Pontifícia Universidade Católica do Rio de Janeiro

Gustavo Alberto Amaral Ayala

Energy and Reserve Scheduling with Post-Contingency Transmission Switching: A Smart Grid Application

TESE DE DOUTORADO

Thesis presented to the Programa de Pós-Graduação em Engenharia Elétrica of the Departamento de Engenharia Elétrica, PUC-Rio as partial fulfillment of the requirements for the degree of Doctor em Engenharia Elétrica.

Advisor: Prof. Alexandre Street Aguiar

Rio de Janeiro

May 2014

Pontifícia Universidade Católica
do Rio de Janeiro

Gustavo Alberto Amaral Ayala

Energy and Reserve Scheduling with Post-Contingency Transmission Switching: A Smart Grid Application

TESE DE DOUTORADO

Thesis presented to the Programa de Pós-Graduação em Engenharia Elétrica of the Departamento de Engenharia Elétrica do Centro Técnico Científico da PUC-Rio, as partial fulfillment of the requerements for the degree of Doutor

Prof. Alexandre Street de Aguiar
Advisor
Departamento de Engenharia Elétrica – PUC-Rio

Dr. Bruno da Costa Flach
IBM Research Brasil

Dr. Sérgio Granville
PSR Soluções e Consultoria em Energia Ltda

Dr. André Luiz Diniz Souto Lima
Cepel

Dr. Geraldo Gil Veiga
RN Tecnologia

Prof. Davi Michel Valladão
Departamento de Industrial - PUC-Rio

Prof. José Eugenio Leal
Coordinator of the Centro Técnico
Científico da PUC-Rio

Rio de Janeiro, May 26th, 2014

Gustavo Alberto Amaral Ayala

Gustavo Ayala is PhD candidate at the Electrical Engineering Department at Pontifical Catholic University of Rio de Janeiro (PUC-Rio), Brazil. He works at ENEVA S.A in the Electrical Energy Regulation and Market Studies department. He worked as an energy researcher at PSR Consultant from 2005 to 2009. He has a Master's degree in Mathematics Economics from IMPA (2008), a Master's degree in Operation Research from PUC-Rio (2008) and a Bachelor degree in Electrical Engineering (2005) also at PUC-Rio.

Bibliographic data

Ayala, Gustavo Alberto Amaral

Energy and reserve scheduling with post-contingency transmission switching: smart grid application / Gustavo Alberto Amaral Ayala; advisor: Alexandre Street Aguiar. – 2014.

99 f.; 30 cm

Tese (doutorado) – Pontifícia Universidade Católica do Rio de Janeiro, Departamento de Engenharia Elétrica, 2014.

Inclui bibliografia

1. Engenharia elétrica – Teses. 2. Despacho de energia e reserva. 3. Ações corretivas. 4. Decomposição de Benders. 5. Programação linear inteira mista. 6. Switchings de linhas. 7. Redes inteligentes. I. Aguiar, Alexandre Street. II. Pontifícia Universidade Católica do Rio de Janeiro. Departamento de Engenharia Elétrica. III. Título.

CDD: 621.3

Acknowledgments

It is with great pleasure that I get to the end of this journey. And the presentation of this thesis would not be possible without the association of great people. And on this occasion I take to express my sincere gratitude.

First, I would like to thank my advisor Alexandre Street, for assistance, support and encouragement to develop this thesis, which is above all a great friend who provided an excellent environment for the development of this work.

I also thank my great friend Raphael Chabar for those endless theoretical discussions could strengthen more and more the present work. Thank you for the attention and the friendship. I expand my thanks to Alexandre Moreira who helped me with implementing the first attempt computational system and who always was willing to help. I also thank other great friends of PUC-Rio: Lucas Freire, Bruno Fânzeres, Joaquim Lacombe and Mario Souto for the friendship, willingness to help and the hours spend especially in the last period of this thesis.

I extend my acknowledgement to Geraldo Veiga with did an important review of this document and sincere willingness to help. I also thank to Mario Veiga and Sergio Granville that do not participates directly on this work but had a relevant importance in my professional and academic formation, the years of working with them was very important to my accomplishment on this thesis.

I also thank Dr. Sen Suvrajeet and Qi Yunwei for the important e-mail communications regarding the decomposition theory.

I also thank my great friends of ENEVA: Eduardo Faria, Evandro Quinaud for the friendship and especially Marco Vieira that give me an unconditional support to finish this PhD thesis.

I also thank my great friend João Paixão for the friendship, willingness to help and profound discussions about the mathematical side of this PhD thesis.

I extend my acknowledgement to the Department of Electrical Engineering of Pontifical Catholic University of Rio de Janeiro which provided an excellent environment for my research. I spent many enjoyable hours with members of the department and colleagues from the laboratory, which I am grateful for this rich environment of learning.

I want to dedicate this thesis to my brother Marcelo Ayala for the fellowship, helpful and support, to my beloved Alexandra Marchi for the comprehension, support, love and fellowship; and finally, I dedicate it to my mother who gave everything I am today. I am eternally grateful to Socorro Ayala.

The most important thing of begin and finish something is the journey. And I would like to express my sincere gratitude to these great people that was present in this part of my life and make this thesis possible.

"If you really want to know a tree, for instance, you've got to climb in the tree, you've got to feel the tree, sit in the branches, listen to the wind blow through the leaves. Then you'll be able to say, 'I know that tree."

Prof. Leo Buscaglia.

Abstract

Ayala, Gustavo Alberto Amaral; Aguiar, Alexandre Street (Advisor). **Energy and Reserve Scheduling with Post-Contingency Transmission Switching: a Smart Grid Application.** Rio de Janeiro, 2014. 99p. PhD Thesis - Departamento de Engenharia Elétrica, Pontifícia Universidade Católica do Rio de Janeiro.

This PhD Thesis is composed by two papers with contributions on operations research applied to smart grid theory. The first paper highlights the economic and security benefits of an enhanced system operation with the advent of a smart grid technology by introducing a novel model, which is a joint energy and reserve scheduling that incorporates the network capability to switch transmission lines as a corrective action to enhance the system capability to circumvent contingency events. The main goal is to reduce operating costs and electric power outages, by adjusting the network connectivity when a contingency occurs. In such a framework, results show that, with a limited number of corrective switches, the system operator is able to circumvent a wider range of contingencies, while resulting in lower operational costs and reserve levels. In our context, a grid that is capable to adjust its generation and also its topology through post-contingency line switching is called a self-healing grid, and its importance in network security and operating costs is demonstrated in this work. The graph structure is explored in the algorithmic solution of the post-contingency transmission switching problem. Numerical results demonstrate a significant reduction in total load shedding and operating cost. It has been also illustrated an expressive improvement in terms of security and operating cost, in comparison to the transmission switching models previously published. The second paper is an application of a modified Benders decomposition to the post-contingency transmission switching problem. The decomposition is an attempt to deal with the NP-hard optimization problem created by the transmission switching and unit commitment variables. The major contribution is the application of a new benders decomposition approach to the problem of transmission switching, in which the first and second stages problems are a mixed-integer program. To deal with this issue, it is used a Branch and Bound (B&B) procedure for the first-stage problem and a sequential convexification procedure for the second-stage problem.

Keywords

Energy and Reserve Scheduling; Corrective Actions; Benders Decomposition; Mixed Integer Linear Programming; line switching; smart grids.

Resumo

Ayala, Gustavo Alberto Amaral; Aguiar, Alexandre Street (Orientador). **Uma aplicação de Smart Grid: Despacho ótimo – Energia e Reserva – com Switch na transmissão Pós-Contingência**. Rio de Janeiro, 2014. 99p. Tese de Doutorado - Departamento de Engenharia Elétrica, Pontifícia Universidade Católica do Rio de Janeiro.

Esta tese de doutorado é composta de dois artigos científicos com contribuições na área de “Smart Grid”. Além disso, a tese também contribui para o desenvolvimento de soluções computacionais eficientes para problemas de programação linear mista e inteira. Outra importante contribuição é o desenvolvimento de método de decomposição benders com segundo estágio inteiro e não convexo aplicado ao problema de “Transmission Switching”. O primeiro artigo científico mostra os benefícios com o advento de uma rede inteligente e o aumento da capacidade do operador do sistema de energia elétrica em tomar ações corretivas em face de ocorrências de contingências. O artigo também analisa consequências práticas na capacidade de “self-healing” da rede pós-contingência. Em nosso contexto, uma rede “self-healing” é uma rede com total flexibilidade para ajustar a geração e as linhas de transmissão antes e depois da ocorrência de alguma contingência. Resultados numéricos mostram significantes reduções no corte de carga para cada contingência e no total. Foi considerado um único período que representa a demanda de pico do sistema, comparou-se o novo método com os utilizados em publicações anteriores. O segundo artigo contribui também para a aplicação da tecnologia de “Smart Grid”, em particular a teoria de “Transmission Switching”. De fato, desenvolvemos uma estratégia de solução para lidar com a complexibilidade NP-Hard criada pelas variáveis de “transmission switching” e “unit commitment” do problema de otimização. Foi desenvolvida uma solução algorítmica baseada na teoria dos grafos. Estudou-se a estrutura topológica desses problemas. Além disso, a maior contribuição foi o desenvolvimento de um novo método de decomposição de benders aplicado para o problema de “transmission switching” com o segundo estágio inteiro e não convexo. Para lidar com este problema de não convexidade, foi desenvolvido um método de convexificação sequencial, implícito a decomposição de benders.

Palavras-chave

Despacho de Energia e Reserva; Ações Corretivas; Decomposição de Benders; Programação linear inteira mista; switchings de linhas; redes inteligentes.

Contents

List of figures

List of tables

1
INTRODUCTION

The electrical system has been operated with the same philosophy at least over the last four decades [1]. This PhD thesis encourages major evolution in this paradigm, more specifically in the use and operation of transmission assets.

Traditionally, System Operators (SOs) treat transmission assets as fixed resources in scheduling models [1]-[4]. Notwithstanding the traditional approach, specific changes in the grid topology can be performed in real time to improve the system reliability [5]-[6]. These variations follow some previously agreed rules (see [7]-[8]) or, in certain cases, are based on the SOs experience. Consequently, current operation standards do not consider the grid topology as a decision in day-ahead scheduling models.

At first glance, switching off a transmission line in a network may be seen as paradox [9]. If a transmission line is available with zero cost, it is reasonable to use it during the system operation. In a tree-network topology, this statement is true. The only rule that matters is Kirchhoff's current law. Nevertheless, if the network has cycles, then Kirchhoff's voltage law must be met as well. Each cycle in the network adds one constraint in the optimization model and, to improve the system operation, switching off a line can be beneficial to avoid the cycle constraint. Therefore, in a meshed network, taking out a line is a tradeoff between Kirchhoff's voltage and current laws.

This paradox can be seen as well in the urban transport traffic field. In 1990, the mayor of New York decided to close the 42nd street for one day [10], as opposed to what many expected, the traffic conditions were improved. Rio de Janeiro city presents a similar example: a street connected to "Estrada Lagoa-Barra" towards "Zona Sul" was permanently closed and the traffic flow in the region has been improved.

Recent research has shown that considering the grid topology as a decision variable improves the system security while reducing operating costs: [11] and [12]-[14]. Fisher et al. [12] developed a transmission switching (TS) and generation dispatch model, by means of a mixed-integer-linear program (MILP), to supply the demand during a single period of time. These authors found a cost reduction of 25 percent for the standard 118-bus IEEE. Hedman et al. [13] analyzed the impact of $n-1$ security criteria in a previous work [12] and found a 15 percent saving for the 118-bus IEEE test system. They also applied this methodology to the 73-bus IEEE tests system and found a cost reduction of eight percent. It is worth mentioning that no cost reduction was verified for the 73-bus system in the absence of a security criterion. Lastly, Hedman et al. [11] considered the unit commitment problem with transmission switching on the day-ahead scheduling. They showed that an optimal network topology exists for each hour of the planning horizon. Hedman et al. [14] proposed the concept of just-in-time transmission, which motivates the use of TS as a corrective action. In [14], the model discussed in [12] is used to accomplish TS and two heuristic approaches were used to tackle the problem.

In [11], the TS benefit is considered as a preventive action in a unit commitment model. The SO determines the optimal pre-contingency scheduling for the generation and transmission assets, but only generators are considered to respond against the loss of system components in post-contingency states. Moreover, a cost-based reserve allocation was not considered in such work, despite its intrinsic dependence on the network topology and the increasing appeal for optimization procedures to schedule optimal reserve levels in a joint energy and reserve market (see [1]-[3] and [15]). As a result, the least cost reserve deliverability was not accounted for in any of the previous TS-related works. Moreover, TS is not considered as a corrective action in any of the previously reported modeling approaches.

Ayala and Street [16] explored exactly the transmission switching as a corrective action. The work in [16] corresponds to the first part of this PhD thesis. The proposed framework is a novel joint energy and reserve scheduling model that accounts for TS in both pre and post-contingency states. This is an application of a smart network in which the system operator incorporates economic and security benefits of fast TS actions into the schedule. The proposed model co-optimizes the joint generation schedule, of energy and reserves, and the transmission topology.

The novel model introduces an immense amount of binary variables to represent three types of grid equipment states: (i) unit commitment; (ii) pre-contingency TS and (iii) post-contingency TS (*PC-TS*). The set of variables (ii) and (iii) increase dramatically the computational complexity of the standard unit commitment models. For every contingency state, the transmission topology can be modified (transmission lines can be switched on or off) before and after an occurrence of a contingency. Therefore the computational time increases significantly. To cope with this type of problem a bender's decomposition is studied. A natural decomposition of post-contingency TS problem includes TS corrective actions (binary variables) on the second-stage. At this point, a difficulty arises, the optimal value recourse objective function could be non-convex and discontinuous [17]. This is a fundamental hypothesis on Benders decomposition that needs to be fulfilled to guarantee the method's convergence [18].

To cope with second-stage mixed-integer variables, Sherali and Fraticelli [17] modified the classical Benders decomposition [18]. They used a particular convex-hull representation of the problem's feasible region, which was explored by Sherali and Adams in [19]. They showed that a convex-hull of a mixed-integer program can be represented only introducing a new set of variables. Therefore, first Sherali and Fraticelli [17] showed that, if this convex hull representation of the problem's feasible region is at hand then the classical Benders decomposition [18] can be applied. Second, they showed that it is not necessary to have the complete convex hull of the problem's feasible region, only an appropriate partial (approximate) convex hull is sufficient. A cutting plane scheme is used to generate cuts that approximate the convex hull of the problem's feasible region. These cuts are function of the first-stage variables, i.e., they are constructed to be valid for all first-stage feasible solution. Therefore, the cuts could be re-used on another first-stage solution. This fact is important and necessary for the method's convergence.

The framework in [17] has some limits. For example, problems with general mixed-integer variables on the second-stage are outside the scope, and only 0-1 mixed-integer variables are contemplated. Other restriction is that only binary variables are allowed on the first-stage problem, therefore even continuous variables are not covered. This restriction is important because it ensures the facial property, i.e., the resultant set by fixing the first-stage component is a face of the particular problem's convex hull. This facial property plays an important role on the method's convergence and finiteness.

Despite these framework's limits, the work in [17] constitutes one of the foundations of Benders decomposition applied to stochastic programs with integer recourse.

The first attempt of the decomposition research developed in this PhD thesis is due to [20]. In their work, it was described a decomposition approach for a two-stage stochastic unit commitment problem where the second-stage binary variables are quick-start generators such as gas turbines and combined-cycle. In a standard unit commitment problem, only the committed units are allowed to produce energy after a contingency. If the unit is not committed, then energy could not be produced. In practice units with quick-start can produce energy after a contingency. In [20], they extended the standard unit commitment problem to a more practical framework in which is considered quick-start and non-quick-start units. The decomposition used in [20] is based on [17].

Although the framework in [20] is broader than in [17], they affirmed that the algorithm converges in finite numbers of steps. A similar but not exactly version of the algorithm established in [20] is applied to the post-contingency TS problem as defined in Ayala and Street [16]. In this case, the convergence is not achieved on the computational experiments and other research way was chosen for the decomposition approach. Two questions are important to be answered in a future research: (i) why the convergence of such method is not achieved and (ii) how to ensure the convergence.

The decomposition approach used in this PhD thesis is due to the algorithm established by Suvrajeet and Yunwei in [21]. The main algorithm's objective is to solve a two-stage problem through a Benders decomposition, which contemplates general mixed-integer variables in both stages. Numerical results of this decomposition approach applied to the post-contingency TS are showed in this PhD thesis, and it also encourages further research on this field.

1.1
Context and System's Operation Historical Evolution

The following section contextualizes the post-contingency TS problem into the energy market and shows important historical evolutions of system operation until the proposed [16] advancement.

1.1.1
Day-ahead and real-time energy markets

This subsection discusses some special characteristics of energy supply and demand balance, describes the dynamics of day-ahead and real-time markets, and also contextualizes the improvement addressed to this PhD thesis.

Special characteristics of electricity supply and demand: notwithstanding of some new improvements in electrical vehicles industry, electricity is a challenging product to store[1] and it has to be produced almost on the same time that is consumed. For this reason, it is a permanent real-time balance of demand and supply.

System operators (SOs) have the objective to balance energy supply and demand with the least operation cost and also fulfill the security of supply requirements. Generally, in the short-term, the system operators plan the day-ahead and manage the electrical system in real-time.

Day-ahead energy market: every day the SOs schedules the next 24-hours of generators' production levels to fulfill the forecast demand throughout the day. To accomplish this task the SOs take into account the system estimated available generation in each hour of the day-ahead. The **Figure 1** below illustrates the supply and demand balance.

[1] Hydro power plants with reservoirs can storage energy but the electricity still needs to be produced in real-time.

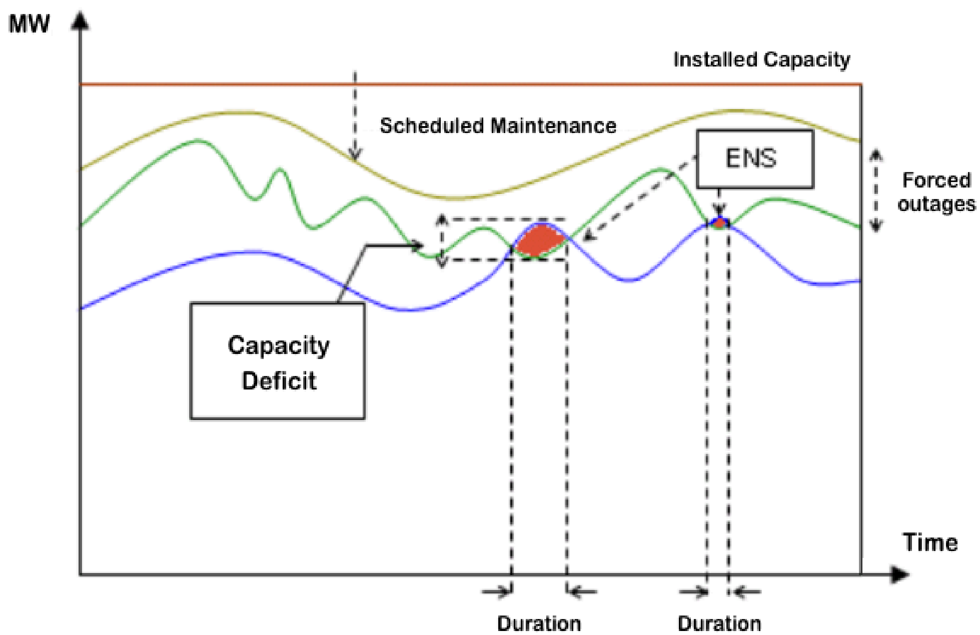


Figure 1 – Energy supply and demand balance.

The yellow line in **Figure 1** shows the estimated available generation regarding the scheduled maintenance of the generators [22]. The blue line exemplifies the forecast demand. The difference between the yellow line and the green line is the forced outages of the generators' units. A forced outage is an unforeseen failure of a grid element (contingency) and it occurs in real-time. The energy not supplied (*ENS*) takes place whenever the system has load shedding. Generally, SOs work with a $n-1$ security level, i.e., the system operator is capable in real time of management the system's operating to deal with at least one network's element failure.

Real-time energy market: in real-time SOs have to cope with some unexpected variation on demand and supply. Forced outages of network's equipment and non-expected variation in demand can oblige SOs to take corrective actions to ensure the security of supply. At this point the research presented in this PhD thesis takes advantage, because it enhances the capability of the system to deal with non-expected events (contingencies). The proposed model can be further explored to deal with wind power producing.

Ayala and Street shown in [16] that the green line in figure 1 can be improved to fulfill the demand only by changing the transmission assets operating mode. For these results it is assumed that the grid has a suitable level of intelligence (fast TS, communication and an adequate real-time system monitoring) to perform these corrective actions.

1.1.2
Historical evolutions of system's operation until TS

Granville and Pereira [4] described and summarized the historical evolution of system operation up to 1987. In 2014, the concept of system operation remains practically the same with the improvements of equipment technology. This subsection extends that historical evolution of system's operation until the post-contingency TS concept.

The first type of system operation is the pure economic dispatch. This type of dispatch determines the minimum cost scheduling of energy production among the available generators' units taking into account the system physical constraints regardless the security of supply.

A natural evolution of this type of dispatch is the concern of security of supply. This safety enhancement can be classified into two periods regarding contingencies' events:

(i) **Pre-contingency:** the system operates with a pre-specified fixed generation level, which is safe for every pre-defined (preventive mode) contingency. The SOs cannot re-dispatch the generators. The result is that the system security of supply is improved although the production costs are increased.

(ii) **Post-contingency:** the system operator can re-dispatch the generation levels after the occurrence of a pre-established set of contingencies. Each generator has a reserve level determined previously by the system operator. Each generator has a range, the reserve level, which is the security level that the generator can accommodate its energy production after a contingency. The system security is improved, and the energy production cost is reduced when compared to the pre-contingency dispatch.

These dispatches are very well-established and explored in [4]. The security improvements take into account only the generators' preventive and corrective actions. Specific changes in the grid topology can be performed in real time to improve the system reliability [5]-[6]. These changes follow some previously agreed rules (see [7]-[8]) or, in certain cases, are based on the SOs experience. Thereby, transmission assets are treated as fixed resources in the optimization models.

In 2010, Fisher et. al. [12] proposed another security and economic improvements on the unit commitment using transmission switching. The unit commitment can be seen as an economic dispatch with additional constraints that intends to represent in more details the units' physical constraints. In 2011, Hedman et al. [11] proposed a co-optimization of generation unit commitment and transmission switching. They treat the transmission assets as a decision variable but only on a preventive mode. Hedman et. al. [13] find an optimal schedule of system operation allowing the transmission assets to vary throughout the periods of the study, and the transmission switching once more is used only in a preventive mode. In 2014, Ayala and Street [16] proposed to model TS as a corrective action and showed numerical results.

For didactic purpose, the models that change the view of transmission assets can be classified into two periods regarding the contingencies' event:

(i) **Pre-contingency:** Hedman et. al. [13];

(ii) **Post-contingency:** Ayala and Street [16].

For the use of a post-contingency TS framework the electrical network should have the capable technology to perform fast communication among the grid elements, detection of contingencies, report and transmission switching.

