Pontifícia Universidade Católica
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Ana Beatriz Ract Pousada

Public Sector and the Allocation of Skills in the Labor Market

Dissertação de Mestrado

Dissertation presented to the Programa de Pós–graduação em Economia of the Departamento de Economia, Centro de Ciências Sociais, PUC-Rio as partial fulfillment of the requirements for the degree of Mestre em Economia.

Advisor: Prof. Gabriel Lopes de Ulyssea

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Resumo

Pousada, Ana Beatriz Ract; Ulyssea, Gabriel Lopes de. **Efeitos do Emprego Público Sobre a Alocação de Habilidades no Mercado de Trabalho**. Rio de Janeiro, 2017. 58p. Dissertação de Mestrado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

Esse artigo investiga como o emprego público afeta a alocação de habilidades na economia. Construímos um modelo de Roy onde trabalhadores se selecionam entre os setores público e privado baseados nas suas produtividades e aversões ao risco. O setor privado é caracterizado por pareamento positivos entre habilidades e tarefas, mas tem retornos incertos, o que pode criar um trade-off para os trabalhadores. Estimamos esse modelo para o Brazil usando dados no nível do trabalhadores para os anos de 2011-2014 e fazemos um exercício contra-factual. Resultados mostram que reduzir o tamanho do setor público aumento a produtividade média do setor privado e reduz o prêmio por fazer ensino superior, mas aumenta a desigualdade salarial.

Palavras-chave

Emprego Público; Alocação de Trabalhadores; Tarefas.

Abstract

Pousada, Ana Beatriz Ract; Ulyssea, Gabriel Lopes de (Advisor). **Public Sector and the Allocation of Skills in the Labor Market**. Rio de Janeiro, 2017. 58p. Dissertação de Mestrado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

This paper investigates how public sector employment affects the allocation of skills in the economy. We develop a Roy model where workers self-select into either public or private sectors based on their productivity and risk aversion. The private sector is characterized by positive assortative matching between skills and tasks, but it has uncertain returns, which potentially creates a trade-off for workers. We estimate the model using Brazilian worker level data for the years of 2011-2014 and use it to perform counter-factual exercises. We show that reducing the size of the public sector increases private sector average productivity, decreases the college wage premium, but increases wage inequality.

Keywords

Public Employment; Worker Allocation; Tasks.

Contents

1
Introduction

There is a large literature that studies resource allocation and its consequences to productivity. For instance, (1) and (2) develop theoretical models of talent allocation between productive and unproductive (rent-seeking) activities and find important effects on growth. Moreover, (3) study the allocation of talent in occupations with frictions specific to each gender and color group, and find that the resulting mis-allocation of workers had high consequences for labor productivity. Considering this, the public sector is an interesting case to analyze the allocation of skills because it is not only a large part of the labor market in many countries[1], but it also has distinct features that distort skill returns. Indeed, (5) finds a positive public wage premium in many European countries. Furthermore, (6) argues that the public sector offers non-pecuniary returns such as life-time tenure and pensions. Therefore, given its magnitude and distinguishing characteristics, the public sector is likely to have first order effects on the allocation of skills in the economy.

To investigate how the public sector affects the allocation of skills in the economy we develop an equilibrium Roy model where individuals are heterogeneous in two dimensions, skill and risk aversion. Workers select themselves into public or private sectors based on their risk aversion and sector specific wages. Consequently, it distorts the distribution of skills available to the private sector. Labor demand and wages at the private sector are a function of skills and determined endogenously based on its supply of each skill. Private sector production allocates workers on tasks based on their skills comparative advantage to perform each task. Thus, private sector labor supply of each skill influences productivity and overall inequality, through skill allocation. In addition, they are subject to a productivity shock that makes wages uncertain.

On the other hand, public sector labor demand and wages are exogenous, deterministic and a function of observed characteristics such as education and experience that are correlated with skills. The idea is to capture three of the main differences in public sector personnel practices that influence worker's

[1]For most European countries more than 10% of the labor force is employed in the public sector (21.47% in the UK; 17.9% in France; 16.03% in Italy and 12.67% in Spain in 2013 according to (4)). This is also true in some Latin American countries, such as Brazil, Mexico and Chile where public sector employees are more than 10% of the labor force.

sector selection. First, most public sector hiring and career paths are highly based on individuals observed characteristics.[2] Second, public sector wages are rigid and do not respond to the supply of workers[3]. Third, the public sector is able to make long term promises to its employees, such as good pensions and job security, what makes public sector wages more certain than private sector ones[4].

We estimate this model for the Brazilian labor market using a national household survey (PNAD) for the years of 2011-2014, which is a representative survey of the whole Brazilian population. Estimation has two stages. In the first stage we estimate the distribution of skills in the private sector through maximum likelihood. We suppose the distribution of wages in the private sector is a finite mixture of normal distributions and estimate average wages for each skill and the probabilities of each individual belonging to each skill. In the second stage we use a minimum distance estimator to approximate those moments estimated in the first stage to their model counterparts. In other words, we match wages and the distribution of skills and observed characteristics in both sectors with wages and labor demand predicted by the model.

With the estimated model we perform a counter-factual exercise reducing the size of the public sector and analyze the effects on wages and productivity through the allocation of skills. Brazil is an interesting case for two reasons. First, there was a sharp increase in public hiring from 2005 to 2014, we use the size of the public sector in 2005 as benchmark to interpret our effects. Second, there are significant wage differences between Brazilians' public and private sectors that vary across skills and demographic characteristics. Therefore, Brazil's public sector has a potential to have first order effects on the allocation of skills between sectors.

What distinguishes this paper from the literature is the inclusion of two dimensions of unobserved heterogeneity combined with observed characteristics. The two dimensions of unobserved heterogeneity creates a trade-off between risk and return for individuals when choosing which sector they are going to work at, that is specific for each demographic characteristic group. Furthermore, the inclusion of observable characteristics allows us to analyze the effects on productivity and wages of increasing public sector employment

[2]In Brazil public sector hiring is done by job specific exams and civil service career paths are clearly set, this is also true for some European countries such as Italy.

[3]Public sector size and wages must be on that year's budget, that must be approved by congress. Furthermore, public hiring is made through competitive exams that take a while to set up.

[4]Several countries offer lifetime tenure to public employees, such as, Brazil, Germany and Spain.

in specific educational groups. In addition, we provide a new framework to estimate models of factor allocation with unobserved skills, that can be adapted to other contexts besides public and private sector selection. We estimate this model for Brazil, however, it can be easily applied to other countries with similar public employment institutions, such as many European countries.

We show that the distribution of risk aversion in the economy is highly concentrated around risk neutrality. However, it has a high dispersion with some individuals being very risk averse[5]. Moreover, the public sector has a higher proportion of high skill workers than low skill ones, thus it is more attractive for high-skill workers. Consequently, private sector distribution of skills is more concentrated around low skill workers. As a result, more than half of private sector tasks are performed by low skill individuals, and a reduction in public sector increases the amount of tasks performed by high-skill workers. Hence, reducing the size of the public sector increases private sector average productivity as a result of a more balanced allocation of skills to tasks.

Furthermore, the public average wage premium is greater for those with higher education than for those with no College degree, in line with other empirical estimations of Brazilian public-private pay differences[6]. Our counterfactual exercise shows that a reduction on the size of the public sector increases the public average wage premium for those with no College and decreases for those with some College. This happens because the increase in the proportion of high-skill workers at the private sector is mostly of individuals with no College degree. As a consequence, average private sector wages for workers with no College degree increases and we observe a sharp reduction in the College wage premium. However, a reduction in public employment increases overall wage inequality. This happens because the rise in the proportion of high-skill individuals that work for the private sector is not accompanied by a same size reduction in high skill wages.

This paper is related to the literature on the implications of the allocation of talent for productivity. (2, 1) discuss the effect of the allocation of talent between productive or unproductive (rent seeking) activities. In my model, the public sector is an unproductive activity, and it distorts the allocation of skills between sectors in the economy[7]. There is also a large literature that analyzes

[5](7) estimates the distribution of risk aversion using insurance choices and also find that individuals are concentrated near risk neutrality and their risk aversion distribution also has heavy tails.

[6]See for example (8).

[7]It is also possible to include the public sector as a productive activity. One way of doing so is assuming that the public good increases private sector productivity. Another way is assuming that the public sector makes a lump sum transfer to workers, thus affecting workers well-fare, but not production.

macroeconomic consequences of talent mis-allocation, for example (9) study occupation decision based on the return for talent and occupational frictions specific to each gender and color group. We use the differences between public and private sectors to analyze skill mis-allocation. In our model, private sector production, is based on models that draw a clear distinction between skills and tasks, such as (10, 11, 12). We apply their framework to a two sector economy (public and private), where individuals are heterogeneous in two dimensions (skills and risk aversion). In this framework, the two dimensions of heterogeneity are relevant, creating a trade-off between risk and return on sector selection.

There are a few articles discussing effects of public employment in the economy. (13, 14) calibrate a general equilibrium model for Brazil to estimate aggregate effects of public employment and find that there are welfare gains reducing the size of public employment in Brazil. With a model estimated for the European economy (15) find that public employment can increase unemployment in response to an increase in economic turbulence and (16) estimate a search model for the British economy. Differently from these papers we are able to assess the effects of public employment on the distribution of skills available to the private sector, labor productivity and wage inequality. Applying a framework that draws a clear distinction between skills and tasks we are able to see how changes in the supply of specific skills affect private sector wages and production. Moreover, those articles do not account for the different hiring practices that makes public sector labor demand more correlated with observed characteristics than unobserved ones, what creates unique interdependencies between public and private labor markets.

This article is also related to the literature that investigates the effects of public sector motivation on the allocation of workers in the public sector. (17, 18, 19, 20) show that choosing civil servants with public sector motivation increases productivity in the public sector which has positive effects on the private sector. However, in my model the public sector is an unproductive activity, therefore public sector production has no effect on private sector productivity or workers well-fare. On the other hand, this paper focus on the effect of public sector employment on the allocation of workers in the private sector. We chose not to account for public sector motivation for simplicity and because it is not relevant for private sector production, although the model can be easily extended to include it.

This article is organized as follows: section 2 presents the data and discusses some key facts about public employment that motivates our modeling choices; section 3 presents the model; section 4 discusses the identification

and estimation of the model; section 5 shows the estimated parameters and model fit; section 6 present a counter-factual exercise; section **??** shows some extensions of the model followed by a conclusion.

2
Data and Background

This section presents our data and some facts about the public sector around the world and in Brazil with the objective of rationalizing the main choices we made when constructing the model. We show how public sector institutions differ from private sector ones around the world and analyze Brazil's case.

2.1
Data

We use data from a National household survey conducted by the Brazilian National Bureau of Statistics (PNAD - IBGE) for the years of 2011 to 2014. This household survey consists in a series of repeated cross sections representative at the National level. The sample is restricted to individuals between 18 and 40 years old, to focus on the age workers usually still transition between sectors. We do not model schooling decisions, thus we use only individuals that have finished their education. In addition we use hourly wages to account for the fact that on average public sector workers work less hours per week than private sector workers. We also exclude rural workers and individuals not economically active. The final estimation sample consists of 269,147 observations divided into 4 years.

We separate workers into 10 groups of observable characteristics, 5 education groups: (1) High-School drop-outs or less; (2) Finished High-School; (3) College drop-outs; (4) College and (5) Grad-School; and 2 age groups: (1) younger than 30 years old and (2) older than 30 years old, as a proxy for experience. We choose education and age to separate the sample into demographic groups because both are highly correlated with skill acquisition. Moreover, they are also the main characteristics taken into consideration for public sector hiring and promotion. Table 2.1 shows some statistics on key variables for these groups. The first two columns shows the distribution of these groups in the population and their public sector participation. Public workers are more concentrated between College educated and those that own a Graduate Degree, indicating that public sector may attract more high-skilled workers than low-skilled ones. Moreover public sector participation in the work

Table 2.1: Descriptive Statistics

	Distribution		log-wages - Pub.		log-wages - Pri.	
	ALL	% Pub.	mean	var.	mean	var.
Young						
HS drop-outs	0.156	0.005	1.403	0.142	1.300	0.379
HS	0.226	0.024	1.731	0.414	1.481	0.352
College drop-outs	0.012	0.046	1.993	0.314	1.814	0.420
College	0.061	0.130	2.474	0.530	2.253	0.546
Grad-School	0.002	0.200	2.820	0.862	2.783	0.628
Old						
HS drop-outs	0.219	0.013	1.492	0.195	1.438	0.469
HS	0.204	0.060	1.848	0.407	1.716	0.477
College drop-outs	0.013	0.085	2.313	0.495	2.150	0.620
College	0.102	0.239	2.620	0.529	2.571	0.671
Grad-School	0.006	0.320	3.174	0.496	3.143	0.760

Notes: the two first columns show the distribution of education levels in the population (first column) and participation of public employment in each education level (second column). The last four columns show average wages and their variances by sector. Data from PNAD-IBGE 2011-4, only individuals between 18 and 40 years old that are not in school.

force is also higher between older people.

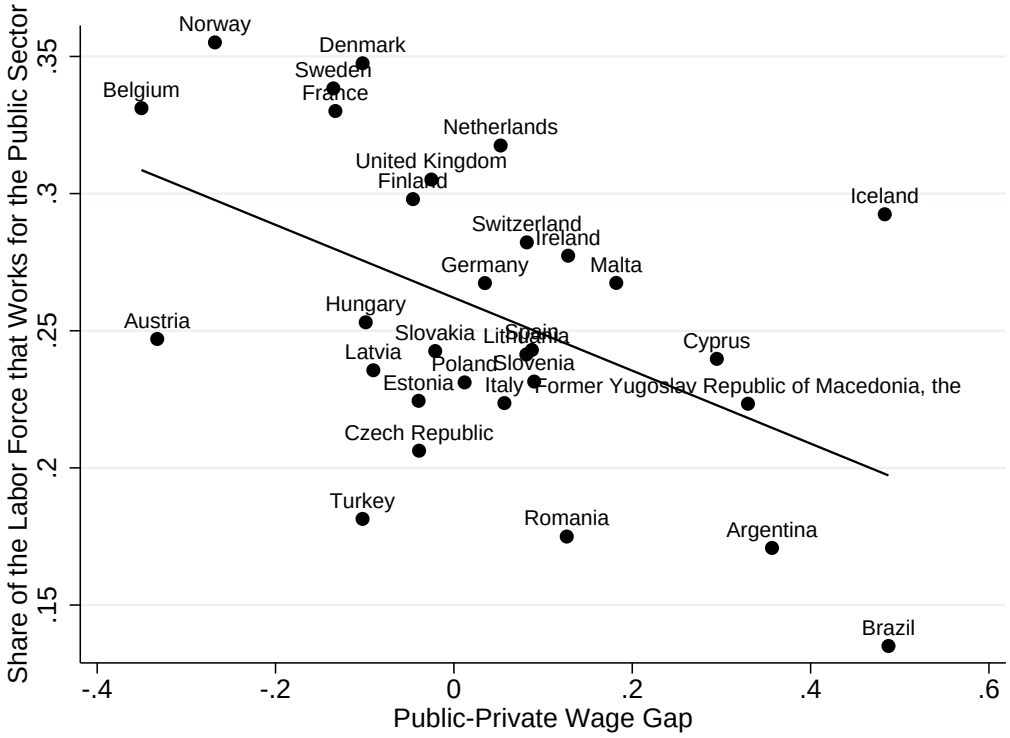
One of our main assumptions in the model is that public sector wages are more rigid and certain than private sector ones. The last four columns of Table 2.1 show evidence that is compatible with this assumption, it presents average log-wages for public and private sector with their variances. Notice that wages increase with educational level and public sector variances are smaller than private sector ones. With the exception of individuals younger than 30 years old with graduate school, who have public sector log-wage variances higher than private sector ones. Furthermore, notice that public sector wages are always higher than private sector ones, suggesting that wage differentials may play an important role regarding sector decision for some skills.

