



Bernardo Kulnig Pagnoncelli

**Sample average approximation for chance
constrained programming**

TESE DE DOUTORADO

Thesis presented to the Postgraduate Program in Mathematics of
the Departamento de Matemática, PUC-Rio as partial fulfillment
of the requirements for the degree of Doutor em Matemática

Adviser : Prof. Carlos Tomei
Co-Adviser: Prof. Shabbir Ahmed
Co-Adviser: Prof. Alexander Shapiro
Co-Adviser: Prof. Humberto José Bortolossi

Rio de Janeiro
February 2009



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Abstract

Pagnoncelli, Bernardo K.; Tomei, Carlos; Ahmed, S.; Shapiro, A.; Bortolossi, Humberto José. **Sample average approximation for chance constrained programming**. Rio de Janeiro, 2009. 53p. TESE DE DOUTORADO — Department of Mathematics, Pontifícia Universidade Católica do Rio de Janeiro.

We study sample approximations of chance constrained problems through the *sample average approximation* (SAA) approach and prove the related convergence properties. We discuss how to use the SAA method to obtain good candidate solutions and bounds for the optimal value of the original problem. In order to tune the parameters of SAA, we apply the method to two chance constrained problems. The first is a linear portfolio selection problem with returns following a multivariate lognormal distribution. The second is a joint chance constrained version of a simple blending problem. We conclude with a more demanding application of SAA methodology to the determination of the minimum provision an economic agent must have in order to meet a series of future payment obligations with sufficiently high probability.

Keywords

Stochastic Programming. Chance Constraints. Sampling Methods. Provisioning problem.

Pagnoncelli, Bernardo K.; Tomei, Carlos; Ahmed, S.; Shapiro, A.; Bortolossi, Humberto José. **Método da aproximação amostral para restrições probabilísticas**. Rio de Janeiro, 2009. 53p. Tese de Doutorado — Departamento de Matemática, Pontifícia Universidade Católica do Rio de Janeiro.

Estudamos aproximações amostrais de problemas com restrições probabilísticas através da *aproximação pela média amostral* (SAA) e demonstramos as propriedades de convergência relacionadas. Utilizamos SAA para obter bons candidatos à solução e cotas estatísticas para o valor ótimo do problema original. Para ajustar corretamente parâmetros, aplicamos o método a dois problemas com restrições probabilísticas. O primeiro é um problema de seleção de portfolio linear com retornos seguindo uma distribuição lognormal multivariada. O segundo é uma versão com restrições probabilísticas conjuntas de um problema da mistura simplificado. Concluimos com uma aplicação mais exigente ao problema de se determinar a provisão mínima que um agente econômico deve ter de forma a satisfazer uma série de obrigações futuras com probabilidade suficientemente alta.

Palavras-chave

Otimização Estocástica. Restrições Probabilísticas. Métodos Amostrais. Problema de Reserva.

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Ah! Doutor! Doutor!... Era mágico o título, tinha poderes e alcances múltiplos, vários, polifórmicos... Era um pallium, era alguma coisa como clâmide sagrada, tecida com um fio tênue e quase imponderável, mas a cujo encontro os elementos, os maus olhares, os exorcismos se quebravam. De posse dela, as gotas de chuva afastar-se-iam transidas do meu corpo, não se animariam a tocar-me nas roupas, no calado sequer. O invisível distribuidor dos raios solares escolheria os mais meigos para me aquecer, e gastaria os fortes, os inexoráveis, com o comum dos homens que não é doutor. Oh! Ser formado, de anel no dedo, sobrecasaca e cartola, inflado e grosso, como um sapo antes de ferir a martelada à beira do brejo; andar assim pelas ruas, pelas praças, pelas estradas, pelas salas, recebendo cumprimentos: Doutor, como passou? Como está, doutor? Era sobre-humano!...

Lima Barreto, *Recordações do Escrivão Isaías Caminha*.