(T)}\right] - P_t^{(T)} h_t Q_t^{(T)}$. Thus, we can write the stochastic revenue in a given period $t \in \mathcal{H}$ as:

$$R_t^{(T)}(\tilde{\pi}_t) = \max\left\{\left(\tilde{\pi}_t - \Lambda\right), 0\right\} h_t Q_t^{(T)} \bar{g}^{(T)} - P_t^{(T)} h_t Q_t^{(T)}. \tag{5-6}$$

According to the option theory [79], by treating energy as "the underlying instrument" (asset from which the option is derived), if $\mathcal{H}$ is the option's maturity, $\tilde{\pi}_t$ represents the asset (energy) spot price, Λ the option's strike and $P_t^{(T)} h_t Q_t^{(T)}$ the option's premium, than $R_t^{(T)}(\tilde{\pi}_t)$ is exactly the revenue of a financial call option written over the commodity energy [79]. In Fig. 5.1, expression (5-6) is depicted as a function of π_t. Note that this graphic has the typical format of the payoff of an agent long in a call option. The only difference between the capacity payment contract with a thermal unit (energy call option) and the classical financial call option is that the energy option has no exercising limit, i.e., the buyer has one call option for each period of the contract horizon (a set of calls).

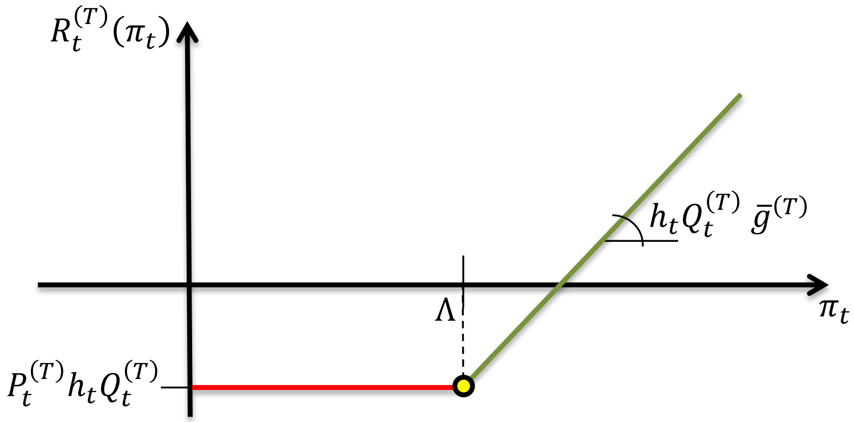

Fig. 5.1: Profit function of an agent long in energy call option as a function of the energy spot price $\tilde{\pi}_t$.

Therefore, in a nutshell, the first term of expression (5-6) represents the payoff of the call option and the second term is its premium with strike price Λ.

5.3 Proposed Business Structure

The business structure proposed in this dissertation involves an Energy Trading Company (ETC) [10] that have a opportunity to sell a standard forward contract to a consumer. To back this contract, the ETC has a set U of capacity payment opportunities with renewable units. In addition, in order to mitigate the exposure to the short-term market, we assume that the ETC has also an opportunity to buy an energy call option with a thermal unit. Fig. 5.2 illustrates the proposed contractual scheme for two renewable plants and a conventional thermal unit.

Note that in this contracting scheme, the ETC bears all of the price-quantity risk because the capacity contracts do not guarantee a fixed energy delivery to fulfill the forward contract with the consumer. In addition, the energy call option acts only as hedge against high spot prices and, typically, have an associated quantity lower than the forward contract. Therefore, in order to mitigate this risk, the ETC has the opportunity to adjust the portfolio by choosing the optimal amount that should be contracted from each opportunity that creates the maximum value for the company, i.e., must optimally define a percentage $\boldsymbol{x} = \left[x^{(F)}, x_1^{(R)}, \ldots, x_{|U|}^{(R)}, x^{(T)}\right]^T$ of the quantity of each contract to compose its portfolio in order to maximize its value

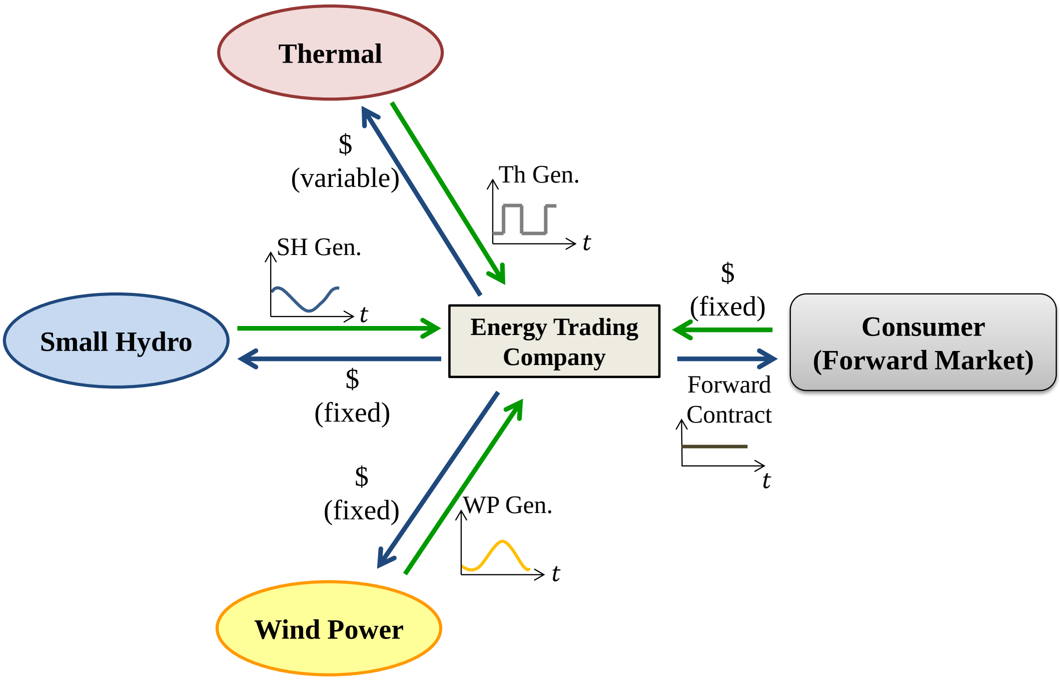

Fig. 5.2: ETC contract scheme: one forward contract, two capacity payment contract (one with a wind farm and one with a small hydro) and one energy call option.

(the certainty equivalent, for instance). In expression (5-7), the net revenue of the ETC under this contract scheme is presented.

$$R_t\left(\boldsymbol{x}, \tilde{\boldsymbol{g}}_t^{(R)}, \tilde{\pi}_t\right) = P_t^{(F)} h_t Q_t^{(F)} x^{(F)} - \sum_{i \in U} P_{i,t}^{(R)} h_t Q_{i,t}^{(R)} x_i^{(R)} -$$
$$\left(P_t^{(T)} - \underline{g}^{(T)} \Lambda\right) h_t Q_t^{(T)} x^{(T)} + \left(\sum_{i \in U} \tilde{g}_{i,t}^{(R)} Q_{i,t}^{(R)} x_i^{(R)} - Q_t^{(F)} x^{(F)}\right) \tilde{\pi}_t h_t +$$
$$\left(\tilde{\pi}_t - \Lambda\right) g_t^{(T)}(\tilde{\pi}_t) h_t Q_t^{(T)}, \quad \forall\, t \in \mathcal{H}. \tag{5-7}$$

Equation (5-7) is basically composed by the sum of the revenue of the individual contracts. Note that, by making $\tilde{\boldsymbol{\xi}}_t = \left[\tilde{\pi}_t,\, \tilde{g}_{1,t}^{(R)}, \ldots, \tilde{g}_{|U|,t}^{(R)}\right]^T$, the ETC's revenue (5-7) has the form of the revenue used to derive all results in Chapter 3, especially the certainty-equivalent maximization problem (3-9). Therefore, a particular application of the theory presented in Chapter 3 is to define the optimal amount of energy to compose the ETC's portfolio in order to maximize its certainty-equivalent measure described in (3.2). In the next section, this optimization model is presented under the framework of the mix between stochastic and robust modeling discussed in Chapter 4.

5.4 Hybrid Robust and Stochastic Model

The hybrid robust and stochastic model which aims to find the optimal portfolio of electricity contracts is presented in the set of equations (5-8)-(5-12):

$$\underset{\substack{x^{(F)},x_i^{(R)},x^{(T)},\\ \delta_\tau(\omega),z_\tau}}{\text{Maximize}} \sum_{\tau\in\mathcal{T}}\sum_{\omega\in\Omega} p(\omega)\left[\lambda\left(z_\tau - \frac{\delta_\tau(\omega)}{1-\alpha}\right) + \right.$$

$$\left.(1-\lambda)R_\tau^{\text{WC}}\big(\boldsymbol{x},\boldsymbol{g}_\tau^{(R)}(\omega)\big)\right](1+J)^{(1-\tau)} \quad (5\text{-}8)$$

subject to:

$$\delta_\tau(\omega) \geq z_\tau - R_\tau^{\text{WC}}\big(\boldsymbol{x},\boldsymbol{g}_\tau^{(R)}(\omega)\big), \qquad \forall\,\tau\in\mathcal{T},\omega\in\Omega; \quad (5\text{-}9)$$

$$Q^{(F)}x^{(F)} \leq \sum_{i\in U} Q_i^{(R)}x_i^{(R)} + Q^{(T)}x^{(T)}; \quad (5\text{-}10)$$

$$\delta_\tau(\omega) \geq 0, \qquad \forall\,\tau\in\mathcal{T},\omega\in\Omega; \quad (5\text{-}11)$$

$$x^{(F)}, x_i^{(R)}, x^{(T)} \in [0,1], \qquad \forall\, i\in U, \quad (5\text{-}12)$$

where the worst-case revenue is obtained from the minimization of the ETC's net revenue (5-7) over the polyhedral uncertainty set (4-1)-(4-7) discussed in Section 4.2, as presented next

$$R_\tau^{\text{WC}}\big(\boldsymbol{x},\boldsymbol{g}_\tau^{(R)}(\omega)\big) = \min_{\boldsymbol{\pi}_\tau(\omega)\in\Pi_\tau^o(\omega)} \sum_{t\in\mathcal{H}_\tau}\left[P_t^{(F)}h_tQ_t^{(F)}x^{(F)} - \sum_{i\in U}P_{i,t}^{(R)}h_tQ_{i,t}^{(R)}x_i^{(R)} - \right.$$

$$\left(P_t^{(T)} - \underline{g}^{(T)}\Lambda\right)h_tQ_t^{(T)}x^{(T)} + \left(\sum_{i\in U} g_{i,t}^{(R)}(\omega)Q_{i,t}^{(R)}x_i^{(R)} - Q_t^{(F)}x^{(F)}\right)\pi_t(\omega)h_t +$$

$$\left.\Big(\pi_t(\omega)-\Lambda\Big)g_t^{(T)}(\pi_t(\omega))h_tQ_t^{(T)}x^{(T)}\right](1+J_\tau)^{-(t-t_\tau^{\text{ini}})}, \quad \forall\,\tau\in\mathcal{T},\omega\in\Omega. \quad (5\text{-}13)$$

In (5-8)-(5-12), the set of available feasible solutions is described as $\mathcal{X} = \{\boldsymbol{x} \in [0,1]^{1\times|U|\times 1} \,|\, Q^{(F)}x^{(F)} \leq \sum_{i\in U} Q_i^{(R)}x_i^{(R)} + Q^{(T)}x^{(T)}\}$. Within the set of parameters, in (5-8), $\lambda \in (0,1)$ has the same interpretation of equation (3-7) and stands for the weight given to the (convex) combination between the expected value and the CVaR of the ETC's revenue, $p(\omega)$ is the probability of the scenario $\omega\in\Omega$ and $\alpha\in(0,1]$ is the level of significance of the CVaR. In practical applications, α generally ranges from 0.90 to 0.99. Constraints (5-9) and (5-11) represent a two-segment piecewise linear function, which computes in $\delta_\tau(\omega)$ only the violations of the scenarios whose worst-case revenue $R_\tau^{\text{WC}}\big(\boldsymbol{x},\boldsymbol{g}_\tau(\omega)\big)$ do not exceed the threshold z_τ, necessary to compute the CVaR [60]. Finally, (5-10) represents the energy balance constraint (the amount of energy bought must cover the amount sold, in avgMW).

In order to represent the ETC's inter-temporal preference, in (5-13), a partial present value accounts for the money value over the time within each subset of periods by means of a discount rate J_τ. In this setting, $R_\tau^{\mathrm{WC}}\big(\boldsymbol{x}, \boldsymbol{g}_\tau(\omega)\big)$ means the worst-case present value of the ETC's revenue for a given scenario $\omega \in \Omega$ at the first period of $\mathcal{H}_\tau$, accounted for by t_τ^{ini} (see Fig. 5.3). In addition, in (5-8), each resultant cash-flow valuated in the first period of each subset is also discounted by a discount rate J in order to obtain the value of the entire cash-flow in a reference date, which we assume to be the very first period of the business (see Fig. 5.4).

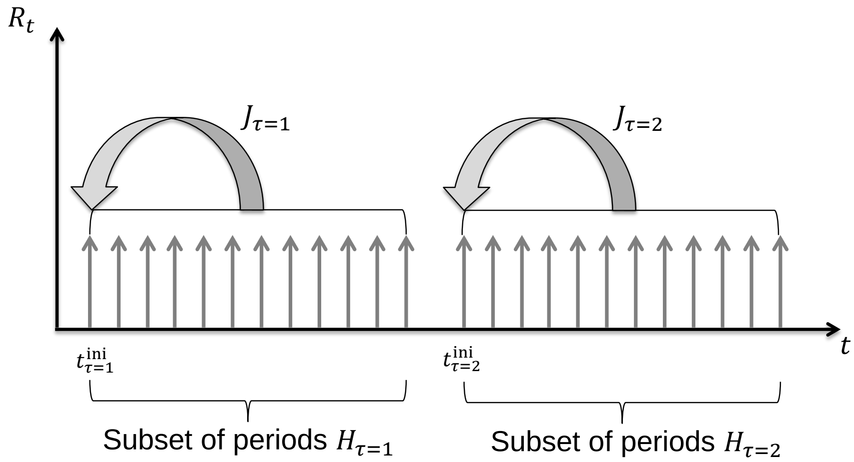

Fig. 5.3: Discount cash-flow inside each subset of periods $\mathcal{H}_\tau$ to the first period t_τ^{ini}.

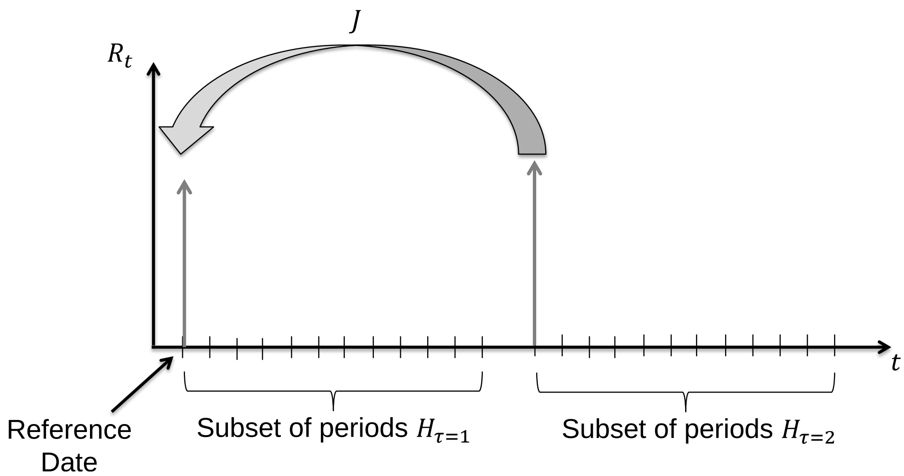

Fig. 5.4: Discount of the resultant cash-flow to the first period of the business.

Finally, the optimal contracting problem presented in (5-8)-(5-12) can be interpreted from two different point of views: stress test [27] and ambiguity averseness [29][30]. In the next Chapter, we discuss both interpretations.

6
Model Interpretations

One of the main contributions of this dissertation is to provide an economic and rational interpretation of the hybrid robust and stochastic contracting model developed in the previous Chapter. In this sense, in this Chapter, we present two different environment in which the optimal contracting problem (5-8)-(5-12) is inserted: stress test [27] and ambiguity averseness [29][30]. In addition, we provide a formal (mathematical) link between robust optimization and ambiguity aversion models [36].

6.1 Stress Test

Stress test is a typically used methodology in financial practice in which stress scenarios are applied to validate the performance of the current portfolio under adverse conditions. These stress scenarios are usually static (in the sense that represent the worst scenario independently of the portfolio), provided externally to the portfolio allocation model and given by a group of experts. It appears in the context of quantification of losses or risks that may appear under special extremal circumstances. The stress scenarios aim to represent some change in macroeconomic, socioeconomic or political factors and how this changes impact a given portfolio. The stress test approach differs among the context, the nature of the tested problem and also the way in which the stress scenarios have been selected [27][80].

