Pontifícia Universidade Católica do Rio de Janeiro

Lucas Freire

On the comparison of computationally efficient quota-sharing methodologies for large-scale renewable generation portfolios

Tese de Doutorado

Thesis presented to the Programa de Pós-Graduação em Engenharia Elétrica of PUC-Rio as a partial fulfillment of the requirements for the degree of Doutor em Engenharia Elétrica.

Advisor: Prof. Alexandre Street de Aguiar

Rio de Janeiro
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Abstract

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Portfolios of renewable electricity sources are interesting risk-management mechanisms for trading in electricity contract markets. When they are formed by players belonging to different companies, their stability relies on the way the risk-mitigation benefit generated by the optimal portfolio is allocated through individual participants. The problem of reaching a stable allocation can be mathematically formulated in terms of finding a quota-sharing vector belonging to the Core of a cooperative game, which can be formulated as a set of linear constraints that exponentially grows with the number of participants. Moreover, the right-hand-side of each constraint defining the Core relies on a given coalition value which, in the present work, is obtained by a two-stage stochastic optimization model. This work presents and compares efficient methodologies mainly based on mixed integer linear programming and Benders decomposition to find quota allocation vectors that belongs to the Core of large-scale renewable energy portfolios. Case studies are presented with realistic data from the Brazilian power system.

Keywords

Renewable energy trading portfolio; cooperative games; Benders decomposition; mixed integer linear programming; Shapley value; Marginal Benefits; large-scale Nucleolus; large-scale Proportional Nucleolus; fractional programming.

Resumo

Freire, Lucas; Aguiar, Alexandre Street (Orientador). **Comparação de metodologias computacionalmente eficientes para rateio de quotas de portfolios de geração de energia renovável de larga escala**. Rio de Janeiro, 2017. 131p. Tese de Doutorado – Departamento de Engenharia Elétrica, Pontifícia Universidade Católica do Rio de Janeiro.

Portfólios de fontes renováveis de energia elétrica são mecanismos de gerenciamento de risco interessantes para comercialização de energia em mercados de negociação bilateral. Quando formados por agentes que pertencem a diferentes companhias sua estabilidade depende da maneira com que os benefícios de mitigação de risco gerados pelo portfólio são alocados individualmente entre os participantes. O problema de se encontrar uma solução estável pode ser matematicamente formulado através da busca de um vetor de alocação de quotas que pertença ao núcleo do jogo cooperativo, que por sua vez pode ser formulado como um conjunto de restrições lineares que aumenta exponencialmente com o número de participantes. Adicionalmente, o lado direito de cada restrição que define o núcleo do jogo cooperativo define o valor de uma determinada coalisão que, no presente trabalho, é obtido através de um modelo de otimização estocástica de dois estágios. Este trabalho compara diferentes metodologias computacionalmente eficientes baseadas em programação linear inteira mista e na técnica de decomposição de Benders para encontrar vetores de alocação de quotas que pertençam ao núcleo de portfólios de larga escala de geradores de energia renovável. São apresentados estudos de casos que utilizam dados reais do sistema elétrico brasileiro.

Palavras-chave

Portfólio de comercialização de energia renovável; jogos cooperativos; decomposição de Benders; programação linear inteira mista; valor Shapley; método de benefícios marginais; Nucleolo de larga escala; Nucleolo proporcional de larga escala; programação fracionária.

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Indexes

i	Component of a vector/set usually related to a player;
$^{(j)}$, $^{(k)}$	Computational iteration;
t	Period (month);
s	Scenario.

Sets

$\emptyset$	Empty set;
C	Set of players / coalition;
I	Set of all players, also represents the grand coalition;
T	Set of periods;
S	Set of discrete scenarios;
$\wp$	Power-set;
$\mathcal{C}$	Set of the power-set of I, excluding the null set and the set I;
$Core$	Core of the cooperative game;
$\mathbb{B}$	Set of the binary numbers;
$\mathbb{R}$	Set of the real numbers.

Vectors

$\boldsymbol{x}$	Sum-one real vector that defines the quotas sharing;
$\boldsymbol{c}$	Binary vector that represents a coalition of players;
$\boldsymbol{c}^{\{i\}}$	Coalition that represents player i individually;
$\mathbb{0}$	Vector of zeros (0); represents the null coalition in vectorial notation;
$\mathbb{1}$	Vector of ones (1); represents the grand coalition in vectorial notation;

Random variables

$\tilde{y}$	Random variable;
$\tilde{\boldsymbol{y}}$	Vector of random variables;
$\tilde{G}_{i,t}$	Electrical energy generation of agent i in period t;
$\tilde{\pi}_t$	Spot price in period t;
$\tilde{R}_t$	Revenue of agent i in period t.

Decision variables

x_i	Quota of player i in the grand coalition;
Q	Contracted amount of energy of a sales contract in the Free Trade Environment;
$\boldsymbol{c}$	Worst-case gain coalition;
δ	Auxiliary variable for a linear convex approximation by parts for the Nucleolus and Proportional Nucleolus algorithms;
δ^{post}	Auxiliary variable for a linear convex approximation by parts for the post-optimization algorithm.
Δ_s	Auxiliary variable for the assessment of scenarios of under the VaR value of a revenue distribution.
z	Auxiliary variable for the assessment of the $CVaR$ value of a revenue distribution;

Dual variables

$\mu_{\boldsymbol{c}}$	Dual variable for constraints that impose the gains of coalitions $\boldsymbol{c} \in \mathcal{C}$;
γ_s	Dual variable relative to the marginal contribution in terms of revenue of player i in the spot market for a given coalition in the scenario s; related to the Marginal Benefits allocation method;
β	Dual variable relative to the marginal contribution in terms of contracting capacity of player i for a given coalition; related to the Marginal Benefits allocation method;

Parameters

p_s	Probability of occurrence of scenario s;
λ	Risk-aversion parameter;
α	Parameter that defines a distribution's percentile level;
h_t	Number of hours of period t;
FEC_i	Firm energy certificate of agent i;
n	Number of players;
P	Fixed price of a quantity sales contract in the Free Trade Environment;
$C_{i,t}^{U}$	Unitary energy generation cost of unit i in period t;
J	Risk-free opportunity cost of money between two periods;
$\mathcal{E}$	Convergence Gap (tolerance);
$UB^{(j)}$	Upper bound of the optimization function in iteration j;
$LB^{(j)}$	Lower bound of the optimization function in iteration j;
M	Parameter for the big-M technique for the linearization of the Proportional Nucleolus;

Functions

$\max_{\boldsymbol{x}}$	Maximization function on the vector of decision variables $\boldsymbol{x}$;
$\min_{\boldsymbol{x}}$	Minimization function on the vector of decision variables $\boldsymbol{x}$;
$\rho_{\alpha,\lambda}(\cdot)$	Certainty equivalent of a revenue distribution under risk-aversion factors α and λ;
$v(C)$	Characteristic function of coalition C;
$v(\boldsymbol{c})$	Characteristic function of coalition $\boldsymbol{c}$ in vectorial notation;
$g^{(j)}$	Worst-case gain function for the Nucleolus in iteration j.
$g^{p(j)}$	Worst-case gain function for the Proportional Nucleolus in iteration j.
$VaR(\cdot)$	Value-at-Risk function;
$CVaR(\cdot)$	Conditional Value-at-Risk function;
$\mathbb{E}(\cdot)$	Expected value function;
Φ_i^m	Function that defines the monetary quota allocation for player i for a given method m;

Operators

*	Optimum value of a given function;
^{T}	Transpose;
10^q	q-th power of 10;
!	Factorial;
$\frac{\partial y}{\partial x}$	Derivation of y in relation to x;
$\sum$	Sum.