1.2
Objective and Contributions

The first objective of this work is to propose a novel joint energy and reserve scheduling model that accounts for TS in both pre and post-contingency states. It is an application of a smart network in which the system operator incorporates economic and security benefits of fast TS actions into the schedule. To accomplish this objective, the model in [2] is extended to consider TS actions by means of binary variables for the pre and post-contingency states. Therefore, the proposed model co-optimizes the joint generation schedule, of energy and reserves, and the transmission topology. The goals of the proposed model are twofold: (i) to reduce electrical power outages by adjusting the network connectivity when a contingency occurs and (ii) ensure the deliverability of reserves through the network at the least cost.

The second objective of this work is to evolve an efficient solution in a modified Benders decomposition approach for the post-contingency TS problem.

The main contributions of this work are the following:

1. the development of a new MILP-based joint energy and reserve scheduling model that accounts for TS actions not only on the pre-contingency state, as done in [11][12][13], but also as a corrective action in all post-contingency states.
2. the demonstration of the model's ability of enhancing power system reliability, while reducing reserve levels and dispatch costs.
3. the application of an efficient algorithm to decompose the post-contingency transmission switching problem.

1.3
Published and under development works related to this thesis

This PhD thesis originated two articles cited below. The first one is already published and the second one is under development.

Article already published in "Electric Power Systems Research" journal: Ayala, Gustavo, and Alexandre Street. "**Energy and reserve scheduling with post contingency transmission switching**" Electric Power Systems Research 111 (2014): 133-140.

Abstract: for security reasons, transmission systems are designed with redundancies. Prior works have identified the benefits to system operations when the transmission assets have a pre-contingency schedule. The system operator chooses the optimal network topology regarding the contingencies, but the transmission system is not capable of performing corrective actions. This thesis highlights the economic and security benefits of an enhanced system operation with the advent of a smart grid technology by introducing a novel model. The proposed model is a joint energy and reserve scheduling one that incorporates the network capability to switch transmission lines as a corrective action to enhance the system capability to circumvent contingency events. The main goal is to reduce operating costs and electric power outages by adjusting the network connectivity when a contingency occurs. In such a framework, results show that with a limited number of corrective switches, the system operator is able to circumvent a wider range of contingencies while resulting in lower operational costs and reserve levels.

Working paper: Ayala, Gustavo, and Alexandre Street. "**Energy and reserves scheduling with post-contingency transmission switching: a modify Benders decomposition approach**".

Abstract: Prior works have identified the benefits to system operations when the transmission assets have a pre and post-contingency schedule. This thesis introduces an efficient algorithm based on Benders decomposition to solve the post-contingency transmission switching problem. The decomposition solves a two-stage stochastic program with mixed-integer program in both stages. Numerical results show computational time improvement compared to standard commercial mixed-integer solvers.

1.4 Organization

The PhD thesis is organized as follows. Chapter 2 introduces the transmission switching in scheduling models. It shows the difference between the preventive and corrective actions regarding the system elements: generator's units and transmission lines. It shows the tradeoff between cycles and security of supply of an electrical energy system. It also shows a technological limit of the transmission switching on the economic and security improvements of an electrical network operation.

Chapter 3 introduces the post-contingency transmission switching model with energy and reserve scheduling. It shows the economic and security benefits of using such technology. Numerical results are presented and encourage further research. This chapter constitutes a work already published.

Chapter 4 explains the decomposition method adopted in this PhD thesis to solve the post-contingency transmission switching problem. A very didactic approach is used, with a general example.

Chapter 5 summarizes the numerical results of the decomposition method explained in chapter 4 and compares with the numerical results obtained in chapter 3.

Chapter 6 discusses about further research involving the achievements of this PhD thesis and indicates some fields that the transmission switching effects can be investigated.

Chapter 7 contains the appendix with the main secondary technical definitions used in this thesis. The chapter also shows the first decomposition attempt of the author and his advisor to solve the post-contingency transmission switching in scheduling models problem.

2
Transmission Switching in Scheduling Models

Transmission switching at first sight can be seen as a paradox. Turning off recourse can be seen as a transition to a more unsafe state. Actually, switching off transmission lines could improve system operating costs and security. From the mathematical point of view, post-contingency transmission switching always enhances system operation. The feasible region of the post-contingency transmission switching mathematical problem contains the feasible region of the optimization models that treats transmission assets as fixed recourses. Therefore, system operators have more flexibility. They can switch on or off transmission lines according to the electrical network necessities.

2.1
Preventive and corrective actions

Two types of actions can be distinguished in relation to the time that the action has been taken to circumvent contingencies: preventive actions, which are made before a contingency occurs – i.e., in the pre-contingency state, and corrective actions, which are employed after the occurrence of a contingency – i.e., in the post-contingency state. Preventive actions aim to pre-schedule expensive but fast resources (e.g., units), which are not actually needed to meet nominal system requirements, to allow for the system to withstand a set of contingencies. In contrast, corrective actions generally make use of resources, preventively committed, to react against contingencies and, therefore, ensuring system stability. For instance, the most representative corrective action that is accounted for in scheduling models is generation re-dispatches [1], [4] and [15]. Most of the recent scheduling models, namely, market or unit commitment planning models that are used to schedule resources within a one-day to one-week time horizon, utilize corrective actions involving generation resources (see [23] and [24]).

In recent works, [11], [12] and [13], TS was proposed as a preventive action to enhance the system capability to circumvent contingencies using generation corrective re-dispatch. However, the utilization of preventive and corrective TS (*PC-TS*) actions in a generation scheduling model may change the pre-contingency operating point to reduced operative costs and mitigate load shedding under a larger set of contingencies. In this framework, from the mathematical perspective, the utilization of *PC-TS* expands the feasible region of the problem and possibly reduces the overall system cost.

2.2
Tradeoff between Kirchhoff's voltage and current laws

Assuming a linearized DC network flow model, Kirchhoff's current law equations provide the mathematical path between surplus and deficit generation buses within their capacity limits. Hence, the impact of the current law in the mathematical model with the introduction of a transmission asset to the system is the incorporation of a new flow decision variable. This variable creates a new path between two buses, enlarging the region of feasible solutions, allowing for a system cost reduction. However, the system power flows must also satisfy Kirchhoff's voltage law. The linearized voltage law imposes a linear relationship between the difference of the connected buses' phase angles and the line power flow. Thus, the impact of the voltage law in the mathematical model with the introduction of a transmission asset to the system is equivalent to the incorporation of a new constraint. Such constraint may reduce the region of feasible solutions and, consequently, may raise the system cost. As a consequence of considering both Kirchhoff's laws, the SO sees a transmission asset as a tradeoff between the new path provided by the current law and the angle constraint imposed by the voltage law.

In a tree-topology network, namely, a connected network with one line less than the number of buses (see [26] and [27] for further details), only the current law applies. The voltage law remains absent because in such topology there are no cycles. A cycle closes a circular dependence between a set of phase angles. This relationship is equivalent to demand orthogonality between the vectors of line flows and the reactances that belong to the cycle, i.e.,

$$\sum_{l \in Cycle} f_l x_l = 0. \quad (1)$$

In this case, each fundamental cycle (see [28] and [29] for details) adds a set of constraints that reduces exactly one degree of freedom of power flow variables. As a side effect, the voltage law can restrict the production of the most economical generators, resulting in an economically inefficient operating point. The system security can also be compromised, even if the system has sufficient capacity to connect generators to loads.

Notwithstanding the limiting characteristic of a cycle, they are used in transmission power systems to create redundancies and improve the system capability to withstand contingencies. Therefore, cycles can be managed by the SO according to their pros and cons, depending on the system state and the network capability to perform reactive switches.

2.3 Illustrative Example

For expository purposes, consider a power system with two connected areas, A and B. Suppose that generators in B are those that provide reserve resources. If there is a redundancy cycle limiting the generation injection into B and a contingency event, e.g., the loss of a tie line, occurs in A, one corrective option is to switch off one line that belongs to the cycle in B, allowing for addition generation injection into this area within committed reserves, to avoid a blackout into A. This simple example shows that the optimal response for a line contingency might be to switch off another line. In this context, the co-optimization of energy and reserves through a joint scheduling model is the appropriated framework to enhance reserves deliverability through the network and minimize the overall system cost.

2.4 Fundamental Cycles and Switching Benefit Limit

Hereinafter, we consider a connected network, i.e., with paths between all buses. If this is not the case, then one can divide the problem in two or more connected systems and then use the following analysis singly. According to (1), each cycle induces a system constraint that reduces by one the number of degrees of freedom of the line flow variables.

If the network is connected, the number of fundamental cycles, Φ, is precisely the number of lines that exceed the tree limit given by the number of buses minus one (see [28] and [29]). Hence, if the cardinality of the set of buses is $|\mathrm{N}|$ and the cardinality of the set of lines is $|\mathcal{L}|$, then the number of fundamental cycles is given by

$$\Phi = |\mathcal{L}| - |\mathrm{N}| + 1. \qquad (2)$$

According to (1), each fundamental cycle imposes a linear constraint for the line flows, reducing by one the degrees of freedom for those variables. Therefore, Φ is an upper bound for the number of switch-off actions with which the SO able to enhance the system operation. Indeed, if the SO switches off more than Φ lines, the system loses connectivity. Moreover, according to the rationale described in subsection 2.2, there is no benefit to switch off a line that does not belong to a fundamental cycle. Thus, the set of transmission lines candidate for TS, $\mathcal{L}^{TS}$, at any state is the union of the sets of lines that belong to any of the fundamental cycles. In the following section, these results will be used to derive valid inequalities to reduce the search space of the line-switch binary variables.

3
Energy and Reserve Scheduling with Post-Contingency Transmission Switching

This chapter presents a co-optimization model for the joint schedule of energy and reserve considering preventive and corrective transmission switching actions. The problem formulation is a mixed integer linear program (MILP). The developed model was inspired on the mathematical formulation presented in [11]. Moreover, the joint energy and reserve scheduling framework proposed in [2] is extended to consider TS in both the pre- and post-contingency states. Similar to [11], the model at hand also considers a preventive TS plan for the pre-contingency state, namely, state zero. This preventive TS plan is accounted for using a binary variable, $z_{l,0}$, that indicates if the transmission line l is on ($z_{l,0} = 1$) or off ($z_{l,0} = 0$) at state zero. The corrective TS is accounted for in the model, following the same logic presented for the preventive actions but using binary variables indexed by the post-contingency state c, namely, $z_{l,c}$.

In optimization, constraint violations are customarily considered by including a penalty function in the objective function (see [23] and [15]). Therefore, the $n-1$ security criterion is considered using soft constraints, i.e., a load shedding term is added into each post-contingency flow-balance equation, and the average load-shed is penalized in the objective function. This is equivalent to a norm-1 penalization for constraints violation. For a sufficiently large load-shed cost coefficient, C^{LS}, the penalized version of the problem is equivalent, in the sense of objective value, to the constrained version if there is at least one feasible scheduling (with zero load shed for every contingency state). This change to the constrained version enables us to assess the reliability benefit, in terms of load shed reduction, of the corrective TS actions in cases where the security criterion cannot be met. According to previous reported works ([1][2][12]), the scheduling model is presented in its single-period version in the following section.

This section presents the *PC-TS* model formulation as a mixed-integer program.

3.1 Nomenclature

3.1.1 Constants

C Total number of contingencies.

c_i^V Variable cost coefficient offered by generator i.

c_i^F The fixed cost coefficient offered by generator i.

c_i^{RD} Cost rate of generator i to provide down-spinning reserve.

c_i^{RU} Cost rate of generator i to provide up-spinning reserve.

d_n Real power load at bus *n*.

I Number of generators.

K Number of transmission lines.

L Number of switchable lines.

M_l Big *M* parameter by line l.

P_{base} Constant used to transform per-unit into real power units.

P_i^{max} Maximum real power output of generator i.

P_i^{min} Minimum real power output of generator i.

F_l^{max} Maximum power flow of transmission line l.

F_l^{min} Minimum power flow of transmission line l.

$\overline{R}_i^U$ Upper bound for the up-spinning reserve of generator i.

$\overline{R}_i^D$ Lower bound for the down-spinning reserve of generator i.

x_l Reactance of line *l*.

$|\mathcal{L}^{TS}|$ Number of switchable lines.

Δz^{max} Maximum number of lines that are switched different of the pre-contingency lines' states.

Γ_{ic} Contingency indicator of generator i, which values 1 if generator i is out in the post-contingency state c, being 0 otherwise.

Γ_{lc} Contingency indicator of line l, which values 1 if line l is out in the post-contingency state c, being 0 otherwise.

Φ Number of fundamental cycles in the network.

θ_n^{max} Maximum Phase angle at bus *n*.

θ_n^{min} Minimum Phase angle at bus *n*.

3.1.2
Variables

$f_{l,c}$ Power flow of line *l* in the post-contingency state *c*.

$f_{l,0}$ Power flow of line *l* in the pre-contingency state.

$p_{i,c}$ Real power output of generator i in the post-contingency state c.

$p_{i,0}$ Real power output of generator i in the pre-contingency state.

r_i^D Down-spinning reserve of generator i.

r_i^U Up-spinning reserve of generator i.

u_i Binary variable that is equal to 1 if generator i is on and is 0 otherwise.

$z_{l,c}$ Transmission switching variable of line l, which is 1 if line l is connected and is 0 if it is open.

$\delta_{n,c}$ Load shedding in bus *n* under post-contingency state *c*.

$\Delta z_{l,c}$ Corrective action variable of line *l* in the post-contingency state c, which values 0 if $z_{l,c} = z_{l,0}$ and 1 otherwise.

$\theta_{n,c}$ Phase angle at bus *n* in the post-contingency state *c*.

3.1.3
Functions

$C_i^P(.)$ Production-cost function offered by generator i.

3.1.4
Sets

$\mathcal{C}$ Set of contingency states (including the pre-contingency state $c = 0$).

$\mathcal{F}_n$ Set of lines with origin at bus n.

$fr(l)$ Origin bus of line l.

I_n Set of generators connected to bus n.

N Set of buses.

$to(l)$ Destination bus of line l.

$\mathcal{T}_n$ Set of lines with destination at bus n.

$\mathcal{L}$ Set of transmission lines.

$\mathcal{L}^{TS}$ Set of switchable transmission lines.

3.1.5
Indexes

c Index of contingency states.

i Index of generators.

l Index of transmission lines.

n Index of buses.

3.2
Problem Formulation

$$Minimize_{\left\{\substack{f_{l,c};p_{i,c};r_i^D;r_i^U;u_i;z_{l,c};\\ \delta_{n,c};\Delta z_{l,c};\theta_{n,c}}\right\}} \sum_{i\in I}\left(C_i^P\left(p_{i,0}\right) + C_i^U r_i^U + C_i^D r_i^D\right) + C^{LS}\sum_{c,n}\delta_{n,c} \tag{3}$$

subject to:

$$\sum_{\forall l \epsilon \mathcal{T}_n} f_{l,c} - \sum_{\forall l \epsilon \mathcal{F}_n} f_{l,c} + \sum_{\forall i \epsilon I_n} p_{i,c} + \delta_{n,c} = d_n;\ \forall\, n \in N, \forall\, c \in \mathcal{C} \tag{4}$$

$$f_{l,c} = \frac{\Gamma_{l,c}}{x_l}\left(\theta_{fr(l),c} - \theta_{to(l),c}\right);\ \forall l \in \mathcal{L}\backslash\mathcal{L}^{TS}, \forall c \in \mathcal{C} \tag{5}$$

$$\frac{1}{x_l}\left(\theta_{fr(l),c} - \theta_{to(l),c}\right) + M_l\left(1 - z_{l,c}\Gamma_{l,c}\right) \geq f_{l,c};\quad \forall l \in \mathcal{L}^{TS}, \forall c \in \mathcal{C} \tag{6}$$

$$\frac{1}{x_l}(\theta_{fr(l),c} - \theta_{to(l),c}) - M_l(1 - z_{l,c}\Gamma_{l,c}) \leq f_{l,c}; \quad \forall l \in \mathcal{L}^{TS}, \forall c \in \mathcal{C} \tag{7}$$

$$F_l^{min} \leq f_{l,c} \leq F_l^{max}; \ \forall l \in \mathcal{L} \backslash \mathcal{L}^{TS}, \forall c \in \mathcal{C} \tag{8}$$

$$z_{l,c}\Gamma_{l,c}F_l^{min} \leq f_{l,c} \leq F_l^{max}\Gamma_{l,c}z_{l,c}; \ \forall l \in \mathcal{L}^{TS}, \forall c \in \mathcal{C} \tag{9}$$

$$u_i\Gamma_{i,c}P_i^{min} \leq p_{i,c} \leq P_i^{max}\Gamma_{i,c}u_i; \ \forall\, i \in I, \forall\, c \in \mathcal{C} \tag{10}$$

$$p_{i,c} \leq p_{i,0} + r_i^U; \ \forall\, i \in I, \ \forall\, c \in \mathcal{C} \tag{11}$$

$$p_{i,0}\Gamma_{i,c} - r_i^D \leq p_{i,c}; \ \forall\, i \in I, \ \forall\, c \in \mathcal{C} \tag{12}$$

$$0 \leq r_i^U \leq \overline{R}_i^U; \ \forall\, i \in I \tag{13}$$

$$0 \leq r_i^D \leq \overline{R}_i^D; \ \forall\, i \in I \tag{14}$$

$$\theta_n^{min} \leq \theta_{n,c} \leq \theta_n^{max}; \ \forall\, n \in N, \forall\, c \in \mathcal{C} \tag{15}$$

$$\Delta z_{l,c} \geq z_{l,c} - z_{l,0}; \ \forall l \in \mathcal{L}^{TS}, \forall\, c \in \mathcal{C} \tag{16}$$

$$\Delta z_{l,c} \geq z_{l,0} - z_{l,c}; \ \forall l \in \mathcal{L}^{TS}, \forall\, c \in \mathcal{C} \tag{17}$$

$$\sum_{\forall l \in \mathcal{L}^{TS}} \Delta z_{l,c} \leq \Delta z^{max}; \ \forall\, c \in \mathcal{C} \tag{18}$$

$$\sum_{\forall l \in (\mathcal{F}_n \cup \mathcal{T}_n) \cap \mathcal{L}^{TS}} z_{l,c} \geq 1; \ \forall\, n \in N \tag{19}$$

$$\sum_{\forall l \in \mathcal{L}^{TS}} z_{l,c} \geq |\mathcal{L}^{TS}| - \Phi; \ \forall\, c \in \mathcal{C} \tag{20}$$

$$\Delta z_{l,c} \geq 0; \ \forall l \in \mathcal{L}^{TS}, \forall\, c \in \mathcal{C} \tag{21}$$

$$z_{l,c} \in \{0,1\}; \ \forall l \in \mathcal{L}^{TS}, \forall\, c \in \mathcal{C} \tag{22}$$

$$u_i \in \{0,1\}; \ \forall\, i \in I \tag{23}$$

The objective function (3) aims to minimize the operative (production) of energy and reserves at the pre-contingency state and the load shedding penalty. Constraint (4) ensures the flow balance for each bus (Kirchhoff's current law), while constraints (5)-(7) impose the Kirchhoff's voltage law – (6) and (7) account for the set of lines that are candidates for TS, and therefore, are presented in their disjunctive linear form. Constraints (8) and (9) impose the maximum and minimum flow for the transmission lines and (10) limits the output of the generating units. Constraints (11) and (12) create the link between the post-contingency generation and the pre-contingency schedule of energy and reserves. Constraints (13) and (14) impose ramp limits for the reserves, and the inequalities (15) constrain phase-angles.

Constraints (16)-(23) represent the switch constraints: (16)-(17) flag if line l has been switched on or off (assuming a value one or zero, respectively) in the post-contingency state c; (18) acts as a conservativeness coefficient that limits the number of corrective post-contingency switches allowed; (19) ensures network connectivity by imposing that at least one line reaches each bus; expression (20) constrains the number of switches to its upper bound, as expression (2). Expressions (21) and (22) define the domains of the switch variables. Lastly, expression (23) imposes the binary nature for the generation on-off scheduling variables. For the sake of simplicity, minimum up and down time constraints are disregarded.

It is important to say that the number of post-contingency switching variables in (23) can be significantly reduced for all post-contingency states in which the line under contingency belongs to, precisely, one cycle. Under such states, all other lines within that cycle are no more suitable for TS, except those that also belong to another cycle. Therefore, the switch variables associated with such lines can be constrained to value 1 and, hence, can be eliminated from the problem. To do that, one needs to run a cycle identification algorithm (see [28]-[29]) to preprocess the optimization model following the aforementioned rationale.

The constraints of ramp-up and ramp-down ensure that system has an appropriate operating reserve, which means that when a contingency occurs the system has to be prepared to respond increasing or decreasing its generation. This capability to vary the energy produced is the system operation reserve. The operation reserve is composed by spinning and non-spinning reserve.

3.3
Case Studies

This section shows numerical examples to highlight the benefits of using TS with both preventive and corrective actions. The case studies compare the operative performance of three scheduling models that are classified by the types of action they consider. The base-case (BC) model is a pure energy and reserve scheduling model described in [2], which does not consider TS. The BC model is a particular case of (3)-(23) and can be found by setting $z_{l,c} = 1$ for all lines and contingency states. Hence, the only corrective action considered in this model is generation re-dispatch within the scheduled reserves. The second model, namely, the P-TS model, builds on the BC model by allowing preventive TS. This is also a particular case of (3)-(23) and can be found by setting $z_{l,c} = z_{l,0}$ for all lines and contingency states. The third model, i.e., the PC-TS model, refers to the complete formulation (3)-(23) and, therefore, constitutes a relaxation for both the BC and P-TS models.

For didactic purposes, two case studies are performed. The first case study makes use of a four-bus system to introduce the results and their main properties.

In both case studies, the generators are assumed to offer a linear cost function of the form $C_i^P(p_{i,0}) = c_i^V p_{i,0} + c_i^F u_i$. The load shedding cost coefficient has been set to a value 10 times the highest variable cost. In addition, a TS penalty cost is considered in the objective function to reduce degenerated solutions with unnecessary switches. It is worth mentioning that the *PC-TS* problem structure allows for many solutions with the same energy and reserve cost but with different network configurations. Moreover, switching off a line might cause system instability and, from the engineering perspective, it is interesting to obtain solutions with the minimum corrective switching actions as possible. Therefore, to solve the model, an objective function penalization was considered to differentiate the cost of degenerated solutions, enhancing the algorithm performance, and also to choose the solution that provides the lowest distance, in terms of number of line switches, between the pre- and post-contingency states. To keep the optimal cost unaffected, the TS penalty coefficient is set to 1% of the lowest variable cost offered by the generators. Such cost is then disregarded after the algorithm terminates and not reported in the results.

3.4
Four-Bus System

In **Figure 2**, the system topology is depicted. Two fundamental cycles were created to highlight the capability of the model to manage the voltage laws in the presence of contingencies. The generators are two identical units, as described in **Table 1**. All lines are assumed to have the same reactance, 0.7 p.u., and the order of the security criterion is $n-1$.

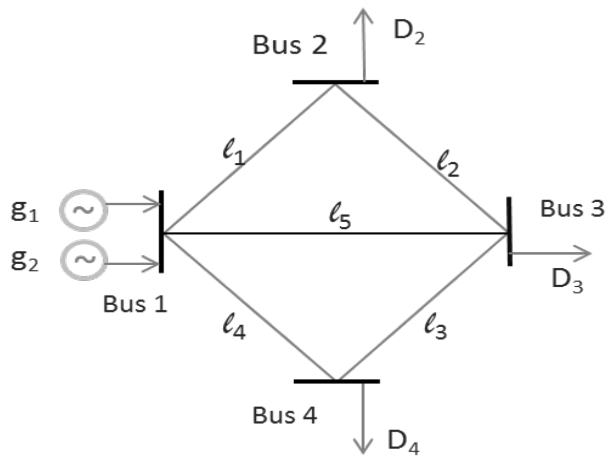


Figure 2- Four-bus system schematic.