In the view of these considerations, we argue that problem (5-8)-(5-12) can be interpreted as a generalization of this technique. Making (4-1)-(4-7) deterministic, $\big(\pi_t^o(\omega), \Delta\pi_t^{+/-}(\omega)\big) \rightarrow (\pi_t^o, \Delta\pi_t^{+/-})$, we have a methodology to evaluate the quality of a proposed portfolio $\boldsymbol{x}$ under high stress since the optimization problem (5-13) recovers the worst financial outcome of this portfolio. Therefore, if on the one hand, stress scenarios used in practice are usually exogenously defined by a group of experts, on the other hand, under the deterministic framework proposed by (5-8)-(5-13), the stress scenarios are obtained endogenously and represent the worst possible spot price realization which affects the ETC's revenue, being thus a step ahead over the stress test technique used in practice.

6.2 Ambiguity Aversion

A second point of view that this methodology can be seen is from aversion to ambiguity. Intuitively, it is easy to see that the decision-making problem using robust optimization has a close relation with the ambiguity-averse model presented in Section 3.3.2, in the sense that both approaches aim to find the worst case uncertainty realization within a given set of possibilities. In recent literature, different approaches treated ambiguity in a decision-making problem using robust optimization [31][32][33][34][35]. In this dissertation, we propose a different approach to characterize ambiguity in the framework of robust optimization [36]. The proposed approach allow us to treat ambiguity from the point of view of the scenarios, which is extremely useful both theoretically as well as for practical applications.

Suppose a probability space $(\Omega, \Sigma, \mathbb{P})$ and a nominal random vector $\tilde{\boldsymbol{\xi}}^o : \Omega \to \mathbb{R}^n$, that belongs to $\mathcal{L}^\infty(\Omega, \Sigma, \mathbb{P})$ [81], as a reference case, and its induced joint-probability distribution function $F_{\tilde{\boldsymbol{\xi}}^o} : \Xi^o \to \mathbb{R}$. This distribution can be interpreted as the best modeling of the uncertainty factors the user is able to make. In this framework, we can define a polyhedral set that defines the inaccuracy that the agent perceives around a reference scenario $\boldsymbol{\xi}^o(\omega)$:

$$\mathcal{E}_K^o\big(\boldsymbol{\xi}^o(\omega)\big) = \Bigg\{ \boldsymbol{\xi}(\omega) = \Big[\xi_1(\omega), \ldots, \xi_n(\omega)\Big]^T \in \mathbb{R}^n \;\Bigg| \tag{6-1}$$

$$\xi_i(\omega) = \xi_i^o(\omega) + \Delta\xi_i^+(\omega) v_i^+(\omega) - \Delta\xi_i^-(\omega) v_i^-(\omega), \qquad \forall\, i \in \{1, ..., n\}; \tag{6-2}$$

$$\sum_{i=1}^{n} \big(v_i^+(\omega) + v_i^-(\omega)\big) \le K; \tag{6-3}$$

$$0 \le v_i^+(\omega) \le 1, \qquad \forall\, i \in \{1, ..., n\}; \tag{6-4}$$

$$0 \le v_i^-(\omega) \le 1, \qquad \forall\, i \in \{1, ..., n\}; \Bigg\}. \tag{6-5}$$

According to this idea, a K-neighborhood of $\tilde{\boldsymbol{\xi}}^o$ is defined for each point of the sample space $\omega \in \Omega$. Therefore, we can define the set of all credible random variables, induced by the set of credible scenarios in $\mathcal{E}_K^o\big(\boldsymbol{\xi}^o(\omega)\big)$, as follows:

$$\mathcal{E}_K^o\big(\tilde{\boldsymbol{\xi}}^o\big) := \Big\{ \tilde{\boldsymbol{\xi}} \in \mathcal{L}^\infty(\Omega, \Sigma, \mathbb{P}) \;\Big|\; \boldsymbol{\xi}(\omega) \in \mathcal{E}_K^o\big(\boldsymbol{\xi}^o(\omega)\big), \; \forall\, \omega \in \Omega \Big\}. \tag{6-6}$$

In this context, a multiplicity of credible distributions around the nominal joint distribution function is induced by (6-6). Then, we can define the following related set of induced distribution functions:

$$\mathcal{F} = \Big\{ F_{\tilde{\boldsymbol{\xi}}} \in \mathcal{D} \;\Big|\; \tilde{\boldsymbol{\xi}} \in \mathcal{E}_K^o\big(\tilde{\boldsymbol{\xi}}^o\big) \Big\}, \tag{6-7}$$

where $\mathcal{D}$ is the set of all joint-probability distribution functions in $\mathbb{R}^n$. Within this set of definitions, the minimization problem in (3-14) can be rewritten in terms of $\tilde{\boldsymbol{\xi}}$ and $\mathcal{E}_K^o\big(\tilde{\boldsymbol{\xi}}^o\big)$ as follows:

$$\min_{F_{\boldsymbol{\xi}} \in \mathcal{F}} \; \Phi_{\alpha,\lambda}\Big(R(\boldsymbol{x}, \tilde{\boldsymbol{\xi}}) \Big) = \min_{\tilde{\boldsymbol{\xi}} \in \mathcal{E}_K^o(\tilde{\boldsymbol{\xi}}^o)} \Phi_{\alpha,\lambda}\Big(R(\boldsymbol{x}, \tilde{\boldsymbol{\xi}}) \Big). \tag{6-8}$$

Moreover, since the set $\mathcal{E}_K^o\big(\tilde{\boldsymbol{\xi}}^o\big)$ is pointwise defined, i.e., the set of all random vectors $\tilde{\boldsymbol{\xi}}$ such that $\boldsymbol{\xi}(\omega) \in \mathcal{E}_K^o\big(\boldsymbol{\xi}^o(\omega)\big)$ for all $\omega \in \Omega$, and we are assuming a coherent risk measure $\Phi_{\alpha,\lambda}$ (as discussed in Section 3.2), which is a non-decreasing function with respect to $R(\boldsymbol{x}, \tilde{\boldsymbol{\xi}})$ [64], the minimization problem in (6-8) is achieved by a pointwise minimum for all $\omega \in \Omega$:

$$\begin{aligned}
\min_{F_{\boldsymbol{\xi}} \in \mathcal{F}} \; \Phi_{\alpha,\lambda}\Big(R(\boldsymbol{x}, \tilde{\boldsymbol{\xi}}) \Big) &= \min_{\tilde{\boldsymbol{\xi}} \in \mathcal{E}_K^o(\tilde{\boldsymbol{\xi}}^o)} \Phi_{\alpha,\lambda}\Big(R(\boldsymbol{x}, \tilde{\boldsymbol{\xi}}) \Big) \\
&= \min_{\boldsymbol{\xi}(\omega) \in \mathcal{E}_K^o\left(\boldsymbol{\xi}^o(\omega)\right)} (1-\lambda) \cdot \sum_{\omega \in \Omega} R(\boldsymbol{x}, \boldsymbol{\xi}(\omega)) \cdot \mathbb{P}\big(\{\omega\}\big) + \\
&\qquad \lambda \cdot \sup_z \left\{ z - \sum_{\omega \in \Omega} \Big(z - R\big(\boldsymbol{x}, \boldsymbol{\xi}(\omega)\big) \Big)^+ \cdot \frac{\mathbb{P}\big(\{\omega\}\big)}{(1-\alpha)} \right\} \\
&= (1-\lambda) \cdot \sum_{\omega \in \Omega} R^{\mathrm{WC}}\big(\boldsymbol{x}, \omega\big) \cdot \mathbb{P}\big(\{\omega\}\big) + \\
&\qquad \lambda \cdot \sup_z \left\{ z - \sum_{\omega \in \Omega} \Big(z - R^{\mathrm{WC}}\big(\boldsymbol{x}, \omega\big) \Big)^+ \cdot \frac{\mathbb{P}\big(\{\omega\}\big)}{(1-\alpha)} \right\},
\end{aligned} \tag{6-9}$$

where

$$R^{\mathrm{WC}}\big(\boldsymbol{x}, \omega\big) = \min_{\boldsymbol{\xi}(\omega) \in \mathcal{E}_K^o\left(\boldsymbol{\xi}^o(\omega)\right)} R(\boldsymbol{x}, \boldsymbol{\xi}(\omega)); \qquad \forall\, \boldsymbol{x} \in \mathcal{X}, \omega \in \Omega, \tag{6-10}$$

and $\Phi_{\alpha,\lambda}$ is the functional presented in (3-15). Therefore, the certainty-equivalent maximization problem can be directly applied to (6-9) resulting in:

$$\underset{\boldsymbol{x}\in\mathcal{X}}{\text{Maximize}} \left\{ \min_{F_{\tilde{\boldsymbol{\xi}}}\in\mathcal{F}} \Phi_{\alpha,\lambda}\Big(R(\boldsymbol{x},\tilde{\boldsymbol{\xi}})\Big)\right\} = \underset{\boldsymbol{x}\in\mathcal{X}}{\text{Maximize}}\ (1-\lambda)\cdot\sum_{\omega\in\Omega} R^{\text{WC}}\big(\boldsymbol{x},\omega\big)\cdot\mathbb{P}\big(\{\omega\}\big) + \lambda\cdot\sup_{z}\left\{ z - \sum_{\omega\in\Omega}\Big(z - R^{\text{WC}}\big(\boldsymbol{x},\omega\big)\Big)^{+}\cdot\frac{\mathbb{P}\big(\{\omega\}\big)}{(1-\alpha)}\right\}. \quad (6\text{-}11)$$

This re-parametrization allow us to treat ambiguity from the point of view of the scenarios $\omega\in\Omega$, which is extremely useful since we can apply the classical Robust Optimization approach described in Section 3.3.3. Moreover, (6-11) has the classical two-stage stochastic programming form, with an equivalent deterministic formulation [55] that can be solved by commercial softwares [82]. Moreover, important to say that such model is also suitable for decomposition techniques, such as L-shaped decomposition. In addition, (6-1)-(6-5) with (6-10)-(6-11) generalizes the traditional stochastic optimization approach. By making $K=0$ in (6-3), the set of random variables $\mathcal{E}_K^o\big(\tilde{\boldsymbol{\xi}}^o\big)$ reduces to a singleton with only the nominal random variable $\tilde{\boldsymbol{\xi}}^o$, which means that the agent considers the reference distribution as the "true" one and thus no ambiguity is considered.

Finally, note that the following partition on the vector of risk factors can be applied $\tilde{\boldsymbol{\xi}} = \Big[\tilde{\boldsymbol{\xi}}_{(\text{amb})}, \tilde{\boldsymbol{\xi}}_{(\text{obj})}\Big]^T$. With this partition, if the agent believes that ambiguity affects only a part of the random vector, lets say $\tilde{\boldsymbol{\xi}}_{(\text{amb})}$, then, with the available information, a stochastic process can not be adjusted to precisely describe the uncertainty on $\tilde{\boldsymbol{\xi}}_{(\text{amb})}$. Therefore, this vector can be treated as "robust" on the CE maximization problem (6-11). On the other hand, if the rest of the vector is assumed to have an objective probability distribution, lets say $\tilde{\boldsymbol{\xi}}_{(\text{obj})}$, then a stochastic process can be well-adjusted and scenarios can be adequately simulated without violate their dynamics. In this case, this vector can be treated as "stochastic" on (6-11) and $F_{\tilde{\boldsymbol{\xi}}^o_{(\text{obj})}} = F_{\tilde{\boldsymbol{\xi}}_{(\text{obj})}}$. Thus, considering a discrete sample space Ω, the maximization problem (6-11) can be written as follows:

$$\underset{\boldsymbol{x}\in\mathcal{X}}{\text{Maximize}}\ (1-\lambda)\cdot\sum_{\omega\in\Omega} R^{\text{WC}}\big(\boldsymbol{x},\boldsymbol{\xi}_{(\text{obj})}(\omega)\big)\cdot\mathbb{P}\big(\{\omega\}\big) + \lambda\cdot\sup_{z}\left\{ z - \sum_{\omega\in\Omega}\Big(z - R^{\text{WC}}\big(\boldsymbol{x},\boldsymbol{\xi}_{(\text{obj})}(\omega)\big)\Big)^{+}\cdot\frac{\mathbb{P}\big(\{\omega\}\big)}{(1-\alpha)}\right\}, \quad (6\text{-}12)$$

where,

$$R^{\mathrm{WC}}\big(\boldsymbol{x}, \boldsymbol{\xi}_{\text{(obj)}}(\omega)\big) = \min_{\boldsymbol{\xi}_{\text{(amb)}}(\omega) \in \mathcal{E}_K^o\left(\boldsymbol{\xi}^o_{\text{(amb)}}(\omega)\right)} R\big(\boldsymbol{x}, \boldsymbol{\xi}_{\text{(obj)}}(\omega), \boldsymbol{\xi}_{\text{(amb)}}(\omega)\big),$$
$$\forall\, \omega \in \Omega, \boldsymbol{x} \in \mathcal{X}. \qquad (6\text{-}13)$$

The equivalent bilevel [83] mathematical programming problem that solves (6-12) is:

$$\underset{\boldsymbol{x},z,\delta(\omega)}{\text{Maximize}} \ \sum_{\omega\in\Omega} p(\omega)\left[(1-\lambda)R^{\mathrm{WC}}\Big(\boldsymbol{x}, \boldsymbol{\xi}_{\text{(obj)}}(\omega)\Big) + \lambda\left(z - \frac{\delta(\omega)}{1-\alpha}\right)\right] \qquad (6\text{-}14)$$

subject to:

$$\boldsymbol{x} \in \mathcal{X}; \qquad (6\text{-}15)$$

$$\delta(\omega) \geq z - R^{\mathrm{WC}}\Big(\boldsymbol{x}, \boldsymbol{\xi}_{\text{(obj)}}(\omega)\Big), \qquad \forall\, \omega \in \Omega; \qquad (6\text{-}16)$$

$$\delta(\omega) \geq 0, \qquad \forall\, \omega \in \Omega, \qquad (6\text{-}17)$$

It is important to mention that, since some variables are represented via scenarios generated externally to the model $\big(\tilde{\boldsymbol{\xi}}_{\text{(obj)}}\big)$, the worst-case revenue (6-13) is defined for each scenario $\omega \in \Omega$, i.e., for each $\boldsymbol{\xi}_{\text{(obj)}}(\omega)$, with $\omega \in \Omega$, a set of $\boldsymbol{\xi}_{\text{(amb)}}(\omega)$ is obtained in order to create the worst possible scenario of revenue in (6-13).

Finally, the decision-making problem with robust optimization is a bilevel programming problem, which cannot be directly solved by commercial solvers [82]. However, for some particular structures of R and $\mathcal{E}_K^o\big(\tilde{\boldsymbol{\xi}}^o\big)$, the results of [26] can be directly applied to transform (6-12) into an equivalent single-level problem. For instance, assuming that $\mathcal{E}_K^o\big(\tilde{\boldsymbol{\xi}}^o\big)$ is a polyhedral set (Fig. 3.3, for example) and the revenue expression is the following linear function on $\tilde{\boldsymbol{\xi}}_{\text{(amb)}}$ and $\boldsymbol{x}$,

$$R\Big(\boldsymbol{x}, \boldsymbol{\xi}_{\text{(obj)}}(\omega), \boldsymbol{\xi}_{\text{(amb)}}(\omega)\Big) = \boldsymbol{c}^T\boldsymbol{x} + f\Big(\boldsymbol{\xi}_{\text{(obj)}}(\omega), \boldsymbol{\xi}_{\text{(amb)}}(\omega)\Big)^T\boldsymbol{x}, \qquad (6\text{-}18)$$

with f also linear on $\tilde{\boldsymbol{\xi}}_{\text{amb}}$, the max-min nonlinear programming problem in (6-12) can be transformed into the following single-level equivalent linear maximization problem:

$$\underset{\substack{\boldsymbol{x}\in\mathcal{X},\\ \boldsymbol{y}(\omega)\in\mathcal{Y}(\omega)}}{\text{Maximize}} \ \Phi_{\alpha,\lambda}\Big(\boldsymbol{c}^T\boldsymbol{x} + \boldsymbol{b}^T\boldsymbol{y}(\omega)\Big). \qquad (6\text{-}19)$$

In (6-19), $\mathcal{Y}(\omega)$ is the correspondent dual feasible region of (6-13) and $\boldsymbol{y}(\omega)$ are the associated vector of Lagrange multipliers. Note that (6-19) has the classical stochastic programming form presented in (3-9), which can be solved using commercial solvers [82]. In the next two Chapters, we present two optimization models for two different business approaches using realistic data from the Brazilian power system to illustrate the applicability of the theory developed in this dissertation.