Glossary of terms and acronyms

ABEEólica	Brazilian Association of Wind Power (in Portuguese)
ANEEL	Electrical Energy National Agency (in Portuguese)
BIO	Biomass generation unit (or player)
CCEE	Electrical Energy Commercialization Chamber (in Portuguese)
CERPCH	National Reference Center in Small Hydropower Plants (in Portuguese)
CVaR	Conditional Value-at-Risk
DisCo	Energy Distribution Company
EPE	Energetic Research Company (in Portuguese)
ETC	Energy Trading Company
FCD	Full Coalitional Dependent formulation for the Nucleolus quota-sharing method
FEC	Firm Energy Certificates
FiTs	Feed-in Tariffs
FTE	Free Trade Environment
GenCo	Energy Generation Company
IEA	International Energy Agency
LP	Linear Programming problem
MB	Marginal Benefits quota-sharing method
MILP	Mixed Integer Linear Programming problem
ONS	National System Operator (in Portuguese)
PAR-p	Periodic Auto Regressive model
PROINFA	Alternative Resources Incentive Program (in Portuguese)
RES	Renewable Energy Sources
RHS	Right-hand-side
RTE	Regulated Trade Environment
SH	Small hydro generation unit (or player)
VaR	Value-at-Risk
VARx	Vector Auto Regressive model with external variables
WP	Wind power generation unit (or player)
WWEA	World Wind Energy Association

1
Introduction

It is visible the globe's urgent call for the increase of sustainable practices. As a consequence of decades of irresponsible and predatory programs, many countries are starting to experience visible impacts on the quality of life of their populations. People are finally pressuring their governments and even the private sector to force a complete redesign of social, economic and environmental policies in defense of practices that will lead us to a more sustainable world. Fortunately, actions are being taken along this path. The International Energy Agency (IEA) recently announced emissions have stalled in 2014 [1] for the first time in 40 years, even with a global economy growth of 3%. This reflects the expansion (and effectiveness) of "green" global efforts. In this context, the development of new technologies in many areas such as transport and energy represents a new shift on the way private and public sectors are now planning their future.

On the energetic side a lot of effort has been done with the goal of decarbonizing the energetic matrices of countries. Hence, renewable energy sources (RES) have been receiving more and more incentives, year by year, since they are much less aggressive for the environment by virtue of being really low in greenhouse emissions (or even neutral depending on the kind of the source). The participation and growth of each technology of renewable power plants varies from country to country. However, the ones that have been leading the investments and, consequently, the penetration on the grids worldwide is the wind power, followed by solar, hydros with small (or even none) reservoirs, and biomass cogeneration plants that use different forms of waste as resource.

Those sources arise as a solution, at least in the short and medium term, for the urgency of electrical energy generation with the less environmental impact as possible. This necessity has even represented a shift on the way the new ventures and enterprises in energy has been globally driven. Even large reservoir hydros, a source that few decades ago was considered renewable, are facing aggressive critics from part of the society and environmentalists, forcing the authorities to opt for the expansion of their grids through the development of hydro plants with small reservoirs, the so called small hydros. Large reservoir hydros demand huge deluged areas, which can lead to more impacts than deforestation in general due to the production of methane as a consequence of organic material decomposition. On the contrary, small hydros have the advantage of producing energy from a renewable source with much less impact but lose, however, their capability of regulating the energy production along periods.

Leading global investments, wind power plants emerge as a cheap [2],[3], reliable and, at certain point, a versatile alternative for producing clean energy. Even though some few problems occur in terms of noise and visual pollution and when windmills are deployed in the route of bird migration, for instance, the technology is relatively easy and fast to build and deploy. Those characteristics make wind power the leading renewable source almost in the whole world with more than 130 gigawatts of installed capacity from 2011 to 2014 [4].

As another option, cogeneration from biomass [5] appears as an alternative that has cheap resource, since it usually makes use of organic waste from different kinds of plants/mills as intake. However, when biomass plants use sugar cane bagasse as resource, for example, the energy production faces a strong seasonality since the availability of the resource is linked to the cane harvest pattern. Outside the harvest period the energy generation of such plants is null. Unfortunately, the uncertainty on energy production is only one of the challenges RES face in the market. In a nutshell it is essential to foster and create fruitful environments for renewables.

Fortunately, RES have been receiving different forms of incentives from the private and public sectors, in favor of the intensification on their penetration on the energetic matrices of many countries. Associations of investors of specific renewable sources, such as the Brazilian Association of Wind Power (ABEEólica, in Portuguese), have been responsible for negotiating with governments for incentives, like financial subsidies and other specific instruments, for example. On the other hand, the public sector is corresponding with incentives that vary on a country to country basis, but are mainly constituted by support mechanisms, such as feed-in tariffs (FiTs) and specific auctions for renewables only (see [6] and

[7]). This kind of cooperation between the public and private sectors has been building up an attractive environment for the development of tenths of gigawatts of RES worldwide.

However, even with the existing incentives, the integration of RES faces difficult challenges, not only on the technical, but also on the commercial side. The inevitable injection of more gigawatts of new intermittent sources, as in the case of RES, in the countries' energetic matrices will bring difficult challenges for the grids on-time operation [8] and expansion planning as well. This has forced debates from part of experts and scientists from all around the world, on how the impact of the large penetration of RES on the grids can be suppressed [9]. Since RES are completely dependent on their natural resources (wind, inflows and waste), they present a strong uncertainty on the short-term generation due to the intrinsic unpredictable weather conditions, and one of the current main challenges is to outline a safe power system expansion and operation planning. Additionally, a lot of research has been done on system reliability for example, as the prominent growth of the market share of RES becomes imminent [10].

Yet, it seems the commercial challenges for RES penetration have not been the major focus of researchers, with less work being done in this area. The main problem to be addressed is to design economic models that can guarantee the integration of RES in power market, which is the exact purpose of this work. Located on an environment where generators face uncertain production and are constantly exposed to market risks, RES need models that safely quantify the market's inherent risks and develop strategies that can guarantee a good horizon for investors.

In the case of Brazil, which serves as basis for this work, two environments are available: (i) the Regulated Trade Environment (RTE) and (ii) the Free Trade Environment (FTE). In the RTE the regulator, looking for the lowest tariffs as possible for the end-users, sets auctions by price-decreasing rounds where investors bids only determines if they are in or out for the given price of each round. This is continued until the total determined volume of energy, interest of the regulator, is fulfilled by investors that bid in within the lowest tariff. In the other environment, the liberalized one, Generation Companies (GenCos) and Energy Trading Companies (ETCs) can freely sign energy contracts between themselves as well as with special consumers. Moreover, in the liberalized market, focus of this work, the competition for contracts with consumers places risks on the spot market for the sellers (and buyers in a lower level). The supply contracts set the obligation, by the part of the seller, to comply with the delivery of the contracted amount of energy in each period of the contracts' time horizon, usually in exchange for a flat rate payment by the buyer (payoff). As in many other countries, this arrangement represents only a financial instrument [11] and the differences between the contracted amount of energy and the real energy production/consumed volume are settled in the spot market. Also, the spot price behaves randomly between lower and upper bounds which are defined annually by the system regulator. As strong characteristics, it is usually kept at low values in average but also very volatile, sometimes floating from the lower to upper bound in few weeks, for instance.