Table 1 - Four-bus system: generator data

Unit	C_i^v $/MWh	C_i^U $/MW	C_i^D $/MW	P_i^{max} MW	P_i^{min} MW	$\overline{R}_i^U$ MW	$\overline{R}_i^D$ MW
{1,2}	100	30	20	140	0	70	70

In Table 2 and Table 3, the remaining four-bus system data are listed:

Table 2 - Four-bus system: circuit data

Line	From Bus	To Bus	x (pu)	F_l^{max} (MW)
l_1	1	2	0.7	100
l_2	2	3	0.7	60
l_3	3	4	0.7	40
l_4	1	4	0.7	40
l_5	1	3	0.7	100

Table 3 - Four-bus: load data

Bus	Load (MW)
1	0
2	40
3	60
4	32

By analyzing the system data, the limit of the number of switches off is determined by the following expression (2): $\Phi = 5 - 4 + 1 = 2$. That is, the maximum operational improvement with TS can be achieved with at most 2 switches. In fact, the results demonstrate that only one switching off is needed.

Figure 3 presents a comparison, in terms of load shedding, for the optimal schedules obtained with the three aforementioned models: *BC*, *P-TS*, and *PC-TS*. The results are shown for each post-contingency state, identified by the line under contingency. Note that, in this case, the schedules provided by the three models are capable of withstanding all of the generation and line #3 contingencies. Therefore, such contingency-states are omitted in Figure below.

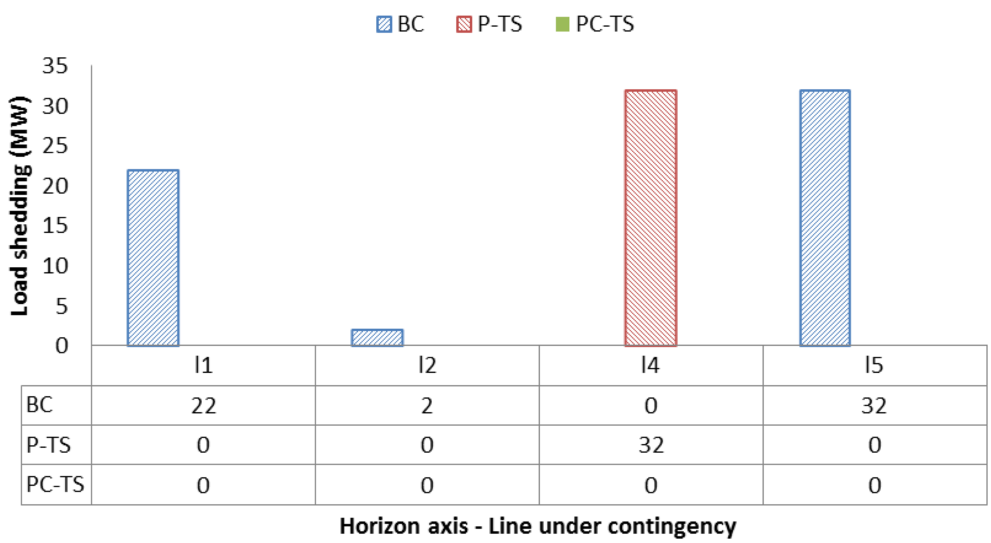


Figure 3 - Load shedding comparison for the four-bus system.

The schedule determined by the *BC* model does not meet the $n - 1$ criterion and sheds load in three of the seven post-contingency states. The cycle formed by lines 3-4-5 limits the pre-contingency schedule and, therefore, the deliverability of reserves is not ensured under the post-contingency states $\{l_1, l_2, l_5\}$ (states identified by the line under contingency). For comparison purposes, the average load shedding observed using the *BC* model (8 MW) can be regarded as an infeasibility measure for the schedule obtained by the *BC* model. If the SO is allowed to perform preventive TS, the system has more flexibility, and the average load shed decreases from 8 MW to 4.57 MW. In this case, the

P-TS model chooses to switch off line l_3, eliminating the cycle formed by lines 3-4-5. However, when line 4 fails, D_3 is islanded and then shed. Lastly, the *PC-TS* model is capable of circumventing the line 4 fault by switching on line 3 and reducing to zero the load shed penalty at the objective function. Moreover, because no load is shed under any post-contingency state, the down reserves are reduced to zero. As a consequence, the operative cost is also reduced relative to the costs found by the other models. The table below depicts the results obtained using the three models.

Table 4 - Four-bus system: main results

		Model		
Generator	Service type	*BC*	*P-TS*	*PC-TS*
1	Energy (MW)	70	70	70
	Up reserve (MW)	62	62	62
	Down Reserve (MW)	0	0	0
2	Energy (MW)	62	62	62
	Up reserve (MW)	70	70	70
	Down reserve (MW)	32	32	0
System	Operative cost ($)	13,200.0	13,200.0	13,200.0
	Avg. load shed cost ($)	8,000.0	4,571.4	0.0
	Total system cost ($)	25,954.0	22,516.1	17,308.9

3.5
Six-Bus System

The six-bus system used in this case study is based on [40]. The Figure 1 shows the six-bus system diagram. The system is composed by 4 generators, 3 loads and 12 transmission lines.

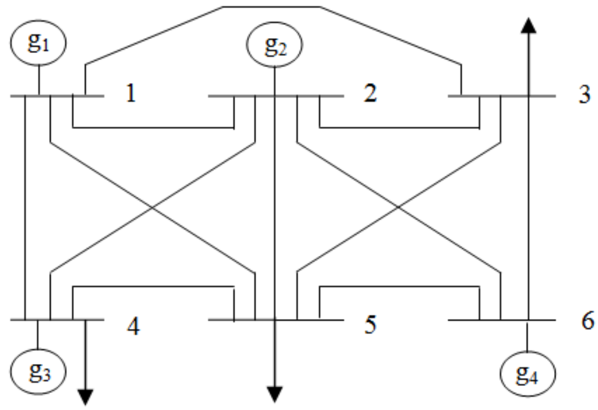


Figure 1 – Six bus system.

The characteristics of transmission lines such as reactance, capacity (flow) limit are shown in the table below:

Table 5 - Six-bus system: circuit data.

Line No.	From Bus	To Bus	X (pu)	Flow Limit (MW)
1	1	2	0,7	30
2	1	4	0,7	100
3	2	3	0,7	100
4	2	4	0,7	70
5	3	6	0,7	70
6	4	5	0,7	75
7	5	6	0,7	80
8	2	6	0,7	85
9	1	5	0,7	90
10	3	5	0,7	95
11	2	5	0,7	100
12	1	3	0,7	105

The characteristics of generation units are shown in the table below:

Table 6 - Six-bus system: generator data

Unit No.	1	2	3	4
Bus No.	1	2	4	6
Cost production ($/MWh)	50	100	300	200
Min. Capacity (MW)	100	0	10	10
Max. Capacity (MW)	100	75	80	80
Startup Cost ($)	50	40	10	10
Shutdown Cost ($)	10	20	20	20
Min. Up Time (h)	1	1	1	1
Min. Down Time (h)	1	1	1	1
Ramp Up Rate (MWh/h)	30	65	70	65
Ramp Down Rate (MWh/h)	20	35	22	30

The demand has the following characteristics showed in the table below:

Table 7 - Six-bus system: demand data.

Bus No.	3	4	5
Percentage (%)	20	40	40
Demand (MW)	35	70	70

The total demand is 175 MW distributed by 20% of load in bus 3, 40% in bus 4 and the remaining in bus 5. The table below summarizes the results found with each model: (i) *BC*; (ii) *P-TS* and (iii) *PC-TS*. The table below summarizes the results with $n-2$ security criterion. The *BC* and *P-TS* model has a maximum load shedding of 57.4% regarding the total demand, while the *PC-TS* model can avoid load shedding in all contingencies.

Table 8 - Six-bus system: maximum load shedding results regarding $n-2$ criterion.

$n-2$ criterion	*BC*	*P-TS*	*PC-TS*
Max-load Shedding (%)	57.4	57.4	0
Max-load Shedding (MW)	100	100	0

The PC-TS model enhances the network security allowing protect the system against contingencies up to two elements.

3.6
Future Research related to Energy and Reserve Scheduling with Post-Contingency Transmission Switching

The model presented in this chapter is unsuitable for a classical Benders decomposition approach because the second-stage problem is not convex. In [17], [20], [21], [36] and [37], decomposition methodologies are developed to solve problems in which the second-stage is a MILP. The design of decomposition methods to solve the *PC-TS* within a tight optimality gap for large-scale power systems is part of this PhD thesis research.

4
Decomposition Methods

The latter part of this doctorate thesis focuses on the improvement of the *PC-TS* model computational efficiency. Benders decomposition with a non-classical framework is the primary purpose of this chapter. For edification purposes, section 4.1 describes a historical evolution of Benders decomposition. Section 4.2 describes the algorithm established in [21] entitled: ancestral Benders cut. Section 4.3 introduces the application of such decomposition method to post-contingency transmission switching problem.

4.1
Historical Evolution of Benders Decomposition

In 1962, J. F. Benders [18] proposed a partitioning procedure for solving mixed-integer variables programming problems. It was, by all accounts, a revolutionary work. Several applications have emerged using Benders decomposition. From a mathematical perspective, to guarantee the Benders decomposition convergence and finiteness, some conditions must be satisfied. One of the requirements is that the recourse function must be convex. Thus, this condition is not always satisfied when integer variables are considered in the second-stage problem [17].

To circumvent this technical challenge, in 2002, Sheralli and Fraticelli [17] proposed a modification of Benders decomposition to tackle integer's sub-problems. The method relies on the use of a cutting plane algorithm to solve 0-1 mixed-integer second-stage problems.

Between the works of Benders in 1962 [18] and Sherali & Fraticelli in 2002 [17], developments with cutting plane algorithms were being performed with different aims. In 1980, Jeroslow [44] developed a cutting plane algorithm based on game theory for a special class of optimization problems, entitled: Disjunctive Programing. In 1993, Balas et. al [45] developed another cutting plane algorithm to solve 0-1 mixed-integer problems. The key role to prove convergence and finiteness of those cutting plane algorithms developed in [44]-[45] is that an extreme point of the feasible set convex-hull is also an extreme point of the partial convex hull (approximation of the convex-hull). This fact is ensured with the facial property[1]. If a problem does not have the facial property then the cutting plane algorithm may not converge.

The cutting plane algorithm used in Sherali and Fraticelli [17] is based on the work developed by Balas et. al [45]. Two conditions are necessary in Sherali and Fraticelli's method [17]: (i) the second-stage problems must have only 0-1 mixed-integer variables; and (ii) the first-stage variables must be binary. The first condition is clearly a requirement of [45]. Therefore, Benders decomposition was extended but still in a limited framework.

In 2006, Sherali et. al [37] proposed a Benders decomposition-based on a Branch and Bound framework. The algorithm was entitled as *DBAB* (decomposition-based Branch and Bound). A Branch and Bound algorithm was introduced in the first-stage problem to guarantee the facial property of the second-stage and, as a consequence, the algorithm developed by Sherali and Fraticelli [17] could be applied.

In 2013, Suvrajeet and Yunwei [21] proposed a novel Benders decomposition that extends this framework to accomplish general mixed-integer problems in both stages. They used a Branch and Bound (B&B) idea for the first-stage problem proposed in [37]. The nodes of the B&B tree inherited the Benders cuts from their ancestry. The decomposition methods applied to the PC-TS problem is based on [21] and [37].

4.1.1
Standard Benders decomposition

The standard approach of a **Benders decomposition algorithm** is the following.

[1] A disjunctive set of type:

$F = \{x \in P: \bigcup_{h \in Q} (A^h x \geq a_0^h, x \geq 0)\}$, where P is a polyhedron, and Q is a set of indexes, has the facial property if every constraint $A^h x \geq a_0^h$ is a face of the polyhedron P. (vide appendix for further details).

Benders decomposition algorithm

Step 0: Decompose the original problem into first and second stage problems.

Step 1: Solve the first-stage problem. Let $\bar{x}$ be the solution found.

Step 2: Solve the second-stage problem using $\bar{x}$ as a fixed parameter.

Step 3: Use the dual variables of the second-stage to generate Benders cut.

Step 4: Add the Benders cut to the first-stage problem and return to the **step 1**.

4.1.2
Finiteness and converge of the standard method – 2nd stage is a linear program

To ensure finiteness and convergence the second-stage objective function has to be convex in respect to the first-stage variables. As long as the second-stage is a linear problem, the convexity condition is satisfied and Benders decomposition converges in a finite number of steps. In some specific cases as shown in [43], even with binary variables in the second stage, the problem continues to fulfill the convexity requirement. In general, technical difficulties emerge when the second-stage problem has integer or binary variables. The convexity requisite may be violated and finiteness and convergence are not guaranteed. The next subsection discusses an algorithm that circumvents this challenge.

4.2
Ancestral Benders cut (2013)

In 2013, Suvrajeet and Yunwei [21] proposed a novel Benders decomposition to solve a general mixed integer program in both stages. From the operations research point of view, the subject is quite interesting because it gathers almost all the important mathematical achievements over the last 60 years such as Benders decomposition, cutting plane algorithms, and Branch and Bound method.

4.2.1
Framework

This section describes the optimization framework addressed in [21] and used in this chapter. The problems are established in the minimization form. The first-stage problem is a general mixed-integer program established as:

$$\min_{x \in X \cap H_1} c^T x + \sum_{s=1}^{S} p_s f_s(x) \quad (1)$$

$$X = \{x \in \mathbb{R}^{n_1} | Ax \leq b, x_i \text{ is integer}, \forall i \in I \subset \{1, \dots, n_1\}\} \quad (2)$$

$$H_1 = \{x \in \mathbb{R}^{n_1} | l_1 \leq x \leq u_1\} \quad (3)$$

Constants

A Matrix $A \in \mathbb{R}^{m_1 \times n_1}$
b Vector $b \in \mathbb{R}^{m_1}$
c Vector $c \in \mathbb{R}^{n_1}$
l_1 Vector $l_1 \in \mathbb{R}^{n_1}$
m_1 Number of constraints represent by matrix A
n_1 Dimension of first-stage solution space
p_s Probability of scenario *s*
S Number of scenarios
u_1 Vector $u_1 \in \mathbb{R}^{n_1}$

Function

$f_s(.)$ Second-stage objective function of scenario *s*

Variable

x First-stage variable

Indexes

i Index of first-stage variable coordinates
s Index of scenarios

Sets

I Set of indexes of first-stage integer variables
H_1 Hyper-rectangle of first-stage variables
X First-stage set
S Scenarios set

The second-stage problem is once again a general mixed-integer problem defined:

$$f_s(x) = \min_{y \in Y \cap H_2} g_s^T y \tag{4}$$

$$subject\ to\ \ W_s y \geq r_s - T_s x \tag{5}$$

$$Y = \{y\ \in\ \mathbb{R}^{n_2} |\ y_j \text{ is integer}, \forall j \in J \subset \{1, \dots, n_2\}\} \tag{6}$$

$$H_2 = \{y\ \in\ \mathbb{R}^{n_2} | l_2 \leq y \leq u_2\} \tag{7}$$

Constants

g_s^T Vector $g_s^T \in \mathbb{R}^{n_2}$
l_2 Vector $l_2 \in \mathbb{R}^{n_2}$
m_2 Number of constraints represent by matrix W_s
n_2 Dimension of second-stage solution space
r_s Vector $r_s \in \mathbb{R}^{m_2}$
T_s Vector $T_s \in \mathbb{R}^{m_2 \times n_1}$
W_s Matrix $W_s \in \mathbb{R}^{m_2 \times n_2}$
u_2 Vector $u_2 \in \mathbb{R}^{n_2}$

Variable

y Second-stage variable $y \in \mathbb{R}^{n_2}$

Indexes

j Index of second-stage variable coordinates
s Index of scenarios

Sets

J Set of indexes of second-stage integer variables
H_2 Hyper-rectangle of second-stage variables
Y Second-stage set

4.2.2
Algorithm general outline

The main objective of the algorithm is to solve the two-stage problem (1)-(7) through a Branch and Bound procedure in which the problem is decomposed into two stages. The Benders decomposition approach is employed to generate lower and upper bounds. The algorithm works only with linear problems, hence it uses only the linear relaxation of both stage problems. In this context, henceforward, the first and second stages are the linear relaxation of the original problems (1)-(3) and (4)-(7).

Following the traditional Benders decomposition, the first-stage receives Benders cuts that approximate the value of the second-stage objective function. To do this, a convex second-stage problem is needed to ensure the recourse function convexity with respect to the first-stage variables. Clearly, this is not the case of the proposed framework. However, based on the findings of [21], in the next subsection it will be apparent how to circumvent this technical difficulty. Each branch step produces a (x, y) set and thus the algorithm iterative approximates the convex hull of these sets.

The algorithm starts performing a Branch and Bound procedure in the first-stage solution space with a lower-liner approximation of the recourse function. The value of the first-stage objective function constitutes a lower bound for each node. The criterion for the node selected is the best node first, i.e., the node with least lower bound. Under this assumption, the lower bound of the selected node provides a global lower bound for the algorithm. If the selected node has a first-stage mixed-integer solution, then the second-stage procedure is started, otherwise the branching procedure is applied following the B&B procedure.

The first step of the second-stage procedure is to solve the corresponding linear problem with the first-stage variable fixed. If the solution is mixed integer, then a Benders cut can be constructed. Using the mixed-integer solution founded in both stages, a global upper bound can be generated. If the solution is not mixed integer, then a cutting plane procedure is used. Following the B&B procedure presented in [21], a cutting plane method is employed to provide the current node with a narrower relaxation valid for the current partition. Within this framework, Benders cuts are generated to approximate the relaxed version in each B&B node. Therefore, such cuts are only valid for the current node and their children. To generate such a cut, a branching procedure in the y-variable is performed with the intent to produce a partition of the (x, y) solution space that does not contain the fractional solution (x^*, y^*), that was previously found by the relaxed second-stage problem. Thus, a cut generation procedure is designed to find a valid inequality for the disjunctive sets generated by the partition so that the violation of the fractional solution is maximized. These cuts are generated and added to the second-stage problem until a mixed-integer solution is found. After that, a new Benders cut can be initiated for the first-stage relaxation problem.

The algorithm stops when the difference between the global upper bound and the global lower bound achieves an optimality tolerance criterion.

4.2.3
Decomposition method - example

This section explains the decomposition method established in [21] with the aid of graphs. The objective is to describe the algorithm step-by-step using charts that will help the general comprehension of the decomposition method. Consider the following integer problem:

$$z^* = \min_{(x,y)} -x - 2y \tag{8}$$

$$y \leq 4.5 - x \tag{9}$$

$$0 \leq x \leq 4.5 \tag{10}$$

$$0 \leq y \leq 3 \tag{11}$$

$$(x, y) \in \mathbb{Z}_+^2 \tag{12}$$

The feasible region is illustrated below.

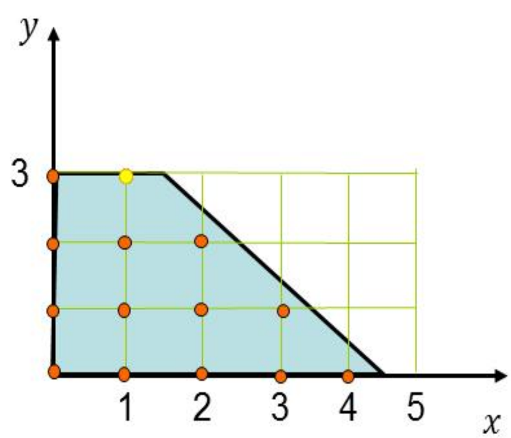


Figure 4 - Feasible region.

The yellow point indicates the optimal solution $(x^* = 1;\ y^* = 3; z^* = -7)$ of the problem (8)-(12). Next, it will be shown step-by-step, how the algorithm converges and achieves the optimal solution. The two-stage decomposition can be stated as:

First-stage:

$$z^* = \min_x -x + \eta \tag{13}$$

$$0 \leq x \leq 4.5 \tag{14}$$

$$\eta \geq -M \tag{15}$$

The original first-stage problem is a general integer problem. The decomposition method relaxed the integer constraints in both stages. Therefore, the second-stage can be stated as:

$$\eta\,(x^*) = \min_y -2y \tag{16}$$

$$y \leq 4.5 - x^* \tag{17}$$

$$0 \leq y \leq 3 \tag{18}$$

The algorithm starts solving the first-stage problem (13)-(15). The solution is $x^{k=1} = 4.5$; $\eta^{k=1} = -M$. The objective value ($z^{k=1} = -M - 4.5$) constitutes a **global lower bound (GLB)** for problem (8)-(12). The **global upper bound (GUB)** can be initiated as $+\infty$. The figure below shows the currents GUB and GLB of problem (8)-(12).

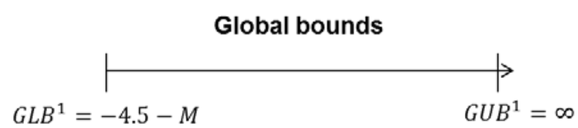


Figure 5 - Global upper and lower bounds of the original MIP.

As the current first-stage solution $x^* = 4.5$ is fractional, the first-stage solution region is partitioned into two nodes: $x \leq 4$ and $x \geq 5$, as shown in figure below.

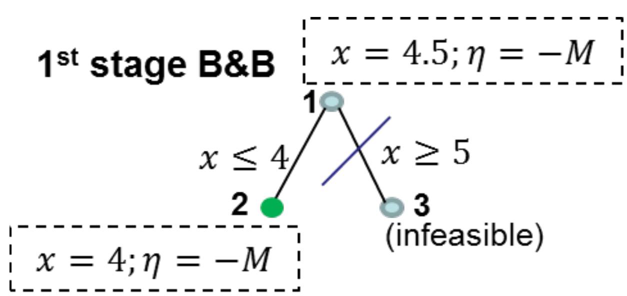


Figure 6 - First-stage Branch and Bound.

The solution of the left node #2 is $x^* = 4$; $\eta^* = -M$ and the right node #3 has no feasible solution, thus the right node can be pruned by infeasibility. The optimal objective function of problem (13)-(15) in each node constitutes a lower bound for respective node, provided the problem (13)-(15) is a linear relaxation of the original problem. The left node has a lower bound of $LB_2 = -4 - M$, and the right node has a lower bound of $LB_3 = \infty$. Therefore, the global lower bound GLB is updated to LB_{2}.

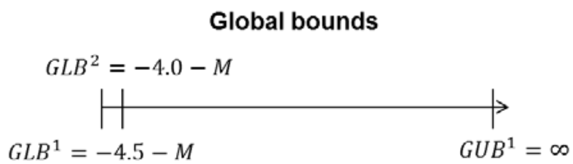


Figure 7 - Current global bounds of the original MIP.