7
Contracting Strategies for Renewable Plants

To illustrate the proposed methodology, we first present two applications of the business structure described in Chapter 5. We assume in this Chapter that the ETC represents a set of renewable generation companies and, therefore, the optimal portfolio must relies only on renewable capacity contracts to back a sale of a standard forward contract in the Brazilian contract market. In this context, the trading revenue (5-7) is simplified to:

$$R_t\big(\boldsymbol{x}, \tilde{\boldsymbol{g}}_t^{(R)}, \tilde{\pi}_t\big) = P_t^{(F)} h_t Q_t^{(F)} x^{(F)} - \sum_{i \in U} P_{i,t}^{(R)} h_t Q_{i,t}^{(R)} x_i^{(R)} + \left(\sum_{i \in U} \tilde{g}_{i,t}^{(R)} Q_{i,t}^{(R)} x_i^{(R)} - Q_t^{(F)} x^{(F)} \right) \tilde{\pi}_t h_t \,, \quad \forall\, t \in \mathcal{H}. \tag{7-1}$$

As a consequence, the worst-case revenue presented in equation (5-13) must be replaced by the following optimization model:

$$R_\tau^{\mathrm{WC}}\big(\boldsymbol{x}, \boldsymbol{g}_\tau^{(R)}(\omega)\big) = \min_{\boldsymbol{\pi}_\tau(\omega) \in \Pi_\tau^o(\omega)} \sum_{t \in \mathcal{H}_\tau} \Bigg[P_t^{(F)} Q_t^{(F)} x^{(F)} - \sum_{i \in U} P_{i,t}^{(R)} Q_{i,t}^{(R)} x_i^{(R)} + \left(\sum_{i \in U} g_{i,t}^{(R)}(\omega) Q_{i,t}^{(R)} x_i^{(R)} - Q_t^{(F)} x^{(F)} \right) \pi_t(\omega) \Bigg] h_t (1 + J_\tau)^{-(t - t_\tau^{\mathrm{ini}})}, \ \forall\, \tau \in \mathcal{T}, \omega \in \Omega. \tag{7-2}$$

Although mathematically precise, the contracting model presented in (5-8)-(5-12) with worst-case revenue (7-2) is a bilevel optimization model [83], which can not be directly solved by commercial linear programming solvers [82]. However, note that the uncertainty sets $\{\Pi_\tau^o(\omega)\}_{\tau \in \mathcal{T}}$ are polyhedral sets and the ETC's revenue (7-1) is a linear function on $\{\tilde{\boldsymbol{\pi}}_\tau\}_{\tau \in \mathcal{T}}$ and on $\boldsymbol{x}$. Therefore, the transformation proposed in [26] and discussed in Section 3.3.3 can be applied to obtain an efficient single-level two-stage linear optimization model. The procedure proposed in [26] can be summarized in three steps:

1. for each $\omega \in \Omega$ and $\tau \in \mathcal{T}$, derive the dual objective function of the linear program defined by the right-hand-side of (7-2). It constitutes a lower bound for the worst-case revenue term, for all dual feasible solution;

2. for each $\omega \in \Omega$ and $\tau \in \mathcal{T}$, derive the dual feasible constraints of the linear program defined by the right-hand-side of (7-2);

3. replace $R_{\tau}^{\mathrm{WC}}\big(\boldsymbol{x}, \boldsymbol{g}_{\tau}^{(R)}(\omega)\big)$ in (5-8) and (5-9) by the dual objective function found in 1. and add in (5-8)-(5-12) the dual feasible constraints derived in 2..

Following the steps, we first need to derive the dual objective function of the right-hand-side of (7-2) and its respective dual feasible constraints. In the next set of equations, we present both developments in which equation (7-3) represents the dual objective function and (7-4)-(7-10) the dual feasible constraints for each $\omega \in \Omega$ and $\tau \in \mathcal{T}$.

$$R_{\tau}^{\mathrm{Dual}}\big(\boldsymbol{x}, \boldsymbol{g}_{\tau}^{(R)}(\omega), \boldsymbol{\Theta}_{\tau}(\omega)\big) = \sum_{t \in \mathcal{H}_{\tau}} \Big(\pi_t^{o}(\omega)\mu_t(\omega) - \eta_t(\omega) - \rho_t(\omega)\Big) - K_{\tau}\beta_{\tau}(\omega) \; + \sum_{t \in H_{\tau}} \left(P^{(F)}Q^{(F)}x^{(F)} - \sum_{i \in U} P_i^{(R)}Q_i^{(R)}x_i^{(R)}\right) h_t(1+J_{\tau})^{-(t-t_{\tau}^{\mathrm{ini}})} \tag{7-3}$$

$$\mu_t(\omega) - (1-r_t^{-})\gamma_t(\omega) + (1+r_t^{+})\theta_t(\omega) \leq \left(\sum_{i \in U} g_{i,t}^{(R)}(\omega)Q_{i,t}^{(R)}x_i^{(R)} - Q^{(F)}x^{(F)}\right) h_t(1+J_{\tau})^{-(t-t_{\tau}^{\mathrm{ini}})}, \quad \forall\, t \in \mathcal{H}_{\tau}, \tau \in \mathcal{T}, \omega \in \Omega \,\big|\, t = 1 + (\tau-1)|\mathcal{H}_{\tau}|; \tag{7-4}$$

$$\mu_t(\omega) + \gamma_{t-1}(\omega) - (1-r_t^{-})\gamma_t(\omega) - \theta_{t-1}(\omega) + (1+r_t^{+})\theta_t(\omega) \leq \left(\sum_{i \in U} g_{i,t}^{(R)}(\omega)Q_{i,t}^{(R)}x_i^{(R)} - Q^{(F)}x^{(F)}\right) h_t(1+J_{\tau})^{-(t-t_{\tau}^{\mathrm{ini}})}, \quad \forall\, t \in \mathcal{H}_{\tau}, \tau \in \mathcal{T}, \omega \in \Omega \,\big|\, t \neq 1 + (\tau-1)|\mathcal{H}_{\tau}| \text{ and } t \neq |\mathcal{H}_{\tau}|\,\tau; \tag{7-5}$$

$$\mu_t(\omega) + \gamma_{t-1}(\omega) - \theta_{t-1}(\omega) \leq \left(\sum_{i \in U} g_{i,t}^{(R)}(\omega)Q_{i,t}^{(R)}x_i^{(R)} - Q^{(F)}x^{(F)}\right) h_t(1+J_{\tau})^{-(t-t_{\tau}^{\mathrm{ini}})}, \quad \forall\, t \in \mathcal{H}_{\tau}, \tau \in \mathcal{T}, \omega \in \Omega \,\big|\, t = |\mathcal{H}_{\tau}|\,\tau; \tag{7-6}$$

$$\Delta\pi_t^{+}(\omega)\mu_t(\omega) + \beta_{\tau}(\omega) + \eta_t(\omega) \geq 0, \qquad \forall\, t \in \mathcal{H}_{\tau}, \tau \in \mathcal{T}, \omega \in \Omega; \tag{7-7}$$

$$\Delta\pi_t^{-}(\omega)\mu_t(\omega) - \beta_{\tau}(\omega) - \eta_t(\omega) \leq 0, \qquad \forall\, t \in \mathcal{H}_{\tau}, \tau \in \mathcal{T}, \omega \in \Omega; \tag{7-8}$$

$$\beta_{\tau}(\omega), \eta_t(\omega), \rho_t(\omega) \geq 0, \qquad \forall\, t \in \mathcal{H}_{\tau}, \tau \in \mathcal{T}, \omega \in \Omega; \tag{7-9}$$

$$\gamma_t(\omega), \theta_t(\omega) \geq 0, \qquad \forall\, t \in \bar{\mathcal{H}}_{\tau}, \tau \in \mathcal{T}, \omega \in \Omega. \tag{7-10}$$

Where $\boldsymbol{\Theta}_{\tau}(\omega) = \big[\mu_t(\omega), \theta_t(\omega), \beta_{\tau}(\omega), \eta_t(\omega), \rho_t(\omega), \gamma_t(\omega)\big]^T$ is the set of dual variables that satisfies (7-4)-(7-10), for each $\omega \in \Omega$ and $t \in \mathcal{H}_\tau$, $\tau \in \mathcal{T}$. Once developed the dual objective function of the right-hand-side of (7-2) and its respective dual feasible constraints, we can proceed to step 3., by replacing $R^{\text{WC}}_\tau\big(\boldsymbol{x}, \boldsymbol{g}^{(R)}_\tau(\omega)\big)$ in (5-8) and (5-9) by $R^{\text{Dual}}_\tau\big(\boldsymbol{x}, \boldsymbol{g}^{(R)}_\tau(\omega), \boldsymbol{\Theta}_\tau(\omega)\big)$ in (7-3) and add to (5-8)-(5-12) the constraints (7-4)-(7-10). By the weak duality, all feasible solution that satisfies (7-4)-(7-10) constitutes a lower bound for the worst-case revenue $R^{\text{WC}}_\tau\big(\boldsymbol{x}, \boldsymbol{g}^{(R)}_\tau(\omega)\big)$, i.e. $R^{\text{WC}}_\tau\big(\boldsymbol{x}, \boldsymbol{g}^{(R)}_\tau(\omega)\big) \geq R^{\text{Dual}}_\tau\big(\boldsymbol{x}, \boldsymbol{g}^{(R)}_\tau(\omega), \boldsymbol{\Theta}_\tau(\omega)\big)$, $\forall\, \boldsymbol{\Theta}_\tau(\omega)$ feasible in (7-4)-(7-10). Therefore, maximizing this revenue (or a coherent measure of this revenue, such as the CVaR) implies in maximize expression (7-3). By making the set of dual variables $\boldsymbol{\Theta}_\tau(\omega)$ as decision variables and using the strong duality theorem [84], in the optimal value $\big(\boldsymbol{\Theta}^*_\tau(\omega)\big)$, the dual objective function recovers the value of the worst-case revenue, i.e., $R^{\text{WC}}_\tau\big(\boldsymbol{x}, \boldsymbol{g}^{(R)}_\tau(\omega)\big) = R^{\text{Dual}}_\tau\big(\boldsymbol{x}, \boldsymbol{g}^{(R)}_\tau(\omega), \boldsymbol{\Theta}^*_\tau(\omega)\big)$. Hence, by using the dual of the worst-case revenue minimization problem, we can transform a bilevel problem into an equivalent single-level linear problem, which can be solved using commercial softwares [82]. In the following, it is presented the single-level equivalent model for the problem (5-8)-(5-12) with worst-case revenue described in (7-2).

$$\underset{\substack{x^{(F)}, x^{(R)}_i, R^{\text{WC}}_\tau(\omega), z_\tau,\\ \delta_\tau(\omega), \mu_t(\omega), \theta_t(\omega),\\ \beta_\tau(\omega), \eta_t(\omega), \rho_t(\omega), \gamma_t(\omega)}}{\text{Maximize}} \quad \sum_{\tau \in \mathcal{T}} \sum_{\omega \in \Omega} p(\omega) \left[\lambda \left(z_\tau - \frac{\delta_\tau(\omega)}{1-\alpha} \right) + (1-\lambda) R^{\text{WC}}_\tau(\omega) \right] (1+J)^{(1-\tau)} \tag{7-11}$$

subject to:

$$R^{\text{WC}}_\tau(\omega) = \sum_{t \in \mathcal{H}_\tau} \Big(\pi^o_t(\omega)\mu_t(\omega) - \eta_t(\omega) - \rho_t(\omega) \Big) - K_\tau \beta_{\tau,s} + \sum_{t \in H_\tau} \left(P^{(F)} Q^{(F)} x^{(F)} - \sum_{i \in U} P^{(R)}_i Q^{(R)}_i x^{(R)}_i \right) h_t (1+J_\tau)^{-(t - t^{\text{ini}}_\tau)}, \quad \forall\, \tau \in \mathcal{T}, \omega \in \Omega; \tag{7-12}$$

$$\delta_\tau(\omega) \geq z_\tau - R^{\text{WC}}_\tau(\omega), \qquad \forall\, \tau \in \mathcal{T}, \omega \in \Omega; \tag{7-13}$$

Constraints (5-10)-(5-12) (7-14)

$$\mu_t(\omega) - (1 - r^-_t)\gamma_t(\omega) + (1 + r^+_t)\theta_t(\omega) \leq \left(\sum_{i \in U} g^{(R)}_{i,t}(\omega) Q^{(R)}_{i,t} x^{(R)}_i - Q^{(F)} x^{(F)} \right) h_t (1+J_\tau)^{-(t - t^{\text{ini}}_\tau)},$$

$$\forall\, t \in \mathcal{H}_\tau, \tau \in \mathcal{T}, \omega \in \Omega \,\Big|\, t = 1 + (\tau - 1)|\mathcal{H}_\tau|; \quad (7\text{-}15)$$

$$\mu_t(\omega) + \gamma_{t-1}(\omega) - (1 - r_t^-)\gamma_t(\omega) - \theta_{t-1}(\omega) + (1 + r_t^+)\theta_t(\omega) \leq \left(\sum_{i \in U} g_{i,t}^{(R)}(\omega) Q_{i,t}^{(R)} x_i^{(R)} - Q^{(F)} x^{(F)}\right) h_t (1 + J_\tau)^{-(t - t_\tau^{\text{ini}})},$$

$$\forall\, t \in \mathcal{H}_\tau, \tau \in \mathcal{T}, \omega \in \Omega \,\Big|\, t \neq 1 + (\tau - 1)|\mathcal{H}_\tau| \text{ and } t \neq |\mathcal{H}_\tau|\,\tau; \quad (7\text{-}16)$$

$$\mu_t(\omega) + \gamma_{t-1}(\omega) - \theta_{t-1}(\omega) \leq \left(\sum_{i \in U} g_{i,t}^{(R)}(\omega) Q_{i,t}^{(R)} x_i^{(R)} - Q^{(F)} x^{(F)}\right) h_t (1 + J_\tau)^{-(t - t_\tau^{\text{ini}})},$$

$$\forall\, t \in \mathcal{H}_\tau, \tau \in \mathcal{T}, \omega \in \Omega \,\Big|\, t = |\mathcal{H}_\tau|\,\tau; \quad (7\text{-}17)$$

$$\Delta\pi_t^+(\omega)\mu_t(\omega) + \beta_\tau(\omega) + \eta_t(\omega) \geq 0, \qquad \forall\, t \in \mathcal{H}_\tau, \tau \in \mathcal{T}, \omega \in \Omega; \quad (7\text{-}18)$$

$$\Delta\pi_t^-(\omega)\mu_t(\omega) - \beta_\tau(\omega) - \eta_t(\omega) \leq 0, \qquad \forall\, t \in \mathcal{H}_\tau, \tau \in \mathcal{T}, \omega \in \Omega; \quad (7\text{-}19)$$

$$\beta_\tau(\omega), \eta_t(\omega), \rho_t(\omega) \geq 0, \qquad \forall\, t \in \mathcal{H}_\tau, \tau \in \mathcal{T}, \omega \in \Omega; \quad (7\text{-}20)$$

$$\gamma_t(\omega), \theta_t(\omega) \geq 0, \qquad \forall\, t \in \bar{\mathcal{H}}_\tau, \tau \in \mathcal{T}, \omega \in \Omega. \quad (7\text{-}21)$$

Note that (7-11)-(7-21) is the implementable version of problem (5-8)-(5-12) using as worst-case revenue the minimization problem presented in (7-2). It is important to mention that exists a vast literature to handle bilevel optimization programs, such as primal-dual or KKT conditions, Benders decomposition, column-constraint generation, among others. However, these methods involve nonlinearities and/or iterative procedures that are undesirable in practice. In this sense, we decided to use the aforementioned methodology to solve the bilevel programming since the single-level equivalent problem is a linear programming, which has efficient algorithms to solve it.

7.1 Case Studies

In order to analyze the accuracy of the proposed methodology, we present two illustrative case studies with different structures and interpretations using realistic data from the Brazilian power sector. In the first case study, we motivate the usage and analyze the results of the optimal portfolio model as a strategic tool to define the optimal medium-term renewable portfolio to back a one-year flat standard forward contract, with monthly short-term settlements. In the second study, we consider a case where an ETC needs to define its optimal participation in a given set of renewable projects to supply a long-term standard forward contract. In this setting, we assume a 10-year business, as typically required by financial institutions to provide competitive interest rates for project financing. Both studies we will analyze from the point of view of stress test, i.e. with a "deterministic" uncertainty set - $(\pi_t^o(\omega), \Delta\pi_t^{+/-}(\omega)) \rightarrow (\pi_t^o, \Delta\pi_t^{+/-})$ in (4-1)-(4-7).