In this conjuncture, the intrinsic intermittent and seasonal resources of RES together with unpredictable spot prices mutually place the well-known price and quantity risk: in the case where the spot prices are in a higher level than the fixed payment price and the generated amount of energy is not enough to satisfy the contract, the seller finds himself exposed in the spot market, which can lead to high financial losses. To prevent this kind of situation, the development of portfolio trading strategies arises as an excellent risk mitigation strategy. Many works have presented different effective proposals for risk mitigation portfolios. In general, those strategies make use of the synergic complementarity of different sources, with different generation profiles, and make use of a joint selling strategy to mitigate the inherent risks of the business. In this setting, it is a common practice to use the linear stochastic programming framework to develop strategic trading models. Moreover, the use of measures of values in which agents express their preference towards risks is also a common practice. For instance, the Value-at-Risk (VaR) and its successor Conditional Value-at-Risk (CVaR) are also widely used in the election of the most adequate certainty-equivalent functional, which constitutes a key tool to define the always very personal risk averseness profile of authors. Furthermore, on an attempt to bring a more foreseeable pattern for the energy trading strategies, the use of sampling methods such as Monte Carlo is a common practice to simulate scenarios that faithfully represent the stochastic processes of energy generation and energy spot prices.

It is also standard works that use these techniques considering their portfolios composed by generators that belong to a particular company, case of the works previews mentioned. However, it becomes interesting as well the analysis of portfolios formed by units of different companies, which in a practical viewpoint represents a plausible condition. The main contribution of this setting, in comparison with a single company portfolio, is that it represents a typical case of cooperative games theory. This area vastly studies circumstances where different companies cooperate, aiming greater benefits with a joint trade strategy then the ones they would receive while operating alone in the market. Moreover, it brings the challenge of sharing the quotas of the joint financial gain among its participants (also called players), which is *as important as* the optimization of the portfolio trading strategy itself. In this setting, the concept of fairness on the quotas allocation methods is a key point for the success of the cooperation among the companies. Otherwise the portfolio (also called pool or grand coalition in this scheme) does not remain stable, ending up with participants' desertion.

In this context, the cooperative games theory [12] plays a central role in the portfolio's stability, since it is a framework capable of producing sharing solutions for the intrinsic synergic benefits of the joint trading strategy. Different works in power systems have already explored the idea of allocating benefits among the participants of a portfolio. For example, [13] has studied different allocation methods for a portfolio composed by 3 renewable units, one small hydro, one wind power plant and one biomass plant. Still in [13], the allocation methods analyzed were: proportional share [14]; Shapley value [15]; and Nucleolus [16]. The former two methods share a common characteristic of having the drawback on the computational side: the problem becomes intractable if the number of participants in the pool increases two much. Nevertheless, they are not the only allocation methods in the cooperative games theory. For example, Aumann-Shapley [17][18] and Marginal Benefits [17] arise as computationally efficient allocation methods. For certain applications, this could place these two methods in advantage when compared to non-scalable methods. However, previews works has shown that the disadvantageous computational burden of the Nucleous method can be favorably resolved. For instance, based on the Benders decomposition technique, [19] have presented an efficient algorithm that recovers the nucleolus solution for quota allocation of a renewable pool composed by a large number of participants. But it is, however, limited to the analysis of the Nucleolus as the only employed method for allocating quotas.

In this context, the present work proposes a robust comparison of different quota allocation methods for sharing the benefits of a joint selling strategy for a RES pool. All concepts regarding to energy commercialization, risk management, decision under uncertainty and cooperative games theory (which includes the allocation methods) are presented. As well, the pool design and the mathematical models that solve the computational intractability for the Nucleolus [20][21] and Proportional Nucleolus [20][22] methods are properly introduced. Furthermore, for the sake of clarification, small and didactical case studies with 3 players only are presented, where the qualitative differences among the allocation methods are well exposed. Finally, a larger case study is presented, where each one of the allocation methods are challenged on their capability of solving large instances. Those case studies are built to evaluate the underlying aspects of the quota sharing methods, with the final goal of depicting the robustness of the presented Nucleolus and Proportional Nucleolus methods.

1.1 Motivation

Aiming the diversification of the Brazilian electrical matrix, it was created in 2004 a program to foster the penetration of RES in the sector. This was made through incentives that promoted a safer environment for new investments in renewables, with small hydros, wind power and biomass plants on its majority. The main incentive was the creation of long-term contracts with distribution companies. Since then, RES have been increasing their penetration on the Brazilian free market, but do not retrieve all possible benefits of such environment yet.

Moreover, according to the market’s official EPE’s (Energetic Research Company, in Portuguese) medium term expansion planning (Decennial Expansion Plan for 2023) there is an expectation of growth of the electrical matrix from about 125 GW in Dec-2013 to 196 GW of total installed capacity in Dec-2023, representing a growth of 72GW (or 58%) in ten years. Figure 1.1 shows this expansion. More impressively, according to the same document, it is also expected a growth of 26 GW in renewables' (including solar) installed capacity in the same period, jumping from 17 GW to around 43 GW, a difference of 153%, as shown in Figure 1.2.

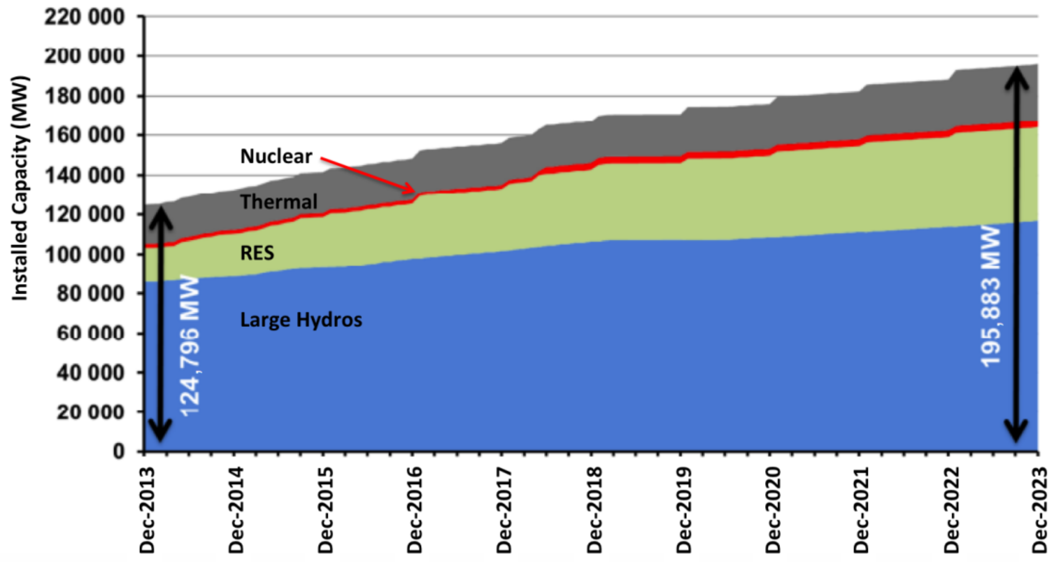

Figure 1.1– Prospected evolution of the Brazilian electrical matrix.