As node #3 is infeasible, the only node active is #2. If there is more than one node active then the criterion adopted to select a node is the best first (the node with the least lower bound). Therefore, the node #2 is selected. As the solution of the left node is integer $x^* = 4$, the second-stage procedure is invoked. In practical terms, the problem (16)-(18) is solved with $x^* = 4$ fixed as the following structure below:

$$\eta\,(x^*) = \min_y -2y \tag{19}$$

$$y \leq 4.5 - (x^* = 4) \tag{20}$$

$$0 \leq y \leq 3 \tag{21}$$

The resultant solution is $x^* = 4; y^* = 0.5$. This solution does not satisfy the integer conditions which may be seen by the green point in the figure below:

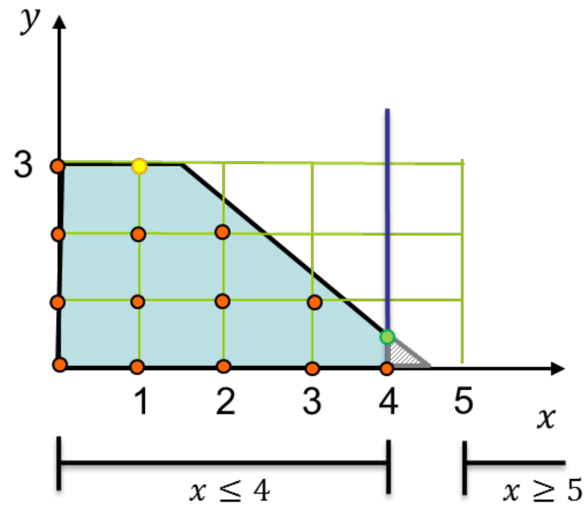


Figure 8 - Second-stage solution regarding the node selected $x \leq 4$ in the first-stage B&B procedure.

Therefore, a Branch and Bound procedure with respect to the second-stage variable is necessary. The *y*-variable partition is described below.

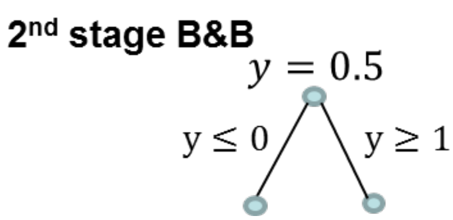


Figure 9 - Second-stage B&B.

The right node problem is infeasible regarding $x^* = 4$. In the left node, an optimal solution is achieved. The Branch and Bound of the second-stage is only used to construct a special partition. The B&B partition constructed contains all feasible *x*-*y*-solutions regarding the first-stage partition and also excludes the fractional solution $x^* = 4; y^* = 0.5$. From the mathematical point of view, it is not necessary to solve completely the B&B. For example, the algorithm can stops after *d* steps of B&B procedure.

The resultant partition $\{y \leq 0\} \cup \{y \geq 1\}$ is represented on the (x, y) feasible region in the figure below:

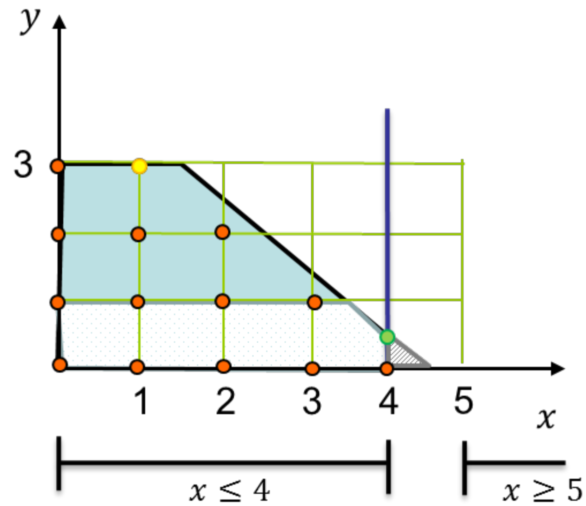


Figure 10 - *(x, y)* partition regarding second-stage B&B partition.

The partition on (x, y) solution space generated by the B&B of the second-stage results into the two following sets:

(i) $R1 = \{(x, y): 0 \leq x \leq 4, y \leq 4.5 - x, 0 \leq y \leq 3, \boldsymbol{y} \leq \boldsymbol{0}\}$; and

(ii) $R2 = \{(x, y): 0 \leq x \leq 4, y \leq 4.5 - x, 0 \leq y \leq 3, \boldsymbol{y} \geq \boldsymbol{1}\}$

The next step is to approximate the convex-hull of the regions *R1* and *R2* through convexification cuts. The figure below shows the cut that has to be added to construct the entire convex-hull of *R1* and *R2*. It is not necessary to construct the entire convex-hull, it is only necessary to produce a valid inequality that cuts off the current fractional solution $x^* = 4; y^* = 0.5$ and approximates the entire convex-hull of *R1* and *R2*. The development of such valid inequality will be shown hereafter.

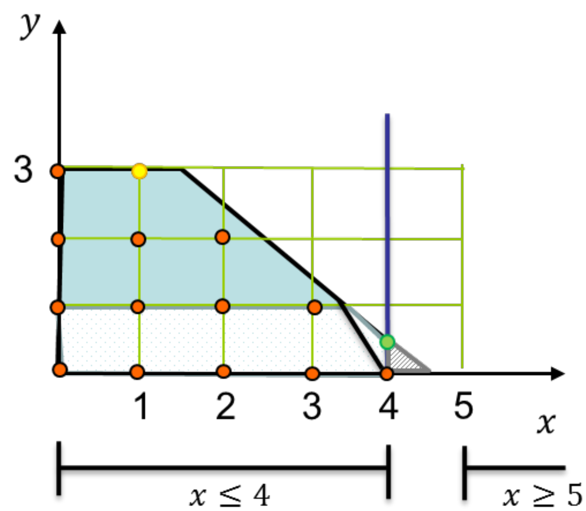


Figure 11 - Cut that has to be added to achieve the convex-hull of *R1* and *R2* regions.

As can be seen above there is only one face of the convex-hull of *R1* and *R2* that cuts off the current fractional solution. For didactic purposes the entire convex-hull of *R1* and *R2* is depicted in the figure below.

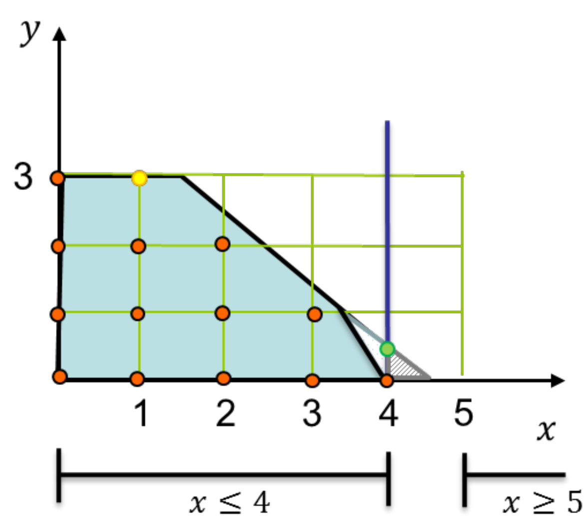


Figure 12 - Convex-hull of regions R1 and R2.

In the cutting plane procedure, the goal is to produce a convexification cut of the type $\pi_1^T x + \pi_2^T y \geq \pi_0$ that approximates the convex-hull of *R1* and *R2* and cuts off the current fractional solution. Following this ideas the cutting plane procedure has to accomplish two conditions:

(i) It has to cut off the current fractional solution $x^* = 4; y^* = 0.5$.

(ii) It has to be valid for all *(x, y)* in *R1* and *R2*.

To accomplish (i) the cut represented by (π_0, π_1, π_2) has to satisfy the following condition $\pi_1^T x^* + \pi_2^T y^* < \pi_0$. As can be seen in

Figure **12**, by construction, there is at least one face of the convex-hull of *R1* and *R2* that cuts off the current fractional solution. One criterion to select one cut among the available convexification cuts is to choose the cut that maximizes the violation $\pi_0 - (\pi_1 x^* + \pi_2 y^*)$.

The conditions imposed by (ii) can be seen as a typical problem of robust optimization [41].The convexification cut is valid for *R1* if and only if $\min_{\pi_1,\pi_2} \pi_1 x + \pi_2 y \geq \pi_0, \forall (x, y) \in R1$. The same occur with the *R2* set. Consequently, the cutting plane procedure can be formulated by a linear program (LP). The resultant problem is entitled as cutting generating linear program (CGLP) and it is defined, in this example, as:

$$\max_{\pi_0,\pi_1,\pi_2} \pi_0 - (\pi_1 x^* + \pi_2 y^*) \tag{22}$$

$$\pi_1 = (-1) \times \lambda_{2,R1} + \mu_{1,R1} - \nu_{1,R1} \tag{23}$$

$$\pi_1 = (-1) \times \lambda_{2,R2} + \mu_{1,R2} - \nu_{1,R2} \tag{24}$$

$$\pi_2 = (-1) \times \lambda_{2,R1} + \mu_{2,R1} - \nu_{2,R1} \tag{25}$$

$$\pi_2 = (-1) \times \lambda_{2,R2} + \mu_{2,R2} - \nu_{2,R2} \tag{26}$$

$$(-4.5)\lambda_{2,R1} + 0 \times \mu_{1,R1} - 4 \times \nu_{1,R1} + 0 \times \mu_{2,R1} - 0 \times \nu_{2,R1} \geq \pi_0 \tag{27}$$

$$(-4.5)\lambda_{2,R2} + 0 \times \mu_{1,R2} - 4 \times \nu_{1,R2} + 1 \times \mu_{2,R2} - 3 \times \nu_{2,R2} \geq \pi_0 \tag{28}$$

$$\pi_0 = -1 \tag{29}$$

$$(\pi_0, \pi_1, \pi_2) \in \mathbb{R}^3 \; ; \; (\lambda_{2,R1}, \mu_{1,R1}, \nu_{1,R1}, \lambda_{2,R2}, \mu_{2,R2}, \nu_{2,R2}) \geq 0 \tag{30}$$

The objective function maximizes the cut violation as explained above. The constraints (23)-(28) guarantee that the valid inequality represented by (π_0, π_1, π_2) does not cut any point in regions *R1* and *R2*. The problem (22)-(30) without the constraint (29) has multiple solutions. Therefore, the constraint (29) is a normalization of (π_0, π_1, π_2). The entire development of the CGLP is in the appendix.

In this case, the CGLP solution is $\pi_0^* = -1, \pi_1^* = -0.25, \pi_2^* = -0.125$, thus the resultant cut is $y \leq 8 - 2x$ (multiplying all π solutions by 8) and it is indeed the last absent face of the convex-hull and also cuts the current fractional solution $x^* = 4; y^* = 0.5$ (see figure below). By construction this convexification cut is only valid for all $\{x \leq 4\}$. Therefore, for the future child nodes of node #2, this convexification cut is still valid. On another hand, there is no guarantee that the convexification cut $y \leq 8 - 2x$ is valid for others nodes.

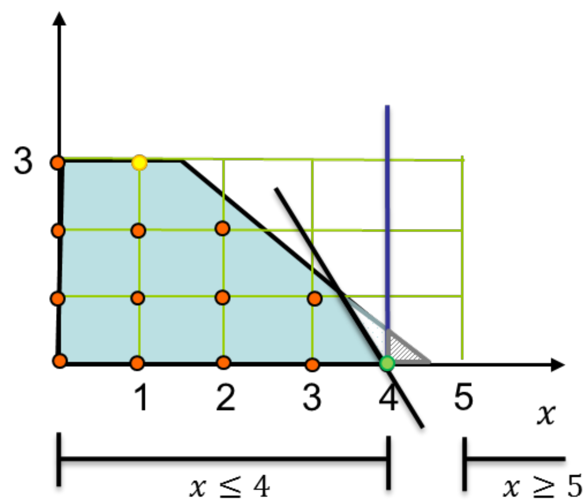


Figure 13 - Convexification cut.

The cut $y \leq 8 - 2x$ is added to the second-stage problem resulting into the following problem:

$$\eta\,(x^*) = \min_y -2y \quad (31)$$

$$y \leq 4.5 - x^* \quad (32)$$

$$0 \leq y \leq 3 \quad (33)$$

$$y \leq 8 - 2x^* \quad (34)$$

The solution of problem (31)-(34) is $x^* = 4, y^* = 0$. In

Figure **13** above the green point represents such solution. The current solution satisfies the integrality constraints. Therefore, a Benders cut can be constructed using the dual variables of problem (31)-(34) and the **global upper bound** can be updated. The new GUB is the original MIP objective function evaluated at $x^* = 4, y^* = 0$, thus $GUB = -(x = 4) - 2(y = 0) = -4$. The figure below shows the global bounds progress.

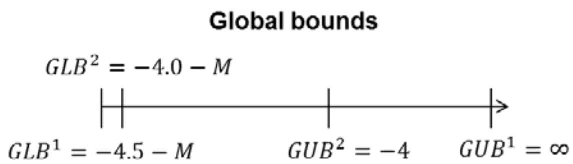


Figure 14 - Global bounds' progress of the original MIP.

The resulting Benders cut $\eta \geq 4x - 16$ is added to the first-stage problem (13)-(15) relative to the node $\{x \leq 4\}$ and such problem is re-solved.

$$z^* = \min_x -x + \eta \quad (35)$$

$$0 \leq x \leq 4.5 \quad (36)$$

$$\eta \geq -M \quad (37)$$

$$\eta \geq 4x - 16 \quad (38)$$

$$x \leq 4 \quad (39)$$

By construction the Benders cut (38) is only valid for node $\{x \leq 4\}$, it is not guarantee that is valid for others first-stages nodes. The figure below shows the Benders cut $\eta \geq 4x - 16$ in the corresponding valid region $\{x \leq 4\}$. The second-stage objective function $\eta(x)$ is approximated.

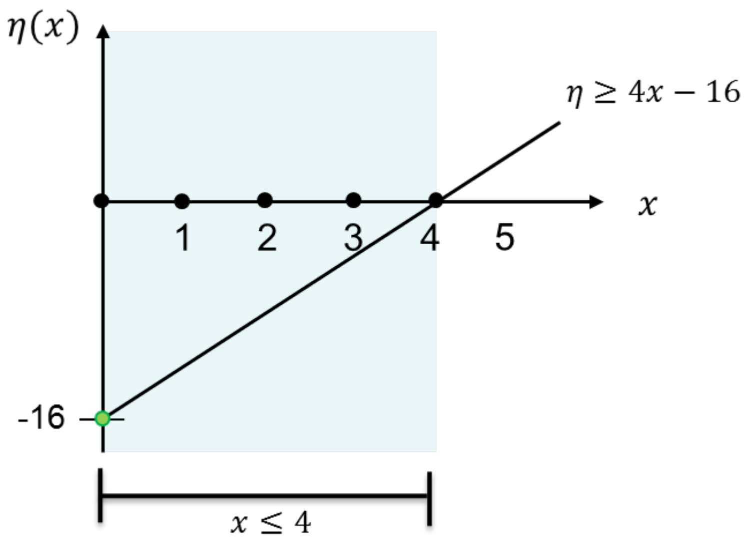


Figure 15 - second-stage objective function approximation by Benders cuts regarding node $\{x \leq 4\}$.

As can be seen in Figure **15**, the solution of problem (35)-(38) is $x^* = 0, \eta^* = -16$. At this point of the algorithm, the lower bound of first-stage node #2 ($x \leq 4$) can be updated to $LB_2 = -16$. In the same manner, the global lower bound is updated to $GLB^3 = -16$. The optimal solution gap is $GAP = [(-4) - (-16)]/16 = 75\%$.

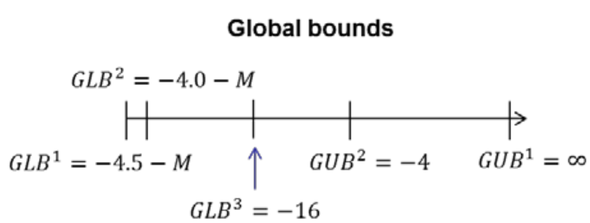


Figure 16 - Global bound progress.

As the first-stage solution $x^* = 0$ is integer, it is not necessary to perform the first-stage B&B thus the second-stage procedure is called directly. The problem (31)-(34) is solved again with $x^* = 0$ fixed. The corresponding solution is $x^* = 0, \boldsymbol{y}^* = \mathbf{3}$, as the second-stage solution is integer, it is not necessary to partition the (x, y) solution space regarding the y-variable. The objective function of the original MIP at $x^* = 0, \boldsymbol{y}^* = \mathbf{3}$ is $z^* = -6$, then the global upper bound is updated, $GUB_3 = -6$.

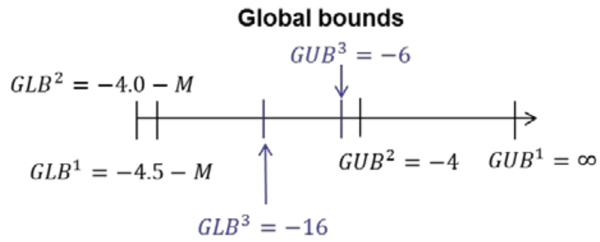


Figure 17 - Global bounds progress.

The resultant gap is $GAP = [(-6) - (-16)]/16 = 62.5\%$. The Benders cut can be constructed directly. In problem (31)-(34) the dual variable of constraint $y \leq 3$ is -2 and others are zero, thereby the resulting Benders cut for the first-stage problem of node $\{x \leq 4\}$ is $\eta \geq -6$ as shown in figure below.

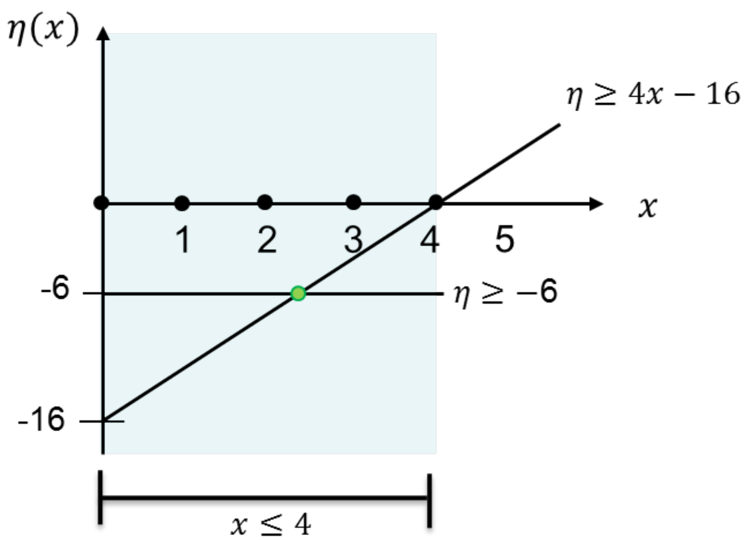


Figure 18 - Second-stage objective function approximation by Benders cuts regarding node $\{x \leq 4\}$.

The first-stage problem is solved again considering the new Benders cut.

$$z^* = \min_x -x + \eta \tag{40}$$

$$0 \leq x \leq 4.5 \tag{41}$$

$$\eta \geq -M \tag{42}$$

$$\eta \geq 4x - 16 \tag{43}$$

$$x \leq 4 \tag{44}$$

$$\eta \geq -6 \tag{45}$$

The solution of problem (40)-(45) is $x^* = 2.5, \eta^* = -6, z^* = -8.5$. The lower bound of node #2 is updated to -8.5. As the first-stage solution $x^* = 2.5$ does not satisfy the integrality constraints, and then the first-stage B&B is necessary. The nodes #4 and #5 are created as depicted in the figure below.

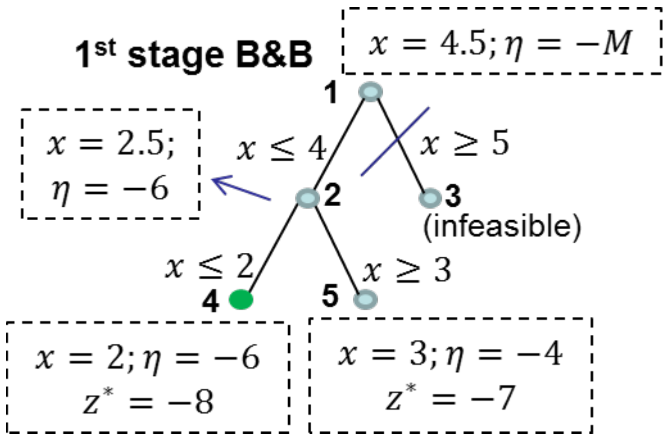


Figure 19 - First-stage B&B.

The first-stage problem is solved in each node. The node $\{x \leq 2\}$ has -8 for the optimal objective function and $x^* = 2, \eta^* = -6$ for optimal solution; the node $\{x \geq 3\}$ has -7 for the optimal objective function and $x^* = 3, \eta^* = -4$ for optimal solution. The lower bounds of node # 4 and node #5 are $LB_4 = -8$ and $LB_5 = -7$ respectively. The criterion adopted to select a node is the best first (the node with the least lower bound), therefore the left node with optimal objective function of -8 is selected and the second-stage procedure is called. The global lower bound is also updated to -8.

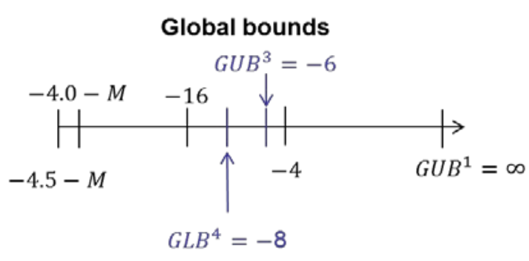


Figure 20 - Global bounds progress.

The figure below shows the *(x, y)* resultant partition $\{x \leq 2\}$ of the first-stage and the solution of the second-stage procedure when $x^* = 2$ is fixed.

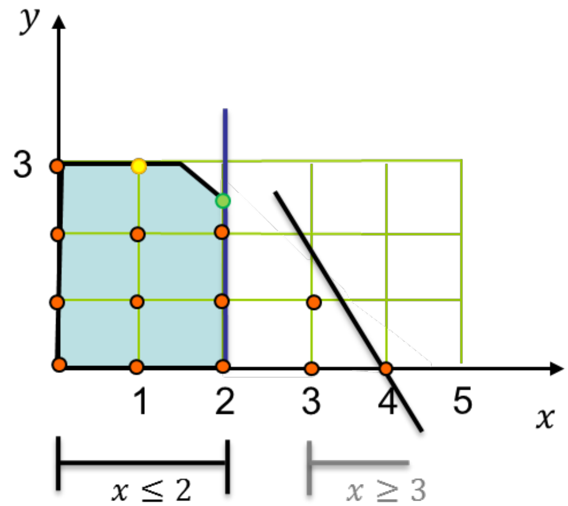


Figure 21 - Second-stage procedure input ($x \leq 2$) and the corresponding second-stage solution $y = 2.5$.

The node represented by $x \leq 2$ inherited the convexification cut $\{y \leq 8 - 2x\}$ generated previously by its ancestor node $x \leq 4$ as can be seen in the figure below. In this case the valid cut $\{y \leq 8 - 2x\}$ is not even a face of the resulting set *(x, y).*

The second-stage solution at this point of the algorithm is $y = 2.5$. Therefore, the second-stage B&B procedure is necessary to construct a partition that excludes $y = 2.5$ and also contains all the original solutions regarding to the current first-stage partition.