For expository purposes, all studies assume J = 10% p.y. with $J_\tau \approx 0.8\%$ p.m. (the equivalent monthly rate). We consider that the energy trading company has no existing portfolio. Two capacity payment contract opportunities with the following renewable plants are assumed to be available: a small hydro (SH) run-of-river plant with 30 MW of installed capacity and 17.4 avgMW of FEC and a Wind Power Plant (WPP) with 54.6 MW of installed capacity and 27.12 avgMW of FEC. Both renewable units agree to sell 100% of their FEC on capacity payment contracts by a flat monthly price $P_t^{(R)} = 90$ R\$/MWh-of-FEC, $\forall\, t \in \mathcal{H}$. We make use of the methodology presented in [24] to jointly simulate a set of 2000 scenarios for the renewable energy generation based on the historical data available for the SH and WPP. Figs. (7.1) and (7.2) depicts the historical data used to simulate the scenarios. Note that both variables present seasonality, intermittent and complementary profile. Moreover, both variables have different variance among the months, motivating thus the use of the model proposed in [24], since the main characteristics of the variables, such as the correlation between the production of the plants and different variance among months, are preserved.

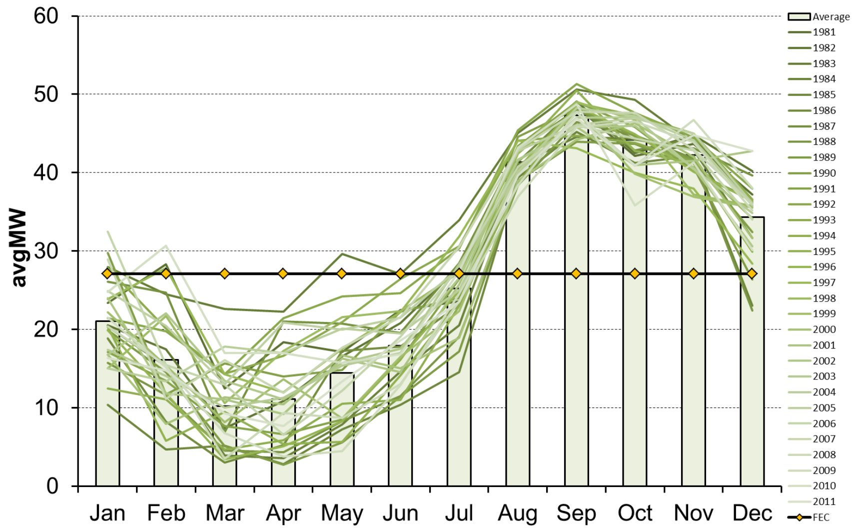

Fig. 7.1: Historical data of a 54.6 MW wind farm in avgMW.

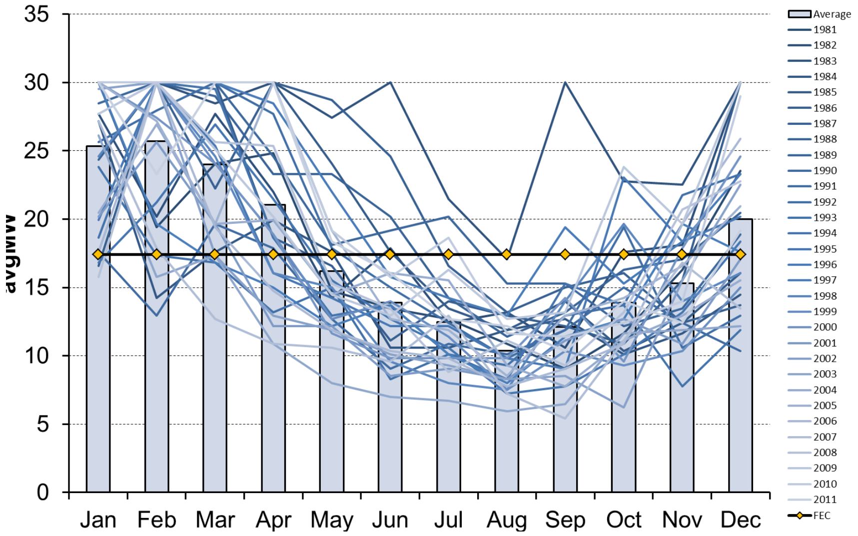

Fig. 7.2: Historical data of a 30 MW small hydro run-of-river plant in avgMW.

The optimization model were implemented in Mosel language under the software FICO Xpress [82] and each problem were solved in less than one minute using a Dell Inspiron 15R Special Edition Laptop.

7.1.1 Medium-Term Portfolio Strategy

The business structure studied in this first case study is the typical energy commercialization trading made in the Brazilian contract market. We consider an energy company selling an one-year flat forward contract of $Q_t^{(F)}$ = 10 avgMW by $P_t^{(F)}$ = 140 R$/MWh, $\forall t \in \mathcal{H}$. The risk-aversion parameters are set to λ = 0.5 (half weight to expected value and half to CVaR) and α = 0.95. We compare the solution obtained by the model proposed in this dissertation, which we call Hybrid Stochastic-Robust (HSR) model, with the Pure Stochastic (PS) model (similar to one proposed in [10]), i.e., using an external spot price scenarios simulated by a least cost dispatch model. In addition, both solutions are benchmarked against actual (observed) market variables (renewable production and short-term market price). The period of the contact opportunities are assumed to range from January 2012 to December 2012. Thus, $\mathcal{T} = 1$ and $\mathcal{H} = \mathcal{H}_\tau = \{1, 2, ..., 12\}$. Using the methodology described in [16] with the official system data from December 2011, we obtain the simulation of the Brazilian system for the contract period, including the short-term market price. Fig. 7.3 shows the statistics for the set of simulated spot-price scenarios for 2012.

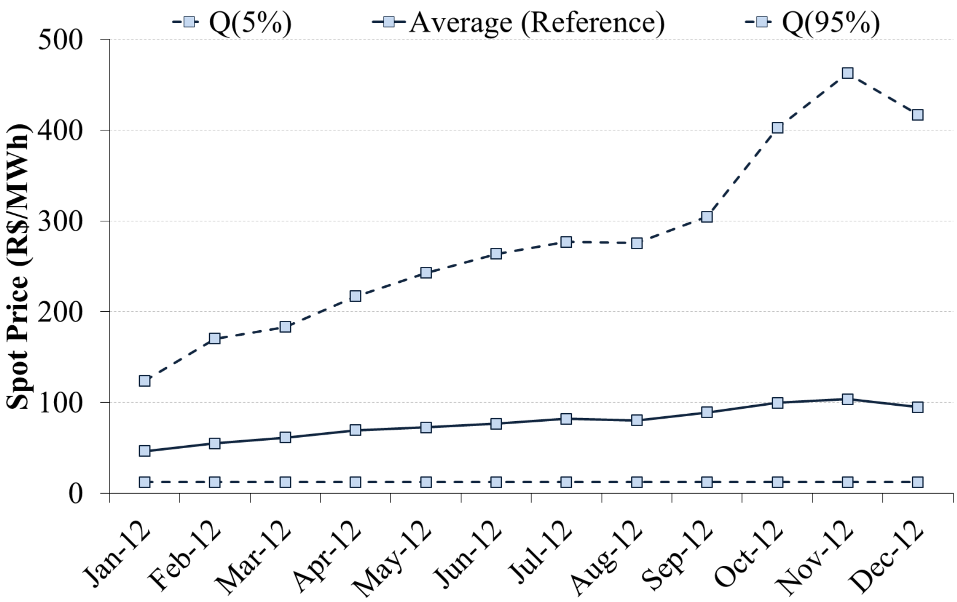

Fig. 7.3: Average and extreme quantiles (5% and 95%) for the simulated spot prices: from January 2012 to December 2012 on a monthly basis.

To construct the "deterministic" uncertainty set, we extract some statistics from the simulated scenarios. The reference time series was set to the simulated average shown in Fig. 7.3 to capture the conditional information of the spot prices throughout the contract horizon. Nevertheless, the maximum positive and negative deviations from the reference scenario were chosen to allow the stress scenario to reach the price cap (730 R$/MWh) and floor (12.1 R$/MWh), respectively, in any period. To constrain the worst-case spot-price scenarios, the maximum and minimum returns were obtained from the historical data for each month. In Table 7.1, the aforementioned input data is summarized.

Our goal in to study the effect of varying the conservatism parameter K_1 on the optimal portfolio. Therefore, we have solved (7-11)-(7-21) for $K_1 = 1, 2, ..., 12$, and the optimal amount sold and contracted of each technology is shown in Table 7.2. We observed that when the spot price is treated as an exogenous variable (second column of Table 7.2), the optimal strategy is to back the forward contract sale solely on the capacity contract with the wind power unit. One way to interpret this result is to observe that the wind production pattern is similar to the spot price one, i.e., high spot price seasons coincide, in general, with high wind power production (see Fig. 7.1 and Fig. 7.3 for better comparison). In contrast, hydro production presents almost full complementarity with these two variables, thus adding no value to the portfolio.

Tab. 7.1: Uncertainty set input data for the medium term study

	π_t^o	$\Delta\pi_t^+$	$\Delta\pi_t^-$		r_t^+	r_t^-
Jan	46.47	683.53	34.37	**Jan** $\rightarrow$ **Feb**	1.29	0.70
Feb	54.93	675.07	42.83	**Feb** $\rightarrow$ **Mar**	1.57	0.61
Mar	61.18	668.82	49.08	**Mar** $\rightarrow$ **Apr**	1.91	0.62
Apr	69.62	660.38	57.52	**Apr** $\rightarrow$ **May**	1.59	0.60
May	72.74	657.53	60.37	**May** $\rightarrow$ **Jun**	1.33	0.50
Jun	76.44	653.56	64.34	**Jun** $\rightarrow$ **Jul**	0.52	0.37
Jul	82.02	647.98	69.92	**Jul** $\rightarrow$ **Ago**	0.41	0.78
Ago	80.30	649.70	68.20	**Ago** $\rightarrow$ **Sep**	2.91	0.17
Sep	89.13	640.87	77.03	**Sep** $\rightarrow$ **Oct**	0.85	0.35
Oct	99.69	630.31	87.59	**Oct** $\rightarrow$ **Nov**	0.44	0.27
Nov	103.69	626.21	91.69	**Nov** $\rightarrow$ **Dec**	0.21	0.56
Dec	94.68	635.32	82.58			

Tab. 7.2: Optimal contracting of each candidate option in the Pure Stochastic model and Hybrid Stochastic-Robust model (avgMW)

	PS	**HSR** ($K_1 = 1$)	**HSR** ($K_1 = 2$)	**HSR** ($K_1 = 3$)
Forward	10.00	10.00	10.00	0.00
WPP	10.00	9.21	9.87	0.00
SH	0.00	4.91	6.88	0.00
Excess	**0.00**	**4.12**	**6.88**	**0.00**

When the model (7-11)-(7-21) is solved with K_1 = 1 and 2, the optimal strategy is also to sell the total amount of the forward contract, but supporting it by a mixed portfolio that is composed of both WPP and SH. This result is due to the stress-scenarios for the spot prices that penalize portfolios with high exposure to the short-term market, which occurs whenever the portfolio generation is below the amount sold. This situation can be observed in Fig. 7.4, which shows an out-of-sample comparison (in energetic terms) for the first three portfolios shown in Table 7.2 with actual renewable production observed in 2012.

It is possible to see that the portfolios found with the robust model mitigate purchases in the short-term market by raising the generation profile throughout the months. The effect of such mitigation is shown in Fig. 7.5, where the impact of the first semester spot-price disturbance (April and May) is attenuated under the robust solutions. For $K_1 \geq 3$, the stress created to the portfolio is so high that it is optimal

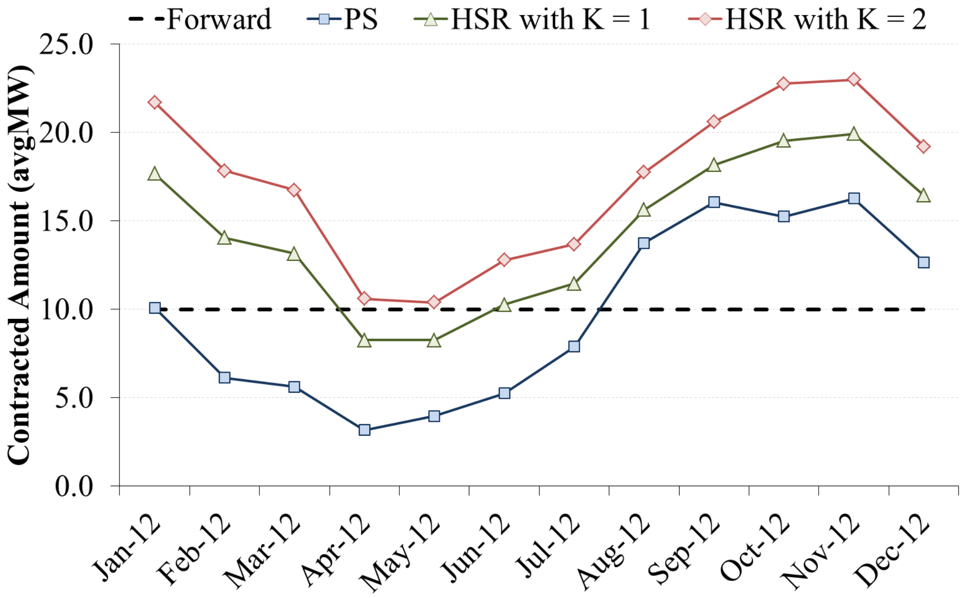

Fig. 7.4: Generation profile of the portfolios found with the stochastic and hybrid stochastic-robust models for the observed data (renewables generation) during the contract horizon: months of 2012.

to not set up the business.

Finally, it is important to mention that it is not always the case that the HSR model outperforms the PS one. In Table 7.3, the stochastic and hybrid stochastic-robust solutions were evaluated under observed generation and prices for two other representative years: 2010 (a typical year with respect to spot price realization - lower values at the beginning of the year and high values at the end) and for 2008 (an unusual year - a price spike in January and low values at the end of the year). We also present the evaluation for 2012.

Tab. 7.3: Back test in the yearly revenue of the ETC (MMR$)

	PS	**HSR** ($\boldsymbol{K_1 = 1}$)	**HSR** ($\boldsymbol{K_1 = 2}$)	**HSR** ($\boldsymbol{K_1 = 3}$)
2008	2.31	3.84	4.65	0.00
2010	5.22	3.83	3.19	0.00
2012	5.12	7.60	9.34	0.00

As shown in Table 7.3, the PS model has the highest yearly revenue in the typical year, mainly because the spot price outcome for this year realized as "expected", i.e., in accordance with the pattern of the simulated scenarios. However, when the spot price realization presents a different pattern with respect to the price scenarios

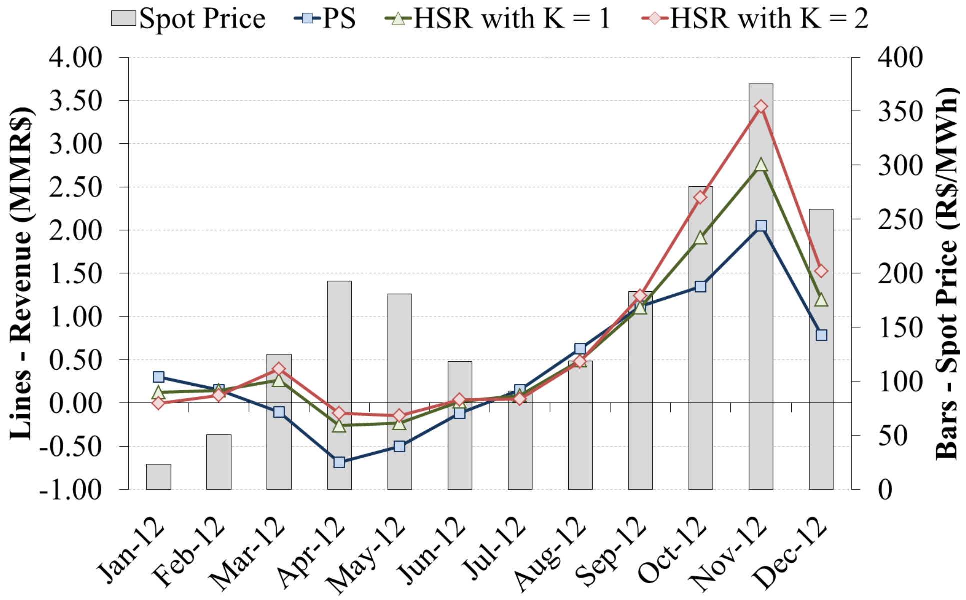

Fig. 7.5: Spot prices and revenue profile of the portfolios found with the stochastic and robust models for the observed data (renewables generation and spot prices) during the contract horizon: months of 2012.

simulated by the stochastic model, the hybrid stochastic-robust model outperforms the pure stochastic one. We also indicate that the HSR model also outperforms the PS one when the spot price increases in the last months of the year, indicating that the robust counterpart was able to capture the typical pattern of prices as well.