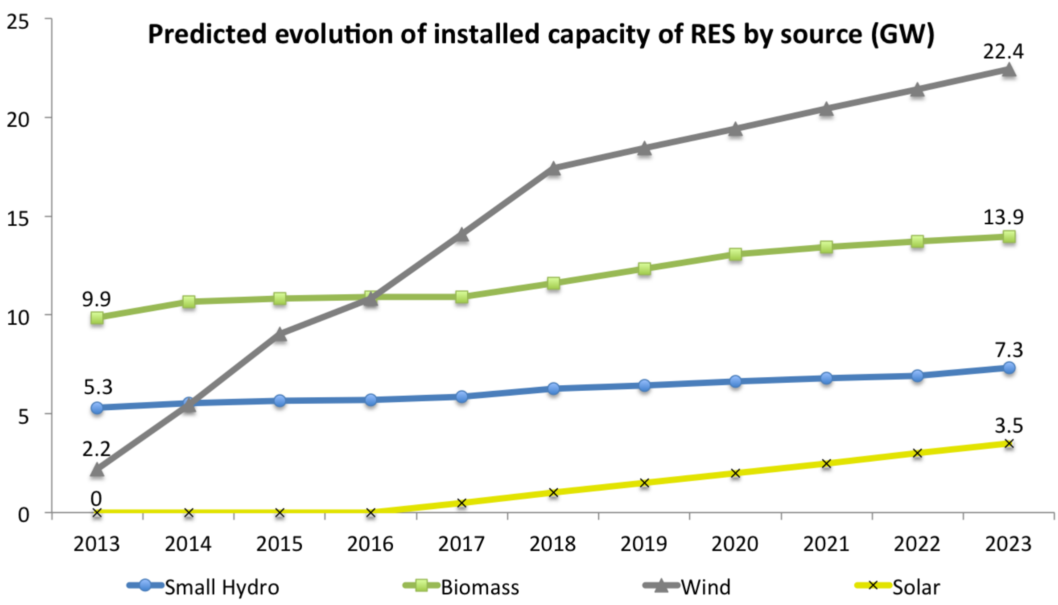

Figure 1.2 – Prospected evolution of RES in Brazil.

However, to exploit such growth, RES will need to overcome hard challenges. For example, the typical seasonal profile on the energy generation of RES brings difficulties on the sale of the very common flat quantity contracts for example. In such contracts, differently from the generation, the energy amount that has to be delivered is the same for all periods in the horizon of contracts. More specifically, among variables that represent risks to RES agents, there is the nature of renewable sources itself and the spot price. These two randomly behaved variables are the main responsible factors for the volatility on contracts' revenue, especially on the free market. They depend basically on the future occurrence of states of nature (e.g., climate factors). And despite obeying a certain pattern, such variables sometimes emerge with a combination of low probability scenarios that can strongly affect the cash flow of intermittent sources, case of RES, inducing high financial losses. Under this framework, it is natural the adequate capture of these low-probability scenarios is of utmost importance. This is exactly what risk measures like CVaR proposes to do.

In the view of these considerations, it is necessary to pursue new energy trading models that can enhance the RES capability of competing with other sources in risky markets, case of the Free Trade Environment. The existing regulatory incentives – such as tariffs discounts for RES – play an important role in this context, although being not enough to mitigate the price-and-quantity risk, inherent to the FTE. It is necessary to make the FTE a real alternative for RES, so they can appreciate the benefits of this environment and break the limitation of trading only (or mainly) via auctions. A smart diversification of an energy contracts portfolio, utilizing the synergic seasonal energy profiles of RES can became a fruitful risk-mitigation option, making the FTE an attractive opportunity for investors. In this context, it is clear the necessity of using the correct framework to treat the inherent risks of the business. Furthermore, a model that mitigates the price-and-quantity risks and consequently aggregates value to the business of RES is crucial for the consolidation of such sources in a sustainable way, not only in the Brazilian electrical matrix, but also worldwide.

1.2 Literature review

The use of portfolio optimization techniques to devise models on electrical markets represents a common practice in the academic area. Some examples can be seen in [23], [24] and [25], where portfolio of pumped storage hydros and wind power plants are studied. Additionally, further examples of recent works that make use of such techniques, as well as the CVaR can be seen in [26], [27], [28], [29] and [30]. Specifically, in [26] it is done an evaluation of sales contracts via option theory. Moreover, optimum electrical energy auction strategies are developed in [27]. Still, [28] presents a model to evaluate the risk for an optimum energy portfolio selection and [29] presents a combined stochastic-robust optimization approach to reduce risks of a renewable energy-trading portfolio.

Furthermore, [30], [31] and [32] can be cited as applied works on energy trading schemes that address the Monte Carlo simulation technique to produce energy generation scenarios as input of a stochastic optimization model that uses the CVaR as risk-averse measure. Specifically, a portfolio composed by a biomass unit and a small hydro to back a contract in the FTE is presented in [30]. As other examples of papers that resort to this scheme, [31] extends the analysis of [30] to a general renewable portfolio of wind, small hydros and biomass plants. Notwithstanding, the main contribution of [32] was to enhance the analysis of [31] by representing correlated wind and inflows and their joint dependence with spot prices. This contributed to a better risk tracking, done via the consideration of uncertainty variables from the Brazilian hydro-thermal dispatch model as explanatory variables. In this same paper, a wind power plant and a small hydro jointly trade their energy in the FTE with results that show a substantial reduction on the price-and-quantity risk due to the synergic complementarity on the source's production. Additionally, the statistical model used in [32] is extensively depicted in another paper, [35], at the same conference.

On games theory side, there are many examples of papers and applications. Particularly, [36] discussed stochastic cooperative games (under uncertainty), addressing many of the aspects of the present work, such as superadditivity, convexity and certainty equivalents. As example of application on Brazilian power sector in a cooperative games theory framework, [37] has presented a Nucleolus-based quotas-sharing scheme for the FEC (Firm Energy Certificates) of the Brazilian large hydros alongside the comparison with various other classic-fashioned methods. Still, [13] proposes a pool composed by different renewable energy generators to jointly trade the pool's equivalent energy in the FTE in order to mitigate the price-and-quantity risk. The main idea of the work is to take advantage on the synergy from the complementary pattern on the energy production of different sources to promote a surplus on the source's financial gains. A cooperative games theory based analysis was made in order to show that the establishment of a renewable pool generates greater benefits than any other subcoalition of players in the pool. However, this work explored a single case study with only three sources and do not address the computational intractability that emerges with the increase on the number of participants.

Such burden aspect is typical of cooperative games theory problems, as depicted in [38] and [39]. Both works focus on the development of computational methods that can capably solve this intractability issue, thus allowing assessment of the core of large-scale problems in a reasonable computational time. Finally, [19] presents a model that, besides establishing a large-scale RES pool in the shape of the one presented in [13], it also solves the Nucleolus method computational intractability. The paper, however, does not make any comparison with different quotas-allocation methods for the proposed renewable pool.

1.3 Publications related to this thesis

The bellow-disposed publications are related to the theme of the present thesis and were produced during the journey of the PhD course.