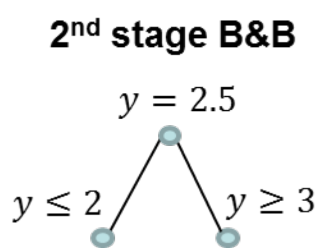


Figure 22 - Second-stage B&B.

The node $y \leq 2$ achieves the optimal solution and the node $y \geq 3$ has no feasible solution. These facts are not important from the mathematical point of view; the only objective of the second-stage B&B is to construct a partition as explained above. The partition $\{y \leq 2\} \cup \{y \geq 3\}$ generated by the B&B of the second-stage results into two sets:

(iii) $R3 = \{(x, y): 0 \leq x \leq 2, y \leq 4.5 - x, 0 \leq y \leq 3, y \leq 8 - 2x, \mathbf{y \leq 2}\}$;

(iv) $R4 = \{(x, y): 0 \le x \le 2, y \le 4.5 - x, 0 \le y \le 3, y \le 8 - 2x, \boldsymbol{y} \ge \mathbf{3}\}$

The resultants sets are shown below.

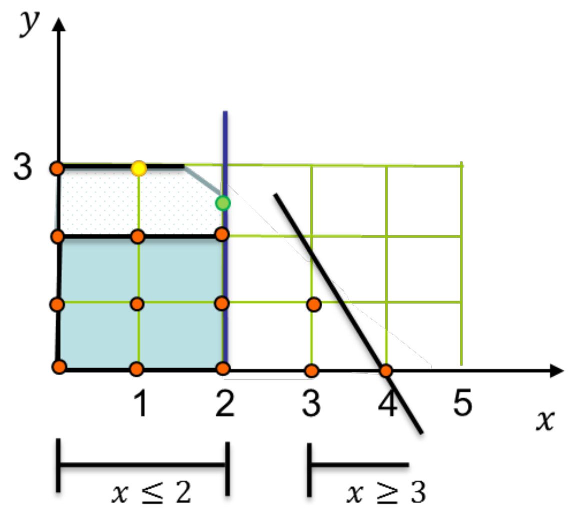


Figure 23 - *(x, y)* solution space partitioned by the second-stage B&B.

The convex-hull of the regions *R3* and *R4* is depicted in figure below. Once more the convex-hull is obtained with only one addition of a valid inequality.

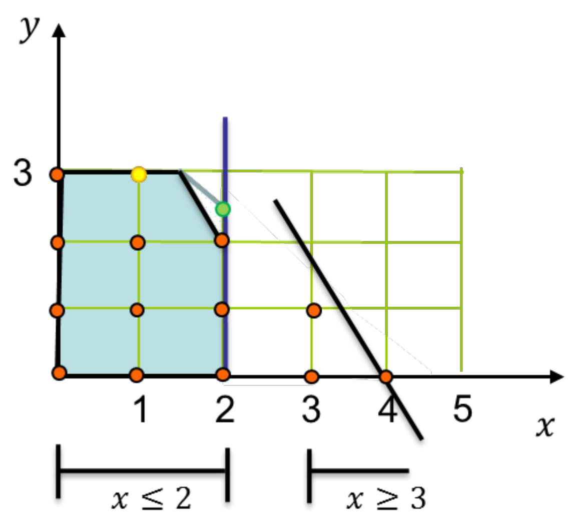


Figure 24 - Convex-hull of the regions *R3* and *R4* generated by the second-stage B&B procedure.

The next step generates a convexification cut that approximate the convex-hull of *R3* and *R4* regions and also cuts off the current fractional solution. The convexification cut generated by the corresponding CGLP. The CGLP has to produce valid inequalities for every *(x, y)* point in *R3* and *R4*. The convexification cut achieved is $y + 2x \leq 6$, and it is depicted in figure below.

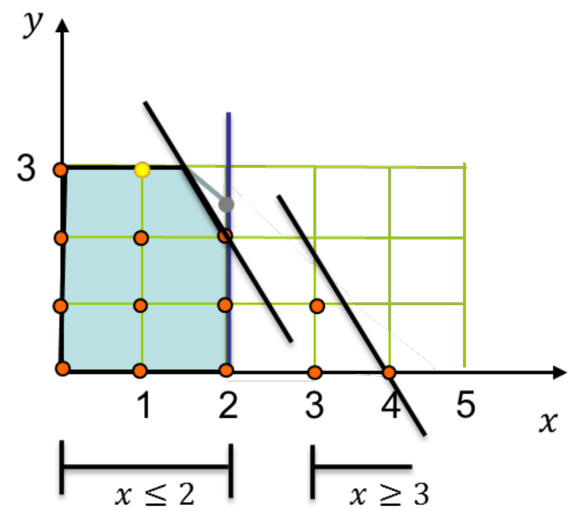


Figure 25 - Convexification cut in the (x, y) solution space.

The next step is to solve the problem (31)-(34) with the additional convexification cut $y + 2x \leq 6$ as shown below.

$$\eta\,(x^*) = \min_y -2y \tag{46}$$

$$y \leq 4.5 - x^* \tag{47}$$

$$0 \leq y \leq 3 \tag{48}$$

$$y \leq 8 - 2x^* \tag{49}$$

$$y \leq 6 - 2x^* \tag{50}$$

The second-stage solution of (46)-(50) is integer $y^* = 2$ then the Benders cut can be constructed. At this point the global upper bound update not improves the overall gap and remains with same value $GUB^3 = -6$. The resulting Benders cut is $\eta \geq 4x - 12$. The next figure shows the second objective function approximation by the current Benders cut and the corresponding region partitioned by first-stage node #4, $\{x \leq 2\}$.

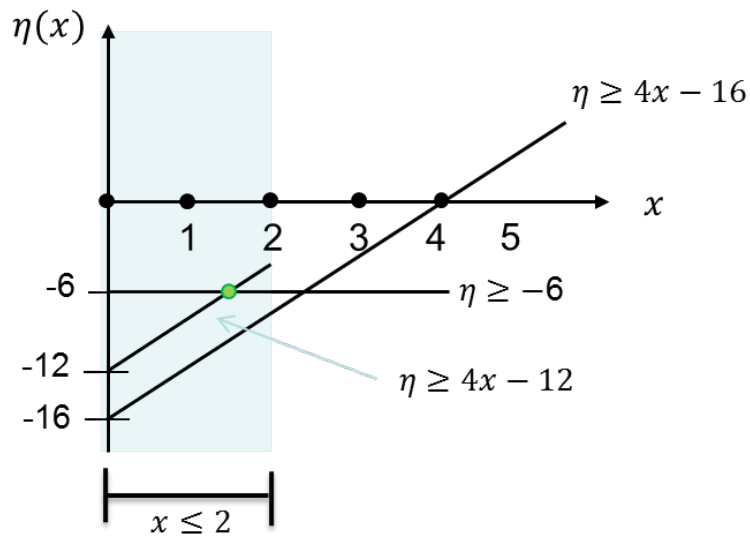


Figure 26 - Benders cuts approximation of the second-stage objective fuction regarding the node represented by $\{x \leq 2\}$.

Thus, the following updated first-stage problem version is solved.

$$z^* = \min_x -x + \eta \quad (51)$$
$$0 \leq x \leq 4.5 \quad (52)$$
$$\eta \geq -M \quad (53)$$
$$x \leq 2 \quad (54)$$
$$\eta \geq 4x - 16 \quad (55)$$
$$\eta \geq -6 \quad (56)$$
$$\eta \geq 4x - 12 \quad (57)$$

The first-stage solution achieved is $x^* = 1.5, \eta^* = -6$, and then the node #4 - lower bound can be updated to $LB_4 = -1.5 - 6 = -7.5$. As the first-stage solution is not integer then a first-stage B&B is necessary. Two branches can be constructed to exclude the solution $x^* = 1.5$. The figure below shows the nodes created by the branching procedure. Node #6 represents the partition $\{x \leq 1\}$ and node #7 represents the partition $\{x \geq 2\}$.

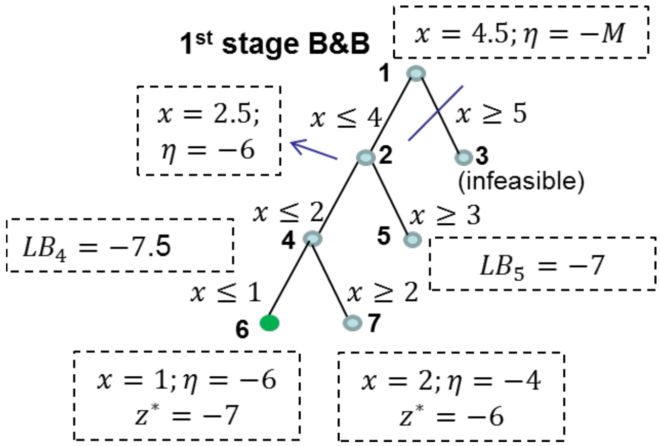


Figure 27 - First-stage Branch and Bound.

Node #6 is selected to call the second-stage procedure. At this point the lower bound of node #6 can be initiated with $LB_6 = -1 - 6 = -7$. The global lower bound can be updated to -7. The second-stage procedure is called and results at $y^* = 3$, therefore the global upper bound can be updated to $GUB_4 = -1 - 2 \times 3 = -7$. Then the resultant gap is zero, there is no need for further iteration, and the algorithm found the optimal solution of the original problem (8)-(12).

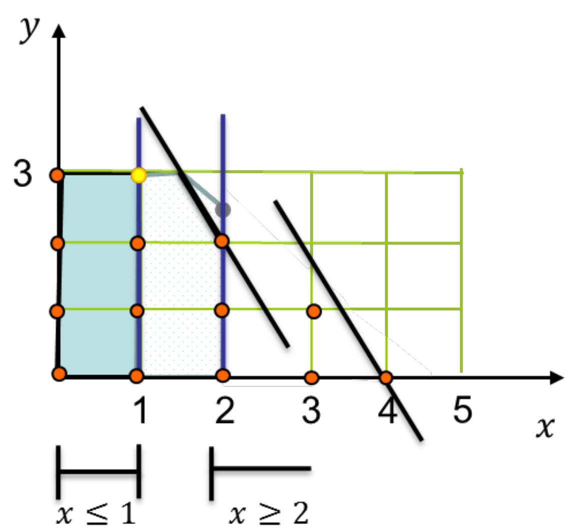


Figure 28 - Second-stage solution is integer and the optimal solution is found.

4.2.4
General case

The main objective of the algorithm is to solve a general mixed-integer two-stage problem using a B&B procedure that allows for the use of Benders decomposition to assess lower and upper bounds.

Initial step: the integer constraints are relaxed then the resulting linear program is solved in the initial node. If the solution is integer then the optimal solution of the first-stage problem is already found. However, if the solution is fractional, then the first-stage space is partitioned into two disjunctive sets.

Branching step: the objective is to produce two disjunctive sets. The current fractional solution cannot belong to either set. Thus the first-stage linear relaxation is solved in each disjunctive set. The optimal values represent lower bounds for each node. At this point, it is important to define the set of active nodes. This set must contain the global optimal solution. Before the first branch (partition), the set of active nodes is only the initial node. After the first branch, the set of active nodes contains two nodes represented by the two disjunctive sets.

Node selection criterion: the node with the least lower bound.

Until this point, the algorithm follows a standard B&B method. After the node selection, if the first-stage variables are integers then the second-stage procedure is used, if not, then the branching procedure continues.

The initial step of the second-stage procedure is to solve the corresponding linear problem given a fixed first-stage solution. If the resultant second-stage solution is integer, then a Benders cut is generated and the global upper bound is updated. If the solution is non-integer, then a cutting plane procedure is used. The Benders cut generated is only valid for the partition defined by the current first-stage B&B node and its offspring.

The algorithm stops when the difference between the global upper and lower bounds reaches an optimal tolerance criterion.

Figure 29 summarizes the decomposition algorithm [21] used in this chapter. The first-stage procedure performs a quasi-standard Branch and Bound algorithm. In each node, the first-stage problem formulated is solved. The non-standard part of the first-stage Branch and Bound occurs when the node selected has an integer's solution. At this juncture, the second-stage procedure is started and the global upper bound is updated. This point of the algorithm is characterized by the "The solution is MIP?" (Figure 29). The inputs of the second-stage procedure are the first-stage solution and the partition represented by $\{l_1^t; u_1^t\}$. l_1^t and u_1^t which represent the inferior and superior bounds respectively of the first-stage partition created in node *t*.

The second-stage procedure starts by solving the corresponding linear problem. In the figure below, the feasible region $\mathcal{Y}_L(x, y)$ represents the relaxation of the original mixed-integer set $\mathcal{Y}(x, y)$ and the $\mathcal{Y}_C(x, y)$ represents the convex-hull of the original set.

If the second-stage solution is integer, then a Benders cut can be constructed through the dual variables of the LP. The Benders cut is only valid for the feasible region determined by the first-stage node. If the second-stage solution is non-integer, then a second-stage Branch and Bound algorithm is performed with the first-stage solution fixed. The leaves of the resultant Branch and Bound form a partition that the current y-solution does not belong. The leaves serve as a foundation for the cutting plane linear program. The CGLP produces a convexification cut that approximates the $\mathcal{Y}_C(x, y)$ set. This procedure is repeated until the y-solution is a mixed-integer. When these conditions are satisfied, then Benders cut is constructed for the first-stage node.

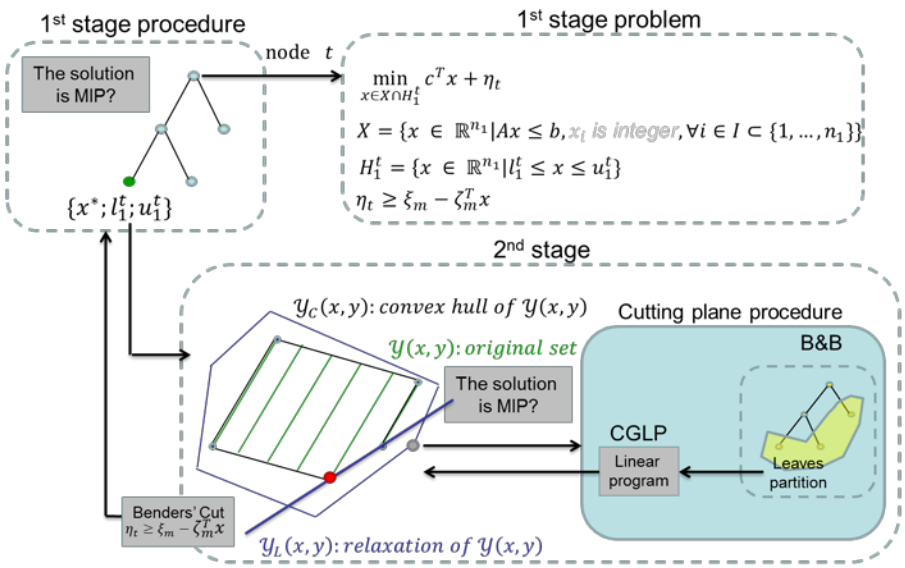


Figure 29 - Decomposition method general scheme.

The following section describes the conditions that a valid inequality must.

4.2.5
Valid inequalities

A cutting plane should have some properties. The valid inequality has to be valid for all points of the original feasible set. Mathematically, it has to accomplish the following condition: $\pi_1^T x + \pi_2^T y \ge \pi_0, \forall (x,y) \in P$. The next proposition uses duality to characterize if an inequality is valid with respect to P.

Proposition 1: Let $P = \{(x,y) \in \mathbb{R}_+^{n_1,n_2} | Wy + Txx \ge r\} \ne \emptyset$ then $\pi_1^T x + \pi_2^T y \ge \pi_0$ is a valid inequality for P if and only if

$$\exists\, u \ge 0 \text{ such as } uT \le \pi_1,\ uW \le \pi_2 \text{ and } ur \ge \pi_0 \tag{58}$$

Proof:

$$\forall (x,y) \in \mathrm{P}, \pi_1^T x + \pi_2^T y \ge \pi_0 \Leftrightarrow \min_{(x,y) \in \mathrm{P}} \pi_1^T x + \pi_2^T y \ge \pi_0 \Leftrightarrow \min_{\substack{Wy+Tx \ge r \\ (x,y) \ge 0}} \pi_1^T x + \pi_2^T y \ge \pi_0 \Leftrightarrow \max_{\substack{uT \le \pi_1 \\ uW \le \pi_2 \\ u \ge 0}} ur \ge \pi_0.$$

The property "for all" is connected to the minimum function, on the same type "exist one" is linked to the maximum function. Therefore,

$$\max_{\substack{uT \le \pi_1 \\ uW \le \pi_2 \\ u \ge 0}} ur \ge \pi_0 \Leftrightarrow \exists\, u \ge 0 \text{ such as } uT \le \pi_1,\ uW \le \pi_2 \text{and } ur \ge \pi_0$$

Remark: observe that if $\pi_1^T x + \pi_2^T y \ge \pi_0$ is a valid inequality for P then $\forall \alpha \in (0,1), \pi_1^T x + \pi_2^T y \ge \alpha\pi_0$ is also a valid inequality for P. To avoid multiple solutions the π_0 can be normalized to “-1” or “1”, depending on the purpose of the valid inequality.

The following proposition characterizes the set of all valid inequalities (π_0, π_1, π_2) for a given polyhedron P. An extension of this proposition can be found in [31] without proof.

Proposition 2: Let $P_i = \{(x,y) \in \mathbb{R}_+^{n_1,n_2} | W^i y + T^i x \ge r^i\} \ne \emptyset,\ i = 1,2$. Then $\pi_1^T x + \pi_2^T y \ge \pi_0$ is a valid inequality for $conv(P_1 \cup P_2)$ if and only if $\exists\, u_i \ge 0$ such as $u_i T^i \ge \pi_1$, $u_i W^i \ge \pi_2$ and $u_i r^i \le \pi_0$, $i = 1,2$.

Proof: $(\Longrightarrow)$ $if\ \pi_1^T x + \pi_2^T y \ge \pi_0, \forall (x,y) \in conv(P_1 \cup P_2) \Longrightarrow \alpha(\pi_1^T x_1, \pi_2^T y_1) + (1 - \alpha)(\pi_1^T x_2, \pi_2^T y_2) \ge \pi_0,\ \forall (x_1,y_1) \in P_1, \forall (x_2,y_2) \in P_2$ and $\forall \alpha \in [0,1]$.

Therefore:

(i) $\pi_1^T x_1 + \pi_2^T y_1 \ge \pi_0, \forall (x_1,y_1) \in P_1 \Leftrightarrow \exists\, u_1 \ge 0$ such as $u_1 T^1 \ge \pi_1$, $u_1 W^1 \ge \pi_2$ and $u_1 r^1 \le \pi_0$; and

(ii) $\pi_1^T x_2 + \pi_2^T y_2 \ge \pi_0, \forall (x_2,y_2) \in P_2 \Leftrightarrow \exists\, u_2 \ge 0$ such as $u_2 T^2 \ge \pi_1$, $u_2 W^2 \ge \pi_2$ and $u_2 r^2 \le \pi_0$

$(\Leftarrow)$ $if\ \pi_1^T x_1 + \pi_2^T y_1 \ge \pi_0, \forall (x_1,y_1) \in P_1$ and $\pi_1^T x_2 + \pi_2^T y_2 \ge \pi_0, \forall (x_2,y_2) \in P_2$ then $\alpha(\pi_1^T x_1, \pi_2^T y_1) + (1 - \alpha)(\pi_1^T x_2, \pi_2^T y_2) \ge \pi_0, \forall \alpha \in [0,1]$.

4.2.6
Cutting plane algorithm

The procedure is the following:

General cutting plane algorithm

Step 0: Relax the integer constraints of the original problem.

Step 1: Solve the relaxed formulation. If the solution x^* satisfies the integer constraints then stop.

Step 2: Generates a valid inequality that cuts off the current fractional solution x^*.

Step 3: Add a valid inequality to the relaxed formulation and return to the **step 1**.

In a general cutting plane method the CGLP produces a valid inequality that cuts off the current fractional solution. Therefore the relaxed problem is solved again with this new valid inequality. This procedure continues until a non-fractional solution is found. Then the algorithm stops.

Following the framework described in section 4.2.1 the CGLP can be defined as:

$$\max_{\pi_1,\pi_2} \pi_0 - \pi_1 x^* - \pi_2 y^* \quad (59)$$

$$\pi_1 = \left(T_i^{t(k)}\right)\lambda_{2,t} + \mu_{1,t} - \nu_{1,t}, \forall t \in \Gamma_2(x^*), \forall i \in \mathrm{I} \quad (60)$$

$$\pi_2 = \left(W_j^{t(k)}\right)\lambda_{2,t} + \mu_{2,t} - \nu_{2,t}, \forall t \in \Gamma_2(x^*), \ \forall j \in \mathrm{J} \quad (61)$$

$$\begin{gathered}\left(r^{t(k)}\right)\lambda_{2,t} + l_1^{t_1(k)}\mu_{1,t} - u_1^{t_1(k)}\nu_{1,t} + \\ + l_2^t \times \ \mu_{2,t} - u_2^t \times \ \nu_{2,t} \geq \pi_0 + x^*\pi_1 + y^*\pi_2, \forall t \in \Gamma_2(x^*)\end{gathered} \quad (62)$$

$$(\pi_1, \pi_2) \in \mathbb{R}^2, (\mu_{1,t}, \nu_{1,t}, \mu_{2,t}, \nu_{2,t}, \lambda_{2,t}) \geq 0 \quad (63)$$

Each vertex of the CGLP defined above represents the convex-hull's facets of particular partitions of the original feasible region [45], more specifically, the first-stage partition $l_1^{t_1(k)}$, $u_1^{t_1(k)}$ and the second-stage partition l_2^t, u_2^t. Therefore, a convexification cut of type $\pi_1(x^*) + \pi_2(y^*) \geq \pi_0$ is obtained. The problem (59)-(63) is the application of the 4.2.5.

$\Gamma_2(x^*)$ represents the set of indexes of the leaves resultant of the second-stage B&B.

$t(k)$ represents the first-stage node selected at the iteration *k* of the algorithm.

4.3
Decomposition method algorithm

This section describes the complete decomposition method algorithm.

First-Stage Problem – ABC Algorithm

Step 0: Initialization.

- ✓ Set iteration $k = 1$ and the global upper bound $V = \infty$.
- ✓ Set the first-stage active nodes $\Gamma_1^k = \{1\}$ and $\mathrm{H}_1^0 = \{x | l_1 \leq x \leq u_1\}$.
- ✓ Set $\mathcal{G}_x^{t,m} = \emptyset$ (is indicator function that is: 1, if the m Benders cut belongs to the first-stage node *t*; and 0, otherwise).
- ✓ Set an optimality tolerance criterion $\varepsilon \geq 0$.
- ✓ Solve the first-stage problem at node $\{1\}$ and obtain its objective value v^1.
- ✓ Set the global lower bound $v = v^1$

while true **do**

update the global lower bound $v = \min_{t \in \Gamma_1^k} v^t$

Γ_1^k is the active node of the first-stage B&B in the iteration *k*.

set $t = \mathrm{argmin}_{t \in \Gamma_1^k} v^t$

if $V - v \leq \varepsilon$ **then**

stop

end if

if $x_i^{t(k)}$ is integer $\forall i \ \in I$ **then**

call the second-stage algorithm and obtain the Benders cut

update $\mathcal{G}_x^t$

if $y_s^d \in \mathcal{Y}_D$ $\forall s$ **then**

update $V$and the incumbent solution (x^*, y_s^*)

end if

for all $t \ \in \ \Gamma_1^k$ **do**

re-solve the first-stage problem

update v^t

end do

else

choose and branch a variable

replace $t(k)$ for the two new child nodes $t^1(k), t^2(k)$ generated above

solve the first-stage problem for the two new nodes and obtain

$v^{t^1(k)}, v^{t^2(k)}$

end if

prune by bound deleting nodes for which $v^t \geq V$

set Γ_1^{k+1} = the current active nodes less the deleting nodes by pruning.