7.1.2 Long-Term Portfolio Strategy

In any business sector, a long-term investment is one of the most difficult challenges for an investing company, not only due to the difficulty to make long-term assumptions, but also because of the assessment of financial support given by financial institutions. Typically, these institutions require a high level of guarantees that, most of the time, make the project infeasible. Therefore, the main goal of investment companies is to achieve low risk projects with (almost) deterministic positive cash-flows. When it happens, a “safe” business is set and a financial support can be obtained. In the view of these considerations, in this study, we consider an ETC deciding its willingness to invest in renewable plants to cover a 10-year forward contract with a consumer in the Brazilian contract market. We use the same WPP and SH considered before and a contract period that ranges from January 2016 to December 2025. We argue that, under this time horizon, the specification of any price model would rely on unpredictable economic and structural data inputs under which almost all rational institutions would not give financial support for investing on these plants due to the high risk of the business. Thus, the proposed approach provides the decision maker with a tool consistent with the stress analysis.

Within the model parameters, we set $\mathcal{T} = \{1, 2, ..., 10\}$ as the set of the years and $\{\mathcal{H}_\tau\}_{\tau\in\mathcal{T}}$ as the collection of sets containing the months of each year $\tau \in \mathcal{T}$, i.e., $\mathcal{H}_1 = \{1, \ldots, 12\}, \mathcal{H}_2 = \{13, \ldots, 24\}, \ldots, \mathcal{H}_{10} = \{109, \ldots, 120\}$ with $\mathcal{H} = \bigcup_{\tau\in\mathcal{T}} \mathcal{H}_\tau = \{1, \ldots, 120\}$. We make use the model proposed in [24] to simulate the joint WPP and SH production scenarios throughout the contract horizon. To construct the "deterministic" uncertainty set, we set to the nominal spot price time series $(\{\pi_t^o\}_{t\in\mathcal{H}_\tau})$, for all years of the study horizon, a typical year of observed (historical) realization: the year of 2010. The maximum positive and negative deviations from the reference scenario were again chosen to allow the stress scenario to reach the price cap (730 R$/MWh) and floor (12.1 R$/MWh), respectively, in all periods. The returns used were the same of the previous study. In Table 7.4, the input data is summarized for a one year (the rest of the years receive the same values):

Tab. 7.4: Uncertainty set input data for the long-term study

	π_t^o	$\Delta\pi_t^+$	$\Delta\pi_t^-$		r_t^+	r_t^-
Jan	12.91	717.09	0.81	**Jan → Feb**	1.29	0.70
Feb	13.82	716.18	1.72	**Feb → Mar**	1.57	0.61
Mar	27.24	702.76	15.14	**Mar → Apr**	1.91	0.62
Apr	21.47	708.53	9.37	**Apr → May**	1.59	0.60
May	32.34	697.66	20.24	**May → Jun**	1.33	0.50
Jun	67.70	662.30	55.60	**Jun → Jul**	0.52	0.37
Jul	89.61	640.39	77.51	**Jul → Ago**	0.41	0.78
Ago	116.66	613.34	104.56	**Ago → Sep**	2.91	0.17
Sep	132.10	597.90	120.00	**Sep → Oct**	0.85	0.35
Oct	137.78	592.22	125.68	**Oct → Nov**	0.44	0.27
Nov	116.68	613.32	104.58	**Nov → Dec**	0.21	0.56
Dec	71.62	658.38	59.52			

To evaluate the results, an efficient frontier curve is created by varying the risk-aversion parameter λ from 0.5 to 0.99 on a 0.05 step-basis. Fig. 7.6, shows the curve for $\{K_\tau\}_{\tau\in\mathcal{T}}$ = 1 and 2. The vertical axis represents the Net Present Value (NPV) of the expected value of the 10-year revenue and the horizontal axis stands for risk, evaluated as the difference between the NPV of the expected value and the

NPV of the CVaR.

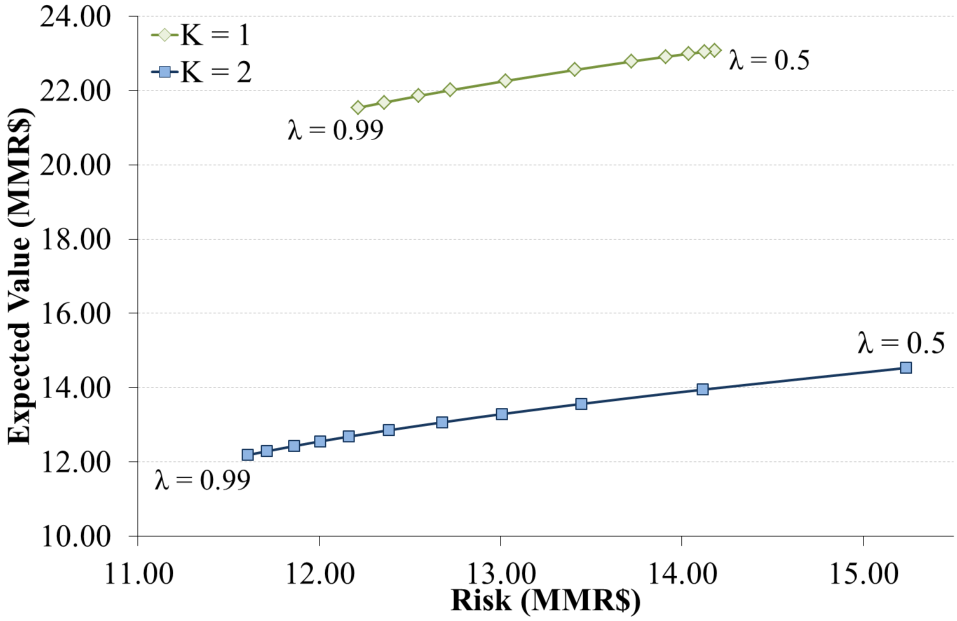

Fig. 7.6: Efficient frontier curve for $\{K_\tau\}_{\tau\in\mathcal{T}}$ = 1 and 2 varying the parameter λ from 0.5 to 0.99 on a 0.05 step-basis.

For $\{K_\tau\}_{\tau\in\mathcal{T}}$ = 2 and λ = $\{0.50, 0.55, 0.60\}$, the risk is higher than the expected return, indicating that the NPV of the CVaR is negative. This fact suggests that under $\{K_\tau\}_{\tau\in\mathcal{T}}$ = 2, the investment decision in renewable plants requires high risk aversion to mitigate the solvency risk (chance of a negative NPV). Table 7.5 depicts the portfolio decisions associated with each point of the efficient frontier curve. Observe that in Table 7.5, as the risk-averse parameter λ grows, the excess of energy bought by the ETC also grows, representing a hedge against the price-quantity risk. These results are expected and follow the same idea explained in the case study of Section 7.1.1. The optimal contracting model aims to create a flatter portfolio, reducing the exposure to the short-term market because this level of conservativeness is highly aggressive against the ETC's revenue.

In general, the efficient frontier varies with respect to the conservativeness level of the robust counterpart, i.e., with $\{K_\tau\}_{\tau\in\mathcal{T}}$. In this sense, to create a long-term portfolio, the decision maker should well-adjust its risk preference to the parameters of the model proposed because the solution obtained is highly dependent on the parameters of the model. For example, a 0.05 variation on λ creates a different portfolio and the relationship between risk and expected revenue. This is also extremely important in a financial support negotiation with a financial institution and may affect the decision of give or not the support.

Tab. 7.5: Optimal contracting of each candidate option for $\{K_\tau\}_{\tau\in\mathcal{T}}$ = 1 and 2 in the 10-year case study for different risk levels of λ (avgMW)

	$\{K_\tau\}_{\tau\in\mathcal{T}} = 1$				$\{K_\tau\}_{\tau\in\mathcal{T}} = 2$			
λ	**Forward**	**WPP**	**SH**	**Excess**	**Forward**	**WPP**	**SH**	**Excess**
0.50	10.00	13.57	2.25	**5.82**	10.00	11.48	5.09	**6.57**
0.55	10.00	13.65	2.29	**5.93**	10.00	11.80	5.18	**6.99**
0.60	10.00	13.74	2.34	**6.08**	10.00	12.00	5.25	**7.25**
0.65	10.00	13.87	2.41	**6.27**	10.00	12.13	5.30	**7.42**
0.70	10.00	14.11	2.47	**6.58**	10.00	12.21	5.35	**7.56**
0.75	10.00	14.43	2.56	**6.99**	10.00	12.34	5.36	**7.70**
0.80	10.00	14.74	2.67	**7.42**	10.00	12.43	5.38	**7.81**
0.85	10.00	14.97	2.76	**7.73**	10.00	12.48	5.41	**7.89**
0.90	10.00	15.08	2.83	**7.90**	10.00	12.55	5.42	**7.97**
0.95	10.00	15.19	2.90	**8.09**	10.00	12.63	5.43	**8.06**
0.99	10.00	15.28	2.95	**8.22**	10.00	12.68	5.44	**8.12**

8
Portfolio Allocation with Energy Call Options

In this second case study chapter, we study the ETC's optimal portfolio allocation considering available an energy call option with a thermal unit. For this purpose, we can directly apply the contract model (5-8)-(5-12) with worst-case revenue described by (5-13). However, one of the problems of the optimal contracting model (5-8)-(5-13), from a computationally point of view, is to assess, endogenously, the thermal generation under the worst-case spot price. As discussed in Section 5.2.2, the thermal dispatch $\left(g_t^{(T)}\right)$ is a deterministic function of the spot price (equation (5-4)). In this sense, since the spot price is obtained endogenously by a minimization problem which aims to minimize the ETC's portfolio revenue, we need to guarantee that the thermal generation follows the rule (5-4) endogenously too. To deal with this problem, we propose a third level problem with the objective to recover the thermal generation rule (5-4) given a spot price "realization" defined by the second level (5-13). This third level can be interpreted as a response by the call option contract against the realization of the worst-case spot price time series, acting as a hedge to the portfolio of the trading company and, then, reflecting the generation of the thermal plant or the call option exercise so that the revenue is maximized. In Fig. 8.1, a scheme of the full contracting model with an interpretation of each level and their input and output is presented to ease the understanding.

Mathematically, the three level optimization problem that describes the scheme in Fig. 8.1 is the following:

$$\underset{\substack{x^{(F)},x_i^{(R)},x^{(T)},\\ \delta_\tau(\omega),z_\tau}}{\text{Maximize}} \sum_{\tau\in\mathcal{T}}\sum_{\omega\in\Omega} p(\omega)\left[\lambda\left(z_\tau - \frac{\delta_\tau(\omega)}{1-\alpha}\right) + \right.$$
$$\left.(1-\lambda)R_\tau^{\text{WC}}\big(\boldsymbol{x},\boldsymbol{g}_\tau^{(R)}(\omega)\big)\right](1+J)^{(1-\tau)} \quad (8\text{-}1)$$

subject to:

$$\delta_\tau(\omega) \geq z_\tau - R_\tau^{\text{WC}}\big(\boldsymbol{x},\boldsymbol{g}_\tau^{(R)}(\omega)\big), \qquad \forall\, \tau\in\mathcal{T},\omega\in\Omega; \quad (8\text{-}2)$$
$$Q^{(F)}x^{(F)} \leq \sum_{i\in U} Q_i^{(R)}x_i^{(R)} + Q^{(T)}x^{(T)}; \quad (8\text{-}3)$$
$$\delta_\tau(\omega) \geq 0, \qquad \forall\, \tau\in\mathcal{T},\omega\in\Omega; \quad (8\text{-}4)$$
$$x^{(F)}, x_i^{(R)}, x^{(T)} \in [0,1], \qquad \forall\, i\in U, \quad (8\text{-}5)$$

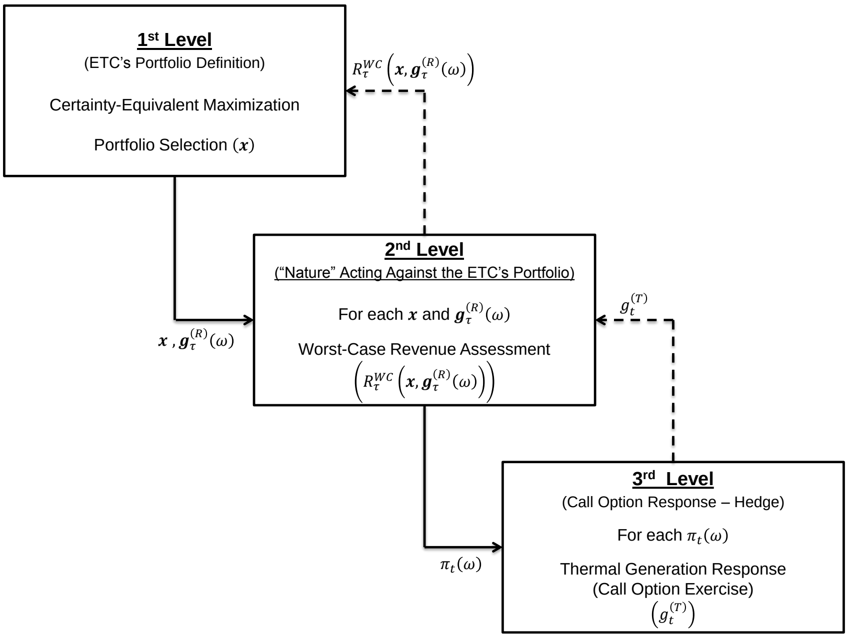

Fig. 8.1: Three-level model scheme.

with worst-case revenue,

$$R_\tau^{\mathrm{WC}}\big(\boldsymbol{x},\boldsymbol{g}_\tau^{(R)}(\omega)\big) = \min_{\boldsymbol{\pi}_\tau(\omega)\in\Pi_\tau^{o}(\omega)} \sum_{t\in\mathcal{H}_\tau}\Bigg[P_t^{(F)}Q_t^{(F)}x^{(F)} - \sum_{i\in U}P_{i,t}^{(R)}Q_{i,t}^{(R)}x_i^{(R)} - \Big(P_t^{(T)} - \underline{g}^{(T)}\Lambda\Big)Q_t^{(T)}x^{(T)} + \Big(\sum_{i\in U} g_{i,t}^{(R)}(\omega)Q_{i,t}^{(R)}x_i^{(R)} - Q_t^{(F)}x^{(F)}\Big)\pi_t(\omega) + \max_{g_t^{(T)}(\omega)\,\in\,\mathcal{G}_t^{(T)}(\omega)}\Big\{\Big(\pi_t(\omega)-\Lambda\Big)g_t^{(T)}(\omega)\Big\}\Bigg] h_t(1+J_\tau)^{-(t-t_\tau^{\mathrm{ini}})}, \quad \forall \tau\in\mathcal{T},\omega\in\Omega. \tag{8-6}$$

where

$$\mathcal{G}_t^{(T)}(\omega) = \left\{ g_t^{(T)}(\omega)\in\mathbb{R}_+ \;\middle|\; \begin{array}{l} g_t^{(T)}(\omega) \le \bar{g}^{(T)}Q_t^{(T)}x^{(T)}; \\ g_t^{(T)}(\omega) \ge \underline{g}^{(T)}Q_t^{(T)}x^{(T)}. \end{array} \right\} \tag{8-7}$$

The optimization problem (8-1)-(8-5) represents the optimal portfolio selection problem (1st level). The optimal portfolio is assessed by maximizing the measure presented in (3-7) of the worst-case revenue (8-6), the 2nd level of the problem, which is accounted for by minimizing the ETC's revenue over the spot price feasible region. Finally, this worst-case revenue considers, endogenously, the call option response by means of the maximization problem in (8-6), which represents the 3rd level of the contracting problem. Writing in its extended form, the third level is the following deterministic optimization problem, one for each scenario $\omega \in \Omega$ and period $t \in \mathcal{H}$:

$$f_t\Big(\pi_t(\omega), x^{(T)}\Big) = \max_{g_t^{(T)}(\omega) \geq 0} \Big(\pi_t(\omega) - \Lambda\Big) g_t^{(T)}(\omega) \tag{8-8}$$

subject to

$$g_t^{(T)}(\omega) \leq \bar{g}^{(T)} Q_t^{(T)} x^{(T)}; \qquad \varphi_t(\omega) \tag{8-9}$$

$$g_t^{(T)}(\omega) \geq \underline{g}^{(T)} Q_t^{(T)} x^{(T)}. \qquad \chi_t(\omega) \tag{8-10}$$