I) Journal Publications

a) **L. Freire**, A. Street, D. Lima, L.A. Barroso, “A Hybrid MILP and Benders Decomposition Approach to Find the Nucleolus Quota Allocation for a Renewable Energy Portfolio”, *IEEE Trans. Power Syst.*, Dec. 2014.

II) Conference Proceedings

a) A. Street, **L. Freire**, D. Lima, J. Contreras, “Sharing Quotas of a Renewable Energy Hedge Pool: A Cooperative Game Theory Approach”, in Proc. *IEEE PowerTech 2011,* Trondheim, Norway, 2011;

b) A. Street, B. Fanzeres, D. Lima, J. Garcia, **L. Freire**, R. Rajagopal, “Mecanismo de Realocação de Energia Renovável: Uma Nova Proposta para Fontes Alternativas” in Proc. *XXII Seminário Nacional de Produção e Transmissão de Energia Elétrica (XXII SNPTEE) 2013*, pp. 1-9, Brasília, Brazil, Oct. 2013;

c) A. Street, A. Veiga, D. Lima, A. Moreira, B. Fanzeres, J. Garcia, **L. Freire**, “Simulação da Geração de Usinas Renováveis Coerentes com os Cenários de Operação do Sistema Elétrico Brasileiro”, in Proc. *XXII Seminário Nacional de Produção e Transmissão de Energia Elétrica (XXII SNPTEE) 2013*, pp. 1-8, Brasília, Brazil, Oct. 2013;

d) A. Street, A. Veiga, D. Lima, B. Fanzeres, **L. Freire**, B. Amaral, “Fostering Wind Power Penetration into the Brazilian Forward-Contract Market”, in Proc. *IEEE PES General Meeting 2012*, pp. 1-8, San Diego, California, USA, Jul. 2012;

e) B. Fanzeres, A. Street; A. Veiga, D. Lima, **L. Freire**, B. Amaral, “Comercialização de Energia Eólica no Ambiente Livre: Desafios e Soluções Inovadoras”, in Proc. *XII Symposium of Specialists in Electric Operational and Expansion Planning (XII SEPOPE) 2012*, pp. 1-10, Rio de Janeiro, Rio de Janeiro, Brazil, May 2012.

1.4
Objectives of this work

The main objective of this work is to present and compare different allocation methods for a risk-averse RES pool formed by a large number of assets belonging to different companies. Also, the analyzed methods are compared in qualitative and quantitative ways. The qualitative comparison is done via analysis of economic signals produced by the allocation methods outcomes. In the quantitative side, the efficiency of an allocation method is “measured” in terms of its capabilities of computationally resolving large instances of the problem in a reasonable execution time. The used framework is the stochastic cooperative games theory [36] and the allocation methods studied are the following: FEC-Proportional Share; Shapley-Value; Marginal Benefits; Nucleolus and Proportional Nucleolus.

Special attention in this work is given for the last two methods, since they pursue the greater stability as possible for the model studied here. As previously mentioned, the goal of the Nucleolus allocation method is to increase the benefit of the absolute monetary worst case advantage within all possible coalitions that can be formed among the participants of the pool. It is based on the idea that if the worst-case gain coalition is satisfied, so it is all the other ones. Under this setting, the so important stability of the pool is assured. Moreover, the Nucleolus and the Proportional Nucleolus allocation methods[1] slightly differ from each other. The difference is that the last one, which is devised from the first, aims the maximization of the worst-case gain in terms of relative gain, i.e., it maximizes the percentage gain of the coalition, in contrast with the maximization in terms of absolute value from its precursor method.

Behind the characteristic of fairness (or justice) around a given allocation method, there is the concept of core. The core of a cooperative game is the set of solutions for quotas allocation of a joint trading strategy portfolio that produces (positive) gains for all possible coalitions. In this context, if a given solution for quotas allocation belongs to the core, no participant (or coalition) has any monetary incentive to leave the pool, securing then the stability of the grand coalition. In practical applications, this arises as a very important characteristic, since a given subset of participants can quit the pool if their cooperation brings more benefits than the ones produced by the grand coalition. Furthermore, the core of a given game can be empty or not. Inside this environment, the exploration of the existence (non-emptiness) of the core of the proposed cooperative game is also a relevant ingredient of the present work.

In this context, it is worth mentioning that the importance of Nucleolus methods is enforced by the fact that both methods always produce solutions that belong to the core of the cooperative game, whenever it is non-empty, which is the case here. This is one of the most important aspects of an allocation method in cooperative games, since it provides a strong economical signal for the players and not all allocation methods benefits from this peculiarity. Hence, any method with this aspect can be used as a "verifier" of the existence of the core: if the method produces positive gains for all coalitions, then the core is non-empty; otherwise, the core is empty.

Moreover, the Nucleolus methods also maximize the gains of the most threatening coalitions, which bring a greater stability for the pool. On the other hand, the inconvenience of Nucleolus methods is the obstacle faced when resolving the optimization problem, which grows exponentially as the number of players in the pool increases [38]. Additionally, at the present work, the right hand

[1] *For the sake of brevity, hereinafter it will be used the term 'Nucleolus methods' (in plural) when referring to both Nucleolus and Proportional Nucleolus methods*

side (RHS) of the set of constraints on the Nucleolus problem defines the optimal trading strategies of each coalition in the pool, representing the synergic gains of the portfolio. In this context, there is an additional difficulty on solving the problem, because each individual optimal strategy is the outcome of an isolated risk-averse stochastic optimization problem. In a real situation of a pool composed by 30 players, the set of constraints imposes that the stochastic optimization problem has to be solved more than one billion times, making the computation of the Nucleolus intractable.

To overcome this obstacle, the present work makes use of the efficient algorithm presented in [19], which is based in Benders decomposition. This interactive algorithm is composed by a primary linear programming (LP) problem and by a secondary Mixed Integer Linear Programming (MILP) problem. The primary (or master) problem is a reduced version of the full Nucleolus problem and produce trial solutions for the Nucleolus allocation. Given a trial solution, the secondary problem, so, finds the worst-case gain coalition among all possible combinations of players in the pool. Next, a cut (constraint) is added on the master problem to force it to satisfy the gains of this just-discovered coalition in the next trial solution. The outcome of this iterative process between the two problems builds the known set of umbrella constraints [40], allowing the algorithm to finitely converge to the Nucleolus solution without the need of evaluating the complete set of coalitions.

However, the work presented in [19] focused exclusively on the analysis of the Nucleolus method and does not incorporate analysis on the Proportional Nucleolus (or any other method). Furthermore, there still exists one more particular complication when such methodology is applied to the Proportional Nucleolus: the auxiliary (secondary) problem, outgrowth of the methodology, becomes nonlinear and cannot be solved by commercial linear programming solvers. This additional drawback is solved for a renewable pool scheme for the first time in the present work. The linearization of the auxiliary problem is done via fractional programming technique (see chapter 4.3.2 of [41]), following the findings of [42].