2.2 Facts About the Public Sector

Public sector workers are a large part of the labor market in most countries, but public-private wage differences vary a lot between them. Figure 2.1 presents public sector employment as a share of the labor market and average log-wage differences for a number of European and Latin American countries in 2014. The data for the European countries was collected at EuroStat, for Brazil at PNAD and for Argentina at INDEC. It is possible

to observe that, for all countries in the sample, public employment represents more than 10% of the labor force. Although Brazil's public sector share of the labor force is not as high as in most European Countries, it is still large[1] and has one of the biggest wage differences suggesting high allocation effects.

Figure 2.1: Public Sector Size and Wage Premium across Countries - 2014



Notes: Relationship between public employment and public-private average log-wage differences for several European and some Latin American countries. Data for the European countries comes from *EuroStat*. Data for Brazil comes from PNAD-2014 individuals between 18-65 years old and data for Argentina comes from INDEC. All values are for 2014.

Furthermore, there are some institutional differences in public sector hiring and job progression practices that influences workers decision to which sector they are going to work at. (6) described five main distinguishing features between public and private sector employment: first, the public sector has a long-horizon; second, there is a limited set of contracts the public sector can offer; third, services provided by the government lack competitive pressure; fourth, the mission of the organization; and fifth, the state has to self-regulate. I will pay further attention to the first and second distinguishing features. The last three distinguishing features are more related to the literature about public sector motivation and how to select public employees to increase state

[1]According to the Brazilian Institute of Geography and Statistics (IBGE - "Perfil dos Municípios Brasileiros 2014) 3.8 million workers were listed as public employees in Brazil.

efficiency[2]. The focus of this article is on how these distinguishing features affect workers sector decision influencing the allocation of skills in the private sector.

First, the public sector has a long horizon, which means that it can make long term promises to its employees. This is why, in most countries, public workers have good pensions and job security. For instance, several European countries offer life-time tenure for public employees, such as Germany, France, Italy and Spain[3]. This is also the case for Brazilian public sector workers. This countries also have preferential pensions schemes for public employees that guarantees much better retirements than for private sector workers. In addition, in most of this countries public sector workers follow clear career paths once they enter the public sector. The combination of all of these institutions makes public sector wages much more certain than private sector ones.

To illustrate this point, figure 2.2 shows current job tenure in years (years working at the current job) by age and sector. Public sector (dashed line) workers are working longer at the same job than private sector (solid line) ones. This is consistent with the fact that public sector workers have job security and, therefore, the older they are the longer they are working at the public sector. On the other hand, if a private sector firm faces an economic shock in the private sector it can adjust wages and labor demand accordingly.

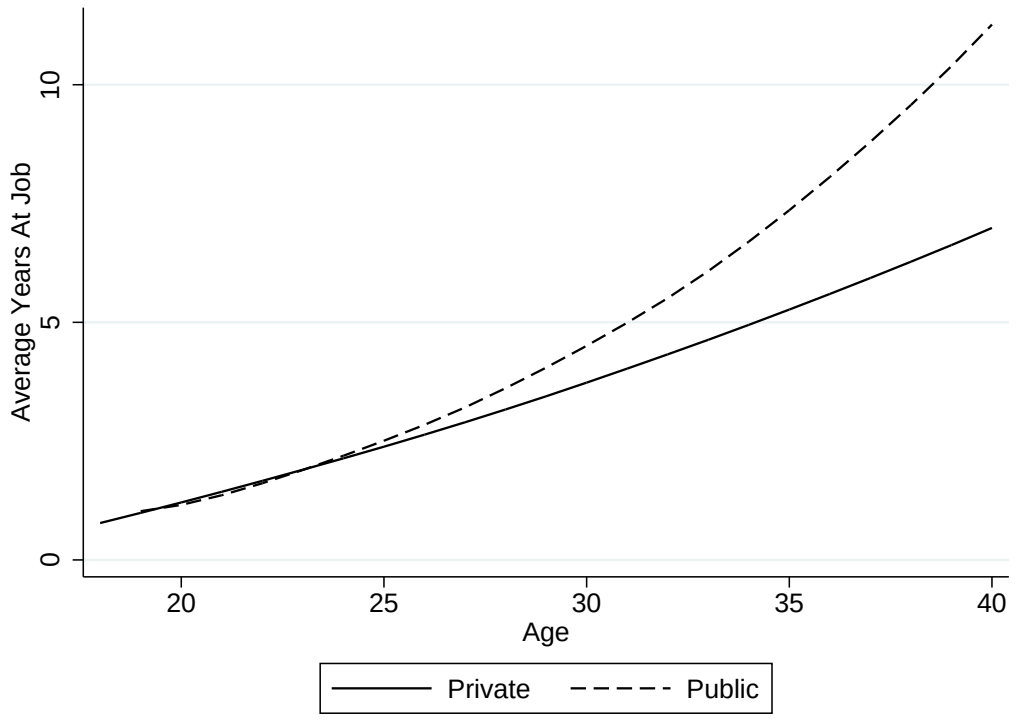
One consequence of this feature is that there is little transition between sectors. Since, PNAD is a series of cross-sections we could not use this data to calculate the transition matrix between sectors. For this statistic we used PNAD-contínua (2012-4), which is also a representative sample of the Brazilian population that follows a household for three quarters and then performs another follow-up interview four quarters after the third quarter. Similarly, we restricted the sample to individuals between 18 and 40 years old that are currently working and not in school. Table 2.2 shows the transition matrix between sectors using this data. It is possible to observe that more than 78% of public sector workers younger than 30 years old do not change sectors in the next period. Furthermore, for workers older than 30 years this probability is larger than 85%. Indicating that most of public sector workers once that have chosen to work at the public sector stay there permanently.

The second main difference between public and private sectors is that there is a limited set of contracts that the public sector can offer to its em-

[2]For more on public sector motivation and the allocation of workers inside the public sector see (17) and (19)

[3]Most of the information about public sector institutions in European countries is from (5)

Figure 2.2: Job Tenure by Sector - Smoothed



Notes: Current job tenure by age and sector smoothed. Data is from PNAD-IBGE (2014), only individuals between 18 and 40 years old that are currently working.

Table 2.2: Public - Private Transition Matrix

	Sector in t-1	
	Public	Private
Public Sector in t		
Younger than 30	0.781	0.012
	(0.002)	(0.000)
Older than 30	0.856	0.015
	(0.001)	(0.000)

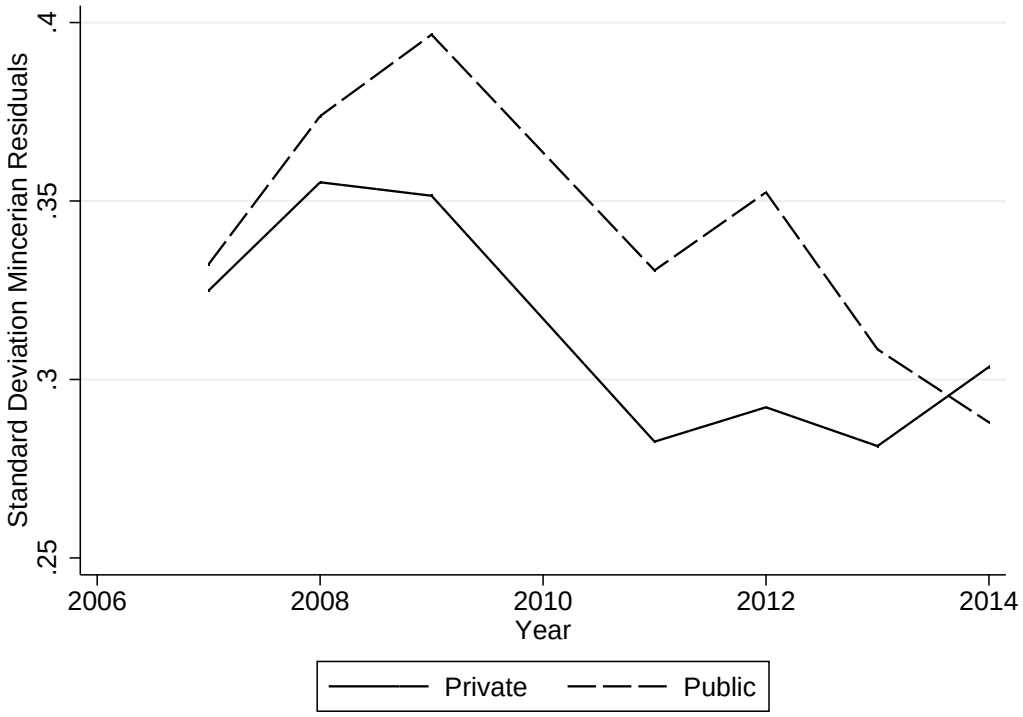
Notes: Proportion of workers that worker at the public sector by age and which sector they come from. Data is from *PNAD-contínua* 2012-2014, only individuals between 18 and 40 years old that are not currently in school.

ployees. To detach public hiring, firing and promotion decisions from the political process, countries have rigid public employment rules. For instance, in many countries public sector hiring is made through open exams which are highly correlated with formal education, this is the case for France, Italy and Spain and also for Brazil. Moreover, in those countries the government sets clear career paths that determine employee's promotion based on education

and experience. This makes public sector wages mostly determined by education and experience.

Figure 2.3 shows the fit, represented by the R^2, of a Mincerian regression of log-hourly-wages on a dummy if the person is male, a dummy if the person is white, dummies for each year of age and dummies for each year of education, by year and sector. Wages at the public sector (dashed line) are more explained by observable variables than at the private sector (solid line). This illustrates the point that public sector wages are more correlated with education and experience than private sector ones.

Figure 2.3: Fit of a Mincerian Regression by Sector



Notes: R^2 of a Mincerian regression of log-hourly-wages on a dummy if the person is male, a dummy if the person is white, dummies for each year of age and dummies for each year of education, by year and sector. Data is from PNAD-IBGE (2002-2014), only individuals between 18 and 40 years old that are currently working.

Furthermore, this rigidity in public sector employment decision allows it little flexibility to respond to fluctuations of labor supply. If the government wants to hire more public employees it has to open a public hiring exam, announce it in the Official State Newspaper with months in advance. Moreover, it has to be on that year's government budget which is approved by Congress. All of this process can take months or even years, that's why we will suppose that public labor demand is determined exogenously. This is also true for public

sector wages, increases in public employees wages must also be approved by Congress in a process that could take months. Therefore, public sector wages do not adjust to promote market clearing at the public sector labor market.

Since public sector wages are exogenously determined, they do not follow the one price rule, in other words, similar individuals may face different wages in each sector. This, in combination with public sector wages being more correlated with observable variables than unobservable ones, creates interesting public-private wage differentials that may influence individuals sector decision. To estimate how the public-private wage differences varies along the wage distribution, we estimated the following quantile regression, with $Q_w(\tau|x, pub)$ being the τth quantile of the wage distribution conditional in a set of controls (x) and a dummy if the person works for the public sector (pub):

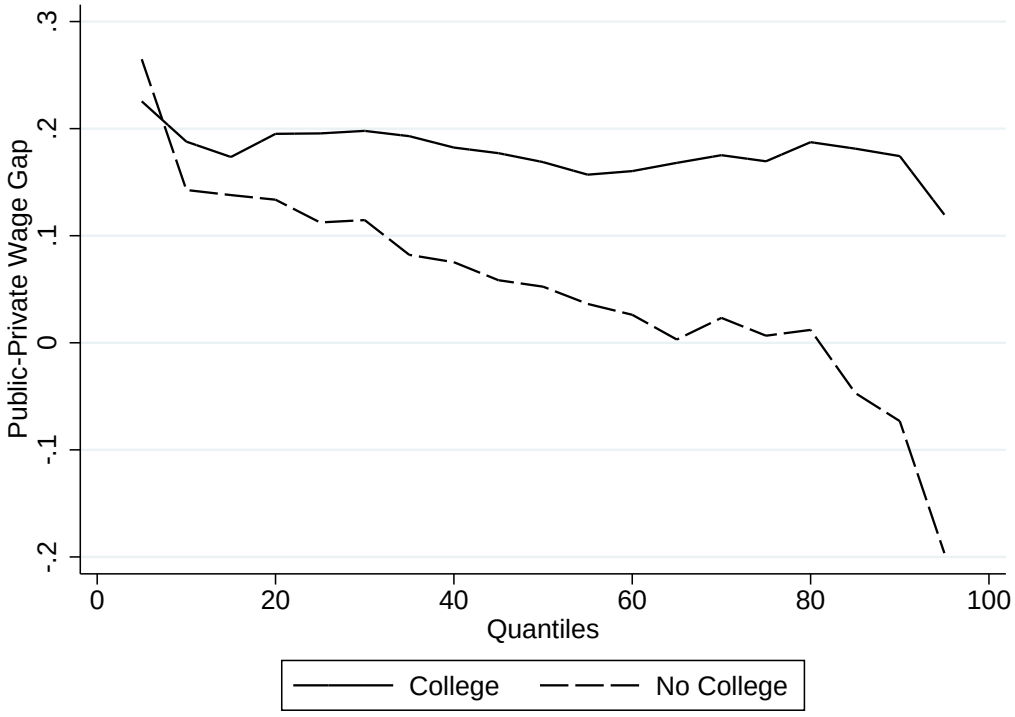
$$Q_w(\tau|x, p) = \alpha(\tau) + \beta(\tau)pub + \theta(\tau)x \tag{2-1}$$

Figure 2.4 shows the $\beta(\tau)$ coefficient of this quantile regression for each quantile. We use as controls a dummy if the person is male, a dummy if the person is white, dummies for each year of age and dummies for each year of education. It shows the large heterogeneity of public-private wage differentials across the wage distribution.