Note that, since (8-8)-(8-10) is a maximization problem, if $\pi_t(\omega) \geq \Lambda$ the optimal solution is $g_t^{(T)}(\omega) = \bar{g}^{(T)} Q_t^{(T)} x^{(T)}$. On the other hand, if $\pi_t(\omega) < \Lambda$, the optimal solution is $g_t^{(T)}(\omega) = \underline{g}^{(T)} Q_t^{(T)} x^{(T)}$. For both cases, the optimal solution recovers the dispatch rule (5-4) and the objective function is exactly the short-term settlement of the call option. However, from a technical point of view, incorporating this third level into the ETC's worst-case revenue problem (8-6) creates an undesirable nonlinearity, leaving the certainty-equivalent maximization problem difficult to be solved. Since we are only interest in the value of the objective function evaluated in the optimal solution (dispatch), we can use the dual problem of (8-8)-(8-10) and apply the transformation proposed in [26] (discussed previously) to overcome this nonlinearity. Since the dual problem of the third level problem is a minimization problem and the second-level (8-6) is also a minimization problem, the argument discussed in Chapter 7 holds and we can "couple" both problem (second and third levels) into a single-level equivalent problem. Thus, writing the dual problem of (8-8)-(8-10), we have:

$$f_t\Big(\pi_t(\omega), x^{(T)}\Big) = \min_{\varphi_t(\omega), \chi_t(\omega)} \Big(\bar{g}^{(T)} \varphi_t(\omega) - \underline{g}^{(T)} \chi_t(\omega)\Big) Q_t^{(T)} x^{(T)} \tag{8-11}$$

subject to

$$\varphi_t(\omega) - \chi_t(\omega) \geq \pi_t(\omega) - \Lambda; \tag{8-12}$$

$$\varphi_t(\omega), \chi_t(\omega) \geq 0. \tag{8-13}$$

By coupling (8-11)-(8-13) into (8-6) we obtain a equivalent second-level linear minimization problem in which the thermal dispatch is considered "correctly", i.e., the generation rule (5-4) is endogenously defined and respects the worst-case spot price realization. This equivalent second-level optimization problem is the following minimization problem, one for each $\tau \in \mathcal{T}$ and $\omega \in \Omega$:

$$R_\tau^{\mathrm{WC}}(\boldsymbol{x}, \boldsymbol{g}_\tau^{(R)}(\omega)) = \min_{\substack{\pi_t(\omega), v_t^+(\omega), v_t^-(\omega), \\ \varphi_t(\omega), \chi_t(\omega)}} \sum_{t \in \mathcal{H}_\tau} \Bigg[P_t^{(F)} Q_t^{(F)} x^{(F)} - \sum_{i \in U} P_{i,t}^{(R)} Q_{i,t}^{(R)} x_i^{(R)} -$$

$$\left(P_t^{(T)} - \underline{g}^{(T)}\Lambda\right) Q_t^{(T)} x^{(T)} + \left(\sum_{i \in U} g_{i,t}^{(R)}(\omega) Q_{i,t}^{(R)} x_i^{(R)} - Q_t^{(F)} x^{(F)} \right) \pi_t(\omega) +$$

$$\left(\bar{g}^{(T)} \varphi_t(\omega) - \underline{g}^{(T)} \chi_t(\omega)\right) Q_t^{(T)} x^{(T)} \Bigg] h_t (1 + J_\tau)^{-(t - t_\tau^{\mathrm{ini}})} \quad (8\text{-}14)$$

subject to

$$\pi_t(\omega) = \pi_t^o(\omega) + \Delta\pi_t^+(\omega) v_t^+(\omega) - \Delta\pi_t^-(\omega) v_t^-(\omega), \quad \forall\, t \in \mathcal{H}_\tau; \quad \mu_t(\omega) \quad (8\text{-}15)$$

$$\sum_{t \in \mathcal{H}_\tau} \left(v_t^+(\omega) + v_t^-(\omega)\right) \leq K_\tau; \quad \beta_\tau(\omega) \quad (8\text{-}16)$$

$$\pi_{t+1}(\omega) \geq (1 - r_t^-)\pi_t(\omega), \quad \forall\, t \in \bar{\mathcal{H}}_\tau; \quad \gamma_t(\omega) \quad (8\text{-}17)$$

$$\pi_{t+1}(\omega) \leq (1 + r_t^+)\pi_t(\omega), \quad \forall\, t \in \bar{\mathcal{H}}_\tau; \quad \theta_t(\omega) \quad (8\text{-}18)$$

$$0 \leq \pi_t^+(\omega) \leq 1, \quad \forall\, t \in \mathcal{H}_\tau; \quad \eta_t(\omega) \quad (8\text{-}19)$$

$$0 \leq \pi_t^-(\omega) \leq 1, \quad \forall\, t \in \mathcal{H}_\tau; \quad \rho_t(\omega) \quad (8\text{-}20)$$

$$\varphi_t(\omega) - \chi_t(\omega) \geq \pi_t(\omega) - \Lambda, \quad \forall\, t \in \mathcal{H}_\tau; \quad \zeta_t(\omega) \quad (8\text{-}21)$$

$$\varphi_t(\omega), \chi_t(\omega) \geq 0, \quad \forall\, t \in \mathcal{H}_\tau. \quad (8\text{-}22)$$

Finally, as discussed in Chapter 7, the contracting model (5-8)-(5-12) is still a non-linear bilevel optimization model. However, note that each feasible region of the problem (8-15)-(8-22), is a polyhedral set and the ETC's revenue (8-14) is a linear function on $\{\tilde{\boldsymbol{\pi}}_\tau\}_{\tau \in \mathcal{T}}$ and on $\boldsymbol{x}$. Therefore, we can apply again the transformation proposed in [26] leading to the following single-level two-stage linear optimization model:

$$\underset{\substack{x^{(F)}, x_i^{(R)}, x^{(T)}, R_\tau^{\mathrm{WC}}(\omega), z_\tau, \\ \delta_\tau(\omega), \mu_t(\omega), \zeta_t(\omega), \theta_t(\omega) \\ \beta_\tau(\omega), \eta_t(\omega), \rho_t(\omega), \gamma_t(\omega)}}{\text{Maximize}} \sum_{\tau \in \mathcal{T}} \sum_{\omega \in \Omega} p(\omega) \Bigg[\lambda \left(z_\tau - \frac{\delta_\tau(\omega)}{1 - \alpha} \right) +$$

$$(1 - \lambda) R_\tau^{\mathrm{WC}}(\omega) \Bigg] (1 + J)^{(1 - \tau)} \quad (8\text{-}23)$$

subject to:

$$R_\tau^{\mathrm{WC}}(\omega) = \sum_{t \in \mathcal{H}_\tau} \left(\pi_t^o(\omega)\mu_t(\omega) - \eta_t(\omega) - \rho_t(\omega) - \Lambda\zeta_t(\omega) \right) - K_\tau \beta_\tau(\omega) +$$

$$\sum_{t \in H_\tau} \left(P^{(F)} Q^{(F)} x^{(F)} - \sum_{i \in U} P_i^{(R)} Q_i^{(R)} x_i^{(R)} \right) h_t (1 + J_\tau)^{-(t - t_\tau^{\text{ini}})} +$$

$$\sum_{t \in H_\tau} \left(\underline{g}^{(T)} \Lambda - P^{(T)} \right) h_t Q^{(T)} x^{(T)} (1 + J_\tau)^{-(t - t_\tau^{\text{ini}})}, \quad \forall \tau \in \mathcal{T}, \omega \in \Omega; \quad (8\text{-}24)$$

$$\delta_\tau(\omega) \geq z_\tau - R_\tau^{\text{WC}}(\omega), \quad \forall \tau \in \mathcal{T}, \omega \in \Omega; \quad (8\text{-}25)$$

Constraints (5-10)-(5-12) (8-26)

$$\mu_t(\omega) - (1 - r_t^-)\gamma_t(\omega) + (1 + r_t^+)\theta_t(\omega) - \zeta_t(\omega) \leq$$
$$\left(\sum_{i \in U} g_{i,t}^{(R)}(\omega) Q_{i,t}^{(R)} x_i^{(R)} - Q^{(F)} x^{(F)} \right) h_t (1 + J_\tau)^{-(t - t_\tau^{\text{ini}})},$$
$$\forall t \in \mathcal{H}_\tau, \tau \in \mathcal{T}, \omega \in \Omega \,\middle|\, t = 1 + (\tau - 1)|\mathcal{H}_\tau|; \quad (8\text{-}27)$$

$$\mu_t(\omega) + \gamma_{t-1}(\omega) - (1 - r_t^-)\gamma_t(\omega) - \theta_{t-1}(\omega) + (1 + r_t^+)\theta_t(\omega) - \zeta_t(\omega) \leq$$
$$\left(\sum_{i \in U} g_{i,t}^{(R)}(\omega) Q_{i,t}^{(R)} x_i^{(R)} - Q^{(F)} x^{(F)} \right) h_t (1 + J_\tau)^{-(t - t_\tau^{\text{ini}})},$$
$$\forall t \in \mathcal{H}_\tau, \tau \in \mathcal{T}, \omega \in \Omega \,\middle|\, t \neq 1 + (\tau - 1)|\mathcal{H}_\tau| \text{ and } t \neq |\mathcal{H}_\tau|\, \tau; \quad (8\text{-}28)$$

$$\mu_t(\omega) + \gamma_{t-1}(\omega) - \theta_{t-1}(\omega) - \zeta_t(\omega) \leq$$
$$\left(\sum_{i \in U} g_{i,t}^{(R)}(\omega) Q_{i,t}^{(R)} x_i^{(R)} - Q^{(F)} x^{(F)} \right) h_t (1 + J_\tau)^{-(t - t_\tau^{\text{ini}})},$$
$$\forall t \in \mathcal{H}_\tau, \tau \in \mathcal{T}, \omega \in \Omega \,\middle|\, t = |\mathcal{H}_\tau|\, \tau; \quad (8\text{-}29)$$

$$\Delta\pi_t^+(\omega)\mu_t(\omega) + \beta_\tau(\omega) + \eta_t(\omega) \geq 0, \quad \forall t \in \mathcal{H}_\tau, \tau \in \mathcal{T}, \omega \in \Omega; \quad (8\text{-}30)$$

$$\Delta\pi_t^-(\omega)\mu_t(\omega) - \beta_\tau(\omega) - \eta_t(\omega) \leq 0, \quad \forall t \in \mathcal{H}_\tau, \tau \in \mathcal{T}, \omega \in \Omega; \quad (8\text{-}31)$$

$$\zeta_t(\omega) \leq \bar{g}^{(T)} Q^{(T)} x^{(T)} h_t (1 + J_\tau)^{-(t - t_\tau^{\text{ini}})}, \quad \forall t \in \mathcal{H}_\tau, \tau \in \mathcal{T}, \omega \in \Omega; \quad (8\text{-}32)$$

$$\zeta_t(\omega) \geq \underline{g}^{(T)} Q^{(T)} x^{(T)} h_t (1 + J_\tau)^{-(t - t_\tau^{\text{ini}})}, \quad \forall t \in \mathcal{H}_\tau, \tau \in \mathcal{T}, \omega \in \Omega; \quad (8\text{-}33)$$

$$\zeta_t(\omega), \beta_\tau(\omega), \eta_t(\omega), \rho_t(\omega) \geq 0, \quad \forall t \in \mathcal{H}_\tau, \tau \in \mathcal{T}, \omega \in \Omega; \quad (8\text{-}34)$$

$$\gamma_t(\omega), \theta_t(\omega) \geq 0, \quad \forall t \in \bar{\mathcal{H}}_\tau, \tau \in \mathcal{T}, \omega \in \Omega. \quad (8\text{-}35)$$

In (8-23)-(8-35), the dual variables of the second level problem defined by (8-14)-(8-22) are decision variables and the set of equations (8-27)-(8-35) is the feasible region of the dual problem of (5-13). Finally, note that (8-23)-(8-35) is the implementable version of problem (5-8)-(5-13). Next we present a set of case studies to illustrate the applicability of the proposed model.

8.1 Pricing Energy Call Option with Ambiguity Aversion on Spot Price Distribution

In this case study, the accuracy of the proposed model (8-23)-(8-35) is illustrated from the point of view of ambiguity aversion. In addition to the contracts considered in the previous Chapter (forward and renewable capacity payment contracts), we consider a thermal call option opportunity in the ETC's portfolio. For expository purpose, we assume a total flexible thermal[1] plant ($\underline{g}^{(T)} = 0$) with VOC (Λ) equals to 90 R\$/MWh. The amount of energy available in the call option is set to be the unit's FEC, assumed to be 5 avgMW, which is also considered to be the maximum production of the plant, i.e. $\forall\, t \in \mathcal{H}$, $\underline{g}^{(T)}Q^{(T)}_t = 0$, for $\tilde{\pi}_t < P^{(T)}_t$ and $\bar{g}^{(T)}Q^{(T)}_t = 5$, for $\tilde{\pi}_t \geq P^{(T)}_t$. We can interpret this opportunity as belonging to a medium-sized gas-fired plant (100 MW of installed capacity), which has low VOC (between 70-100 avgMW) and have 5% of its installed capacity "uncontracted" and agreed to sell it in the market. In this case, the thermal unit is willing to receive a fixed payment by the hedge when it generates (dispatches).

Our goal in this study is to assess the value of this call option from the point of view of a trading company that already sold a one-year flat forward contract with quantity $Q^{(F)}_t$ = 10 avgMW and price $P^{(F)}_t$ = 140 R\$/MWh backed on the same WPP unit considered in the last Chapter (54.6 MW of installed capacity and 27.12 avgMW of FEC). The amount bought in energy from the renewable capacity payment contract to cover the forward contract was $Q^{(R)}_{\text{WPP},t}$ = 10 avgMW and $P^{(R)}_{\text{WPP},t}$ = 90 avgMW, i.e., the ETC bought the minimum necessary to cover the sell. In this sense, we make a sensitivity analysis on the call option price (the option premium) and compare the expected value and the CVaR of the trade revenue with and without the option. We assume that the contract period ranges from January 2013 to December 2013, thus $\mathcal{T} = \{1\}$ and $\mathcal{H} = \mathcal{H}_\tau = \{1, 2, ..., 12\}$. The risk-aversion parameters are set to α = 0.95 and λ = 0.50 as usual.

In order to obtain the nominal spot price scenarios ($\{\pi^o_t(\omega)\}_{t\in\mathcal{H}}$) for the contract period (January 2013 to December 2013), we make use of the methodology described in [16] with the Brazilian official system data from December 2011. The maximum positive and negative deviations from the reference scenario were chosen to allow the endogenous spot price to reach the price cap (730 R\$/MWh) and floor (12.1 R\$/MWh), respectively, in any period. The return constraints (4-4) and

[1] Typically, the cost of the inflexible generation is embedded in the contract price.

(4-5) are relaxed in this study since the simulation scenarios already capture the inter-temporal relationship between consecutive periods. The expected value and the CVaR of the trading without the call option in the portfolio is presented Table 8.1 for K_1 = 0 (stochastic), 1 and 2.

Tab. 8.1: Expected value and CVaR of the trading without considering the call option in the portfolio for K_1 = 0 (stochastic), 1 and 2 (MMR$)

	K_1 = 0	K_1 = 1	K_1 = 2
Exp. Value	3.49	0.45	-2.15
CVaR	-1.19	-3.49	-6.55

Note that, as expected, this revenue measures (risk and return) decreases as we increase the value of K_1, since the spot price became more aggressive against the company's portfolio. In addition, the CVaR of the trading, which, roughly speaking, represents the average of worst revenue scenarios, is negative for every $K_1 = 0, 1$ and 2, reaching -6.55 MMR$ in one year, if the spot price deviates for "2 months" with respect to the simulated scenarios. Thus, a hedge against this possibility is of utmost importance for a risk-averse trading company in this business (amount sold in contracts equal to the amount bought from capacity contract). In Fig. 8.2 a sensitivity analysis in the call option premium is presented. Note that, for K_1 = 0, equivalent to solve a pure stochastic problem, is optimal to purchase the full call option until its premium reach 30 R$/MWh. After that, the optimal amount is reduced in the portfolio, reflecting the increase in the cost of the option. After 130 R$/MWh, the call is so expensive that brings no value to the portfolio.

Likewise, we can make the same analysis for K_1 = 1 and 2. The option has full value until a premium of 45 R$/MWh and 85 R$/MWh, respectively, reducing the optimal amount in the portfolio for values after that, being valueless after 160 R$/MWh and 180 R$/MWh, respectively. Therefore, the graphs presented in Fig. 8.2 prices the value of a call option with a thermal unit in a portfolio for each possible premium available and represent an important decision tool in real negotiations.

At last, in order to illustrate the importance to consider multiple possible probability distributions when defining the optimal strategy, we perform a back test on this portfolio with observed data of wind production and spot price for the contract year (January 2013 to December 2013). This back test is motivated due to major change occurred in the Brazilian spot price formation methodology in September 2013 when the Brazilian system operator incorporate a risk averse operation methodology (instead of a risk neutral operation) to assess the future cost function of the system (see [85] for more details). Thus, even if the probability distribution induced by the simulation of the system in December 2011 for the contract year is assumed to be the real distribution of the spot prices, after this methodology change, it is hard to believe that this assumption remains true and represents, minimally, the real outcome. The main idea therefore is to study how the portfolio behaves with this "structural change" in the spot price formation methodology.

