

Mauro Quiles de Oliveira Lustosa

IRA Codes: Design and Evaluation

Dissertação de Mestrado

Dissertation presented to the Postgraduate Program in Communication Systems of the Departamento de Engenharia Elétrica, PUC-Rio as partial fulfillment of the requirements for the degree of Mestre em Sistemas de Comunicações

Advisor: Prof. Weiler Alves Finamore

Rio de Janeiro
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Prof. Weiler Alves Finamore

Advisor

Departamento de Engenharia Elétrica — PUC-Rio

Prof. Eduardo Sany Laber

Departamento de Informática — PUC-Rio

Prof. Jaime Portugheis

Departamento de Comunicações — UNICAMP

Prof. José Eugenio Leal

Coordinator of the Centro Técnico Científico — PUC-Rio

Rio de Janeiro — August 21, 2009

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Mauro Quiles de Oliveira Lustosa

Mauro Lustosa graduated from Pontifícia Universidade Católica do Rio de Janeiro (RJ, Brazil) in Electrical Engineering with emphasis on Telecommunications, enrolling immediately in the University's graduate programme on Telecommunications Systems.

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Abstract

Lustosa, Mauro Quiles de Oliveira; Alves Finamore, Weiler. **IRA Codes: Design and Evaluation**. Rio de Janeiro, 2009. 84p. MSc. Dissertation — Departamento de Engenharia Elétrica, Pontifícia Universidade Católica do Rio de Janeiro.

Irregular Repeat-Accumulate codes are motivated by the challenge of providing a class of codes that use linear-time encoding and decoding while communicating reliably at rates close to channel capacity. They were introduced by Hui Jin, Khandekar & McEliece in 2000, their article proves that IRA codes achieve channel capacity for the binary erasure channel and exhibit remarkably good performance on the AWGN channel. The theoretical developments supporting IRA codes stem from the efforts at the development of capacity achieving Low-Density Parity-Check codes. LDPC codes were first proposed by Robert Gallager in 1963 and became the subject of intense research during the past decade after being dormant for a long period since its conception. Efforts by many researchers have developed its potential for channel coding in applications as diverse as satellite communications, wireless networks and streaming over IP, as well as studies on its usage in Distributed Source Coding. The goal of this dissertation is the evaluation of IRA codes and the effects of different graph construction methods in its performance. The use of the many variations of the Progressive Edge-Growth algorithm with IRA codes was tested in simulations on the AWGN channel.

Keywords

Irregular LDPC codes. sparse graphs. factor graphs. modern coding theory. capacity achieving codes.

Resumo

Lustosa, Mauro Quiles de Oliveira; Alves Finamore, Weiler. **Códigos IRA: Projeto e Avaliação**. Rio de Janeiro, 2009. 84p. Dissertação de Mestrado — Departamento de Engenharia Elétrica, Pontifícia Universidade Católica do Rio de Janeiro.

Os códigos IRA (Irregular Repeat-Accumulate) são uma classe de códigos criada com o objetivo de permitir codificação e decodificação em tempo linear garantindo comunicação robusta a taxas próximas à capacidade do canal. Eles foram introduzidos por Jin, Khandekar & McEliece em 2000. O artigo no qual foram apresentados provou que os códigos IRA alcançavam a capacidade do canal de apagamento e mostravam desempenho comparável ao dos códigos Turbo no canal AWGN (Additive White Gaussian Noise). Os desenvolvimentos teóricos por trás dos códigos IRA vieram da busca pelos primeiros códigos LDPC (Low Density Parity Check), ou códigos em grafos, que atingiriam a capacidade do canal AWGN. Os códigos LDPC — propostos originalmente por Robert Gallager em 1963 — se tornaram objeto de grande interesse nas últimas décadas após um longo período de ostracismo desde sua concepção, desenvolvendo seu potencial para codificação de canal em aplicações tão diversas quanto comunicações por satélite, redes sem fio e streaming via IP, bem como codificação distribuída de fonte. O objetivo desta dissertação é a avaliação dos códigos IRA e os efeitos de diferentes métodos de construção de grafos em seu desempenho. O uso das muitas variações do algoritmo PEG (Progressive Edge-Growth) foi testado em simulações no canal AWGN.

Palavras-chave

Códigos LDPC irregulares. grafos esparsos. grafos fatores.

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Never trust the brute-force power of a computer network to do the job of a combinatorialist.

Ed Pegg, Jr., *Math Games*.