$k = k + 1$

end while

The second-stage problem is solved by the following algorithm:

Second-Stage Problem – ABC algorithm

Step 0 (Initialization): set iteration $d = 1$.

Step 1 (Creation of a unit partition): given the first-stage variable x^k, solve the original sub problem $f_s(x^k)$ using a B&B procedure. Extract the resulting leaf nodes set Γ_2.

while true **do**

Step 3: Solve the relaxed sub problem $f_{L,s}^d(x^k)$ to obtain y_s^d

if $y_s^d \in \mathcal{Y}_D$ $(original\ set\ with\ mixed-integer\ constraints)$ **then**

return $y_s^k = y_s^d$

stop

else

solve the CGPL with the set H_1^t (first-satge partition) and the unit partition Γ_2 to obtain convexification cut.

Update $\Pi_0^t(s), \Pi_1^t(s)$ and $\Pi_2^t(s)$

end if

Extend the indicator function $\mathcal{G}_y^{t(k),s}$ (indicator function that tracks if a convexification cut belongs to the partition represents by the node t(k) of the first-stage) to store in memory that the cut generated just before is active for node $t(k)$

$d = d + 1$

end while

4.3.1
Decomposition method applied to transmission switching

The benders decomposition method shown below is based on the works [21] and Sherali et. al. in [17]. Therefore, the decomposition of TS problem has a non-convex integer second stage. To deal with this problem we apply a sequential convexification procedure based in a B&B of the first and second stages.

The original problem for a single period:

$$\text{Min} \sum_{g} \{C_g(p_{g,0})\} + \sum_{g} \{C_g^{RU} r_g^U + C_g^{RD} r_g^D\} + \sum_{c,n} C_n \delta_{n,c} + \sum_{c,l} C_l \left(z_{l,c}^+ + z_{l,c}^-\right) \quad (64)$$

subject to:

$$\sum_{\forall l \epsilon \mathcal{F}(n)} f_{l,c} - \sum_{\forall l \epsilon \mathcal{T}(n)} f_{l,c} + \sum_{\forall g \epsilon \mathcal{G}(n)} p_{g,c} + \delta_{n,c} = d_n \quad ; \forall\, n, c \quad (65)$$

$$\Gamma_{l,c} z_{l,c} f_l^{min} \leq f_{l,c} \leq f_l^{max} \Gamma_{l,c} z_{l,c} \quad ; \forall\, l, c \quad (66)$$

$$\frac{1}{x_l}\left(\theta_{fr(l),c} - \theta_{to(l),c}\right) - f_{l,c} + M_l\left(1 - \Gamma_{l,c} z_{l,c}\right) \geq 0 \quad ; \forall\, l, c \quad (67)$$

$$\frac{1}{x_l}\left(\theta_{fr(l),c} - \theta_{to(l),c}\right) - f_{l,c} - M_l\left(1 - \Gamma_{l,c} z_{l,c}\right) \leq 0 \quad ; \forall\, l, c \quad (68)$$

$$\Gamma_{g,c} u_g P_g^{min} \leq p_{g,c} \leq P_g^{max} \Gamma_{g,c} u_g \quad ; \forall\, g, c \quad (69)$$

$$p_{g,c} - p_{g,0} \leq r_g^U \quad ; \forall\, g, c \quad (70)$$

$$r_g^U \leq R_g^+ \quad ; \forall\, g \quad (71)$$

$$\Gamma_{g,c}\left(p_{g,0}\right) - p_{g,c} \leq r_g^D \quad ; \forall\, g, c \quad (72)$$

$$r_g^D \leq R_g^- \quad ; \forall\, g \quad (73)$$

$$z_{l,c} - z_{l,0} \leq z_{l,c}^+ \quad ; \forall\, l, c \quad (74)$$

$$z_{l,0} - z_{l,c} \leq z_{l,c}^- \quad ; \forall\, l, c \quad (75)$$

$$\sum_{l=1}^{NL} \left(z_{l,c}^+ + z_{l,c}^-\right) \leq L \; ; \forall\, c \quad (76)$$

$$r_g^U, r_g^D \geq 0 \quad (77)$$

$z_{l,c}^{+}, z_{l,c}^{-} \geq 0 \; ; \; \forall \, l, c$ (78)

$u_g \in \{0,1\}; \;\; \forall \, g$ (79)

$z_{l,c} \in \{0,1\}; \;\; \forall \, l, c$ (80)

The original problem can be decomposed into two problems as follows:

$$\min \sum_{g} C_g(p_{g,0}) + \sum_{g} \{C_g^{RU} r_g^{U} + C_g^{RD} r_g^{D}\} + \sum_{c,n} C_n \delta_{n,c} + \sum_{c,l} C_l (z_{l,c}^{+} + z_{l,c}^{-})$$

$$f^{c}(u_g, p_{g,0}, z_{l,0}, r_g^{U}, r_g^{D}) = \sum_{n} C_n \delta_{n,c} + \sum_{l} C_l (z_{l,c}^{+} + z_{l,c}^{-})$$

The first stage problem can be written as:

$[RMP]$ $\quad \min_{\{u_g, P_{g,0}, f_{l,0} z_{l,0}, r_g^{U}, r_g^{D}, f^{c}, \theta_{n,0}\}} \sum_g C_g(p_{g,0}) + \sum_g \{C_g^{RU} r_g^{U} + C_g^{RD} r_g^{D}\} + \sum_c f^{c}$

subject to:

$$\sum_{\forall l \epsilon \mathcal{F}(n)} f_{l,0} - \sum_{\forall l \epsilon \mathcal{T}(n)} f_{l,0} + \sum_{\forall g \epsilon \mathcal{G}(n)} P_{g,0} = d_n \quad ; \forall \, n \tag{81}$$

$$z_{l,0} f_l^{min} \leq f_{l,0} \leq f_l^{max} z_{l,0} \quad ; \; \forall \, l \tag{82}$$

$$\frac{1}{x_l}\left(\theta_{fr(l),0} - \theta_{to(l),0}\right) - f_{l,0} + M_l\left(1 - z_{l,0}\right) \geq 0 \quad ; \; \forall \, l \tag{83}$$

$$\frac{1}{x_l}\left(\theta_{fr(l),0} - \theta_{to(l),0}\right) - f_{l,0} - M_l\left(1 - z_{l,0}\right) \leq 0 \quad ; \; \forall \, l \tag{84}$$

$$u_g P_g^{min} \leq p_{g,0} \leq P_g^{max} u_g \quad ; \; \forall \, g \tag{85}$$

$$r_{\mathrm{g}}^{U} \leq R_g^{+} \quad ; \; \forall \, g \tag{86}$$

$$r_{\mathrm{g}}^{D} \leq R_g^{-} \quad ; \; \forall \, g \tag{87}$$

$$u_g \in [0,1]; \;\; \forall \, g \tag{88}$$

$$z_{l,0} \in [0,1]; \;\; \forall \, l \tag{89}$$

+ Benders cut.

For each contingency the optimal corrective actions regarding generation and transmission rescheduling are the solutions of the following MIP problems.

$$\forall c, \text{let } [SMIP^c]: \qquad \min_{z_{l,c}} f^c(u_g, p_{g,0}, z_{l,0}) = min \sum_n C_n \delta_{n,c} + \sum_l C_l (z_{l,c}^+ + z_{l,c}^-)$$

$$\sum_{\forall l \epsilon \mathcal{F}(n)} f_{l,\mathrm{c}} - \sum_{\forall l \epsilon \mathcal{T}(n)} f_{l,\mathrm{c}} + \sum_{\forall g \epsilon \mathcal{G}(n)} P_{g,c} + \delta_{n,c} = d_n \quad ; \forall\, n \tag{90}$$

$$\Gamma_{l,c} z_{l,\mathrm{c}} f_l^{min} \leq f_{l,c} \leq f_l^{max} \Gamma_{l,c} z_{l,\mathrm{c}} \quad ; \forall\, l \tag{91}$$

$$\frac{1}{x_l}\left(\theta_{fr(l),\mathrm{c}} - \theta_{to(l),\mathrm{c}}\right) - f_{l,\mathrm{c}} + M_l\left(1 - \Gamma_{l,c} z_{l,c}\right) \geq 0 \quad ; \forall\, l \tag{92}$$

$$\frac{1}{x_l}\left(\theta_{fr(l),\mathrm{c}} - \theta_{to(l),\mathrm{c}}\right) - x_l f_{l,\mathrm{c}} - M_l\left(1 - \Gamma_{l,c} z_{l,c}\right) \leq 0 \quad ; \forall\, l \tag{93}$$

$$\Gamma_{g,c} \hat{u}_g P_g^{min} \leq P_{g,c} \leq P_g^{max} \Gamma_{g,c} \hat{u}_g \quad , \forall\, g \tag{94}$$

$$p_{\mathrm{g,c}} \leq \hat{r}_{\mathrm{g}}^U + \hat{p}_{\mathrm{g,0}} \quad ; \forall\, g \tag{95}$$

$$p_{\mathrm{g,c}} \geq \Gamma_{g,\mathrm{c}} \hat{p}_{\mathrm{g,0}} - \hat{r}_{\mathrm{g}}^D \quad , \forall\, g \tag{96}$$

$$z_{l,c} - \hat{z}_{l,0} \leq z_{l,c}^+ \quad ; \forall\, l \tag{97}$$

$$\hat{z}_{l,0} - z_{l,c} \leq z_{l,c}^- \quad ; \forall\, l \tag{98}$$

$$\sum_{l=1}^{NL} \left(z_{l,c}^+ + z_{l,c}^-\right) \leq L \tag{99}$$

5
Results

This section shows the results for the IEEE 30-bus system data [32]. It is compared the results obtained by the post-contingency transmission switching model with two formulations: (i) as a mixed-integer program as in [16]; and (ii) applying the decomposition method [21] presented in chapter 4. The case study highlights the operational benefits of a PC-TS scheduling model. The experiments were performed using a 2.67 GHz 8-Core (TM) i7 PC with 16 GB of RAM. The optimization problems were programmed and solved under the MOSEL-Language/Xpress-MP 7.2 framework [30].

5.1
IEEE 30-bus system

The IEEE 30-bus system data can be found in [32]. The system is composed of 41 transmission lines, 10 generator units connected to 6 buses. To illustrate the capability of the model to reduce load shedding and operative costs, the capacity of line 15 is reduced from 65 to 50 MW to stress the system. Under such a configuration, the system is not compliant with the $n-1$ criterion for any of the models. To avoid load shedding under the generation contingencies, the first three generators of buses 1, 2, and 5 were divided into seven units. The generators data are presented in the table below.

Table 9 - IEEE - 30-bus system: generator data.

Unit	Bus	C_i^v	C_i^U	C_i^D	P_i^{max}	P_i^{min}	$\overline{R}_i^U$	$\overline{R}_i^D$
		$/MWh	$/MW	$/MW	MW	MW	MW	MW
{1,2,3}	1	200	3.02	1.94	67	17	50	50
{4,5}	2	175	3.09	1.99	40	10	30	30
{6,7}	5	100	2.98	1.92	25	7.5	35	35
8	8	325	2.96	1.90	35	10	25	25
9	11	300	3	1.93	30	10	20	20
10	13	150	5	3.76	40	12	28	28

5.1.1
PC-TS solution with as mixed-integer program

The commercial solver of XPRESS performs a Branch and Bound (B&B) algorithm (see [31]) to solve the BC, P-TS and PC-TS models. For the integer preprocessing of XPRESS the Newton-barrier method was selected. This method proves to be more efficient with the IEEE 30-bus system; all other XPRESS-MIP options were the default. The unit commitment (on/off) is the same for all the models: they commit all the units except generators 9 and 10. The main results are listed in Table 10, which compares the performance of the models, and the generation scheduling is described in the appendix.

Table 10 – IEEE 30-bus system: computational results.

Parameters	BC	P-TS	PC-TS
Energy cost ($)	52,381.4	50,930.0	50,930.0
Up-reserve ($)	1,351.4	1,496.5	1,496.5
Down-reserve ($)	1,085.7	1,206.0	417.2
Total system ($)	65,660.9	62,653.3	54,620.5
Avg. load shed (MW)	3.1	2.6	0.3
Worst load shed (MW)	33.7	44.2	6.7
Worst load shed (%D*)	11.9	15,6	2.3
GAP (%)	0.0	0.0	1.8
Computational Time (s)	0.9	15.8	444.9

*values in % of the system demand

The consideration of corrective TS actions in the scheduling of the energy and reserve assets enable a substantial system-cost reduction, as demonstrated in line 5 of Table 10. The scheduling determined by the PC-TS model provides a cost reduction of 13% compared to the scheduling determined by the *P-TS* model and a cost reduction of 17% compared to the scheduling determined by the *BC* model. The system cost savings are due to a reduction in the down-spinning reserve levels and due to an increase in the system reliability. The total down-spinning reserve cost determined by the *PC-TS* provides a reduction of 65% and 62% compared to the results of the *P-TS* and *BC* models respectively (see line 4 of Table 10), while the average load shedding is reduced by 88%

and 90% compared to the results of the *P-TS* and *BC* models respectively (see line 6 of Table 10).

In Table 11, the load shed incurred in each contingency state is compared for the scheduling obtained by the three models. The *PC-TS* model substantially reduces the load shed in every contingency state, except when line 37 is under contingency, which disconnects bus 26 from the rest of the grid.

Table 11 - IEEE 30-Bus System: Load shed per contingency.

Contingency	BC	P-TS	PC-TS
l2	28.9	11.0	0.0
l3	27.0	9.0	0.0
l4	33.7	67.0	0.0
l6	26.2	7.6	0.0
l7	0.0	22.8	0.0
l8	15.6	0.0	0.0
l12	3.3	3.3	1.1
l14	6.5	6.8	0.5
l37	3.5	3.5	3.5
g8	12.4	0.0	0.0

Because the *PC-TS* model can be regarded as a relaxation of the constraints of both of the *P-TS* and *BC* models, it outperforms both alternatives in terms of the economic and reliability benefits for the system operation. Clearly, the side effect of considering such a more widely responsive capacity in the scheduling model is the computational effort required to determine a solution within a tight optimality gap. This drawback notwithstanding, note that the Branch and Bound method is able to determine good solutions that outperform both alternative methods within a reasonable computation time. In Figure 30, the objective function value is shown for each integer solution found by the Branch and Bound algorithm for the *PC-TS* model. The value is presented in monetary values (left axis) and in terms of cost reduction regarding the *P-TS* system cost (see Table 10). The algorithm stops when a feasible solution is determined within the selected optimality criterion, which was set to 2% in this case study. Of the Branch and Bound method solutions determined, 80% outperformed the *BC* and *P-TS* solutions. This high percentage suggests that despite a higher computational burden to solve the *PC-TS* model within a tight optimality gap, the *PC-TS* model is effective to provide high-quality solutions that outperform solutions obtained with existing methodologies.

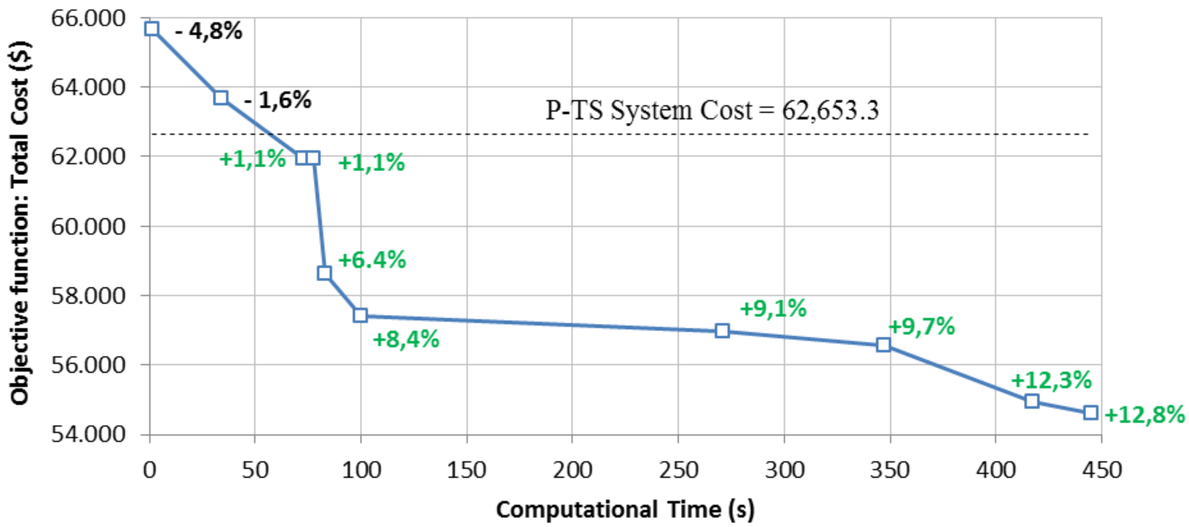


Figure 30 - Branch and Bound solutions for the IEEE 30-bus system: objective function cost ($) and the difference (%) between the PC-TS current Total Cost and P-TS optimal Total Cost.

The Figure 30 shows the evolution of *PC-TS* solutions (cost reduction with respect to the *P-TS* model) throughout the algorithm computing time. The time required to achieve the *PC-TS* optimal solution with 2% of optimality gap is higher to the time to reach the *P-TS* solution. Nevertheless, the *PC-TS* model finds a solution that outperforms in more than 8% the *P-TS* optimal cost in 100 seconds. Moreover, the substantial cost reduction of 12.8% encourages further algorithmic developments to reduce the computational time.

Finally, note that the system upper bound for switch actions is $\Phi = 41 - 30 + 1 = 12$ within the set of 37 lines candidates for TS. Nevertheless, Table VIII revels that only 5 lines are used to perform corrective actions, i.e., post-contingency switches. Furthermore, only in 14 of the 51 post-contingency states (27%) does the optimal solution make use of a switching action.

Table 12 - IEEE 30-Bus System: Preventive and Corrective Switches.

Line	Contingency state label (line under contingency)														
	pre	l_2	l_3	l_4	l_7	l_8	l_9	l_{12}	l_{13}	l_{14}	l_{17}	l_{19}	l_{20}	l_{30}	l_{32}
l_1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
l_5	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1
l_6	1	0	0	1	1	0	1	1	1	0	1	1	1	0	0
l_{11}	0	1	1	1	1	1	0	0	1	0	0	1	1	1	1
l_{23}	0	1	1	1	1	1	1	1	0	1	1	0	0	0	1

The observations described above can be explored in off-line studies aiming to determine the critical lines to be considered as candidates for TS. For illustrative purposes, if the set of candidate lines $\mathcal{L}^{\text{TS}}$ is restricted to the set of lines that perform any switch in the optimal solution, the *PC-TS* model running time reduces to 29 seconds when the same solution depicted in Table 10, Table 11 and Table 12 is determined. These off-line studies can substantial reduce the TS space helping the model convergence for industrial sized systems.

5.1.2
PC-TS solution with the decomposition algorithm

This section shows the decomposition algorithm performance regarding the *PC-TS* model. The method was applied to the post-contingency transmission switching problem developed in the first part of this PhD thesis. The decomposition method performance is compared to the standard MIP algorithm from commercial solver (XPRESS).

The results in terms of computational efficiency are quite productive and encorauge future research. The blue line, in figure below, represents the solutions achieved by a standard MIP commercial solver and the red line represents the decomposition method performance.

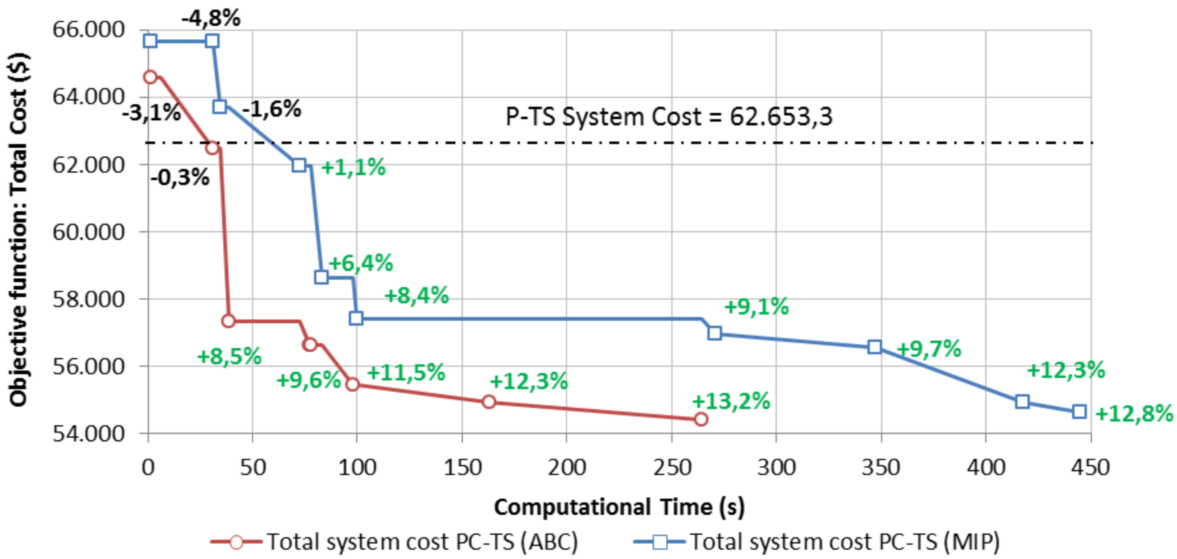


Figure 31 – computational time of a commercial solver solution versus decomposition method for the IEEE 30-bus system.

As a result the decomposition method, entitled as ABC outperforms the *P-TS* final solution in 31 s while the MIP commercial algorithm spends 72 s to outperform the same solution. The time spend to achieve the final solution achieved with the decomposition algorithm has considerable less time than with MIP-commercial solver. The gap in both methods was 2%. Final solution was achieved in 264 s for the decomposition method and in 445 s for MIP commercial solver.

5.2 Conclusions

The first paper that compounds this PhD thesis co-optimizes the energy and reserves taking into account the benefits of a network capable of performing corrective TS actions in a scheduling model. The model is capable of enhancing the deliverability of reserves through the network while minimizing the system overall reserve levels. This improved system performance is achieved using a co-optimization of the generation, energy and reserves, and transmission assets in both pre- and post-contingency states.

In the proposed framework, the SO manages cycles in the transmission grid according to the tradeoff between the additional transmission capacity benefit of a line and the power flow constraint due to the voltage law. Therefore, the topological aspects of the network limit the economic and security benefits of the proposed approach. The number of optimal switches is limited above by the number of fundamental cycles in the network and the set of line candidates for TS are identified as the set of lines participating in a cycle.