In Table 8.2, we present the yearly revenue of the trading company for K_1 = 0, 1 and 2 assuming that the amount of call option obtained using the proposed model was implemented for a premium ranging from 50 to 100 R$/MWh.

Tab. 8.2: Back test in the yearly revenue of the ETC for a range of premiums varying from 50 to 100 R$/MWh (MMR$) and the optimal amount contracted (avgMW) for K_1 = 0, 1 and 2

$P^{(T)}$	K_1 = 0		K_1 = 1		K_1 = 2	
	Rev. (MMR$)	Qt. Thrm. (avgMW)	Rev. (MMR$)	Qt. Thrm. (avgMW)	Rev. (MMR$)	Qt. Thrm. (avgMW)
50	6.74	2.83	8.82	4.76	9.08	5.00
55	6.23	2.45	8.15	4.31	8.86	5.00
60	5.99	2.32	7.65	4.00	8.64	5.00
65	5.73	2.16	7.11	3.61	8.42	5.00
70	5.54	2.05	6.70	3.34	8.20	5.00
75	5.39	1.99	6.45	3.22	7.98	5.00
80	5.18	1.83	6.09	2.95	7.76	5.00
85	5.00	1.70	5.77	2.70	7.54	5.00
90	4.73	1.44	5.48	2.47	7.24	4.88
95	4.38	1.01	5.30	2.36	6.76	4.49
100	4.21	0.82	5.15	2.28	6.38	4.20

Note that stochastic model has the lowest value for different possible option premiums, reflecting, among other things, the impact of changing the spot price formation rules on the optimal hedge level. This result reflects the adjustment that the model can perform in the spot distribution in order to seek the worst-case revenue within a set of credible probability distributions. As a result, the optimization model robustifies the ETC's portfolio against unexpected variations on the scenarios considered as nominal or typical and mitigates this intrinsic risk inherent to spot price variables, being thus averse to ambiguity.

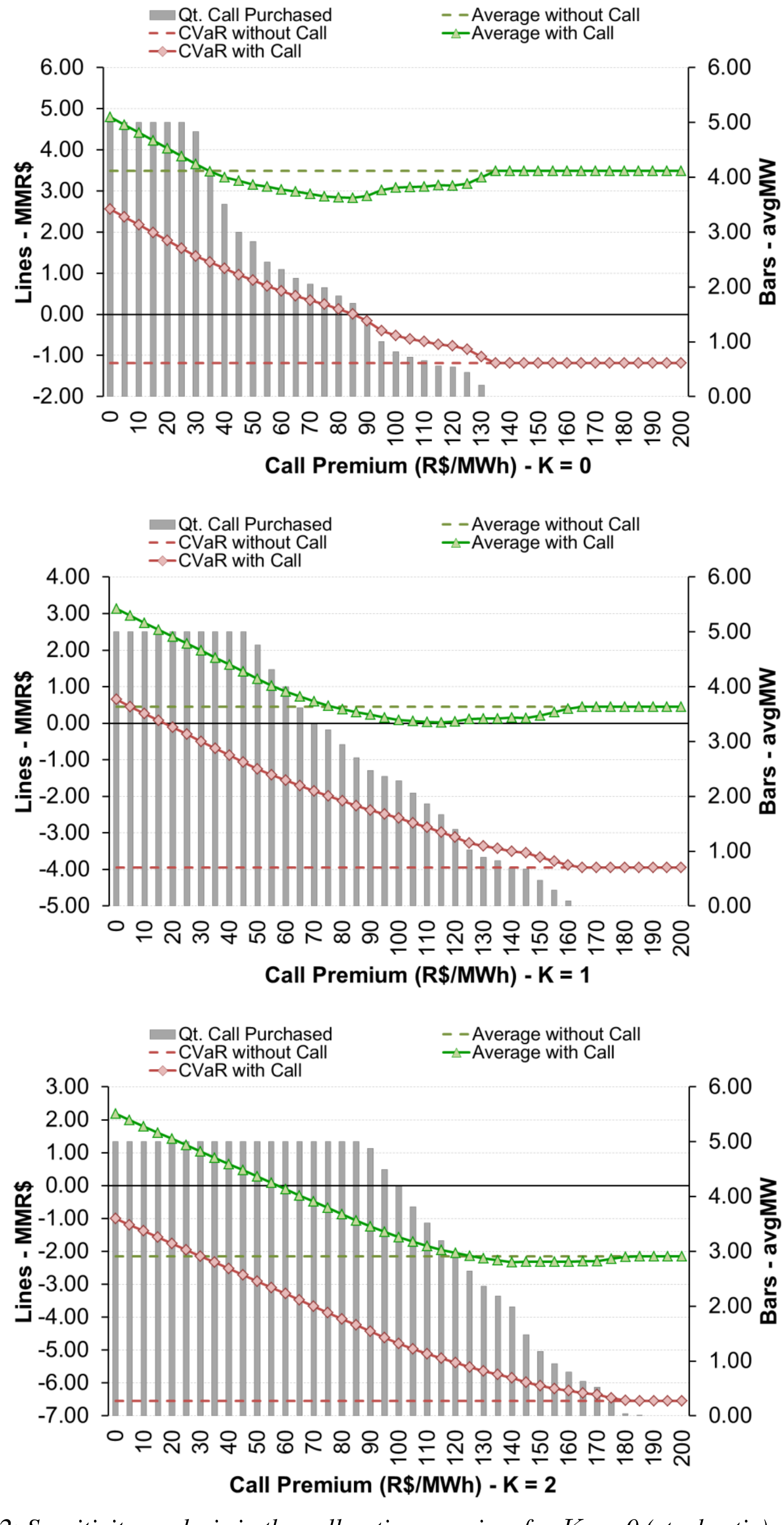

Fig. 8.2: Sensitivity analysis in the call option premium for K_1 = 0 (stochastic), 1 and 2.

8.2 Full Portfolio Allocation with Aversion to Ambiguity

This last case study comprises the full portfolio allocation problem. The contract opportunities considered in this study are the same of the previous studies: (i) one flat forward contract with quantity $\{Q_t^{(F)}\}_{t\in\mathcal{H}} = 10$ avgMW and price $\{P_t^{(F)}\}_{t\in\mathcal{H}} = 140$ R$/MWh; (ii) an energy call option with a total flexible thermal plant ($\underline{g}^{(T)} = 0$) with VOC (Λ) equals to 90 R$/MWh and $\{Q_t^{(T)}\}_{t\in\mathcal{H}} = 5$ avgMW; (iii) a renewable capacity contracts with a wind power of 54.6 MW of installed capacity and 27.12 avgMW of FEC which agreed to sell 100% of its FEC for $\{P_{\text{WPP},t}^{(R)}\}_{t\in\mathcal{H}} = 90$ R$/MWh; and (iv) a renewable capacity contracts with a small hydro of 30 MW of installed capacity and 17.4 avgMW of FEC which agreed to sell 100% of its FEC for $\{P_{\text{SH},t}^{(R)}\}_{t\in\mathcal{H}} = 90$ R$/MWh. The contract period is the same of the previous study (January 2013 to December 2013) thus being affected by the change in spot price formation methodology after September. Hence, $\mathcal{T} = \{1\}$ and $\mathcal{H} = \mathcal{H}_\tau = \{1, 2, ..., 12\}$. The risk-aversion parameters are set to α = 0.95 and λ = 0.50 as usual.

To evaluate the results, a sensibility analysis is performed again on the call option price (call premium). Fig. 8.3 shows a curve in which the x-axis represents the call premium (in R$/MWh), the primary (left) y-axis measures the expected value and the CVaR in MMR$ and the secondary (right) y-axis shows the amount of energy contracted, in avgMW, from the available opportunities. Note that, for $K_1 = 0$ (pure stochastic model), the call option is bought in full until its premium reaches 45 R$/MWh. For $\{P_t^{(T)}\}_{t\in\mathcal{T}} \in \{50, 55, 60\}$, the optimal amount brought to the portfolio is a percentage of the contract. After that, the thermal plant has no value to the portfolio due to the expensive price and the forward contract is covered only by the renewable plants. Note that, for all prices, the total amount bought in contracts is higher than the quantity of the contract, reflecting a hedge against low production scenarios.

Likewise, for $K_1 = 1$, which represents a single deviation on the nominal scenarios, the optimal decision is to fully buy the call option until a premium of 45 R$/MWh. However, its value has a slower decay with respect to the pure stochastic, being valueless only after 90 R$/MWh. In addition, the total amount bought in contracts is higher than the quantity of the contract, reflecting a hedge against low production scenarios too, but the total net of energy in the portfolio is higher than $K_1 = 0$, since the aggressiveness of the spot price is higher. At last, in the $K_1 = 2$ graph in Fig. 8.3, a different pattern can be seen. Note that the optimal portfolio without the call option is to not set up the business due to the high aggressiveness of the spot price against the ETC's portfolio. Therefore, the trading has value only if the call option have value in the portfolio. This happens for call premiums below 70 R$/MWh.

As a final comment, the graphs presented in Fig. 8.3 expresses the optimal portfolio that should be contracted to back up a one-year forward contract in the Brazilian contract market with: (i) aversion to price-quantity risk associated with the renewable production; (ii) hedge against price spikes by means of thermal call options; and (iii) ambiguity averseness using a worst-case approach within a multiple credible probability distribution set. Again we argue that these figures are important decision tool in real negotiations.

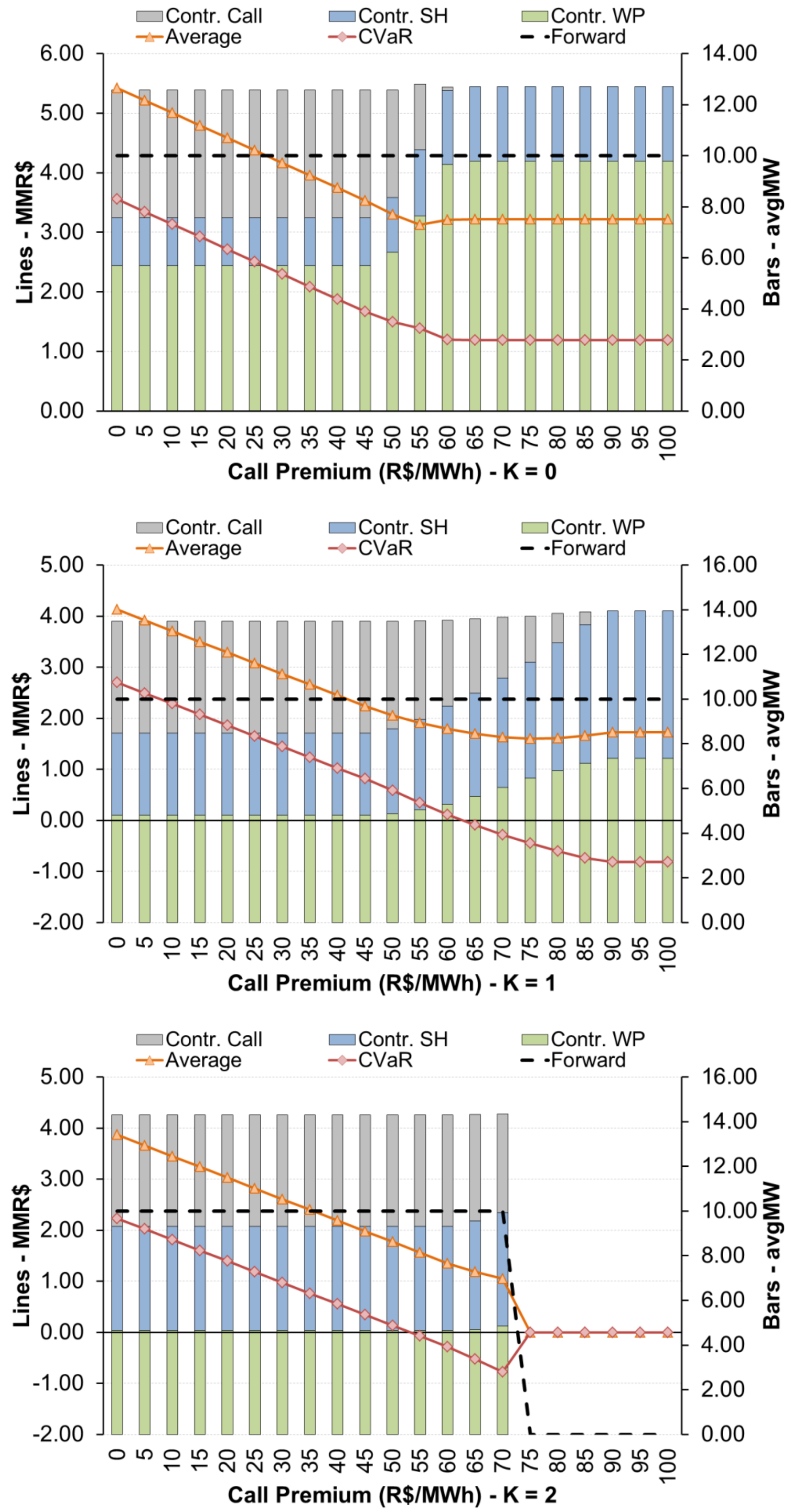

Fig. 8.3: Sensitivity analysis in the call option premium for K_1 = 0 (stochastic), 1 and 2 in the full problem study.

9
Conclusion

Energy commercialization is one of the most important challenges of a generation company in a liberalized market. This process typically involves the definition of the optimal strategy that should be made in order to maximize the company's value with an adequate modeling of all important risk factors that affect the business and the agent's risk profile. In the case of trading renewable generation, this process became worsened due to the uncertainty inherent to its production that, associated to the irregularity of the short-term market price, creates a major risk for the trading company, known as price-quantity risk. In this dissertation, we present a novel approach to determine the risk-constrained optimal portfolio of an energy trading company that sells standard forward contracts to end-users in the contract market backed on renewable generation. Such a model assesses the optimal amount to trade in contracts by the ETC, given their respective specifications (prices, starting dates, durations, etc.), which is robust with respect to unexpected variations in spot prices but that also considers the stochastic nature of the production of renewable assets in a risk-constrained setting. In addition, we include into the optimal portfolio model, the possibility to consider an energy call option contract to hedge the portfolio against price spikes. This approach provides an alternative to current models based on the simulation of prices. The main motivation for this approach comes from the recognition that the simulation of short-term market prices is a very difficulty task due to the complex formation process which this variable is derived. It involves complex interactions between participants in the market and largely depend on unpredictable market conditions.

Two different points of view derived from the model were discussed: (i) stress test, in which the agent considers the worst-case realization of the spot price against its cash-flow endogenously to the formation of the portfolio; and (ii) ambiguity aversion, where the optimal portfolio is constructed considering a set of credible distribution functions around a nominal random variable. One of the main results of this dissertation is a relation between the solution obtained from the classical ambiguity-averse model and the robust optimization model. We provide this link by making use of a re-parametrization of the problem, that shows that the proposed methodology that evolves robust optimization is equivalent to consider ambiguity on some risk factors distribution.

We illustrated the applicability of the methodology by means of a set of case studies in which an ETC must define the portfolio of renewable sources (wind and small hydros) in the Brazilian contract market to back up a mid- and long-term forward contract. In addition, we apply the proposed model to price the value of an energy call option in such renewable portfolio. We show, by means of back testing on the ETC's revenue and using realistic data from the Brazilian power system, that the proposed methodology outperforms the classical stochastic approach whenever the observed prices deviates from the simulated scenarios. Since the majority of the trading decisions are set with at least 6 months in advance, we argue that this deviations are very typical due to the "time uncertainty" on the simulation (six months in advance) and the aforementioned difficulty to predict the short-term market price. Therefore, the methodology proposed in this dissertation is a powerful tool in risk managing of renewable energy commercialization.

Ongoing research related to this work includes a definition of a stochastic conservatism parameter correlated with the generation of the plants as well as its estimation and the generalization of the methodology to include other types of contracts, such as flexible and collar contracts.

Bibliography

[1] "Operador Nacional do Sistema - ONS, Brasil." `http://www.ons.org.br/`.

[2] S. Gsanger and J.-D. Pittleloud, "World Wind Energy Association: Annual Report 2012." `http://www.wwindea.org/webimages/WorldWindEnergyReport2012_final.pdf`, May 2013.

[3] G. Rothwell and T. Gómez, *Electricity Economics: Regulation and Deregulation*. New York, NY: Wiley, 1st ed., 2003.

[4] S. Stoft, *Power System Economics: Designing Markets for Electricity*. New York, NY: Wiley, 1st ed., 2002.

[5] N. Hadjsaïd and J.-C. Sabonnadière, *Power Systems and Restructuring*. New York, NY: Wiley, 1st ed., 2013.

[6] E. G. Kardakos, C. K. Simoglou, and A. G. Bakirtzis, "Short-Term Electricity Market Simulation for Pool-Based Multi-Period Auctions," *IEEE Trans. Power Syst.*, vol. 28, pp. 2526–2535, Aug. 2013.