This figure does not show the relative wages individuals actually face, because it does not include a series of unobserved characteristics such as skills and risk aversion. Nevertheless, it still provides suggestive evidence of the trade-offs some individuals face when deciding which sector to work for. First notice that individuals without College degree (dashed line) face a greater heterogeneity of wage differentials across the wage distribution than individuals with a College degree (solid line). This can be explained by the fact that public sector wages are more correlated with education than private ones. In private sector firms, individuals with no College degree, but high skills, may get high wages. However, in the public sector even high-skills individuals do not get high-wages without a College degree. Therefore, the public-private wage gap is very high at the beginning of the wage distribution, but it becomes negative at the end of the wage distribution, for individuals without College.

Wage differentials for workers with a College degree follow a similar pattern, decreasing on the quantiles, but with a smaller inclination. Moreover, for most quantiles, the public-private wage gap is greater for workers with College than for workers without College. This suggests that the trade-offs individuals face when choosing a sector are also correlated with education and unobserved skills. Nevertheless, this estimation does not account for individuals specific risk aversion. Some individuals may work at the public sector even

Figure 2.4: Public-Private Wage Gap Using Quantile Regression



Notes: Coefficient of a dummy if an individual works at the public sector of a quantile regression of log-hourly-wages on a dummy if the person is male, a dummy if the person is white, dummies for each year of age and dummies for each year of education, by education and sector. Data is from PNAD-IBGE (2014), only individuals between 18 and 40 years old that are currently working.

with a negative public-private wage gap, because public sector wages are more certain than private sector ones. Therefore, to consistently estimate the public-private wage premium we would need to account for this two unobserved variables, skills and risk aversion.

There is a large literature studying the public private wage gap for several countries, but most of the evidence is not causal. (21) study the public-private wage gap in France using quantile regression in a panel dataset with individual fixed effects and find a positive wage premium in the beginning of the wage distribution, and a negative for the end of the distribution, both for men and women. (22) do a similar study for Brazil, but not using worker fixed effects, and find a positive public-private wage gap at low percentiles of the distribution and negative at the high percentiles. (23) also uses quantile decomposition, this time for several European countries[4] and find similar results with positive premium at the mean for all countries and decreasing premium along quantiles. Overall, those articles show evidence in line with our estimations shown in

[4]Austria, Belgium, Germany, Spain, France, Greece, Ireland, Italy, Portugal and Slovenia.

figure 2.4, even after controlling for unobserved heterogeneity using fixed effects.

Furthermore, there are some papers that estimate the average public wage premium and across education and experience groups. (5) studies the public-private premium for several Europeans countries[5], but this time accounting for lifetime earnings and find a positive premium for all countries. For Brazil, (24) studies the public wage premium by education in 2005 and find a public-private wage gap decreasing with years of schooling. However, (8) estimates the public wage premium for Brazil using the privatization of some State owned companies at the end of the 1990's as an exogenous shock and find it positive and increasing with years of schooling. Their estimations control for both observed and unobserved heterogeneity, because they are able to observe the same individual before and after the privatization.

[5]Germany, Netherlands, France, Italy and Spain.

3
The Model

3.1
Set Up

The model is a two-sector equilibrium static Roy model, where individuals must choose between public and private sector jobs. Besides demographic characteristics, which are observed by the econometrician, workers are heterogeneous in two unobserved variables their skills and risk aversion. Public sector wages are certain and a function of these demographic characteristics, while private sector wages are uncertain and a function of skills. Therefore, each individual faces different trade-offs when deciding which sector to work at, depending on their skills, demographic characteristics and risk aversion. The private sector labor demand makes a clear distinction between skills and tasks. Changes in the public sector may affect the distribution of skills available to the private sector through workers sector decision. The skill distribution available to the private sector affects the matching function between skills and tasks having consequences on wages and productivity.

There is a continuum of workers of size one, with a finite discrete set of skills $s = 1, ..., S$ and a continuous and unbounded set of risk aversions $\beta \in [0, +\infty)$. Skills and risk aversion are independent and known by the worker and private sector firms, but not observed by the econometrician and the public sector. There is a set of discrete and finite characteristics $e = 1, ..., E$, such as education, that are jointly distributed with skills by $P(s, e)$[1], such characteristics are common knowledge. Because the public sector has rigid hiring and promotion rules, public sector wages and labor demand depend on these observable characteristics (e). On the other hand, private sector wages and labor demand are a function of skills (s).

We can interpret e as a proxy for skills. Education decisions here are taken as exogenous to simplify the analysis. However, those with higher skills may self-select into higher education. To control for this factor we allow for a non parametric specification of the joint distribution of skills and demographic

[1]One possible extension is to set skills as continuous, in this case each e will have a different skill distribution. In the model developed by (12) skills are continuous.

characteristics. Therefore, in this case skill can be something that is acquired through education and it is not necessarily an intrinsic characteristic, but some individuals may also never be able to reach some skill types. Another issue with taking education decisions as exogenous is that individuals may choose to stay more years in school with the objective of applying to a public job. Although this may be true, when an individual decides to earn another degree she does not know what will be the size and wages of the public sector when she finishes it. Since the individual can not predict public sector wages and employment at their graduation, they do not influence education decision.

Even though dynamics is an important factor on an individuals' decision to work at the public sector, the model is static for simplicity. Our objective is to model skill allocation and its effects on productivity and inequality, not necessarily to examine dynamic choices. There are two possible interpretations of the model that explains its static nature. First, that when the individual chooses sector she makes a permanent decision taking into account her lifetime earnings in each sector. Second, that each period the worker chooses which sector she is going to work for, taking into account wages at both sectors at this period.

The model does not account for the extensive margin, in other words, individuals cannot choose if they will participate in the labor market. One interpretation is that individuals must work and cannot choose to stay out of the labor market. Another interpretation is that we are modeling the decisions of the subpopulation that have already chosen to participate in the labor market. However, this is an issue if the decision to participate depends on individuals skill and observable characteristic through public or private returns. Appendix A discusses a model extension that includes the decision to participate in the labor market and its consequences to our results.

We do not account for public sector production, since our objective is to analyze the effects on private sector production. To rationalize it the public sector can be seen as an unproductive or rent-seeking activity. Another possibility is that public workers produces a public good that is a lump-sum transfer to workers. Therefore, this public good does not influence workers sector decision, but increases their welfare. In this interpretation a reduction on the size of the public sector reduces workers welfare through the public good. Appendix B discusses a possible extension that includes this lump-sum transfer.

3.2

Labor Supply

We assume that individuals consume everything they earn, therefore, their utility depends only on wages. Bernoulli utility is $u(w|\beta)$, where β is the risk aversion parameter which is specific to each individual and distributed according to $G(\beta)$. Individuals know their risk aversion parameter, but the econometrician does not observe it. We assume that everyone is risk averse, but they differ in their intensities ($\beta \geq 0$)[2].

Wages in the public sector are a function of observable characteristics e, while wages in the private sector are a function of skills s. This captures the fact, discussed earlier, that wages at the public sector are more rigid to be exempt from political influence, thus public sector hiring is made through exams that are highly correlated with formal education and promotion is due mostly to experience and education. On the other hand, the private sector can have a more flexible hiring and promotion process being able to identify the skill of the worker. In other words, the public sector knows only observable characteristics of the worker (e), while the private sector can observe skills (s). Public sector wages are certain, because the public sector can make long-term promises to its employees such as job security and clear career paths. Private sector wages are uncertain, since private sector firms are subject to productivity shocks and can adjust wages and/or employment accordingly. The determination of private sector wages will be explained in subsection 3.3, but it is influenced mostly by the distribution of skills available to the private sector.

Since public sector labor demand and wages are exogenous, there may be more people applying to public jobs than there are job openings. Therefore, there must exist a selection rule that chooses which of the workers that applied will be hired. Once an individual chooses to work in the public sector there is a probability $b(e)\tau_s$ that she will be accepted to work there. With probability $(1 - b(e)\tau_s)$ she is not accepted and her wage is zero. This probability depends on how many individuals with the same observable characteristics are applying, through $b(e)$, but also accounts for the fact that individuals with higher skill have a higher probability of being accepted ($\tau_{s'} > \tau_s \Leftrightarrow s' > s$). For instance, Brazilian public sector selection uses exams, in this case τ_s represents the fact that individuals with higher skill do better on these exams.

If we interpret sector decision as a lifetime decision, this probability can be seen as a cost of entering after choosing the public sector. In Brazil's case, this cost could be rationalized as the number of months that the worker will

[2]This hypothesis is not necessary to close the model and can be relaxed, but it is here for computational purposes.

be without wage, due to exam preparations. On the other hand, if the decision is made every period it is the probability that the worker will get a public service position if she chooses the public sector this period, in the case she does not get hired her wage will be zero for this period. Notice that this attaches some uncertainty to the public sector choice, but this uncertainty depends on individuals' skill level. To sum up, the utility of a worker who chooses the public or private sector is respectively:

$$U_b(s,e) = b(e)\tau_s u(w_b(e)|\beta) \tag{3-1}$$

$$U_r(s) = E[u(w_r(s)|\beta)|s,\beta] \tag{3-2}$$

The proportion of workers with skill s and observable characteristic e who choose the private sector is:

$$P_r(s,e) = P\left[U_b(s,e) < U_r(s)|s,e\right] \tag{3-3}$$

In other words, this is the probability that if we select a worker at random in the population her private sector expected utility ($U_r(s)$) is higher than her public sector one ($U_b(s,e)$). Notice that workers, when deciding which sector to work, must account not only for average wages for her skill level, but also for uncertain private sector wages and the probability of being selected to work for the public sector.

3.3 Labor Demand

Private sector labor demand is based on the models developed by (12) and (11), which draw a clear distinction between skills and tasks. The private sector produces one final good using a continuum of tasks of size one ($t \in [0,1]$). The idea is that tasks produce output and workers are hired to perform such tasks and specific worker's skills have comparative advantage to perform specific tasks. Firm's problem in the private sector yields a matching function from skills to tasks.

The private sector is subject to a productivity shock v, that is distributed according to $F(v)$ with $E(v) = 0$. This productivity shock creates uncertainty in private sector wages. Since the model is static, this productivity shock can be interpreted as the effect of many shocks that will happen throughout worker's lifetime, if we interpret sector decision as a permanent decision made when young. Another explanation, is that the worker may face a transitory shock if she chooses the private sector in that period. The production function of the final good is a C.E.S. aggregation of the different tasks ($y(t)$), where A is a

productivity increasing parameter.

$$Y = A \cdot v \cdot \left[\int_0^1 y(t)^{\frac{\eta-1}{\eta}} dt \right]^{\frac{\eta}{\eta-1}} \tag{3-4}$$

The production of each task uses only labor and different skills are perfect substitutes in the production of tasks. However, each skill has its own productivity in producing each task, represented by the function $B(s,t)$. The demand for skill s to produce task t is $L(s,t)$ and determined endogenously in the model.

$$y(t) = \sum_{s=1}^{S} B(s,t)L(s,t) \tag{3-5}$$

The final good firm, which is a representative firm, chooses the demand for tasks and the task producers choose labor demand for their specific task. In this case, $p(t)$ is the task price and $w_r(s)$ are the equilibrium wages. The problems of the final good producer and the task producer are respectively the following:

$$\underset{y(t)}{\text{Max}}\, A \cdot v \cdot \left[\int_0^1 y(t)^{\frac{\eta-1}{\eta}} dt \right]^{\frac{\eta}{\eta-1}} - \int_0^1 p(t)y(t)dt \tag{3-6}$$

$$\underset{\{L(s,t)\}_s}{\text{Max}}\, p(t) \cdot \sum_{s=1}^{\bar{S}} B(s,t)L(s,t) - \sum_{s=1}^{\bar{S}} w_r(s)L(s,t) \tag{3-7}$$

We can interpret tasks as intermediate goods, that are aggregated to produce the final good. However, another, more general interpretation is to see tasks as a work activity that produces output[3]. Under this interpretation tasks are not necessarily intermediate goods, but what workers actually do to produce the final good. For instance, some workers are hired to clean the firm, others are hired to operate machines while some are hired to develop the line of production. The main advantage of this framework is that workers with different skills can produce all tasks and the mapping between skills and tasks depends on market conditions. Therefore, if the supply of high skill workers decreases the firm will hire some middle skill workers to develop the line of production, but this middle skill workers will be less productive to do this task than high-skill workers. The framework where labor demand for each skill appears directly in the production function of the final good producer is a special case of this model in which there is a one-to-one mapping between skills and tasks.

Notice that wages can only vary across skills, but not tasks, because of the law of one price. If the same skill has two possible wages everyone would choose to do the task with the higher wage and in equilibrium the wages will be the same. Skill specific productivity function $B(s,t)$ resumes the comparative

[3]This interpretation comes from (11).

advantage each skill has to produce specific tasks. Task skill intensity grows with t, hence tasks defined by smaller t are less skill intense than tasks defined by higher t. We assume workers with low skill have comparative advantage to do less intensive tasks, and vice-versa, which is the same as saying that $B(s,t)$ is log-supermodular:

Assumption 3.1 $B(s',t')B(s',t) \geq B(s,t')B(s,t), \ \ \forall\, s' > s \ and \ t' > t.$

Which implicates that there is positive assortative matching between skills and tasks in equilibrium. Under this assumption private sector labor demand is determined by the following lemma[4]:

Lemma 3.2 $\exists\{T_s\}_{s=0}^{\bar{S}} \mid T_s \in [0,1] \ \forall s \ , \ T_{\bar{S}} = 1 \ e \ T_0 = 0$, *such that,* $\forall t \in [T_{h-1}, T_h], \ L(h,t) > 0 \ e \ L(s,t) = 0 \ \forall s \neq h$

In other words, there is a sequence of thresholds $\{T_s\}_{s=0}^{\bar{S}}$ that determine which tasks each skill is going to be demanded to perform. This sequence of thresholds is a matching function that links skills to tasks, which represents the allocation of skills to tasks. It is determined by market clearing and depends on the skill distribution available to the private sector.