1.5 Contributions

The contributions of the present work are listed below:

i. Present and compare different quota allocation methods for a large-scale risk-averse cooperative RES pool;

ii. Present an MILP (Mixed-Integer Linear Programming problem) approach that, given a trial solution for the quota allocation, finds the coalition with the worst-case gain for the Proportional Nucleolus method via fractional programming technique. This procedure generates strong cuts for the relaxed problem, used as Benders cuts to locally approximate the worst-case gain function in a reformulated version of the full problem. Moreover, this procedure can be used as an “oracle” that verifies if a given solution, produced by any method, are always inside the core of the cooperative game or not;

iii. Present an algorithm based on Benders decomposition and MILP that efficiently finds the Proportional Nucleolus quota allocation for a large-scale RES pool. This algorithm makes use of an MILP problem that is similar to the one described in the previews item;

iv. Present the proof of existence of the non-empty core for the proposed cooperative renewable portfolio.

Additionally, it is important to highlight that the contribution above disposed in item (iii) arise as natural development of the publication I.a of Section 1.3.

To corroborate the effectiveness of the proposed methodology, realistic data from the Brazilian power system is used to build a large-scale dataset of test instances. Due to the algorithmic nature of the contribution, results will focus on the capability of solving large-scale problems with acceptable computational effort.

1.6 Organization

Chapter 2 introduces the reader the main RES of the Brazilian power market. Chapter 3 discourses about the peculiarities of the market where those sources trade their energy, as well as the uncertainties regarding the energy trade in the FTE. Closing the chapter, the mathematical expression that rules their revenue in the FTE is presented, which is promptly modified to reflect the pool's revenue. Chapter 4 initially presents the concepts of the cooperative games theory, supporting the cooperative game characterization of the pool. Then, the adopted value function (also known as certainty equivalent) for the RES pool is presented, prior to the proof of the existence of the non-empty core of the game. Chapter 5 is responsible for presenting all the allocation methods used in this work, discussing their characteristics and presenting their respective formulation. Still, a small example of quotas sharing is presented for a 3-players case pool in order to compare and motivate the underlying allocation methods. Chapter 6 highlights the combinatory explosion of the pool, a problem that affects almost all the presented methods, and proposes the MILP-based Benders decomposition approach to solve this issue for the two Nucleolus Allocation based methods. The chapter then presents a methodology that ensures the equalization of individual gains for the Nucleolus methods, which is not always guaranteed by the original formulations. Chapter 7 presents realistic case studies that compare all proposed allocation methods, firstly in a qualitative optics, and then testing their performance on solving large-scale instances of the pool, showing the effectiveness of the proposed MILP-based approaches. Finally, Chapter 8 closes the work with conclusions and suggestions on immediate and eventual future researches.

2
The main RES in Brazil

Since the global oil crisis of 1973, Brazil has focus on investments on non-oil-dependent energy sources, such as the production of alcohol from sugar cane to substitute gasoline, and also expanding the hydroelectric share on its electrical matrix. As a consequence, currently in Brazil around 80% of the grid's installed capacity come from renewable energy sources, including 84.6 GW of large reservoir hydros (63%), 4.8 GW of small hydros (3.5%), 12.4 GW of biomass (9%) and 6.2 GW of wind power (4.6%). Under this environment, the three main RES in the Brazilian power sector, which is used as basis for this work, are small run-of-river hydros, wind power plants and biomass cogeneration plants. These sources had a boom after the Brazilian regulator established the PROINFA program, in 2002, which was focused in the promotion of RES in the power sector. During the active period of the program, which was closed in 2011, more than 1,152 MW of installed capacity of small hydros and 553 MW of biomass were added to grid [43]. Adversely, wind power technology was yet very expensive in Brazil during the 2000's decade, so that its current protagonist role on the penetration of RES in the Brazilian grid started to show up by the beginning of the 2010's decade only. Further information about the individual aspects of these sources will be discussed in the next sections.

2.1
Small hydro plants

Brazil's main alternative to foster investment policies on renewables was exploring the large number of basins on its vast territory. This promoted an expansion on investments of hydro plants throughout the national territory, with exception for amazon forest areas that, due to environmental and technical barriers, experienced less investment on this side. In this context, Brazil's South and Southeast regions currently concentrate the major number of hydro plants, followed by the Central and Northeast regions, as seen in Figure 2.1. On the contrary, the majority of unexplored hydraulic potential of Brazil's is concentrated in the amazon. This contrast is seen in Figure 2.2, where dark colored areas in the map constitute regions with greater unexplored potential than the light colored ones, with pizza graphs indicating the total estimated unexplored potential in gray.

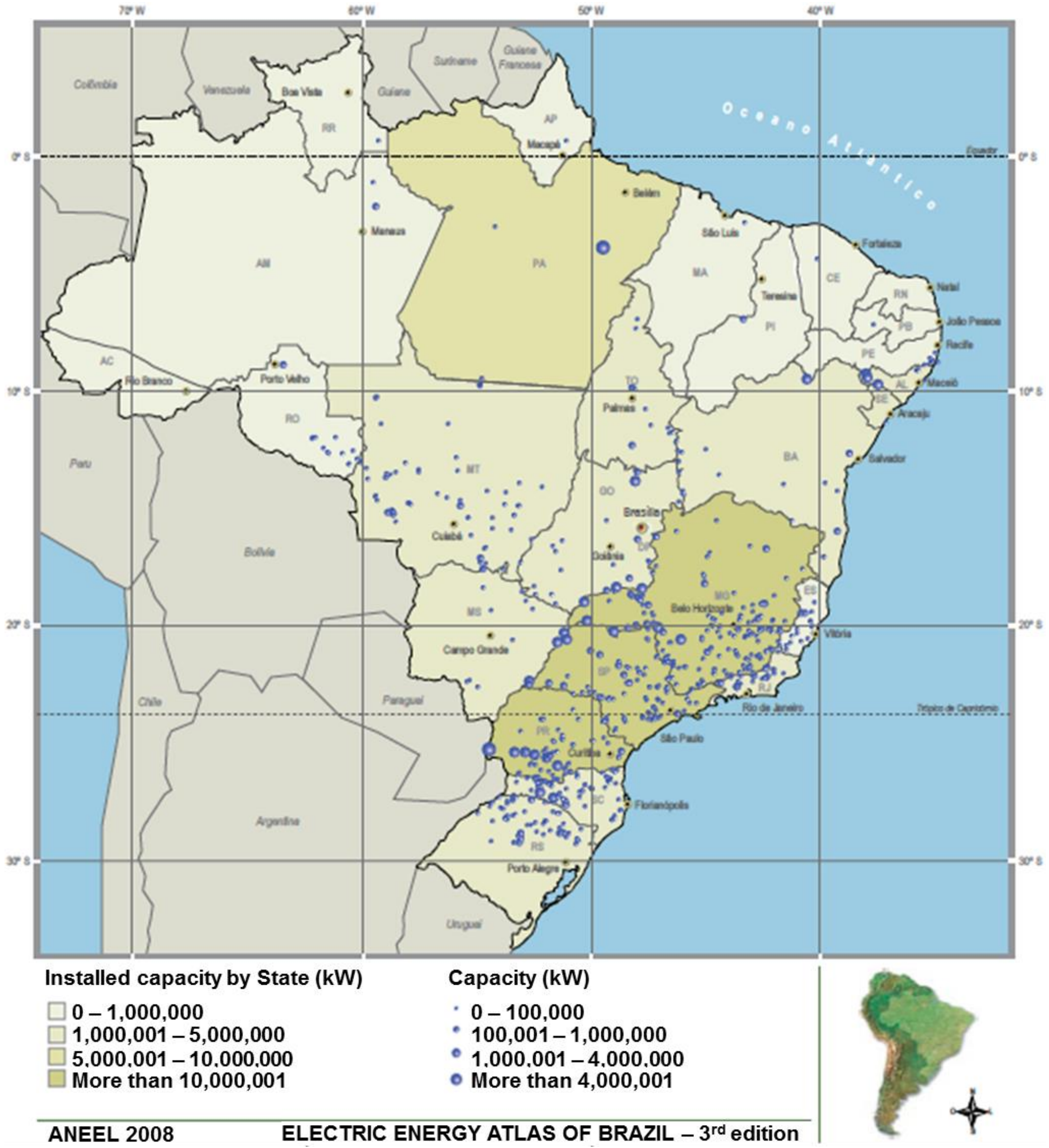

Figure 2.1 – Explored hydraulic potential – hydraulic plants – of Brazil.