[7] J. Contreras, O. Candiles, J. L. de la Fuente, and T. Gómez, "Auction Design in Day-Ahead Electricity Markets," *IEEE Trans. Power Syst.*, vol. 16, pp. 88–96, Feb. 2001.

[8] G. B. Sheblé, *Computational Auction Mechanisms for Restructured Power Industry Operation*. Norwell, MA: Kluwer Academic, 2nd ed., 2002.

[9] J. MacCormack, H. Zareipour, and W. D. Rosehart, "Long-Term Market Equilibrium Model with Strategic, Competitive, and Inflexible Generation," *IEEE Trans. Power Syst.*, vol. 27, pp. 2291–2292, Nov. 2012.

[10] A. Street, L. A. Barroso, B. Flach, M. V. Pereira, and S. Granville, "Risk Constrained Portfolio Selection of Renewable Sources in Hydrothermal Electricity Markets," *IEEE Trans. Power Syst.*, vol. 24, pp. 1136–1144, Aug. 2009.

[11] J. Cabero, A. Baíllo, S. Cerisola, M. Ventosa, A. García-Alcalde, F. Perán, and G. Relaño, "A Medium-Term Integrated Risk Management Model for a Hydrothermal Generation Company," *IEEE Trans. Power Syst.*, vol. 20, pp. 1379–1388, Aug. 2005.

[12] L. A. Barroso, M. V. Pereira, and J. Rosenblatt, "Ensuring Energy Supply Adequacy in Market-Based Systems: The Brazilian Experience," *IEEE PES General Meeting 2003*, vol. 1, pp. 529–531, Jul. 2003.

[13] L. A. Barroso, J. Rosenblatt, A. Guimarães, B. Bezerra, and M. V. Pereira, "Auctions of Contracts and Energy Call Options to Ensure Supply Adequacy in the Second Stage of the Brazilian Power Sector Reform," *IEEE PES General Meeting 2006*, pp. 1–8, Jul. 2006.

[14] B. Bezerra, L. A. Barroso, S. Granville, A. Guimarães, A. Street, and M. V. Pereira, "Energy Call Options Auctions for Generation Adequacy in Brazil," *IEEE PES General Meeting 2006*, pp. 1–8, Jul. 2006.

[15] P. Lino, L. A. Barroso, M. V. Pereira, R. Kelman, and M. Fampa, "Bid-Based Dispatch of Hydrothermal Systems in Competitive Markets," *Annals of Operations Research*, vol. 120, pp. 81–97, Apr. 2003.

[16] M. V. Pereira and L. M. Pinto, "Multi-Stage Stochastic Optimization Applied to Energy Planning," *Mathematical Programming*, vol. 52, pp. 359–375, May. 1991.

[17] M. V. Pereira, L. A. Barroso, and J. Rosenblatt, "Supply Adequacy in the Brazilian Power Market," *IEEE PES General Meeting 2004*, vol. 1, pp. 1016–1021, Jun. 2004.

[18] G. Boyle, *Renewable Energy: Power for a Sustainable Future*. Oxford, UK: Oxford University Press, 2nd ed., 2004.

[19] A. Street, L. A. Barroso, S. Granville, and M. V. Pereira, "Bidding Strategy under Uncertainty for Risk-Averse Generator Companies in a Long-Term Forward Contract Auction," *IEEE PES General Meeting 2009*, vol. 1, pp. 1–8, Jul. 2009.

[20] Y. Oum, S. Oren, and S. Deng, "Hedging Quantity Risks with Standard Power Options in a Competitive Wholesale Electricity Market," *Naval Research Logistics*, vol. 53, pp. 697–712, Oct. 2006.

[21] A. Street, D. Lima, L. Freire, and J. Contreras, "Sharing Quotas of a Renewable Energy Hedge Pool: A Cooperative Game Theory Approach," *IEEE PowerTech 2011*, vol. 1, pp. 1–6, Jun. 2011.

[22] L. Freire, “Modelo de Comercialização de Energia Renovável no Ambiente de Contratação Livre via Teoria de Jogos Cooperativos,” Master’s thesis, Pontifical Catholic University, Rio de Janeiro, Brazil, 2013.

[23] L. A. Barroso, A. Street, S. Granville, and M. V. Pereira, “Offering Strategies and Simulation of Multi-Item Iterative Auctions of Energy Contracts,” *IEEE Trans. Power Syst.*, vol. 26, pp. 1917–1928, Nov. 2011.

[24] A. Street, D. A. Lima, A. Veiga, B. Fanzeres, L. Freire, and B. Amaral, “Fostering Wind Power Penetration into the Brazilian Forward-Contract Market,” *IEEE PES General Meeting 2011*, pp. 1–8, Jul. 2011.

[25] D. Bertsimas and M. Sim, “Robust Discrete Optimization and Network Flows,” *Mathematical Programming*, vol. 98, pp. 49–71, Sep. 2003.

[26] D. Bertsimas and M. Sim, “The Price of Robustness,” *Operations Research*, vol. 52, pp. 35–53, Jan.–Feb. 2004.

[27] J. Dupacova and J. Polivka, “Stress Testing of VaR and CVaR,” *Quantitative Finance*, vol. 7, Aug. 2007.

[28] D. Ellsberg, “Risk, Ambiguity and the Savage Axioms,” *The Quartely Journal of Economics*, vol. 75, pp. 643–669, Nov. 1961.

[29] I. Gilboa and D. Schmeidler, “Maxmin Expected Utility with Non-Unique Prior,” *Journal of Mathematical Economics*, vol. 18, pp. 141–153, Aug. 1989.

[30] F. Maccheroni, M. Marinacci, and A. Rustichini, “Ambiguity Aversion, Robustness, and the Variational Representation of Preferences,” *Econometrica*, vol. 74, pp. 1447–1498, Nov. 2006.

[31] G. C. Calafiori and L. E. Ghaoui, “On Distributionally Robust Chance-Constrained Linear Programs,” *Journal of Optimization Theory and Applications*, vol. 130, pp. 1–22, Jul. 2006.

[32] E. Erdogan and G. Iyengar, “Ambiguous Chance Constrained Problems and Robust Optimization,” *Mathematical Programming*, vol. 107, pp. 37–61, Jun. 2006.

[33] X. Chen, M. Sim, and P. Sun, “A Robust Optimization Perspective on Stochastic Programming,” *Oper. Res.*, vol. 55, pp. 1058–1071, Nov–Dec. 2007.

[34] E. Delage and Y. Ye, "Distributionally Robust Optimization under Moment Uncertainty with Application to Data-Driven Problems," *Oper. Res.*, vol. 58, pp. 595–612, May–Jun. 2010.

[35] A. Ben-Tal, D. Bertsimas, and D. Brown, "A Soft Robust Model for Optimization under Ambiguity," *Oper. Res.*, vol. 58, pp. 1220–1234, Jul–Aug. 2010.

[36] B. Fanzeres, A. Street, and L. A. Barroso, "Contracting Strategies for Generation Companies with Ambiguity Aversion on Spot Price Distribution," *XVIII Power System Computational Conference (XVIII PSCC) 2014*, pp. 1–8, Aug. 2014.

[37] "Agência Nacional de Energia Elétrica - ANEEL, Brasil." `http://www.aneel.gov.br/`.

[38] "Brazilian Energy Expansion Plan for 2020 - MME, brasil." `http://www.mme.gov.br/mme/galerias/arquivos/noticias/2011/SUMARIO-PDE2020.pdf`.

[39] "Câmara de Comercialização de Energia Elétrica - CCEE, Brasil." `http://www.ccee.org.br/`.

[40] E. Faria, L. A. Barroso, R. Kelman, S. Granville, and M. V. Pereira, "Allocation of Firm-Energy Rights Among Hydro Plants: An Aumann-Shapley Approach," *IEEE Trans. Power Syst.*, vol. 24, pp. 541–551, May. 2009.

[41] M. J. Alzate, A. I. Cadena, and J. M. Benavides, "Analysis to the Firm Energy Market Proposal for the Colombian Electricity Market," *IEEE PES Transm. and Dist. Conference and Exposition 2008*, vol. 1, pp. 1–5, Aug. 2008.

[42] "Resolução Normativa Nº 109, de 26 de Outubro de 2004 - ANEEL, Brasil." `http://www.aneel.gov.br/cedoc/ren2004109.pdf`.

[43] C. Vazquez, M. Rivier, and I. J. Perez-Arriaga, "A Market Approach to Long-Term Security of Supply," *IEEE Trans. Power Syst.*, vol. 17, pp. 349–357, May. 2002.

[44] F. Porrua, B. Bezerra, L. A. Barroso, P. Lino, F. Ralston, and M. V. Pereira, "Wind Power Insertion through Energy Auctions in Brazil," *IEEE PES General Meeting 2010*, pp. 1–8, Jul. 2010.

[45] "Resolução Normativa Nº 271, de 3 de Julho de 2007 - ANEEL, Brasil." `http://www.aneel.gov.br/cedoc/ren2007271.pdf`.

[46] C. Ruiz and A. J. Conejo, "Pool Strategy of a Producer with Endogenous Formation of Locational Marginal Prices," *IEEE Trans. Power Syst.*, vol. 24, pp. 1855–1866, Nov. 2009.

[47] R. Fernández-Blanco, J. M. Arroyo, and N. Alguacil, "A Unified Bilevel Programming Framework for Price-Based Market Clearing under Marginal Pricing," *IEEE Trans. Power Syst.*, vol. 27, pp. 517–525, Feb. 2012.

[48] Q. Wang, J.-P. Watson, and Y. Guan, "Two-Stage Robust Optimization for N-k Contingency-Constrained Unit Commitment," *IEEE Trans. Power Syst.*, vol. 28, pp. 2366–2375, Aug. 2013.

[49] A. Street, F. Oliveira, and J. M. Arroyo, "Contingency-Constrained Unit Commitment with n-K Security Criterion: A Robust Optimization Approach," *IEEE Trans. Power Syst.*, vol. 26, pp. 1581–1590, Aug. 2011.

[50] M. Carrión and J. M. Arroyo, "A Computational Efficient Mixed-Integer Linear Formulation for the Thermal Unit Commitment Problem," *IEEE Trans. Power Syst.*, vol. 21, pp. 1371–1378, Aug. 2006.

[51] J. M. Arroyo and A. J. Conejo, "Optimal Response of a Thermal Unit to an Electricity Spot Market," *IEEE Trans. Power Syst.*, vol. 15, pp. 1098–1104, Aug. 2000.

[52] ABEEólica, "Brazil Winp Power 2012 - Conference and Exhibition, rio de janeiro." `http://www.brazilwindpower.org`, 2012.

[53] "Resolução Nº 652, de 9 de Dezembro de 2003 - ANEEL, Brasil." `http://www.aneel.gov.br/cedoc/res2003652.pdf`.

[54] A. K. Dixit and R. S. Pindyck, *Investment Under Uncertainty*. Princeton, NJ: Princeton University Press, 1st ed., 1994.

[55] J. Birge and F. Louveaux, *Introduction to Stochastic Programming*. New York, NY: Springer, 2nd ed., 2011.

[56] C. Acerbi, "Coherent Measure of Risk in Everyday Market Practice," *Quantitative Finance*, vol. 7, pp. 359–364, Aug. 2007.

[57] G. J. Alexander and A. M. Baptista, “A Comparison of VaR and CVaR Constraints on Portfolio Selection with the Mean-Variance Model,” *Management Science*, vol. 50, pp. 1261–1273, Sep. 2004.

[58] P. Artzner, F. Delbaen, J.-M. Eber, and D. Heath, “Coherent Measures of Risk,” *Mathematical Finance*, vol. 9, pp. 203–228, Jul. 1999.

[59] P. Jorion, *Value at Risk: The New Benchmark for Managing Financial Risk*. New York, NY: McGraw Hill, 3rd ed., Oct. 2006.

[60] R. T. Rockafellar and S. P. Uryasev, “Optimization of Conditional Value-at-Risk,” *Journal of Risk*, vol. 2, pp. 21–41, Apr. 2000.

[61] G. Casella and R. L. Berger, *Statistical Inference*. Stamford, CT: Cengage Learning, 2nd ed., Jun. 2001.

[62] A. J. Kleywegt, A. Shapiro, and T. Homem-De-Mello, “The Sample Average Approximation Method for Stochastic Discrete Optimization,” *SIAM Journal of Optimization*, vol. 12, no. 2, pp. 479–502, 2002.

[63] G. C. Pflug, “Some Remarks on the Value-at-Risk and the Conditional Value-at-Risk,” *Nonconvex Optimization and Its Applications*, vol. 49, pp. 272–281, 2000.

[64] A. Street, “On the Conditional Value-at-Risk Probability-Dependent Utility Function,” *Theory and Decision Journal*, vol. 68, pp. 49–68, Feb. 2010.

[65] A. Street, *Equivalente Certo e Medidas de Risco em Decisões de Comercialização de Energia Elétrica*. PhD thesis, Pontifical Catholic University, Rio de Janeiro, Brazil, 2008.

[66] A. Shapiro and A. Nemirovski, “On Complexity of Stochastic Programming Problems,” *Continuous Optimization: Applied Optimization*, vol. 99, pp. 111–146, 2005.

[67] P. Klibanoff, M. Marinacci, and S. Mukerji, “A Smooth Model of Decision Making under Ambiguity,” *Econometrica*, vol. 73, pp. 1849–1892, Nov. 2005.

[68] L. I. Castro and A. Chateauneuf, “Ambiguity Aversion and Trade,” *Economic Theory*, vol. 48, pp. 243–273, Oct. 2011.

[69] A. L. Soyster, “Convex Programming with Set-Inclusive Constraints and Applications to Inexact Linear Programming,” *Operations Research*, vol. 21, pp. 1154–1157, Sep.–Oct. 1973.

[70] D. Bertsimas and D. B. Brown, “Constructing Uncertainty Sets for Robust Linear Optimization,” *Operations Research*, vol. 57, pp. 1483–1495, Nov.–Dec. 2009.

[71] D. Bertsimas and M. Sim, “Tractable Approximations to Robust Conic Optimization Problems,” *Mathematical Programming*, vol. 107, pp. 5–36, Jun. 2006.

[72] Wenqing, M. Sim, J. Sun, and C.-P. Teo, “From CVaR to Uncertainty Set: Implications in Joint Chance-Constrained Optimization,” *Operations Research*, vol. 58, pp. 470–485, Mar.–Apr. 2010.

[73] V. Gonzalez, J. Contreras, and D. W. Bunn, “Forecasting Power Prices using a Hybrid Fundamental-Econometric Model,” *IEEE Trans. Power Syst.*, vol. 27, pp. 363–372, Feb. 2012.

[74] A. J. Conejo, M. A. Plazas, R. Espinola, and A. B. Molina, “Day-Ahead Electricity Price Forecast using the Wavelet Transform and ARIMA Models,” *IEEE Trans. Power Syst.*, vol. 20, pp. 1035–1042, May. 2005.

[75] F. Olsina and C. Weber, “Stochastic Simulation of Spot Power Prices by Spectral Representation,” *IEEE Trans. Power Syst.*, vol. 24, pp. 1710–1719, Nov. 2009.

[76] H. Niu, R. Baldick, and G. Zhu, “Supply Function Equilibrium Bidding Strategies with Fixed Forward Contracts,” *IEEE Trans. Power Syst.*, vol. 20, pp. 1859–1867, Nov. 2005.

[77] L. P. Garces and A. Conejo, “Weekly Self-Scheduling, Forward Contracting, and Offering Strategy for a Producer,” *IEEE Trans. Power Syst.*, vol. 25, pp. 657–666, May. 2010.

[78] T. Trezos, “Risk and Benefits Assessment of Long and Short Term Capacity Contract Strategies,” *IEEE Trans. Power Syst.*, vol. 4, pp. 981–987, Aug. 1989.

[79] J. C. Hull, *Options, Futures, and Other Derivatives*. Upper Saddle River, NJ: Prentice Hall, 8th ed., Feb. 2011.

[80] P. H. Kupiec, “Stress Testing in a Value at Risk Framework,” *The Journal of Derivatives*, vol. 6, pp. 7–24, Sep. 1998.

[81] C. S. Kubrusly, *Introduction to Measure Theory: A First Course*. Oxford, UK: Elsevier Academic Press, first ed., 2006.

[82] Xpress Optimization Suite. [Online]. Available: http://www.fico.com/.

[83] S. Dempe, *Foundations of Bilevel Programming*. Norwell, MA: Kluwer, first ed., 2002.

[84] D. Bertsimas and J. Tsitsiklis, *Introduction to Linear Programming*. Nashua, NH: Athena Scientific, first ed., 1997.

[85] A. Shapiro, W. Tekaya, J. P. da Costa, and M. P. Soares, “Risk Neutral and Risk Averse Stochastic Dual Dynamic Programming Method,” *European Journal of Operational Research*, vol. 224, pp. 375–391, Jan. 2013.