To close the model it is necessary to assume that, at the thresholds, both skill types are equally profitable, given the market wages and prices.

$$\begin{aligned} p(T_s) \cdot B(s, T_s) \cdot L(s, T_s) - w_r(s) \cdot L(s, T_s) = \\ p(T_s) \cdot B(s+1, T_s) \cdot L(s+1, T_s) - w_r(s+1) \cdot L(s+1, T_s) \end{aligned} \tag{3-8}$$

Which implies that:

$$\frac{L(s, T_s)}{L(s+1, T_s)} = \frac{w_r(s+1)}{w_r(s)} = \frac{B(s+1, T_s)}{B(s, T_s)} \tag{3-9}$$

Therefore market clearing wages must be equal to the ratio of comparative advantage at the thresholds to exist an equilibrium. The first order condition of the final good producer is:

$$y(t) = \frac{Y \cdot (A \cdot v)^{\eta-1}}{p(t)^{\eta}} \Rightarrow \int_0^1 p(t)^{1-\eta} dt = (A \cdot v)^{1-\eta} \tag{3-10}$$

Wages are equal to the marginal productivity of labor, $w_r(s) = p(t) \cdot B(s,t)$. Combining this with the arbitrage condition 3-9 and the first order condition of the final good 3-10 we can calculate wages as a function of the thresholds. The labor demand for each skill is:

$$l_r(s) = \int_{T_{s-1}}^{T_s} L(s,t) dt \ \text{ with } \ L(s,t) = \frac{H \cdot (A \cdot v)^{\eta} B(s,t)^{\eta-1}}{w_r(s)^{\eta}} \tag{3-11}$$

[4]demonstration follows (11).

Where $H = \left[\int_0^1 y(t)^{\frac{\eta-1}{\eta}} dt\right]^{\frac{\eta}{\eta-1}}$ is the average product of the private sector discounting the effects of productivity parameter A.

Public sector demands workers exogenously based on observable characteristics, where $l_b(e) \in [0,1]$ is the public sector labor demand for worker characteristic e and $w_b(e)$ is the public sector wage also determined exogenously. This captures the fact that variations on public labor demand and wages has to go through bureaucratic issues that does not allow them to respond immediately to market movements.

3.4 Equilibrium

Public sector labor demand depends on observable characteristics (e) and private sector labor demand depends on skills (s). Therefore, we define the public and private labor supply as:

$$P_b(e) = \sum_{s=1}^{S} (1 - P_r(s,e)) \cdot P(s,e) \tag{3-12}$$

$$P_r(s) = \sum_{e=1}^{E} P_r(s,e) P(s,e) \tag{3-13}$$

In equilibrium public and private labor markets clear, $P_b(e) = l_b(e), \forall e = 1, ..., E$ and $P_r(s) = l_r(s), \forall s = 1, ..., S$. Notice that while private sector market clearing depends on skills public sector depends on observable characteristics. This generates different public-private wage gaps for individuals with the same observable characteristic. Lastly, equilibrium can be defined as:

Definition 3.3 (Equilibrium) *Equilibrium is a sequence of thresholds* $\{T_s\}_{s=0}^{\bar{S}}$*, a private sector average product* H *and a vector of probabilities of being accepted to work at the public sector* $b(e)$ *such that private sector and public sector labor markets clear (*$P_b(e) = l_b(e)$ *and* $P_r(s) = l_r(s)$*).*

Since private sector wages depend on skills, even individuals with the same demographic characteristics face different trade-offs when choosing a sector. Therefore changes in public employment for specific demographic groups have different effects on the private sector distribution of skills and, hence, on productivity and wages. Even aggregate changes in public hiring have different consequences for each demographic group. This generates interesting interdependencies between skills and observable characteristics that will be further explored when we conduct counter-factual exercises.

4
Estimation and Identification

4.1
Functional Forms

In order to be able to estimate the model we are going to assume some functional forms. The functions described in the model that need parametric assumptions are: the Bernoulli utility $u(w|\beta)$; the distribution of risk aversion $G(\beta)$; the distribution of the productivity shock $F(v)$; and the function that defines the skill comparative advantage $B(s,t)$[1].

First we assume that Bernoulli utility has a C.R.R.A. form $u(w|\beta) = w^{1-\beta}$, thus the individuals with high risk-aversion have higher β. Which means that individuals have decreasing absolute risk aversion, in other words, with this utility function workers accept more risk if the premium is higher. In addition, we assume that the distribution of risk aversion follows a Weibull distribution with shape parameter k and scale parameter λ ($\beta \sim \text{Weibull}(k,\lambda)$). The main reason for choosing this distribution is that it is very flexible.

Finally, we assume that the distribution of the productivity shock is log-normal ($v \sim \text{lognormal}(0,\sigma^2)$). This implies that the distribution of wages for each skill is also log-normal, as it is consensus in the literature. Furthermore, we assume that skill comparative advantage is exponential $B(s,t) = \exp(\delta_s t)$, which implies that wages for each skill has also an exponential form. The parameter δ_s resumes the comparative advantage and assumption 3.1 implies that $\delta_{s'} > \delta_s$, $\forall s' > s$.

The parameters of the model to be estimated are the skill specific comparative advantage (δ_s), public selection advantage (τ_s), the elasticity of the private sector C.E.S. production function (η), the productivity parameter (A), the variance of the economic shock (σ^2), the parameters that define the distribution of risk aversion (k,λ) and skill distribution ($P(s|e)$). Moreover, the endogenous variables are the probabilities of being selected to work at the public sector ($b(e)$), the task thresholds (T_s) and the average product discounting the effect of the productivity parameter (H).

[1]Some of these functional hypotheses can be relaxed in further versions.

4.2 Estimation Method

We use a two stage estimator. First, we estimate private sector supply and average wages for each skill. In the first stage we also estimate the joint distribution of skills and observable characteristics in the private sector, the distribution of observable characteristics in the public sector and public sector average wages for each demographic characteristic. Then, in the second stage, we use those moments to estimate the parameters of the model.

4.2.1 First Stage

First, notice that the model implies that the distribution of wages in the private sector ($f(\log(w)|r)$) is a finite mixture of normal distributions with same variance.

$$f(\log(w_i)|r) = \sum_{s=1}^{S} P(s|r)\phi\left(\frac{\log(w) - \log(\omega_r(s))}{\sigma^2}\right), \quad \text{with } \log(\omega_r(s)) = E[\log(w_r(s))|s] \tag{4-1}$$

Using maximum likelihood we can estimate the private sector distribution of skills $P(s|r)$, the variance of the productivity shock σ^2 and average wages of each skill $\log(\omega_r(s))$. To maximize the likelihood function we use the E.M. algorithm as it is described in (25), chapter 2. The E.M. algorithm formulates the finite mixture estimation as a missing variable problem, in other words, we observe wages but we do not observe which skill each individual belongs to. Furthermore, the E.M. algorithm has two steps: the E-Step and the M-Step. The E-Step takes an initial guess of parameters $(P_0(s|r), \sigma_0^2, \log(\omega_r(s)_0))$ and estimates for each observation the posterior probability that this observation is of skill s:

$$P_0(i \in s) = \frac{P_0(s|r)\phi\left(\frac{log(w_i) - \log(\omega_r(s)_0)}{\sigma_0^2}\right)}{\sum_{h=1}^{S} P_0(h|r)\phi\left(\frac{log(w_i) - \log(\omega_r(h)_0)}{\sigma_0^2}\right)} \tag{4-2}$$

Then the M-Step maximizes the following function obtaining a new set of guesses:

$$(\sigma_1^2, \log(\omega_r(s)_1)) = \underset{\sigma^2, \log(\omega_r(s))}{\operatorname{argmax}} \sum_{i=1}^{n}\sum_{s=1}^{S} P_0(i \in s)\phi\left(\frac{\log(w) - \log(\omega_r(s))}{\sigma^2}\right), \quad P_1(s|r) = \sum_{i=1}^{n} P_0(i \in s) \tag{4-3}$$

The algorithm iterates the E-Step and then the M-Step until the likelihood function converges.

To estimate the number of skills (S), we maximize the likelihood function for all the possible number of skills (S) where the model is still identified - in the next subsection we will describe this set - and choose the one with the lowest B.I.C. information criterion. In addition, the application of the E.M. estimator gives us the optimal posterior probability that each observation is of skill s ($P_\infty(i \in s)$). Taking the averages of this posterior probabilities allows us to estimate $P(s|e,r)$. Therefore, using the E.M. estimator to maximize the likelihood function of the private sector distribution of wages, we estimate the distribution of skills in the private sector $P(s|r)$ and across demographic groups $P(s|r,e)$, the number of skills S, average private sector wages for each skill $\log(\hat{\omega}_r(s))$ and the variance of the productivity shock σ^2.

Furthermore, in the first stage we also estimate moments that define the labor market in the public sector. In particular, we estimate the proportion of individuals of each demographic characteristic in the public sector ($P(e|b)$) and the proportion of individuals that choose the private sector and belongs to skill s, given observed characteristic e ($P(s,r|e) = P(s|e,r) * P(r|e)$). In addition, we estimate the proportion of workers that choose the private sector and belongs to skill s ($P(s,r) = P(s|r) * P(r)$). We also estimate average public sector wages for each demographic characteristic ($w_b(e)$) and the distribution of observable characteristics ($P(e)$) that are exogenous variables of the model.

4.2.2 Second Stage

In the second stage we use the moments estimated in the first stage to calculate parameters and endogenous variables of the model. In other words, we have a vector of moments estimated in the first stage ($\hat{m}$) and their model counterparts ($m(\theta, y)$) that are a function of the parameters (θ) and endogenous variables (y). We use a minimum distance estimator to approximate those vectors, solving the following problem[2]:

$$(\hat{\theta}, \hat{y}) = \underset{\theta,y}{\operatorname{argmin}} \; (\hat{m} - m(\theta, y))' W (\hat{m} - m(\theta, y)) \tag{4-4}$$

Where,

$$\hat{m} = (P(\hat{s}, r), \log(\hat{\omega}_r(s)), P(\hat{s, r}|e), P(\hat{e}|b)) \tag{4-5}$$

We assume that the data represents an equilibrium. Furthermore, market clearing for public and private labor markets is a linear transformation of vector $\hat{m}$. First, in the private sector we use the market clearing condition to directly as a moment $P(s,r) = l_r(s)$. Then, in the public sector, market clearing is a

[2]In this version we assume that W is an identity matrix, but in later versions we will estimate using a more efficient weight matrix.

linear combination of $P(s,r|e)$ and the exogenous variable $P(e)$:

$$P(b,e) = \left[1 - \sum_{s=1}^{S} P(s,r|e)\right] P(e) = l_b(e) \tag{4-6}$$

Thus, because market clearing conditions are used during the estimation procedure, we are able to estimate both the parameters and endogenous variables of the model.

The vector of model counterparts is:

$$m(\theta, y) = (l_r(s), \log(\omega_r(s)), P(r|s,e), l_b(e)/P(b)) \tag{4-7}$$

First, in the labor demand side $l_r(s)$ and $\log(\omega_r(s))$ are the following functions:

$$l_r(s) = \frac{H}{\omega_r(s)^{\eta}} \int_{T_{s-1}}^{T_s} \exp(\delta_s t(\eta - 1))dt \ , \forall \ s = 1, ..., S \tag{4-8}$$

$$\log(\omega_r(s)) - \log(\omega_r(1)) = \sum_{h=2}^{s} (\delta_h - \delta_{h-1}) T_{h-1} \ , \forall \ s = 2, ..., S \tag{4-9}$$

$$\omega_r(1) = A \left[\sum_{s=1}^{\bar{S}} \exp \left(\sum_{h=2}^{s} (\delta_h - \delta_{h-1}) T_{h-1} \right)^{1-\eta} \int_{T_{s-1}}^{T_s} \exp(\delta_s t)^{\eta - 1} dt \right]^{\frac{1}{\eta - 1}} \tag{4-10}$$

To allow identification we will calibrate the CES parameter of the private sector production function $\eta = 1.4$ and will set $\delta_1 = 1$. The elasticity of substitution of the private sector production function (η) follows the consensus on the literature of wage inequality and skill premium (see for example (26) and (27))[3]. The interpretation of the comparative advantage parameter (δ_s) is now always relative to the first skill. This allows us to estimate all the parameters of the private sector labor demand $\delta_2, ..., \delta_S$ and A, and the endogenous variables of the labor demand side $T_1, ..., T_{S-1}$ and H.

On the supply side of the model, $P(r|s,e) = F\left(\tau_s, b(e), k, \lambda | w_b(e), \omega_r(s), \sigma^2\right)$ where k and λ are, respectively, the shape and scale parameters of the Weibull distribution. Furthermore,

$$P(e|b) = \frac{\left[1 - \sum_{s=1}^{S} P(r|s,e)P(s|e)\right] P(e)}{\sum_{e=1}^{E} \left[1 - \sum_{s=1}^{S} P(r|s,e)P(s|e)\right] P(e)} = \frac{P(e,b)}{P(b)} \tag{4-11}$$

In other words, to estimate the distribution of skills along with the parameters of the labor supply, we are using the distribution of observable characteristics inside the public sector and workers sector decision. Therefore, we are able to estimate the parameters of the supply side of the model (τ_s, k, θ and $P(s|e)$) and the endogenous variables ($b(e)$).

Notice that the model counter-parts used in the labor demand and labor

[3]Next versions will include robustness checks with other values of the C.E.S. parameter of the private sector production(η).

supply sides are independent of each other. As a consequence, we can estimate the parameters and endogenous variables of the labor demand side separately from the labor supply side. To estimate standard-errors of all parameters and endogenous variables we use non-parametric bootstrap. The bootstrap includes both estimation stages and uses the same number of skills for all bootstrap samples. To calculate standard errors we draw 50 bootstrap samples of the data.

4.3 Identification

In the first stage, identification of the distribution of skills in the private sector comes primarily from parametric assumptions on the distribution of private sector wages. In other words, we are able to identify the distribution of skills in the private sector because we assume that it is a finite mixture of normal distributions with the same variance. In the second stage, the distinction between labor demand and labor supply comes from the fact that we estimate private sector labor demand, depending only on skill, while on the supply side we estimate public sector labor supply that needs a set of observable characteristics.