Still in the graphs of Figure 2.2, the known unexplored potentials of each region are indicated in dark blue with the explored potentials under construction or under operation being represented by blue and light blue colors, accordingly.

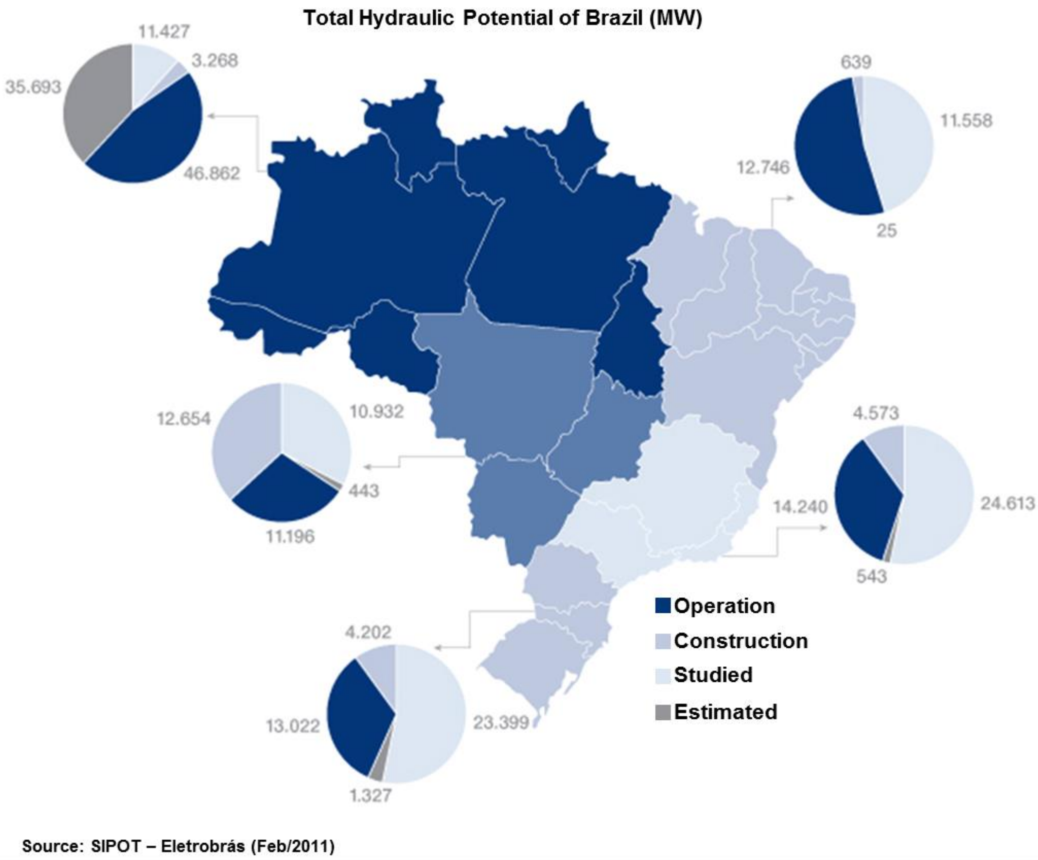

Figure 2.2 – Total hydraulic potential of Brazil (including unexplored).

Despite the great unexplored hydraulic potential, as previously discussed, on the environmental side small hydros are practically seen as the only option to produce electrical energy on areas where the Brazilian basins are still unexplored. Nevertheless, in early 2012, the small hydros represented 3.3% (3.86 GW) of the 117 GW of total installed capacity in Brazil (including the whole national electrical grid). According to ANEEL (Electrical Energy National Agency, in Portuguese), 2.3 GW from small hydros are already licensed making total installed capacity from this source reach nearly 7 GW by the end of 2019. Considering the extension of the basin areas of the country, it is possible that this share increases even more in the coming decades [44]. Additionally, prospections of National Reference Center in Small Hydropower Plants (CERPCH, in Portuguese) are more optimists and anticipate the total installed capacity from small hydros, considering the explored and unexplored potential, can reach 12 GW.

More specifically, following the resolution 394 from 04/12/1998 from ANEEL, a small run-of-river hydro is any small hydro plant with at least 1 MW and up to 30 MW of installed capacity that respects the maximum of 13 km^2 of deluged area. In this context, a small run-of-river hydro plant operates without the regulation of its production, i.e., small reservoirs do not allow the unit's operators to control the water flow through the turbines. Additionally, these kinds of plants are usually installed in small to medium size rivers that have a minimum significant altitude variation on the watercourse, in order to achieve enough hydraulic head to move the plant's turbines. On situations where the river inflow exceeds turbines' capacity, the water has to be spilled. Contrarily, in dry periods, the lack of water can even let turbines idle. Finally, another strong characteristic of hydro plants in Brazil is the seasonal pattern of energy production, with well-defined periods of high and low production at months around summer and winter, respectively. This typical profile is depicted in Figure 2.3, represented by terms of the average and the quantiles of 5% and 95% of de historical production of a unit placed at Paraibuna river (south-east of Brazil).

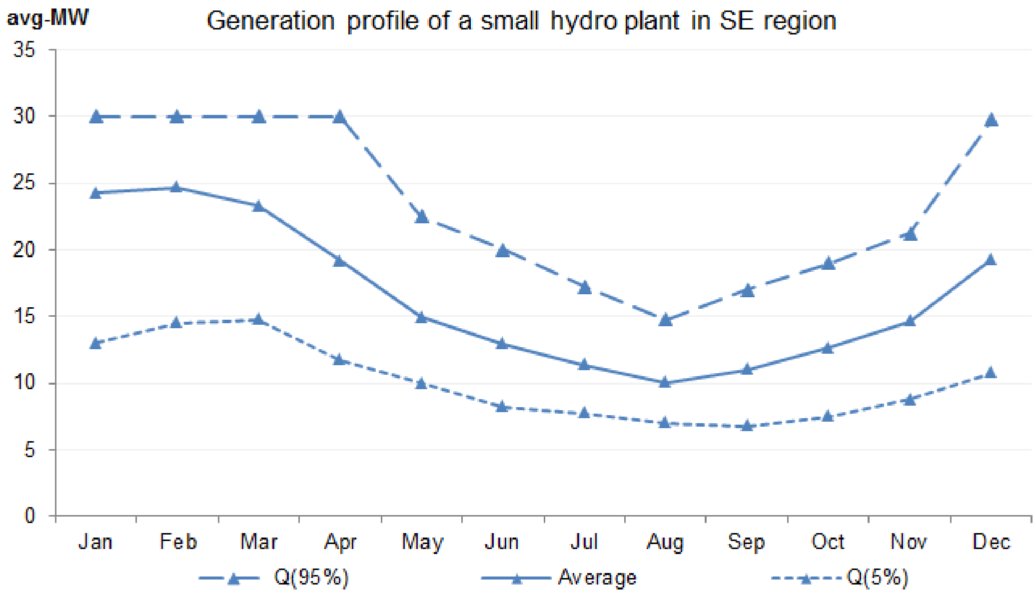

Figure 2.3 – Generation profile of a typical small hydro.