Computational experiments highlight the benefits of using TS as a corrective action. The results are presented in comparative fashion to characterize the synergic benefit of accounting for *PC-TS* actions in the scheduling model. The results demonstrate significant operative benefits for the test system used compared to the *P-TS* model: security is enhanced under the proposed approach and the model is capable of capturing such security improvements while reducing the reserve levels. The drawback of the TS-based models lies in its computational burden, which has been indicated in previous works (see [11], [14], [25], [33] and [34]), the work in [35] suggests that further research should be focused on developing decomposition techniques to tackle large-scale systems as the decomposition method [21] presented in chapter 4. Notwithstanding the computational burden, two empirical results were found:

(i) the number of optimal corrective switches is reasonably low and are concentrated in a small subset of critical lines, which can be understood as a virtue of the model – the number of binary variables can be reduced and, from an engineering perspective, solutions are more likely to be implementable;

(ii) the *PC-TS* model is effective to provide high-quality solutions that outperform solutions obtained with existing methodologies.

Both (i) and (ii) encourage further research in the subject of TS. In the next section, a list of future research is presented.

6
Future Work

For future work, it would be worthwhile analyzing practical consequences of certain theoretical concepts of graph theory applied to the transmission switching problem. Spanning tree algorithms seem to be quite fruitful when they are applied to a transmission switching problem, especially because of the Kirchhoff's laws trade-off discussed above and the dynamic of network's cycles.

Another subject is the consequences of smart grid (transmission switching) on Financial Transmission Rights (FTRs). How the developing in transmission switching area will affect the Financial Transmission Rights (FTRs) market, the agents market power and total market welfare.

To solve the transmission switching problems for network real size cases, it is necessary to conduct further research into the algorithmic solution in an attempt to deal with the NP-hard constraints and variables that unit commitment and transmission switching produce. Preliminary results shown in this thesis demonstrate that Benders decomposition can improve the computationally efficiency, but for real size cases it is necessary to further improve the solution technology. In this PhD thesis it was shown that the Benders decomposition is a promising field for future research. In [35], a Dantzig-Wolfe reformulation and column generation are applied to achieve the optimal dispatch with transmission switching and unit commitment. This technique reduced the computational efforts considerably. Therefore, it seems a promising field for further investigation. Furthermore, column generation can be used to enhance B&B procedures through a branch-and-cut-and-price algorithm. The development of valid inequalities to reduce the search space based on topological and/or electrical properties of the network constitute an interesting and promising topic of research.

Additionally, [38] provides a PTDF-based formulation in which TS is implementable. Such formulation has the advantage of reducing the number of constraints and decision variables in the problem. Moreover, heuristics methods developed in [25], [33] and [34] constitute a relevant area for further research, since the consideration of TS in scheduling models significantly impacts in the complexity of the problem. Lastly, the extension of the proposed model to consider an AC load flow model constitutes another important research topic that deserves attention.

7
References

[1] A. Street, A. Moreira, and J. M. Arroyo, “Energy and Reserve Scheduling Under a Joint Generation and Transmission Security Criterion: An Adjustable Robust Optimization Approach,” IEEE Trans. Power Syst., vol. 24, no. 1, pp. 3-14, Jan. 2014.

[2] Arroyo, J. M., & Galiana, F. D. (2005). Energy and reserve pricing in security and network-constrained electricity markets. Power Systems, IEEE Transactions on, 20(2), 634-643.

[3] Galiana, F. D., Bouffard, F., Arroyo, J. M., & Restrepo, J. F. (2005). Scheduling and pricing of coupled energy and primary, secondary, and tertiary reserves. Proceedings of the IEEE, 93(11), 1970-1983.

[4] A. Monticelli, M. V. Pereira, and S. Granville, “Security-constrained optimal power flow with post-contingency corrective rescheduling,” IEEE Trans. Power Syst., vol. PWRS-2, no. 1, pp. 175-180, Feb. 1987.

[5] G. Schnyder and H. Glavitsch, “Security enhancement using an optimal switching power flow,” Conference Papers: 1989 Power Industry Computer Application Conference, pp. 25–32, 1989.

[6] H. Glavitsch, “Switching as means of control in the power system,” International Journal of Electrical Power and Energy Systems, vol. 7, no. 2, pp. 92–100, 1985.

[7] PJM Manual 03: Transmission Operations: Revision 44 Nov. 1 2013, pp. 66 and 88.

[8] PJM. (November 01, 2013). PJM - Switching Solutions. In PJM. Retrieved January 24, 2014, from http://www.pjm.com/markets-and-operations/etools/oasis/system-information/switching-solutions.aspx.

[9] M. Bjorndal and K. Jornsten, “Paradoxes in networks supporting competitive electricity markets,” Department of Finance and Management Science, NHH, Tech. Rep., 2000.

[10] What if They Closed 42d Street and Nobody Noticed? (1990). Retrieved May 01, 2014, from http://www.nytimes.com/1990/12/25/health/what-if-they-closed-42d-street-and-nobody-noticed.html

[11] K. W. Hedman, M. C. Ferris, R. P. O’Neill, E. B. Fisher, and S. S. Oren, “Co-optimization of generation unit commitment and transmission switching with n-1 reliability,” IEEE Trans. Power Syst., vol. 25, no. 2, pp. 1052-1063, May 2010.

[12] Fisher, E. B., O'Neill, R. P., & Ferris, M. C. (2008). Optimal transmission switching. Power Systems, IEEE Transactions on, 23(3), 1346-1355.

[13] Hedman, K. W., O'Neill, R. P., Fisher, E. B., & Oren, S. S. (2009). Optimal transmission switching with contingency analysis. Power Systems, IEEE Transactions on, 24(3), 1577-1586.

[14] Hedman, Kory W., et al. "Smart flexible just-in-time transmission and flowgate bidding." Power Systems, IEEE Transactions on 26.1 (2011): 93-102.

[15] A. Street, F. Oliveira, and J. M. Arroyo, “Energy and reserve scheduling under an N-K security criterion via robust optimization,” in Proc. 17th Power Systems Computation Conference (PSCC’11), Stockholm, Sweden, Aug. 2011.

[16] Ayala, Gustavo, and Alexandre Street. "Energy and reserve scheduling with post-contingency transmission switching." Electric Power Systems Research 111 (2014): 133-140.

[17] Sherali, Hanif D., and Barbara MP Fraticelli. "A modification of Benders decomposition algorithm for discrete subproblems: An approach for stochastic programs with integer recourse." Journal of Global Optimization 22.1-4 (2002): 319-342.

[18] Benders, J. F. "Partitioning procedures for solving mixed-variables programming problems." Computational Management Science 2.1 (2005): 3-19.

[19] Sherali, Hanif D., and Warren P. Adams. A reformulation-linearization technique for solving discrete and continuous nonconvex problems. Vol. 31. Springer, 1998.

[20] Zheng, Qipeng P., et al. "A decomposition approach to the two-stage stochastic unit commitment problem." Annals of Operations Research 210.1 (2013): 387-410.

[21] Qi, Yunwei, and Suvrajeet Sen. "Ancestral Benders Cuts and Multi-term Disjunctions for Mixed-Integer Recourse Decisions in Stochastic Programming." (2013).

[22] Stoft; S,Power System Economics.Designing Markets for Electricity New York: Wiley., 2002

[23] A. Street, F. Oliveira, and J. M. Arroyo, "Contingency-constrained unit commitment with n-K security criterion: A robust optimization approach," IEEE Trans. Power Syst., vol. 26, no. 3, pp. 1581-1590, Aug. 2011.

[24] F. Bouffard, F. D. Galiana, and J. M. Arroyo, "Umbrella contingencies in security-constrained optimal power flow," in Proc. 15th Power Systems Computation Conf. (PSCC'05), Liège, Belgium, Aug. 2005.

[25] P. A. Ruiz, J. M. Foster, A. Rudkevich, and M. C. Caramanis, "On fast transmission topology control heuristics," IEEE PES General Meeting, July 2011.

[26] A. J. Wood and B. F. Wollenberg, Power Generation, Operation, and Control, 2nd ed. New York: Wiley, 1996.

[27] R. K. Ahuja, T. L. Magnanti, and J. B. Orlin, Network Flows: Theory, Algorithms, and Applications, Prentice Hall, 1993.

[28] M. Syslo, "On the fundamental cycle set graph," IEEE Trans. Circuits and Syst., vol. 29, no. 3, pp. 136 - 138, Mar. 1982.

[29] Kavitha, Telikepalli, and Kurt Mehlhorn. "A polynomial time algorithm for minimum cycle basis in directed graphs." STACS 2005. Springer Berlin Heidelberg, 2005. 654-665

[30] Xpress Optimization Suite. [Online]. Available: http://www.fico.com/.

[31] L. Wolsey, Integer Programming. New York: Wiley Inter-Science, 1998.

[32] Power System Test Case Archive, Univ. Washington, Dept. Elect. Eng., 2007. [Online]. Available: https://www.ee.washington.edu/research/ps-tca/index.html.

[33] J. D. Fuller, R. Ramasra, and A. Cha, "Fast heuristics for transmission line switching," IEEE Trans. Power Syst., vol. 27, no. 3, pp. 1377-1386, Aug. 2012.

[34] Soroush, Milad, and J. David Fuller. "Accuracies of Optimal Transmission Switching Heuristics Based on DCOPF and ACOPF." 1-9.

[35] J. C. Villumsen and A. B. Philpott, “Column Generation for Transmission Switching of Electricity Networks with Unit Commitment,” Lecture Notes in Engineering and Computer Science, vol. 2189, pp. 1440-1443, 2011.

[36] Sen, Suvrajeet, and Hanif D. Sherali. "Decomposition with branch-and-cut approaches for two-stage stochastic mixed-integer programming." Mathematical Programming 106.2 (2006): 203-223.

[37] Sherali, Hanif D., and Xiaomei Zhu. "On solving discrete two-stage stochastic programs having mixed-integer first-and second-stage variables." Mathematical Programming 108.2-3 (2006): 597-616.

[38] P. A. Ruiz, A. Rudkevich, M. C. Caramanis, E. Goldis, E. Ntakou, and C. R. Philbrick, "Reduced mip formulation for transmission topology control," in 50th Annual Allerton Conference on Communication, Control, and Computing, 2012.

[39] R. P. O’Neill, R. Baldick, U. Helman, M. H. Rothkopf, and W. Stewart, “Dispatchable transmission in RTO markets,” IEEE Trans. Power Syst., vol. 20, no. 1, pp. 171–179, Feb. 2005.

[40] A. Khodaei and M. Shahidehpour, “Transmission switching in security-constrained unit commitment,” IEEE Transactions on Power Systems, vol. 25, no. 4, pp. 1937–1945, 2010.

[41] Bertsimas, Dimitris, and Melvyn Sim. "The price of robustness." *Operations research* 52.1 (2004): 35-53.

[42] Sen, S., and Hanif D. Sherali. "On the convergence of cutting plane algorithms for a class of nonconvex mathematical programs." Mathematical Programming 31.1 (1985): 42-56.

[43] L. Freire, “Modelo de Comercialização de Energia Renovável no Ambiente de Contratação Livre via Teoria de Jogos Cooperativos,” Master’s thesis, PUC-Rio, Rio de Janeiro, 2013;

[44] Jeroslow, Robert G. "A cutting-plane game for facial disjunctive programs." SIAM Journal on Control and Optimization 18.3 (1980): 264-281.

[45] Balas, Egon, Sebastián Ceria, and Gérard Cornuéjols. "A lift-and-project cutting plane algorithm for mixed 0–1 programs." Mathematical programming 58.1-3 (1993): 295-324.

8
Appendix A

8.1
Basic definitions

Polyhedron

- P is a polyhedron if $P = \{x \in \Re^n | Ax \geq b\}$.

Valid inequality of a polyhedron P:

- If $\alpha^T x \leq \beta, \forall x \in P$ then $\alpha^T x \leq \beta$ is a valid inequality for P.

Supporting hyperplane of a polyhedron P:

- It is a valid inequality with a non-empty intersection with P.

Face of a polyhedron P:

- It is the intersection of $\{x \mid \alpha^T x = \beta\}$ and the polyhedron P such that $\alpha^T x \leq \beta$ is a valid inequality for P.

Extreme point or vertex of a polyhedron P:

- It is a face with dimension 0.

Edge of a polyhedron P

- It is a face with dimension 1.

Facet of a polyhedron P:

- It is a face with dimension n-1.

Ray of polyhedron:

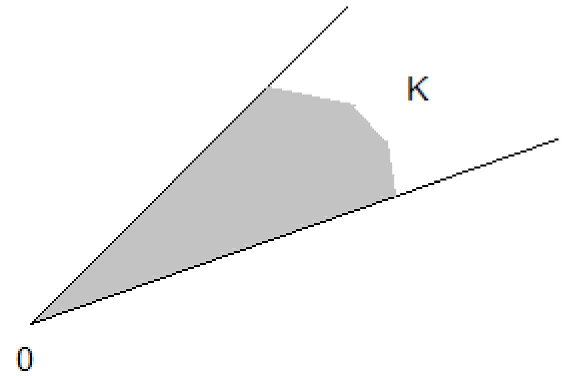
- r is a ray of polyhedron P if $\forall x \in P \rightarrow \{x + tr\} \in P, t \geq 0$.

Extreme ray of polyhedron:

- r is an extreme ray of P if is not a linear combination of the rays of P.

Cone:

- K is a cone if $\forall x \in K \rightarrow tx \in K, \forall t \in \Re, t \geq 0$.



Convex set

- $\mathcal{C}$ is a convex set if $\forall x, y \in \mathcal{C} \rightarrow tx + (1-t)y \in \mathcal{C}, \forall t \in [0,1]$.

Farkas' lema

$$Ax \leq 0, b^T x > 0, x \in \Re^n$$

or

$$A^T y = b, y > 0, x \in \Re^n$$

Face of a polyhedron

- $F = \{x \in P \mid \alpha^T x = \beta\}$ is a face of the polyhedron P if $P \subseteq \{x \mid \alpha^T x \leq \beta\}$.

Some insights:

If the inequality $\{x \mid \alpha^T x \leq \beta\}$ is not tight then the face F is empty.

Facial property

An inequality $\alpha^T x \leq \beta$ is facial regarding P if the set $P \cap \{x \mid \alpha^T x \leq \beta\}$ is a face of P.

8.2
A computational attempt to solve PC-TS efficiently

In 2012, a decomposition approach to the two-stage stochastic unit commitment problem was introduced by [20].

8.2.1
A Benders decomposition

The benders method decomposes the original model in two problems. Suppose the following problem.

$$Min\ cx + dy \tag{100}$$

$$subject\ to:$$

$$Ax + Dy \geq b \tag{101}$$

$$x \in X \tag{102}$$

$$y \in Y \tag{103}$$

$$Min\ dy \tag{104}$$

$$subject\ to:$$

$$Dy \geq b - A\bar{x} \tag{105}$$

$$\Gamma y \leq \gamma \tag{106}$$

$$\alpha_t y \geq \beta_t - \varphi_t \bar{x}; \qquad \forall t = 1, \cdots, T \tag{107}$$

8.2.2
Example

This example is based in Sherali and Fraticelli work [17].

Example:

P:

$$\underset{x_1, y_1}{\text{Minimize}} - x_1 - 2y_1$$

$$subject\ to:$$

$$-4x_1 - 3y_1 \geq -6$$

$$(x_1, y_1) \in \{0,1\}^2$$

The idea is to use the Reformulation-Linearization Technique to reformulate the feasible region of P in an equivalent polyhedral with one dimension above that contains only the first stage variable as binary. Benders decomposition can be applied in the problem reformulation.

First step: define Z_1 as a linear relaxation of the problem P feasible region with respect the first stage variables.

$$Z_1 = \{(x_1, y_1): -4x_1 - 3y_1 \geq -6;\ \ 0 \leq x_1 \leq 1;\ y_1 \in \{0,1\}\}$$

Second step: define Z as the convex hull of Z_1.

$$Z = conv(Z_1) = conv\{(x_1, y_1): -4x_1 - 3y_1 \geq -6;\ \ 0 \leq x_1 \leq 1;\ y_1 \in \{0,1\}\}$$

Apply the Reformulation step for the subset Z to obtain a new formulation of Z without the binary constraint of variable y_1.

In this case, the polynomial factors of the Reformulation step are: y_1 and $(1 - y_1)$. For didactic purposes, we will multiply one by one as follows:

The reformulation of constraint $-4x_1 - 3y_1 \geq -6$ produces two new constraints as follows:

- Product y_1:

$y_1 \times (-4x_1 - 3y_1 \geq -6) \rightarrow\ -4x_1y_1 - 3y_1y_1 \geq -6y_1 \rightarrow\ -4w - 3y_1 \geq -6y_1 \rightarrow$

$3y_1\ - 4w \geq 0$

- Product $(1 - y_1)$:

$(1 - y_1) \times (-4x_1 - 3y_1 \geq -6) \rightarrow -4x_1(1 - y_1) + (-3y_1)(1 - y_1) \geq -6(1 - y_1) \rightarrow$

$-4x_1 + 4x_1y_1 - 3y_1 + 3y_1y_1 \geq -6 + 6y_1 \rightarrow$

$-4x_1 - 6y_1 + 4w \geq -6$

The reformulation of the constraint $x_1 \leq 1$ produces more two new constraints as follows:

- Product y_1:

$y_1 \times (x_1 \leq 1) \rightarrow y_1x_1 \leq y_1 \rightarrow w \leq y_1 \rightarrow y_1 - w \geq 0$

- Product $(1 - y_1)$:

$(1 - y_1) \times (x_1 \leq 1) \rightarrow (1 - y_1)x_1 \leq (1 - y_1)) \rightarrow x_1 - y_1x_1 \leq 1 - y_1$

$\rightarrow x_1 - w \leq 1 - y_1 \rightarrow -x_1 - y_1 + w \geq -1$

The reformulation of the constraint $0 \leq x_1$ produces more two new constraints as follows:

- Product y_1:

$y_1 \times (0 \leq x_1) \rightarrow 0 \leq x_1y_1 \rightarrow w \geq 0$

- Product $(1-y_1)$:

$(1-y_1)\times(0\le x_1)\rightarrow 0\le(1-y_1)x_1\rightarrow 0\le x_1-y_1x_1\rightarrow x_1-w\ge 0$

$\rightarrow x_1-w\le 1-y_1\rightarrow -x_1-y_1+w\ge -1$

Then Z can be written as a high degree equivalent feasible region:

$$
\begin{aligned}
Z=\{(x_1,y_1):\quad & 3y_1-4w\ge 0;\\
& -4x_1-6y_1+4w\ge -6\\
& y_1-w\ge 0\\
& -x_1-y_1+w\ge -1\\
& w\ge 0\\
& x_1-w\ge 0\}
\end{aligned}
$$

Then the problem P can be reformulated as:

P': $\underset{x_1,y_1,w}{\text{Minimize}} -x_1-2y_1$

Subject to: Dual variable

$$3y_1-4w\ge 0; \quad \pi_1 \quad (108)$$

$$-4x_1-6y_1+4w\ge -6 \quad \pi_2 \quad (109)$$

$$y_1-w\ge 0 \quad \pi_3 \quad (110)$$

$$-x_1-y_1+w\ge -1 \quad \pi_4 \quad (111)$$

$$w\ge 0 \quad \pi_5 \quad (112)$$

$$x_1-w\ge 0 \quad \pi_6 \quad (113)$$

$$x_1\in\{0,1\} \quad (114)$$

Finally, we apply benders decomposition to the problem *P'* as follows:

$G^T=[0\quad -4\quad 0\quad 1\quad -1\quad 0]^T$; $H^T=[3\quad -6\quad 1\quad 0\quad -1\quad 0]^T$;

$F^T=[-4\quad 4\quad -1\quad -1\quad 1\quad 1]^T$; $f^T=[0\quad -6\quad 0\quad 0\quad -1\quad 0]^T$.

$$\underset{x_1\in\{0,1\}}{\text{Min}}\{-x_1+\underset{y_1,w}{\text{Min}}\{-2y_1: Hy+Fw\ge f-Gx\}\}$$

$$\underset{x_1\in\{0,1\}}{\text{Min}}\{-x_1 + \underset{\pi}{\text{Max}}\{\pi_2(-6+4x_1) - \pi_4 x_1 + \pi_5(-1+x_1) : \pi \in \Psi\}\}$$

$$\Psi = \{3\pi_1 - 6\pi_2 + \pi_3 - \pi_5 \geq -2; -4\pi_1 + 4\pi_2 - \pi_3 - \pi_4 + \pi_5 + \pi_6 = 0; \pi \geq 0\}$$

$$\min_{x\in X\cap\{0,1\}^n}\{cx + \min_{y,w}\{dy : Hy + Fw \geq f - Gx\}\} \qquad (115)$$

$$\min_{x\in X\cap\{0,1\}^n}\{cx + \max_{\pi}\{\pi(f - Gx) : \pi H = d; \pi F = 0; \pi \geq 0\}\} \qquad (116)$$

For computational employments, it is not need to resolve the benders master problem to optimality, instead of that a branch-and-cut approach could be used.

8.2.3
Benders decomposition using a sequential convexification

In this section, it will be shown a convexification process of the integer feasible region Z. This fact is important to guarantee the convex condition that is necessary to benders decomposition works.

$$\alpha_t y \geq \beta_t - \varphi_t \bar{x}; \qquad \forall t = 1, \cdots, T \qquad (117)$$

8.2.4
Benders decomposition applied to TS

The benders decomposition method is based on [20]. The problem has an integer and non-convex second stage. To deal with this problem we apply a convexification sequential procedure.