On the labor demand side we need to estimate $2S + 2$ parameters and variables (δ_1, ..., δ_S, η, A, T_1, ..., T_{S-1}, H), with $2S$ equations ($l_r(1)$, ..., $l_r(S)$, $\log(\omega_r(1))$, ..., $\log(\omega_r(S))$). Therefore, the labor demand side is identified because we calibrate the C.E.S. parameter of the private sector production and we set the comparative advantage parameter of the first skill equal to one. On the other hand, in the labor supply side we have $SE + E$ equations ($P(s, e, r)$, $P(e|b)$) to estimate $SE + S + 2$ parameters and endogenous variables ($P(s|e)$, τ_s, $b(e)$, k, λ). Therefore, we need $E \geq S + 2$ to identify the supply side of the model. It is the set of observable characteristics, combined with the estimation of the distribution of skills and observable characteristics at the private sector that allows the estimation of a non-parametric distribution of skills and observable characteristics.

5
Results

5.1
Estimation Results

The estimated parameters, with bootstrap standard errors, are shown in table 5.1. First and second rows show private sector comparative advantage in production (δ_s) and access to public jobs advantage (τ_s) for each skill, respectively.

Table 5.1: Estimated Parameters - Skills

	s=1	s=2	s=3	s=4	s=5	s=6	s=7
δ_s	1.000	16.660	16.662	16.666	16.678	18.185	20.234
	-	(0.177)	(0.177)	(0.177)	(0.176)	(0.178)	(0.186)
τ_s	116.89	183.83	242.35	259.67	303.62	541.24	545.32
	(2.986)	(8.409)	(8.481)	(8.738)	(23.448)	(18.138)	(96.184)

Notes: Bootstraped standard erros in parenthesis.

Individuals with the lowest skill ($s = 1$) have a very strong comparative disadvantage in private sector production relative to other skills. Moreover, skills 2 through 5 comparative advantage is very close in magnitude, and their differences is not statistically significant. However, the difference between skills five, six and seven is significant. Therefore, skills can be divided into four groups regarding their comparative advantage parameter in private sector production (δ_s): the first one composed only of skill one, a second group with skills 2 through 5, the third and forth groups composed of skills six and seven respectively[1].

Even though private sector production comparative advantage is very similar from skill 2 to 5, access to public jobs advantage is quite different. Only the difference between skills four and five and the difference between skills six and seven are not statistically significant. Moreover, skills 6 and 7

[1] Albeit the private sector production comparative advantage parameter can be divided into four statistically different groups, the number of skills was optimally estimated.

have a very strong advantage when compared to other skills. Similar with the private sector production comparative advantage parameter, skill 1 has a very strong comparative disadvantage in access to public jobs. This pattern of access to public jobs advantage strongly favors high-skill individuals, and generates a high proportion of skills six and seven working for the public sector.

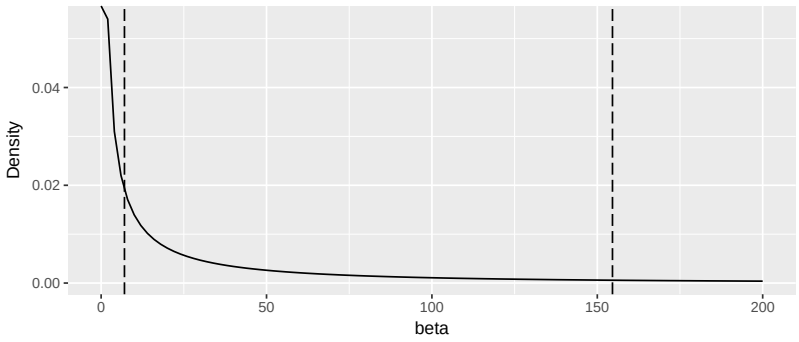
Table 5.2: Estimated Parameters - Distributions

k	λ	σ^2
0.389	18.10	0.221
(0.1060)	(0.297)	(0.002)

Notes: Bootstraped standard erros in parenthesis.

Table 5.2 shows the parameters of the distribution os risk aversion (k, λ) and the variance of the economic shock (σ^2). The parameters that define the distribution of risk aversion imply that most of the population is concentrated at a risk aversion parameter (β) close to one, which would imply a log utility. However, since the shape parameter is very small, the distribution has heavy tails with some individuals having a very high risk aversion. Figure 5.1 shows the estimated distribution of risk aversion and illustrates this point. The vertical lines represent the median and the 90th percentile respectively. These results are consistent with the literature that estimates risk aversion based on insurance choices and find that the distribution of risk aversion is concentrated near risk neutrality, but has a high dispersion[2]

Figure 5.1: Distribution of Risk Aversion



Notes: Estimated distribution of risk aversion p.d.f.. Vertical lines indicate the median and the 90th percentile.

[2]See for example (7) and (28).

The variance of the economic shock (σ^2) is 0.22 which is smaller than the observed variance of log-hourly-wages for all demographic groups (according to table 2.1). Moreover, it is higher than what was estimated previously in the literature, (29, 30, 31) estimate a variance of the transitory shock on income smaller than 0.1 for the U.S.. However, those papers are able to consistently separate the transitory and the permanent component of income shocks. Since our data is series of repeated cross-sections, we are not able to separate those components of income shocks.

Table 5.3: Distribution of Skills Conditional on Observed Characteristics

	s=1	s=2	s=3	s=4	s=5	s=6	s=7
younger than 30 years							
HS	0.033	0.220	0.250	0.246	0.210	0.034	0.008
drop-outs	(0.002)	(0.018)	(0.019)	(0.019)	(0.029)	(0.078)	(0.013)
HS	0.013	0.215	0.244	0.241	0.206	0.069	0.011
	(0.002)	(0.011)	(0.011)	(0.011)	(0.020)	(0.044)	(0.015)
College	0.002	0.187	0.213	0.211	0.183	0.191	0.013
drop-outs	(0.000)	(0.016)	(0.015)	(0.018)	(0.019)	(0.070)	(0.016)
College	0.001	0.127	0.164	0.169	0.156	0.361	0.023
	(0.000)	(0.004)	(0.004)	(0.004)	(0.005)	(0.017)	(0.000)
Grad-	0.000	0.077	0.094	0.094	0.085	0.589	0.060
School	(0.000)	(0.005)	(0.005)	(0.005)	(0.005)	(0.026)	(0.026)
older than 30 years							
HS	0.027	0.211	0.240	0.237	0.203	0.071	0.011
drop-outs	(0.002)	(0.019)	(0.020)	(0.020)	(0.033)	(0.080)	(0.018)
HS	0.010	0.190	0.217	0.214	0.186	0.167	0.016
	(0.000)	(0.007)	(0.009)	(0.009)	(0.000)	(0.129)	(0.132)
College	0.003	0.145	0.165	0.164	0.162	0.330	0.032
drop-outs	(0.000)	(0.006)	(0.007)	(0.007)	(0.007)	(0.026)	(0.000)
College	0.001	0.098	0.119	0.120	0.108	0.505	0.049
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Grad-	0.001	0.039	0.046	0.046	0.041	0.709	0.119
School	(0.000)	(0.002)	(0.002)	(0.002)	(0.003)	(0.035)	(0.038)

Notes: Bootstrap standard erros in parenthesis.

Table 5.3 shows the distribution of skills conditional on observed characteristics for the whole population with standard errors in parenthesis. As expected, higher skills are concentrated between those with higher educational levels. Even in the highest educational levels (graduate degree) the proportion of individuals with skill 7 is still very small and most of the individuals of that group are concentrated on skill 6. In addition, even at the highest educational

levels there are still some individuals of skills 1 and 2. This indicates that, even though skill here is something that can be acquired through education, there are still some people that will never be high skill types. Furthermore, workers with high skill (6 and 7) are also more prevalent among those older than 30 years old, which is also the group with a higher share of public employment. This suggests that skill can be something learned through experience.

Table 5.4: Skill types in the private sector

	s=1	s=2	s=3	s=4	s=5	s=6	s=7
T_s	0.108	0.689	0.801	0.864	0.903	0.970	1.000
	(0.000)	(0.003)	(0.002)	(0.002)	(0.002)	(0.001)	(0.000)
$log(\omega_r(s))$	−0.255	1.436	1.438	1.441	1.451	2.812	4.800
	(0.021)	(0.002)	(0.001)	(0.001)	(0.001)	(0.008)	(0.031)
P(s|r)	0.016	0.200	0.227	0.224	0.190	0.128	0.011
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Notes: T_s are the thresholds that represent factor allocation; $\log(\omega_r(s))$ are average private sector log-wages; and $P(s|r)$ is the skill distribution in the private sector. Bootstrap standard errors in parenthesis.

The variables that define the equilibrium for the private sector are shown in table 5.4. The tasks thresholds T_s indicate a convex matching function of tasks to skills, implicating a very high dispersion of wages. The skill distribution at the private sector is concentrated on skills 3 and 4, which, combined with the comparative advantage parameter, implicates a high demand for tasks with middle intensity.

Table 5.5: Probability of Being Accepted to Work at the Public Sector ($b(e)$)

	HS drop-outs	HS	College drop-outs	College	Grad-School
younger	0.000	0.001	0.001	0.002	0.002
	(0.001)	(0.001)	(0.113)	(0.012)	(0.012)
older	0.000	0.002	0.002	0.002	0.003
	(0.001)	(0.124)	(0.021)	(0.004)	(0.003)

Notes: $b(e)$ is the probability of being accepted in the public sector, bootstrap standard errors in parenthesis. First line shows the probability for those younger than 30 years old and second line for those older than 30.

Table 5.5 shows the probability of being accepted to work at the public sector for each demographic group. These probabilities are quite small and most of them are not statistically significant, indicating that the major

factor affecting the probability of being accepted in public employment is the individuals' skill level. Table 5.6 shows the skill distribution in the population (row 1), in the private sector (row 2) and in the public sector (row 3) and illustrates this point. The strong comparative advantage in access to public jobs for skills 6 and 7 generates a skill distribution in the public sector highly concentrated among high-skill workers. More than half of public employees are of skills 6 and 7, while those skill types are less than 20% of the workers in the population. Therefore, the public sector attracts mostly high-skill workers and shifts the distribution of skills in the private sector in the direction of low skill workers.

Table 5.6: Skill Distribution

	s=1	s=2	s=3	s=4	s=5	s=6	s=7
$P(s)$	0.016	0.190	0.218	0.216	0.187	0.156	0.017
$P(s\|r)$	0.016	0.200	0.228	0.225	0.191	0.129	0.011
$P(s\|b)$	0.012	0.028	0.065	0.073	0.122	0.587	0.112

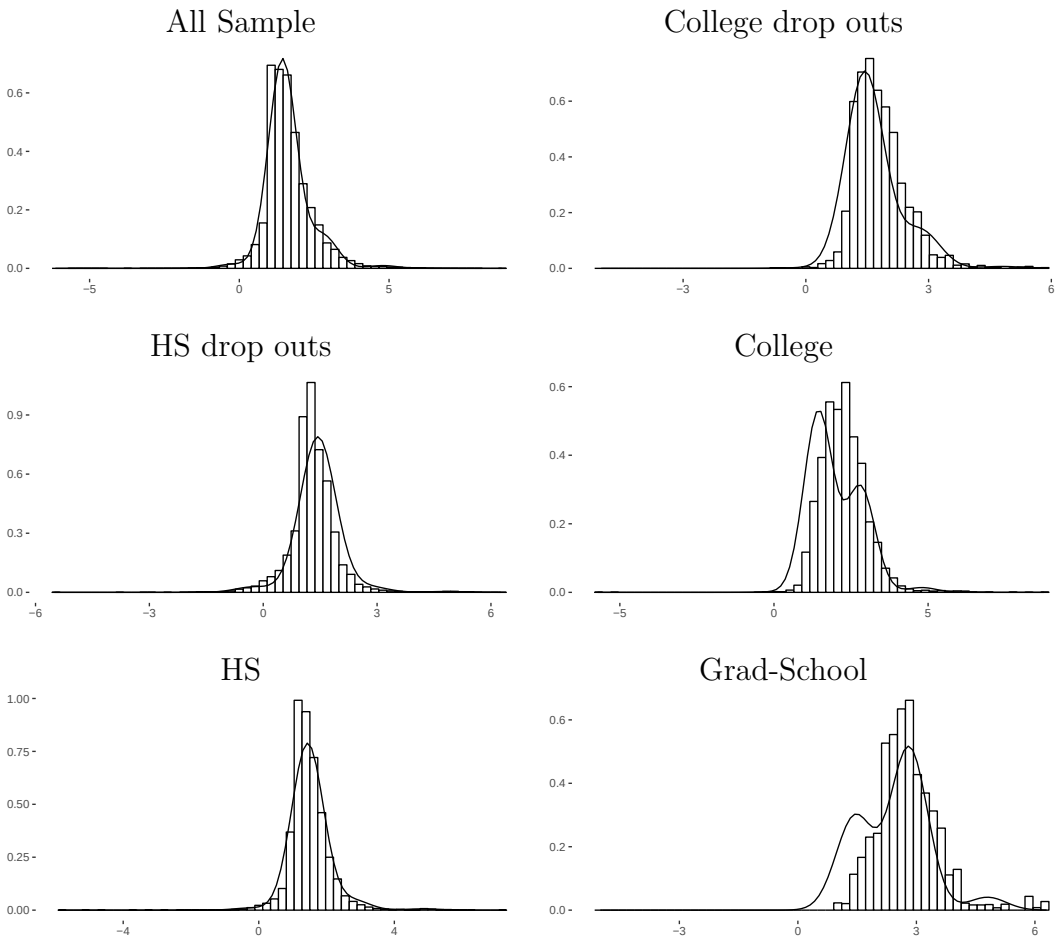
Notes: Bootstrap standard errors in parenthesis. $P(s)$ is the skill distribution in the whole population, $P(s|r)$ is the skill distribution for private sector workers and $P(s|b)$ is the skill distribution for public sector workers.

5.2 Model Fit

Figures 5.2 and 5.3 show the distribution of private sector wages in the data and calculated by the model for individuals younger than 30 years old and older than 30 years old respectively. The model fits well the distribution of wages for the whole population. Notice that to estimate the distribution of skills we use the distribution of wages in the whole population, not dividing it in demographic groups. These non-targeted moments will be used to evaluate model fit. The model fits well the distribution of wages for High-School drop-outs and high-school graduates for both age groups. The fit remains good for College drop-outs and College graduates for those younger than 30 years old. However, the fit is not that good for those with graduate school at both skill levels and for college drop-outs and graduates older than 30 years old.

Table 5.7 shows public sector employment for each observed characteristic in the data and predicted by the model. To guarantee public sector equilibrium these two variables need to be equal, as is shown in the table. Table 5.8 shows private sector log-wages for each demographic group in the data and estimated by the model and brings similar conclusions to that of figures 5.2 and 5.3.

Figure 5.2: Distribution of wages in the Private Sector - younger than 30 years old



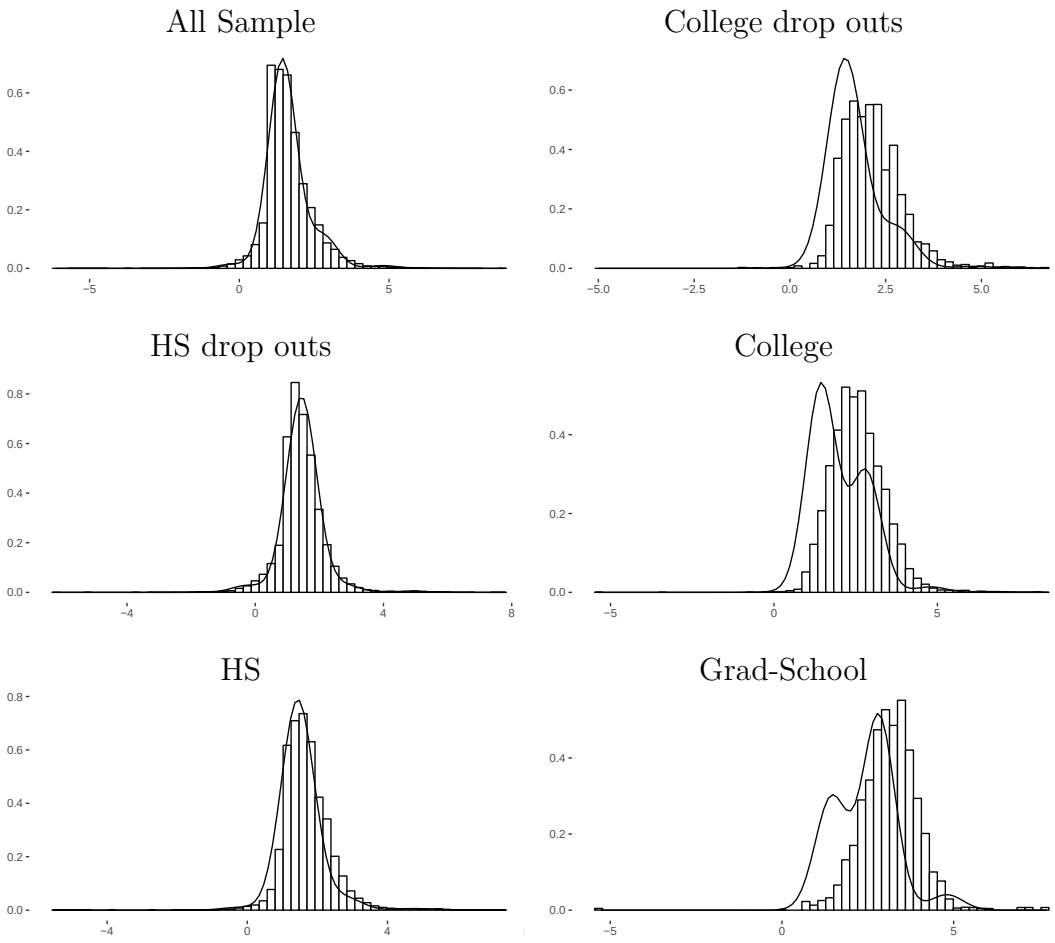
Notes: Distribution of wages in the private sector according to PNAD 2011-4 (bars) and estimated private sector distribution of wages (lines) by education.

Table 5.7: Proportion of public sector workers for each observed characteristic

	younger than 30 years		older than 30 years	
	Data	Model	Data	Model
HS drop-outs	0.001	0.001	0.003	0.003
HS	0.005	0.005	0.012	0.015
College drop-outs	0.001	0.001	0.001	0.001
College	0.008	0.008	0.024	0.024
Grad-School	0.000	0.000	0.002	0.002

Notes: Proportion of public sector workers for each age and education group in the data and estimated by the model. Data from PNAD 2011-4 individuals between 18-40 years old not in school.

Figure 5.3: Distribution of wages in the Private Sector - older than 30 years old



Notes: Distribution of wages in the private sector according to PNAD 2011-4 (bars) and estimated private sector distribution of wages (lines) by education.

The model fits better the data for those with less than 30 years and lower educational levels.

Table 5.9 presents the public-private wage premium calculated by the estimated model $(\log(w_b(e)) - \sum_{s=1}^{S} \log(\omega_r(s))P(s|e))$. The public-private wage gap increases with education, with the exception of individuals with graduate degree and younger than 30 years old. This results are consistent with previous estimations of the public-private wage gap. Particularly, (8) estimates the public wage premium using the privatization of some state owned companies as an exogenous shock. Similar to our results, they find that the public wage premium is bigger for higher educational levels. Nonetheless, their estimations are slightly bigger than ours, they find that those with College education have a public-private wage gap of 0.544 and those in high-school have a public wage premium of 0.491.

Table 5.8: Private sector average log-wages for each observed characteristic

	younger than 30 years		older than 30 years	
	Data	Model	Data	Model
HS drop-outs	1.300	1.447	1.438	1.505
HS	1.481	1.516	1.716	1.634
College drop-outs	1.814	1.681	2.150	1.922
College	2.253	1.988	2.571	2.259
Grad-School	2.783	2.431	3.143	2.773

Notes: Private sector average log-wage for each age and education group in the data and estimated by the model. Data from PNAD 2011-4 individuals between 18-40 years old not in school.

Table 5.9: Public-Private Wage Gap for Each Demographic Group

	Younger	Older
HS drop-outs	−0.056	−0.038
HS	0.178	0.138
College drop-outs	0.250	0.318
College	0.462	0.322
Grad-School	0.369	0.363

Notes: Public-private wage gap estimated by the model $\log(w_b(e)) - \sum_{s=1}^{S} \log(\omega_r(s))P(s|e)$.

In conclusion, the estimated model represents reasonably well the patterns and moments characterized in the data. Although it underestimates slightly wages for higher education groups it still preserves most of the relationships seen in the data. Wages grow with education and age and most of public sector employees are older than 30 years old. Some extensions are possible to improve the results, however this specification provides a parsimonious good fit with the data.

6
Counter-Factual Exercises

This section shows the effects of a reduction in the size of the public sector, which means reducing the public sector labor demand by the same percentage for each demographic group[1]. In our sample the public sector accounts for 5.88% of the labor force, if we reduce in 10% the size of the public sector it will employ 5.29% of the labor force. Brazil experienced a large increase in public sector hiring from 2005 until 2014, going back to 2005 represents a decrease of 15.64% in the number of public employees. Furthermore, a reduction of 11.30% in the number of public employees would mean going back to the size of the public sector in 2007. We will show results of a reduction in public sector using this trend observed in the last decade as a benchmark. First we present the effect of reducing the size of the public sector on the distribution of wages and then on productivity and skill allocation.

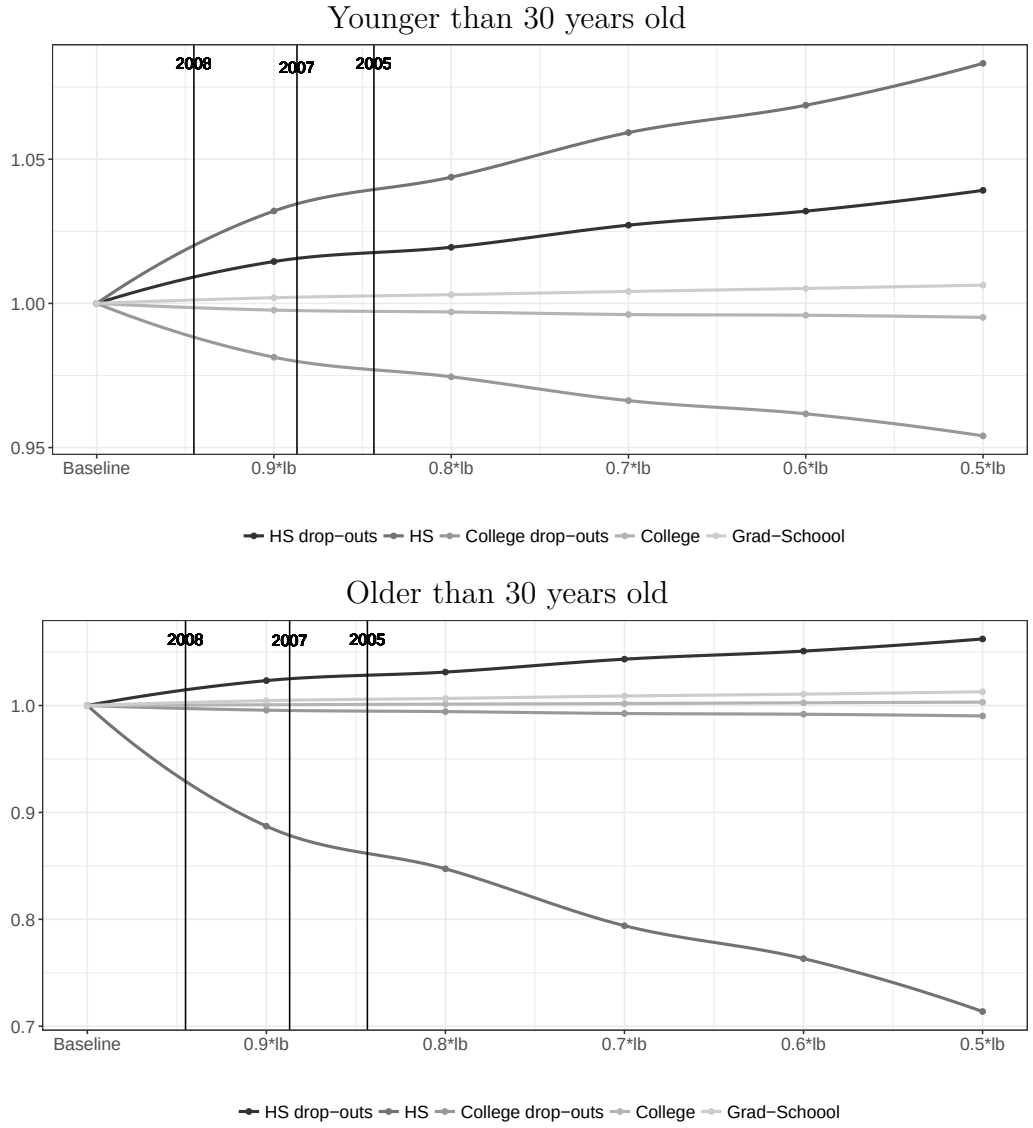
6.1
Effects on the Distribution of Wages

Figure 6.1 shows the growth in the public-private wage gap. For those younger than 30 years old, a reduction in public labor demand increases the public premium for lower education levels (High-School drop-outs and completed) and decreases for higher education levels (College drop-outs and completed), while for graduate students we do not see any change. Differently, for those older than 30 years old, it is possible to observe a sharp decline in the public-private wage differentials for those who completed High-School and a rise for High-School drop-outs. Similarly, the wage premium for College drop-outs and individuals who completed College older than 30 years old declines and nothing happens for grad-students.

Since the only variable in the public sector wage premium that changes is the private sector average wage for each skill, it summarizes the changes in wages for each demographic group not accounting for the change in the skill distribution available to the private sector. On the other hand, figure 6.2 shows the growth in average private sector log-wages for each observable

[1]This exercise will be made for the different demographic groups separately in next versions.

Figure 6.1: Public Sector Wage Premium



Notes: Effects of a reduction on the size of the public sector on the public-private wage gap $\log(w_b(e)) - \sum_{s=1}^{S} \log(\omega_r(s))P(s|e)$, for each demographic group in relation to the baseline. Vertical lines represent the size of the public sector in 2008, 2007 and 2005 respectively in Brazil for our sample.

characteristic taking into account the change in the skill distribution available to the private sector ($\sum_{s=1}^{S} P(s|r,e)log(\omega_r(s))$). For both age groups High-School drop-outs, those who completed High-School and College drop-outs average private sector wages increases, while individuals with a College degree and Grad-School wages decline or show no movement. This is the opposite of what was observed in the public-sector wage premium. The rise in the public-private wage gap for High-School drop-outs and those who completed High-School would indicate a decline in private sector wages, on the contrary of what we observe. Therefore, changes in the skill distribution available to the private sector play a big role on the determination of average wages.

As a result, the effects of a decrease in public sector labor demand on average wages for each demographic group are very similar to the proportion of high skill workers in the private sector, in this case we define high skill those individuals with skills 5, 6 and 7. Figure 6.3 shows the proportion of high skill workers in the private sector for each demographic group. It is similar to log average wages in the private sector, with High-School drop-outs, High-School and College drop-outs increasing with a reduction on the size of the public sector. Table 6.1 shows a summary of what happens to wages by demographic group after a 10% reduction on the size of the public sector.