The original problem for a single period:

$$\text{Min} \sum_g \{C_g(p_{g,0})\} + \sum_g \{C_g^{RU} r_g^U + C_g^{RD} r_g^D\} + \sum_{c,n} C_n \delta_{n,c} + \sum_{c,l} C_l \left(z_{l,c}^+ + z_{l,c}^-\right) \qquad (118)$$

$subject\ to$:

$$\sum_{\forall l\epsilon\mathcal{F}(n)} f_{l,c} - \sum_{\forall l\epsilon\mathcal{T}(n)} f_{l,c} + \sum_{\forall g\epsilon\mathcal{G}(n)} p_{g,c} + \delta_{n,c} = d_n \ \ ; \forall\, n, c \qquad (119)$$

$$\Gamma_{l,c} z_{l,c} f_l^{min} \leq f_{l,c} \leq f_l^{max} \Gamma_{l,c} z_{l,c} \quad ; \forall\, l, c \tag{120}$$

$$\frac{1}{x_l}\left(\theta_{fr(l),\mathrm{c}} - \theta_{to(l),\mathrm{c}}\right) - f_{l,\mathrm{c}} + M_l\left(1 - \Gamma_{l,c} z_{l,c}\right) \geq 0 \quad ; \forall\, l, \mathrm{c} \tag{121}$$

$$\frac{1}{x_l}\left(\theta_{fr(l),\mathrm{c}} - \theta_{to(l),\mathrm{c}}\right) - f_{l,\mathrm{c}} - M_l\left(1 - \Gamma_{l,c} z_{l,c}\right) \leq 0 \quad ; \forall\, l, \mathrm{c} \tag{122}$$

$$\Gamma_{g,\mathrm{c}} u_g P_g^{min} \leq p_{g,c} \leq P_g^{max} \Gamma_{g,\mathrm{c}} u_g \quad ; \forall\, g, \mathrm{c} \tag{123}$$

$$p_{\mathrm{g,c}} - p_{\mathrm{g,0}} \leq r_{\mathrm{g}}^U \quad ; \forall\, g, \mathrm{c} \tag{124}$$

$$r_{\mathrm{g}}^U \leq R_g^+ \quad ; \forall\, g \tag{125}$$

$$\Gamma_{g,\mathrm{c}}\left(p_{\mathrm{g,0}}\right) - p_{\mathrm{g,c}} \leq r_{\mathrm{g}}^D \quad ; \forall\, g, \mathrm{c} \tag{126}$$

$$r_{\mathrm{g}}^D \leq R_g^- \quad ; \forall\, g \tag{127}$$

$$z_{l,c} - z_{l,0} \leq z_{l,c}^+ \quad ; \forall\, l, c \tag{128}$$

$$z_{l,0} - z_{l,c} \leq z_{l,c}^- \quad ; \forall\, l, c \tag{129}$$

$$\sum_{l=1}^{NL}\left(z_{l,c}^+ + z_{l,c}^-\right) \leq L \; ; \forall\, \mathrm{c} \tag{130}$$

$$r_{\mathrm{g}}^U, r_{\mathrm{g}}^D \geq 0 \tag{131}$$

$$z_{l,c}^+, z_{l,c}^- \geq 0 \; ; \forall\, l, c \tag{132}$$

$$u_g \in \{0,1\}; \;\; \forall\, g \tag{133}$$

$$z_{l,c} \in \{0,1\}; \;\; \forall\, l, c \tag{134}$$

The original problem can be decomposed into two problems as follows:

$$\min \sum_g C_g\left(p_{g,0}\right) + \sum_g \left\{C_g^{RU} r_g^U + C_g^{RD} r_g^D\right\} + \sum_{c,n} C_n \delta_{n,c} + \sum_{c,l} C_l\left(z_{l,c}^+ + z_{l,c}^-\right)$$

$$f^c\left(u_g, p_{g,0}, z_{l,0}, r_g^U, r_g^D\right) = \sum_n C_n \delta_{n,c} + \sum_l C_l\left(z_{l,c}^+ + z_{l,c}^-\right)$$

The first stage problem can be re-written as:

$$[RMP] \quad \min_{\{u_g,P_{g,0},f_{l,0}z_{l,0},r_g^U,r_g^D,f^c,\theta_{n,0}\}} \sum_g C_g(p_{g,0}) + \sum_g \{C_g^{RU} r_g^U + C_g^{RD} r_g^D\} + \sum_c \eta^c$$

subject to:

$$\sum_{\forall l \epsilon \mathcal{F}(n)} f_{l,0} - \sum_{\forall l \epsilon \mathcal{T}(n)} f_{l,0} + \sum_{\forall g \epsilon \mathcal{G}(n)} P_{g,0} = d_n \quad ; \forall\, n \tag{135}$$

$$z_{l,0} f_l^{min} \le f_{l,0} \le f_l^{max} z_{l,0} \quad ; \forall\, l \tag{136}$$

$$\frac{1}{x_l}\left(\theta_{fr(l),0} - \theta_{to(l),0}\right) - f_{l,0} + M_l\left(1 - z_{l,0}\right) \ge 0 \quad ; \forall\, l \tag{137}$$

$$\frac{1}{x_l}\left(\theta_{fr(l),0} - \theta_{to(l),0}\right) - f_{l,0} - M_l\left(1 - z_{l,0}\right) \le 0 \quad ; \forall\, l \tag{138}$$

$$u_g P_g^{min} \le p_{g,0} \le P_g^{max} u_g \quad ; \forall\, g \tag{139}$$

$$r_{\mathrm{g}}^U \le R_g^+ \quad ; \forall\, g \tag{140}$$

$$r_{\mathrm{g}}^D \le R_g^- \quad ; \forall\, g \tag{141}$$

$$u_g \in \{0,1\}; \ \forall\, g \tag{142}$$

$$z_{l,0} \in \{0,1\}; \ \forall\, l \tag{143}$$

+ Benders cut.

For each contingency the optimal corrective actions regarding generation and transmission rescheduling are the solutions of the following MIP problems.

$\forall c$, let $[SP^c]$: $\quad \min_{z_{l,c}} \eta^c\left(u_g, p_{g,0}, z_{l,0}\right) = min \sum_n C_n \delta_{n,c} + \sum_l C_l\left(z_{l,c}^+ + z_{l,c}^-\right)$

$$\sum_{\forall l \epsilon \mathcal{F}(n)} f_{l,c} - \sum_{\forall l \epsilon \mathcal{T}(n)} f_{l,c} + \sum_{\forall g \epsilon \mathcal{G}(n)} P_{g,c} + \delta_{n,c} = d_n \quad ; \forall\, n \tag{144}$$

$$\Gamma_{l,c} z_{l,c} f_l^{min} \le f_{l,c} \le f_l^{max} \Gamma_{l,c} z_{l,c} \quad ; \forall\, l \tag{145}$$

$$\frac{1}{x_l}\left(\theta_{fr(l),c} - \theta_{to(l),c}\right) - f_{l,c} + M_l\left(1 - \Gamma_{l,c} z_{l,c}\right) \ge 0 \quad ; \forall\, l \tag{146}$$

$$\frac{1}{x_l}\left(\theta_{fr(l),c} - \theta_{to(l),c}\right) - x_l f_{l,c} - M_l\left(1 - \Gamma_{l,c} z_{l,c}\right) \le 0 \quad ; \forall\, l \tag{147}$$

$$\Gamma_{g,c}\hat{u}_g P_g^{min} \leq P_{g,c} \leq P_g^{max}\Gamma_{g,c}\hat{u}_g \quad ,\forall\, g \tag{148}$$

$$p_{\mathrm{g,c}} \leq \hat{r}_{\mathrm{g}}^{U} + \hat{p}_{\mathrm{g,0}} \quad ;\ \forall\, g \tag{149}$$

$$p_{\mathrm{g,c}} \geq \Gamma_{g,c}\hat{p}_{\mathrm{g,0}} - \hat{r}_{\mathrm{g}}^{D} \quad ,\forall\, g \tag{150}$$

$$z_{l,c} - \hat{z}_{l,0} \leq z_{l,c}^{+} \quad ;\ \forall\, l \tag{151}$$

$$\hat{z}_{l,0} - z_{l,c} \leq z_{l,c}^{-} \quad ;\ \forall\, l \tag{152}$$

$$\sum_{l=1}^{NL} \left(z_{l,c}^{+} + z_{l,c}^{-}\right) \leq L \tag{153}$$

These MIP problems are the second stage problems of benders decomposition. The convex condition for benders convergence not holds. To deal with this issue we will propose a convex hull formulation for the $[SP^c]$ second stage problem feasible region.

If the binary variables $\{z_{l,0} = \hat{z}_{l,0}\}$ and $\{z_{l,c} = \hat{z}_{l,c}\}$ of the first and second stage respectively are fixed then can be reduced to linear problems as follows:

For every contingency c:

$[LP^c]$ $\quad Min\ \sum_n C_n \delta_{n,c}$

Subject to: dual variable

$$\sum_{\forall l \epsilon \mathcal{F}(n)} f_{l,\mathrm{c}} - \sum_{\forall l \epsilon \mathcal{T}(n)} f_{l,\mathrm{c}} + \sum_{\forall g \epsilon \mathcal{G}(n)} p_{g,c} + \delta_{n,c} = d_n \quad ;\forall\, n \qquad \pi_{d_n}^{c} \tag{154}$$

$$\Gamma_{l,c} z_{l,c} f_l^{min} \leq f_{l,c} \leq \Gamma_{l,c} z_{l,c} f_l^{max} \quad ;\ \forall\, l \qquad \pi f_{l,c}^{\pm} \tag{155}$$

$$\frac{1}{x_l}\left(\theta_{fr(l),\mathrm{c}} - \theta_{to(l),\mathrm{c}}\right) - f_{l,\mathrm{c}} \geq -M_l\left(1 - \Gamma_{l,c} z_{l,c}\right) \quad ;\ \forall\, l \qquad \pi_{\theta_{l,c}}^{+} \tag{156}$$

$$\frac{1}{x_l}\left(\theta_{fr(l),\mathrm{c}} - \theta_{to(l),\mathrm{c}}\right) - f_{l,\mathrm{c}} \leq M_l\left(1 - \Gamma_{l,c} z_{l,c}\right) \quad ;\ \forall\, l \qquad \pi_{\theta_{l,c}}^{-} \tag{157}$$

$$\Gamma_{g,c}\hat{u}_g P_g^{min} \leq P_{g,c} \leq P_g^{max}\Gamma_{g,c}\hat{u}_g \quad ,\forall\, g \qquad \pi_{g,c}^{\pm} \tag{158}$$

$$p_{\mathrm{g,c}} \leq \hat{p}_{\mathrm{g,0}} + \hat{r}_{\mathrm{g}}^{U} \quad ;\ \forall\, g \qquad \pi_{r_g^U}^{c} \tag{159}$$

$$p_{\mathrm{g,c}} \geq \Gamma_{g,c}\hat{p}_{\mathrm{g,0}} - \hat{r}_{\mathrm{g}}^{D} \quad ;\ \forall\, g \qquad \pi_{r_g^D}^{c} \tag{160}$$

$$p_{\mathrm{g,c}} \geq 0 \tag{161}$$

With the solutions of $[LP^c]$, we can produce cuts to the feasible region of SP.

$$\eta^c \geq \sum_l \left[\hat{\pi}^+_{f_{l,c}} f_l^{max} + \hat{\pi}^-_{f_{l,c}} f_l^{min} + \left(\hat{\pi}^+_{\theta_{l,c}} - \hat{\pi}^-_{\theta_{l,c}}\right) M_l\right] \Gamma_{l,c} \hat{z}_{l,c}$$

$$+ \sum_g \left(\hat{\pi}^+_{g,c} P_g^{max} + \hat{\pi}^-_{g,c} P_g^{min}\right) \Gamma_{g,c} \hat{u}_g +$$

$$+ \sum_g (\hat{\pi}^c_{r_g^U} + \hat{\pi}^c_{r_g^D} \Gamma_{g,c}) \hat{p}_{\mathrm{g},0} + \sum_g \hat{\pi}^c_{r_g^U} \hat{r}_{\mathrm{g}}^U - \sum_g \hat{\pi}^c_{r_g^D} \hat{r}_{\mathrm{g}}^D$$

$$+ \sum_n d_n \hat{\pi}^c_{d_n} + \sum_l \left(\hat{\pi}^-_{\theta_{l,c}} - \hat{\pi}^+_{\theta_{l,c}}\right) M_l \qquad (162)$$

This cut can be written in a vector format:

$$b^c z^c + \eta^c \geq f^c + \alpha^c \hat{u} + \beta^c \hat{p}_0 + \gamma^c \hat{r}^U + \hat{\vartheta}^c \hat{r}^D \qquad (163)$$

$$b^c = \left\{\left[\pi^+_{f_{l,c}} f_l^{max} + \pi^-_{f_{l,c}} f_l^{min} + \left(\pi^+_{\theta_{l,c}} - \pi^-_{\theta_{l,c}}\right) M_l\right] \Gamma_{l,c}\right\}_{l=1}^{NL} \qquad (164)$$

$$f^c = \textstyle\sum_n d_n \pi^c_{d_n} + \sum_l \left(\pi^-_{\theta_{l,c}} - \pi^+_{\theta_{l,c}}\right) M_l \qquad (165)$$

$$\alpha^c = \{\left(\pi^+_{g,c} P_g^{max} + \pi^-_{g,c} P_g^{min}\right) \Gamma_{g,c}\}_{g=1}^{NG} \qquad (166)$$

$$\beta^c = \left\{\pi^c_{r_g^U} + \pi^c_{r_g^D} \Gamma_{g,c}\right\}_{g=1}^{NG} \qquad (167)$$

$$\gamma^c = \left\{\pi^c_{r_g^U}\right\}_{g=1}^{NG} \qquad (168)$$

$$\hat{\vartheta}^c = \left\{-\pi^c_{r_g^D}\right\}_{g=1}^{NG} \qquad (169)$$

$$\hat{p}_0 = \{\hat{p}_{\mathrm{g},0}\}_{g=1}^{NG}; \hat{r}^U = \{\hat{r}_{\mathrm{g}}^U\}_{g=1}^{NG}; \hat{r}^D = \{\hat{r}_{\mathrm{g}}^D\}_{g=1}^{NG}; z^c = \{z_{l,c}\}_{l=1}^{NL} \qquad (170)$$

Then the relaxed linear program of $[SP^c]$ is as follows:

$[RLP^c]$

$$Min \sum_l C_l \left(z^+_{l,c} + z^-_{l,c}\right) + \eta^c$$

Subject to: dual variable

$$z_{l,c} - z^+_{l,c} \leq \hat{z}_{l,0} \quad ; \forall\, l \qquad \pi^+_{z_{l,c}} \qquad (171)$$

$$z^-_{l,c} + z_{l,c} \geq \hat{z}_{l,0} \quad ; \forall\, l \qquad \pi^-_{z_{l,c}} \qquad (172)$$

$$\pi^z_{l,c} \qquad (173)$$

$0 \leq z_{l,c} \leq 1 \quad ; \forall\, l$

$$\sum\nolimits_{l=1}^{NL} \left(z_{l,c}^{+} + z_{l,c}^{-}\right) \leq L \qquad \pi_c^L \qquad (174)$$

$$b_m^c z^c + \eta^c \geq\ f_m^c + \alpha_m^c \hat{u} + \beta_m^c \hat{p}_0 + \gamma_m^c \hat{r}^U + \vartheta_m^c \hat{r}^D;\ \forall\, m \in M^c \qquad \pi_c^m \qquad (175)$$

The solutions of $[RLP^c]$ provide a lower bound for the main MIP problem.

$$\chi^c \geq \sum_l \left(\pi_{z_{l,c}}^{+} + \pi_{z_{l,c}}^{-}\right)\hat{z}_{l,0} + \\ + \sum_m \pi_c^m \{f_m^c + \alpha_m^c \hat{u} + \beta_m^c \hat{p}_0 + \gamma_m^c \hat{r}^U + \vartheta_m^c \hat{r}^D\} + \sum_l \pi_{l,c}^z + L\pi_c^L \qquad (176)$$

$$\chi^c \geq \psi^c +\ \nu^c z_0 + \rho^c u + \lambda^c p_0 + \mu^c r^U + \xi^c r^D \qquad (177)$$

$$\nu^c = \left\{\pi_{z_{l,c}}^{+} + \pi_{z_{l,c}}^{-}\right\}_{l=1}^{NL} \qquad (178)$$

$$\psi^c = \sum_m \pi_c^m f_m^c + \sum_l \pi_{l,c}^z + L\pi_c^L \qquad (179)$$

$$\rho^c = \left\{\sum_m \pi_c^m\, \alpha_m^{g,c}\right\}_{g=1}^{NG} \qquad (180)$$

$$\lambda^c = \left\{\sum_m \pi_c^m\, \beta_m^{g,c}\right\}_{g=1}^{NG} \qquad (181)$$

$$\mu^c = \left\{\sum_m \pi_c^m\, \gamma_m^{g,c}\right\}_{g=1}^{NG} \qquad (182)$$

$$\xi^c = \left\{\sum_m \pi_c^m\, \vartheta_m^{g,c}\right\}_{g=1}^{NG} \qquad (183)$$

8.2.5 Intersection benders cuts

$[LP^c]$ $\qquad Min\ \sum_n C_n \delta_{n,c}$

Subject to: dual variable

$$\sum_{\forall l \epsilon \mathcal{F}(n)} f_{l,c} - \sum_{\forall l \epsilon \mathcal{T}(n)} f_{l,c} + \sum_{\forall g \epsilon \mathcal{G}(n)} p_{g,c} + \delta_{n,c} = d_n \quad ; \forall\, n \qquad \pi_{d_n}^c \qquad (184)$$

$$\Gamma_{l,c}\hat{z}_{l,c} f_l^{min} \leq f_{l,c} \leq \Gamma_{l,c}\hat{z}_{l,c} f_l^{max} \quad ; \forall\, l \qquad \pi f_{l,c}^{\pm} \qquad (185)$$

$$\pi_{\theta_{l,c}}^{+} \qquad (186)$$

$$\frac{1}{x_l}\left(\theta_{fr(l),\mathrm{c}}-\theta_{to(l),\mathrm{c}}\right)-f_{l,\mathrm{c}}\geq -M_l\left(1-\Gamma_{l,c}\hat{z}_{l,\mathrm{c}}\right)\ ;\ \forall\ l$$

$$\frac{1}{x_l}\left(\theta_{fr(l),\mathrm{c}}-\theta_{to(l),\mathrm{c}}\right)-f_{l,\mathrm{c}}\leq M_l\left(1-\Gamma_{l,c}\hat{z}_{l,\mathrm{c}}\right)\ ;\ \forall\ l \qquad \pi_{\theta_{l,c}}^{-} \qquad (187)$$

$$\Gamma_{g,c}\hat{u}_g P_g^{min}\leq P_{g,c}\leq P_g^{max}\Gamma_{g,c}\hat{u}_g\quad ,\forall\ g \qquad \pi_{g,c}^{\pm} \qquad (188)$$

$$p_{\mathrm{g,c}}\leq \hat{p}_{\mathrm{g,0}}+\hat{r}_{\mathrm{g}}^{U}\ ;\ \forall\ g \qquad \pi_{r_g^U}^{c} \qquad (189)$$

$$p_{\mathrm{g,c}}\geq \Gamma_{g,c}\hat{p}_{\mathrm{g,0}}-\hat{r}_{\mathrm{g}}^{D}\ ;\ \forall\ g \qquad \pi_{r_g^D}^{c} \qquad (190)$$

$$p_{\mathrm{g,c}}\geq 0 \qquad (191)$$

The dual problem of $[LP^c]$ is as follows:

$[LP^c]$ $\qquad Max\ \sum_n d_n\pi_{d_n}^{c}+\sum_l \Gamma_{l,c}\hat{z}_{l,\mathrm{c}}\left[f_l^{max}\pi f_{l,c}^{+}-f_l^{min}\pi f_{l,c}^{+}\right]+$
$\sum_l \pi_{\theta_{l,c}}^{+}+\sum_g \pi_{g,c}^{\pm}+\pi_{r_g^U}^{c}+\pi_{r_g^D}^{c}$

Subject to: dual variable

$$\sum_{\forall l\epsilon\mathcal{F}(n)} f_{l,\mathrm{c}}-\sum_{\forall l\epsilon\mathcal{T}(n)} f_{l,\mathrm{c}}+\sum_{\forall g\epsilon\mathcal{G}(n)} p_{g,c}+\delta_{n,c}=d_n\ ;\forall\ n \qquad \pi_{d_n}^{c} \qquad (192)$$

$$\Gamma_{l,c}\hat{z}_{l,\mathrm{c}}f_l^{min}\leq f_{l,c}\leq \Gamma_{l,c}\hat{z}_{l,\mathrm{c}}f_l^{max}\ ;\ \forall\ l \qquad \pi f_{l,c}^{\pm} \qquad (193)$$

$$\frac{1}{x_l}\left(\theta_{fr(l),\mathrm{c}}-\theta_{to(l),\mathrm{c}}\right)-f_{l,\mathrm{c}}\geq -M_l\left(1-\Gamma_{l,c}\hat{z}_{l,\mathrm{c}}\right)\ ;\ \forall\ l \qquad \pi_{\theta_{l,c}}^{+} \qquad (194)$$

$$\frac{1}{x_l}\left(\theta_{fr(l),\mathrm{c}}-\theta_{to(l),\mathrm{c}}\right)-f_{l,\mathrm{c}}\leq M_l\left(1-\Gamma_{l,c}\hat{z}_{l,\mathrm{c}}\right)\ ;\ \forall\ l \qquad \pi_{\theta_{l,c}}^{-} \qquad (195)$$

$$\Gamma_{g,c}\hat{u}_g P_g^{min}\leq P_{g,c}\leq P_g^{max}\Gamma_{g,c}\hat{u}_g\quad ,\forall\ g \qquad \pi_{g,c}^{\pm} \qquad (196)$$

$$p_{\mathrm{g,c}}\leq \hat{p}_{\mathrm{g,0}}+\hat{r}_{\mathrm{g}}^{U}\ ;\ \forall\ g \qquad \pi_{r_g^U}^{c} \qquad (197)$$

$$p_{\mathrm{g,c}}\geq \Gamma_{g,c}\hat{p}_{\mathrm{g,0}}-\hat{r}_{\mathrm{g}}^{D}\ ;\ \forall\ g \qquad \pi_{r_g^D}^{c} \qquad (198)$$

$$p_{\mathrm{g,c}}\geq 0 \qquad (199)$$

8.2.6 Cut Generation Linear Program

$[RLP^c]$

$$Min\sum_l C_l\left(z_{l,c}^{+}+z_{l,c}^{-}\right)+\eta^c$$

Subject to: dual variable

$$z_{l,c} - z_{l,c}^{+} \leq \hat{z}_{l,0} \quad ; \forall\, l \qquad \pi_{z_{l,c}}^{+} \tag{200}$$

$$z_{l,c} + z_{l,c}^{-} \geq \hat{z}_{l,0} \quad ; \forall\, l \qquad \pi_{z_{l,c}}^{-} \tag{201}$$

$$0 \leq z_{l,c} \leq 1 \quad ; \forall\, l \qquad \pi_{l,c}^{z} \tag{202}$$

$$\sum_{l=1}^{NL} \left(z_{l,c}^{+} + z_{l,c}^{-}\right) \leq L \qquad \pi_{c}^{L} \tag{203}$$

$$b_m^c z^c + \eta^c \geq f_m^c + \alpha_m^c \hat{u} + \beta_m^c \hat{p}_0 + \gamma_m^c \hat{r}^U + \vartheta_m^c \hat{r}^D ;\ \forall\, m \in M^c \qquad \pi_c^m \tag{204}$$

$$A_1 = [I_{NL\times NL} \quad -I_{NL\times NL} \quad 0_{NL\times NL} \quad 0_{NL\times 1}]$$
$$A_2 = [I_{NL\times NL} \quad 0_{NL\times NL} \quad I_{NL\times NL} \quad 0_{NL\times 1}]$$
$$A_3 = [I_{NL\times NL} \quad 0_{NL\times NL} \quad 0_{NL\times NL} \quad 0_{NL\times 1}]$$
$$A_4 = [0_{1\times NL} \quad 1_{1\times NL} \quad 1_{1\times NL} \quad 0]$$
$$A_5 = [1_{1\times NL} - e_c \quad 0_{1\times NL} \quad 0_{1\times NL} \quad 1]$$
$$c = [0_{1\times NL} \quad C_l \quad C_l \quad 1]$$

$$A_1^T = \begin{bmatrix} I_{NL\times NL} \\ -I_{NL\times NL} \\ 0_{NL\times NL} \\ 0_{1\times NL} \end{bmatrix}; A_2^T = \begin{bmatrix} I_{NL\times NL} \\ 0_{NL\times NL} \\ I_{NL\times NL} \\ 0_{NL\times 1} \end{bmatrix}; A_3^T = \begin{bmatrix} I_{NL\times NL} \\ 0_{NL\times NL} \\ 0_{NL\times NL} \\ 0_{1\times NL} \end{bmatrix}; A_4^T = \begin{bmatrix} 0_{NL\times 1} \\ 1_{NL\times 1} \\ 1_{NL\times 1} \\ 0 \end{bmatrix}; A_5^T = \begin{bmatrix} 1_{NL\times 1} - e_c^T \\ 0_{NL\times 1} \\ 0_{NL\times 1} \\ 1 \end{bmatrix}$$