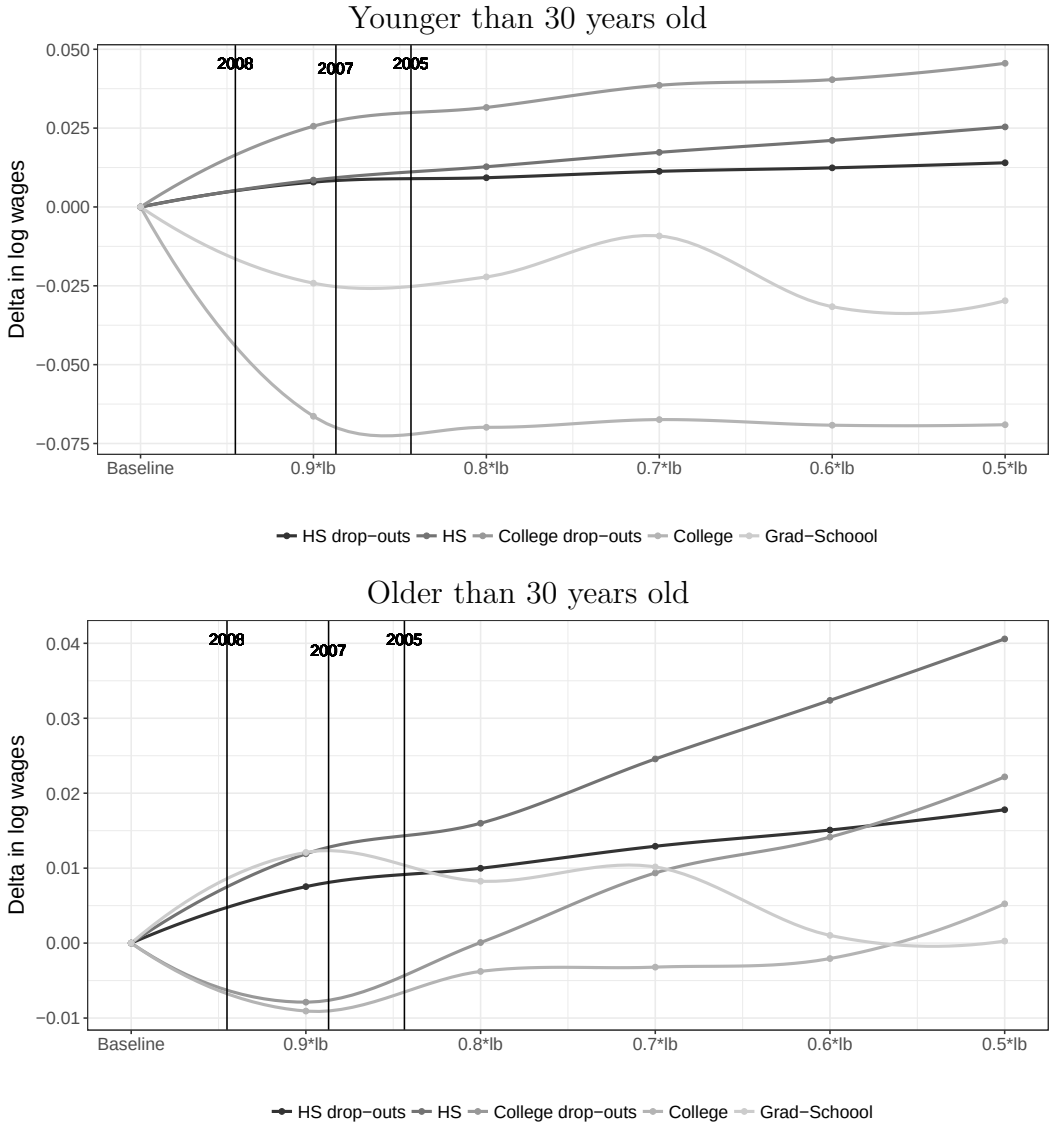
Table 6.1: Effects of a 10% Reduction on the Size of the Public Sector

	Public Sector Wage Premium	Private Sector Average Log Wages
Younger than 30 years old		
HS drop-outs	1.45%	0.79%
HS	3.21%	0.85%
College drop-outs	-1.87%	2.56%
College	-0.23%	-6.64%
Grad-School	0.20%	-2.42%
Older than 30 years old		
HS drop-outs	2.33%	0.75%
HS	-11.29%	1.19%
College drop-outs	-0.43%	-0.79%
College	0.07%	-0.91%
Grad-School	0.46%	1.21%

Notes: Percent growth in the public sector wage premium and private sector average log-wages after a 10% reduction on the size of the public sector, by demographic group.

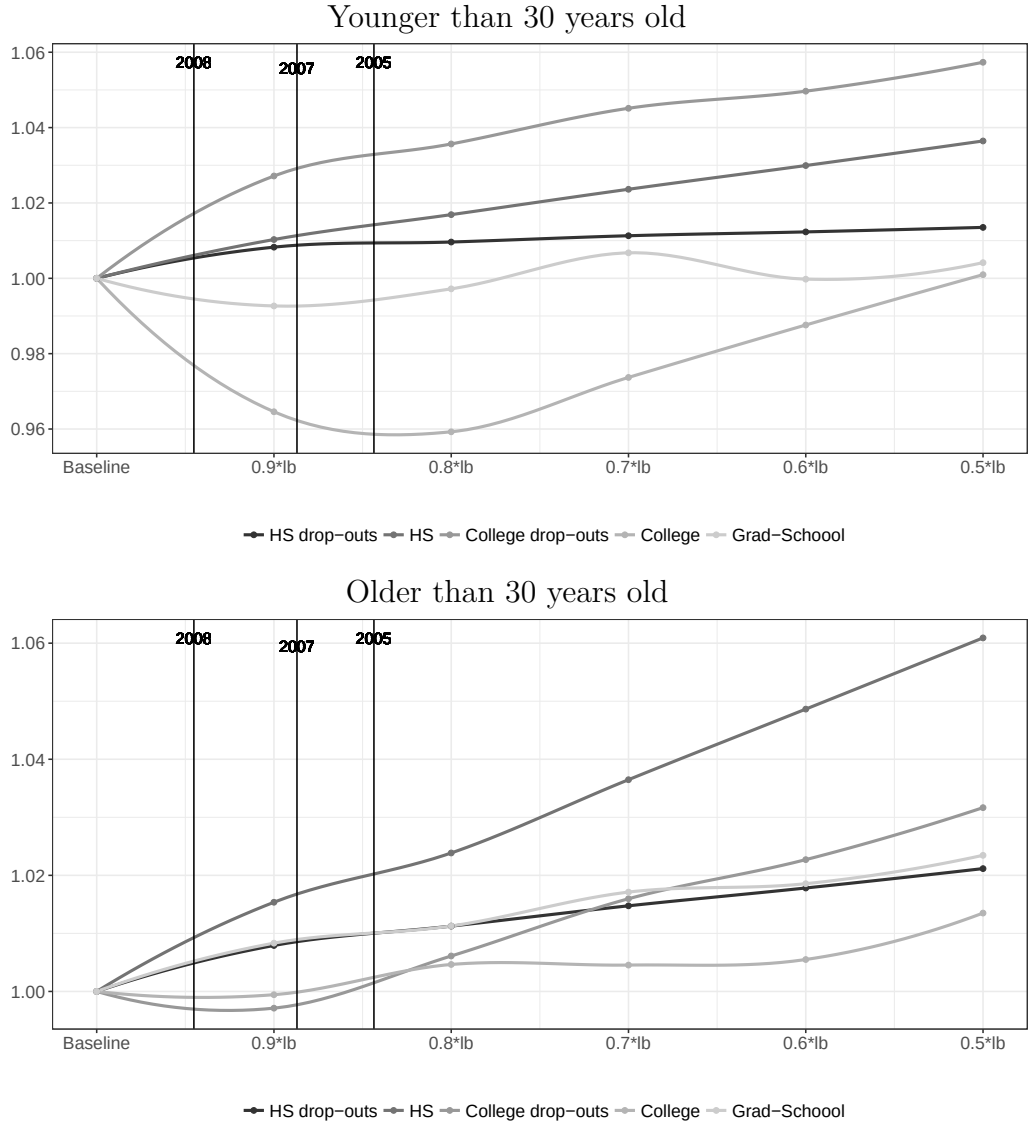
Since the increase in private sector average log wages was higher for those without a College degree in both age groups, the College wage premium must decline. Figure 6.4 shows the effects of a reduction in public employment on

Figure 6.2: Private Sector Average Log-Wages



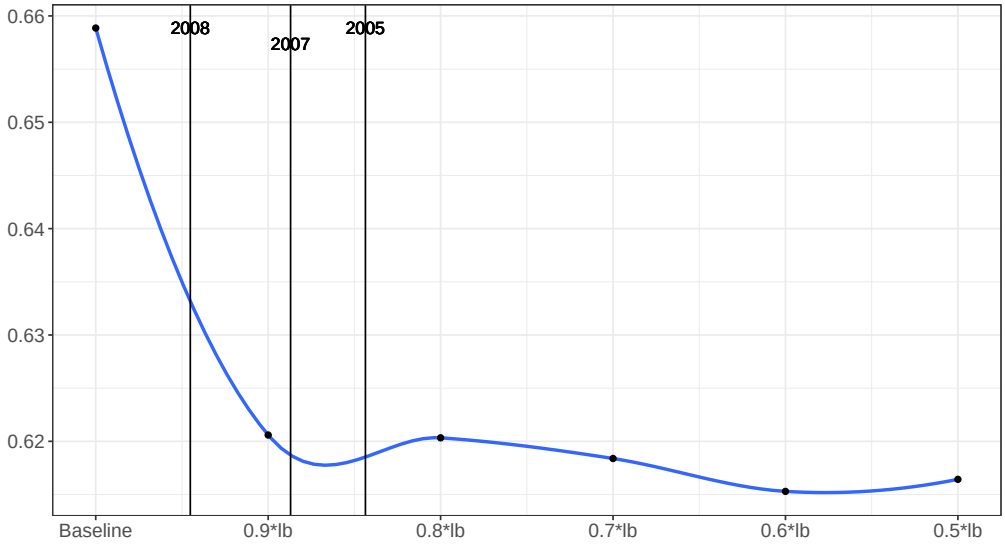
Notes: Percent growth in private sector average wages for each demographic group, after a reduction on the size of the public sector. Vertical lines represent the size of the public sector in 2008, 2007 and 2005 respectively in Brazil for our sample.

Figure 6.3: Private Sector Proportion of High Skill Workers ($s \in \{5, 6, 7\}$)



Notes: Percent growth in private sector proportion of high-skill individuals for each demographic group, after a reduction on the size of the public sector. Vertical lines represent the size of the public sector in 2008, 2007 and 2005 respectively in Brazil for our sample.

Figure 6.4: College Wage Premium in the Private Sector



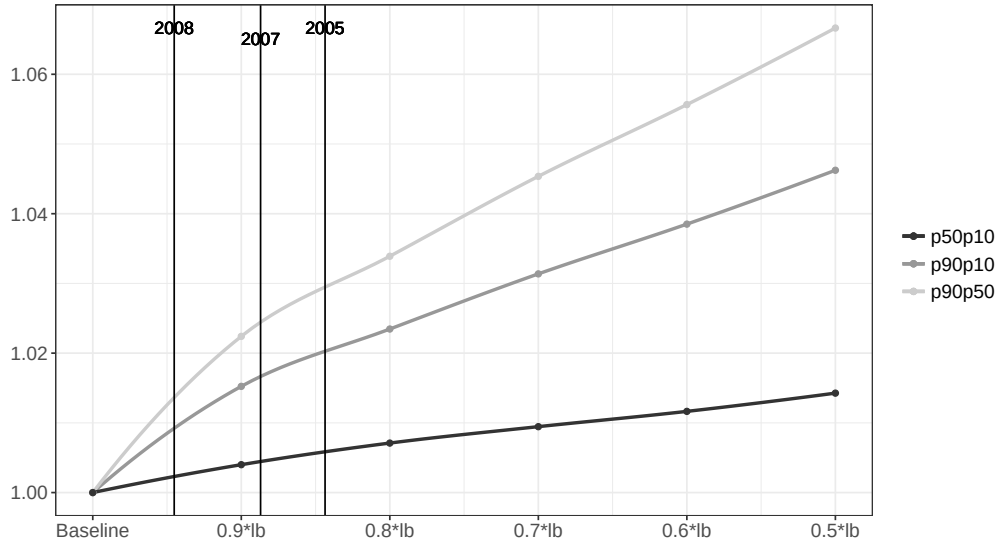
Notes: Effects of a reduction in public employment on the College wage premium in the private sector. Vertical lines represent the size of the public sector in 2008, 2007 and 2005 respectively in Brazil for our sample.

the private sector College wage premium. As expected a decline on the size of the public sector reduces the College wage premium. More specifically a 10% reduction on public labor demand implies on a 5.81% decline on the College wage premium, this is due primarily to an increase on the proportion of high-skill for those with no College degree.

Because the public sector does not set wages based on skills, but based on observable characteristics, high-skill individuals with low educational level prefer mostly the private sector. On the other hand, low-skill individuals with high educational levels prefer the public sector, because the public sector pays based on their education and not on their skill. Therefore, we would expect that a reduction on the size of the public sector would increase the proportion of high-skill individuals for low-education groups and decrease the same proportion for high-education groups. This generates the observed effect on the College wage premium.

Nonetheless, a decline on the size of the public sector generates an increase in inequality. Figure 6.5 shows the growth on the ratios $p90/p10$, $p90/p50$ and $p50/p10$ of the log wage distribution at the private sector. It is possible to see that a reduction on the size of the public sector increases wage inequality measured by those ratios. More specifically, a 10% reduction in public labor demand increases the ratios $p90/p10$, $p90/p50$ and $p50/p10$ by 1.52%, 2.24% and 0.40% respectively. This happens mainly because the public

Figure 6.5: Private Sector Wage Inequality



Notes: Percent growth in $p90/p10$, $p90/p50$ and $p50/p10$ wage ratios during a reduction in public employment. Vertical lines represent the size of the public sector in 2008, 2007 and 2005 respectively in Brazil for our sample.

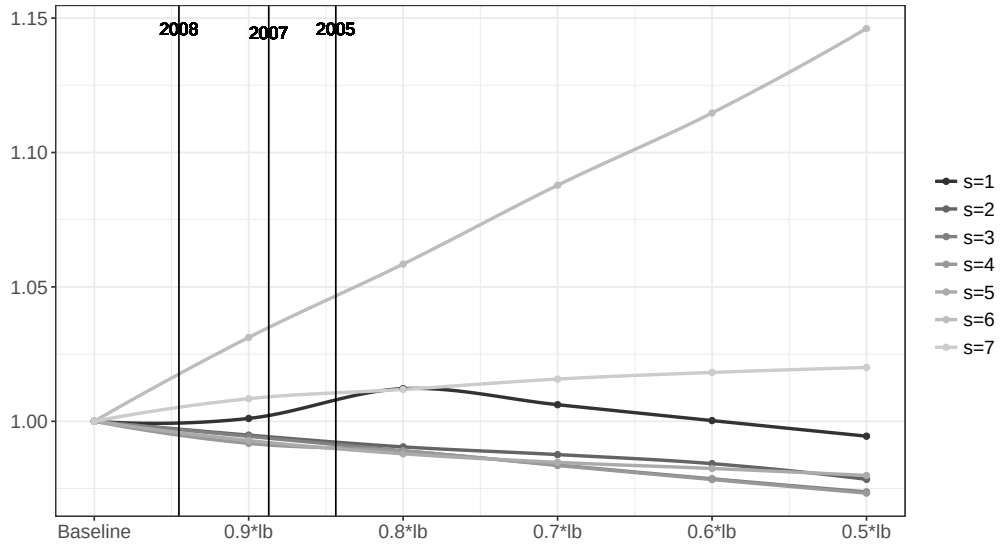
sector employs mostly high-skill workers.

For instance, figure 6.6 shows the growth in the probability of belonging to skill s given that the worker chose the private sector ($P(s|r)$). It is possible to see that the proportion of individuals of skills 6 and 7 on the private sector rises a lot, while the same proportion declines for the other skills. Therefore, there is an increase in the supply of high-skill workers in the private sector. This increase in supply generates a slight decrease in wages for those skills, as can be seen in figure 6.7 that shows average wages for each skill in the private sector. However, this decrease in wages was not enough to suppress the increase in the proportion of high skill individuals that rose wage inequality.

6.2 Allocation and Productivity Effects

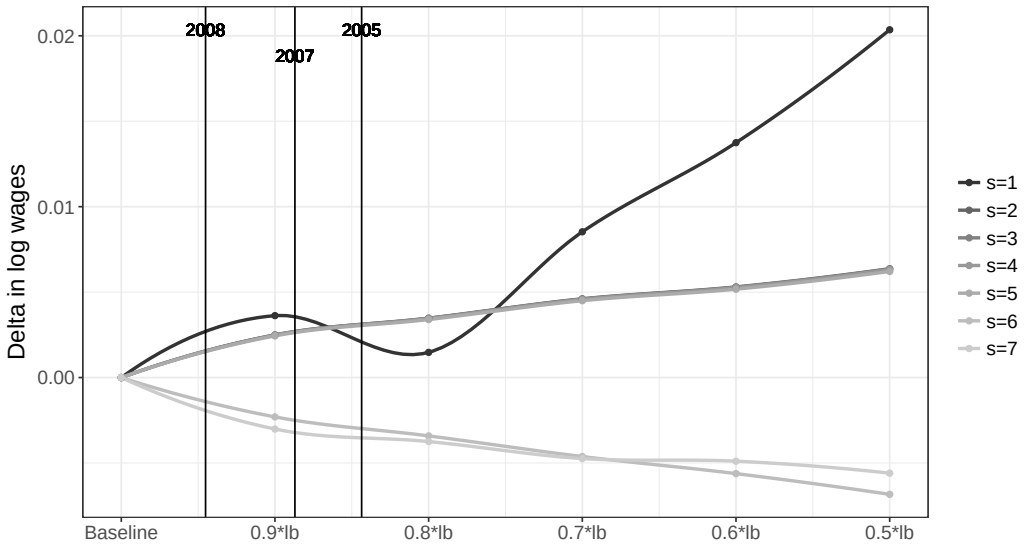
From table 5.4 it is possible to see that the allocation of skills to tasks imply that more than 60% of tasks are performed by skills 1 and 2. In other words, the matching function of tasks to skills is convex, as can be seen in figure 6.8. It shows the matching function of tasks to skills first in our baseline (blue) and reducing the public sector (red). It is also possible to see that as the proportion of high skill individuals increases with the reduction in the size of the public sector, the private sector goes to a more balanced matching between skills and tasks. In other words, the reduction in the size of the public sector

Figure 6.6: Skill Distribution available to the Private Sector



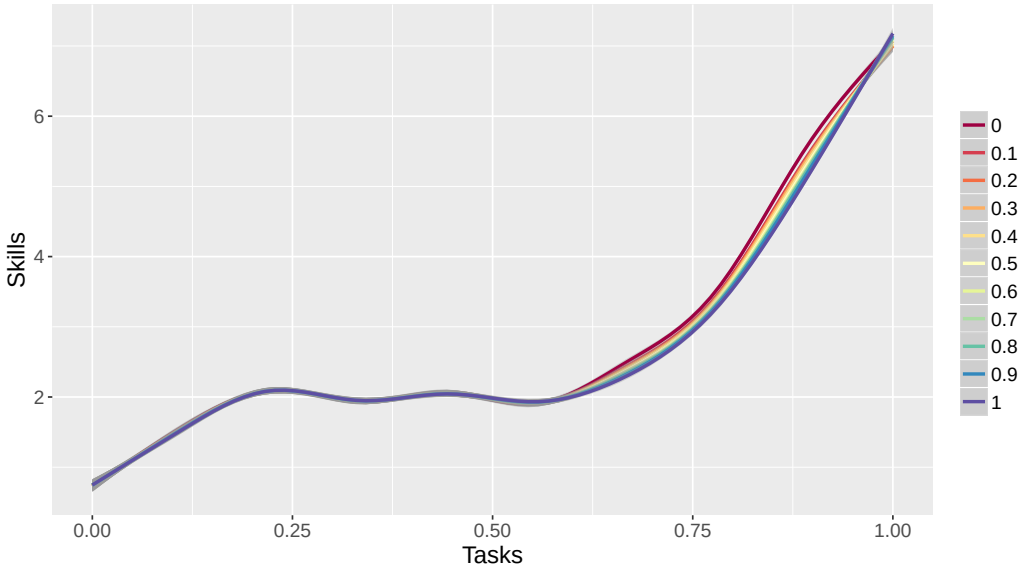
Notes: Percent growth in the probability of each skill type given the worker chose the private sector ($P(s|r)$) during a reduction in public employment. Vertical lines represent the size of the public sector in 2008, 2007 and 2005 respectively in Brazil for our sample.

Figure 6.7: Average Private Sector Wages for Each Skill



Notes: Percent growth in average log-wages for each skill, after a reduction on the size of the public sector. Vertical lines represent the size of the public sector in 2008, 2007 and 2005 respectively in Brazil for our sample.

Figure 6.8: Skills to Tasks Matching Function (Smoothed)

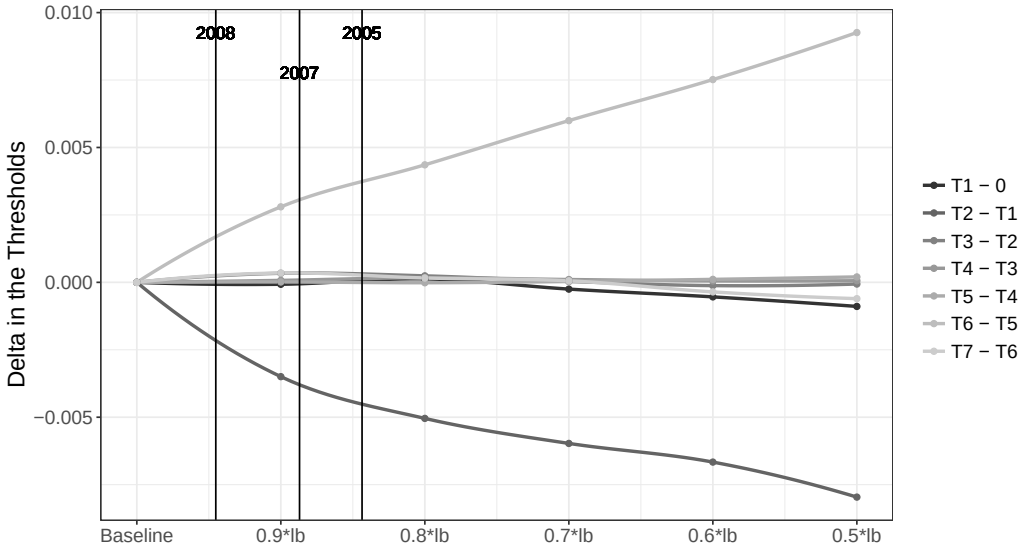


Notes: Skill to tasks matching function smoothed, in our baseline (blue) and while reducing the size of the public sector (red). Vertical lines represent the size of the public sector in 2008, 2007 and 2005 respectively in Brazil for our sample.

increases the amount of tasks performed by high skill individuals.

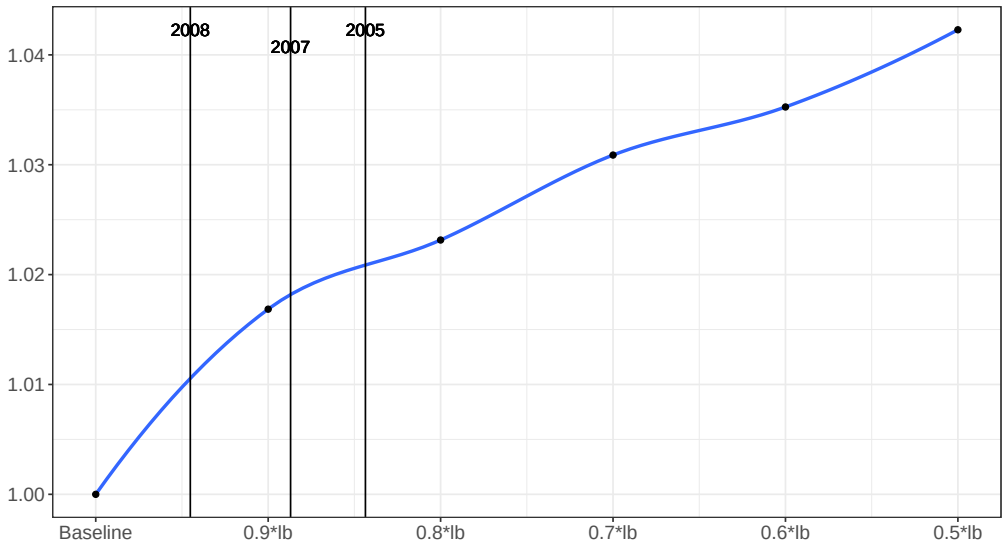
This pattern is also possible to see in figure 6.9, it shows the proportion of tasks performed by each skill as measured by the difference from the baseline. There was a growth in the amount of tasks performed by skill 6 and a sharp decline in tasks produced by skill 2. Although the magnitudes here seem very small, this change in the allocation of skills to tasks has a strong effect on private sector average productivity. Figure 6.10 shows the growth on private sector average productivity, as measured by the product divided by the amount of private sector workers (Y/l_r), when we decrease the size of the public sector. It is possible to see that decreasing the size of the public sector increases private sector productivity mostly as a result of a more balanced matching between skills and tasks. Estimations show that a 10% reduction on the size of the public sector increases private sector average productivity by 1.69%.

Figure 6.9: Private Sector Skill Thresholds



Notes: Difference from the baseline in the delta of skill thresholds ($T_s - T_{s-1}$), after a reduction in the size of the public sector. Vertical lines represent the size of the public sector in 2008, 2007 and 2005 respectively in Brazil for our sample.

Figure 6.10: Average Private Sector Productivity



Notes: Growth in average productivity (Y/l_r) if we decrease the size of the public sector. Vertical lines represent the size of the public sector in 2008, 2007 and 2005 respectively in Brazil for our sample.

7
Conclusion

We construct a static Roy model to assess if public sector employment negatively affects productivity and wage inequality in the economy through skill allocation. In the model individuals are heterogeneous in their skills and risk aversion, private sector demands endogenously based on skills and public sector demands exogenously based on education which is correlated with skills. Public sector wages are certain while private sector wages are uncertain. The combination of different public-private wage gaps for each skill and uncertainty makes the size of the public sector labor market influence the skill distribution available to the private sector. Changes in the skill distribution available to the private sector affects productivity and wages through skill allocation.

We estimate this model for the Brazilian labor market using PNAD 2011-4. Results show that a reduction in public labor demand increases private sector productivity, because it raises the proportion of tasks carried out by high skill individuals. In addition, the same decrease on the size of the public sector reduces the College wage premium and increases private sector wage inequality. Although the hike in the supply of high skill workers reduced high skill wages, it was not enough to overcome the rise in the proportion of high skill workers that contributed to increase wage inequality. However, the rise in the proportion of high-skill individuals was stronger for those without a College degree generating a reduction on the College wage premium.

Furthermore, there are some interesting counter-factual exercises that we intend to perform in future versions. First, changing employment and wages at the public sector for specific education groups. In addition, it would be interesting to analyze what would happen if we make public sector wages uncertain, and see how changes in this uncertainty affects individuals sector decision. Moreover, this is an interesting model to analyze the effects of the rise of the proportion of public workers in the labor force of the last decade, and its consequence on the allocation of skills in the economy.

Although the model is of general equilibrium, it assumes that the public sector is an unproductive sector. However, a reduction in the size of the public sector would also implicate a reduction in the amount of public goods supplied to the consumers and firms. Therefore, our counter-factual exercises must be

interpreted taking into account this limitation. Further contributions should analyze the impacts of public employment institutions on the allocation of skills both in public and private sectors and its implications to the supply of public and private goods and, consequently, welfare.

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A
Participation Decision

This section will describe a possible extension to the model that models the extensive margin of the labor market. Workers can choose which sector they will work for and if they will enter the labor market, so they have three choices. If they choose not to participate in the labor market their utility will be a reservation utility of $\bar{U}$, which is observed by the worker, but not by the econometrician. Workers are heterogeneous in their reservation utilities and this heterogeneity follows a normal distribution $\bar{U} \sim N(\mu, \sigma_U^2)$. Therefore, workers choose to work for the private sector if $U_r > U_b$ and $U_r > \bar{U}$, public sector if $U_b > U_r$ and $U_b > \bar{U}$ and to stay at home if $\bar{U} > U_b$ and $\bar{U} > U_r$. Which implies that the probabilities that a worker with skill s and demographic characteristic e chooses the private sector, the public sector or not to work is:

$$P_r(s,e) = P(U_b < U_r \cap \bar{U} \leq U_r | s, e) \tag{A-1}$$

$$P_b(s,e) = P(U_b \geq U_r \cap \bar{U} \leq U_b | s, e) \tag{A-2}$$

$$P_n(s,e) = P(U_b < \bar{U} \cap U_r < \bar{U} | s, e) \tag{A-3}$$

A reduction on the size of the public sector decreases U_b through a reduction in $b(e)$, it becomes harder to be accepted to work at the public sector. Workers may respond to it choosing to work at the private sector or to stay out of the labor market. Notice that with this formulation the proportion of workers that choose to stay out of the labor market depends on the skill and on the observed characteristic (e). Therefore, it may influence the distribution of skills available to the private sector, thus, average productivity and wage inequality.

Since we have data on individuals that choose to stay out of the labor market it is possible to estimate the model including this extension. To do it we need to include E more moments in the second stage that are the proportion of individuals that choose to stay out the labor market for each demographic characteristic. Consequently we will be able to identify all the parameters of the model and the parameters of the distribution of the reservation utility (μ, σ_U^2).

B
Public Good

Even though we interpret the public sector as a unproductive sector, the model also allows to interpret that the public sector produces a public good. The production of the public good depends on the labor demand for each skill at the public sector ($l_b(s)$) and a productivity parameter D:

$$G = D\left[\sum_{s=1}^{S} l_b(s)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}} \tag{B-1}$$

This public good enters as a lump sum transfer in the workers utility, thus not affecting workers sector decision. In this framework, reducing the size of the public sector increases private sector productivity and, consequently, wages, but reduces the public good. Therefore, it is not clear its impact on welfare. However, since we don't observe individuals utility or the provision of public goods we can not estimate its production function. For this reason, we can not discuss welfare effects of reducing the size of the public sector.