2.2 Biomass plants

The use of sugarcane bagasse for electricity generation is a practice used in several places in the world. This production is done via biomass plants: thermal units that generate electrical energy through organic materials (the so called biomass resources). Depending on the particular conditions of each country, the variation is found in the energetic efficiency the bagasse provides. Currently, the best results are achieved in Hawai'i and Reunion Island. In Brazil, the world's largest cane sugar producer, the cogeneration at sugar cane and alcohol plants is also a common practice with the generation of between 20 and 30 kWh per ton of crushed cane. With the adoption of higher pressure levels on steam and more efficient turbines, steam cycles can produce more than 80 kWh per ton of crushed cane. Additionally, according to [5], this value can be further increased with the energetic use of cane leaves and tips currently left in the field during harvesting.

Moreover, since the beginning of the colonization, in century XI, sugar cane production was always a strong product of the Brazilian economy, with huge plantations spread throughout the Southeast and Northeast regions of the country. Furthermore, also due to investments policies on non-oil-dependent energy sources, Brazil launched a program to promote investments on the production of sugar cane alcohol. This kind of alcohol became an alternative to gasoline, with the first cars moved 100% by the new fuel being produced by the end of the 1970's decade.

The huge number of sugar cane plantations in Brazil creates also the opportunity of cogeneration of electrical energy through burning of the sugar cane bagasse. Although sugar cane bagasse is the main resource for biomass plants in Brazil, as input they can also have other kinds of organic resources, such as wood combings, manure, food waste and other agricultural waste that produce methane gas. Additionally, recent investments point out another promising biomass resource: eucalypt combings, which also have strong availability in Brazil, due to the numerous eucalypt plantations that produces cellulose, intake of paper plants. . Moreover, according to a report of the Pew Environment Group (non-governmental organization based in the US), in 2011 Brazil was for the first time the world leader in installed capacity for power generation from biomass (including generation from dung, wood, agricultural residues and food waste), with total of 1.9 GW at that year.

Finally, as the sugar cane harvest periods are yearly well defined, it can be observed an almost unique generation profile of sugar-cane biomass plants, with little variations among years. In this context, Figure 2.4 presents the typical energy generation profile of a biomass plant placed in the Southeast region of Brazil.

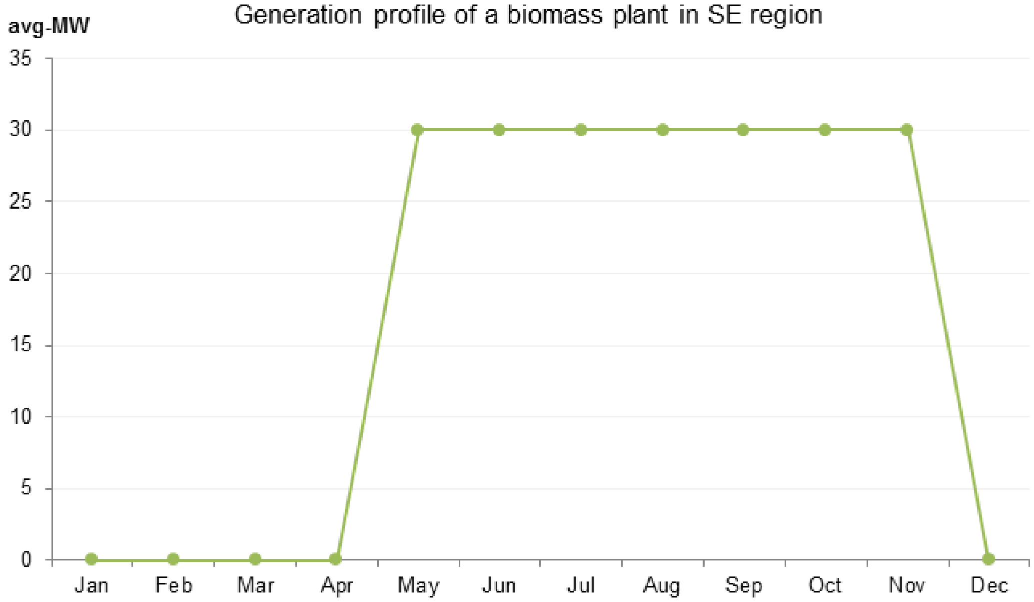

Figure 2.4 – Deterministic biomass plant generation profile.

2.3 Wind power plants

Wind power plants are those that use wind turbines to convert the kinetic energy from wind into electrical energy. Such kinds of plants are experiencing an aggressive growth worldwide. According to WWEA (World Wind Energy Association), the total installed capacity of wind power around the world jumped from 236 GW in 2011 to more than 370 GW by the end of 2014, a growth of 56% in 3 years, average of 18.8% per year, as seen in Figure 2.5.

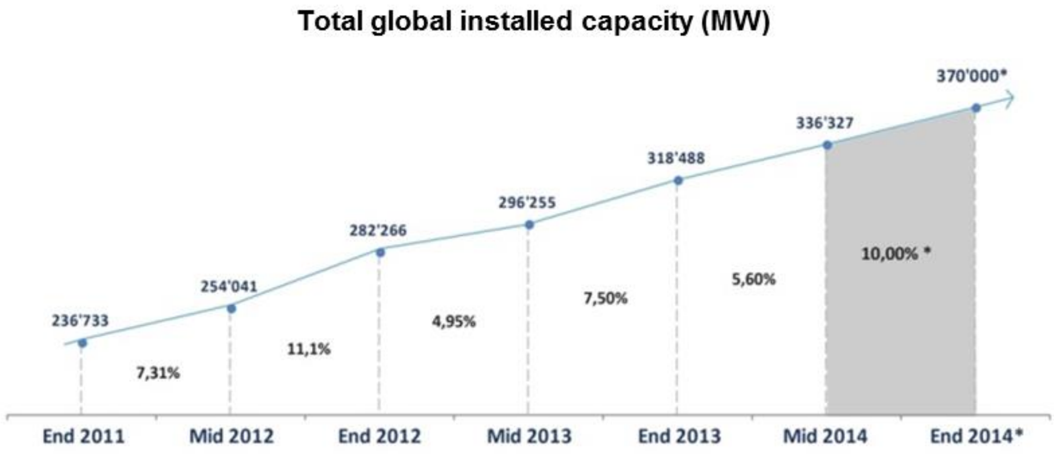

Figure 2.5 – Total installed capacity of wind power plants worldwide.

This situation has not been different in Brazil, where wind power has been receiving concrete investments, with a promising prospective growth for the next years. The upsurge in installed capacity of this technology from 27 MW in 2005 to more than 6.2GW in 2014 corroborate with this fact. Furthermore, as seen in Figure 2.6, the prospection is wind power plants meet solids 16 GW of installed capacity in Brazil by the end of 2019.

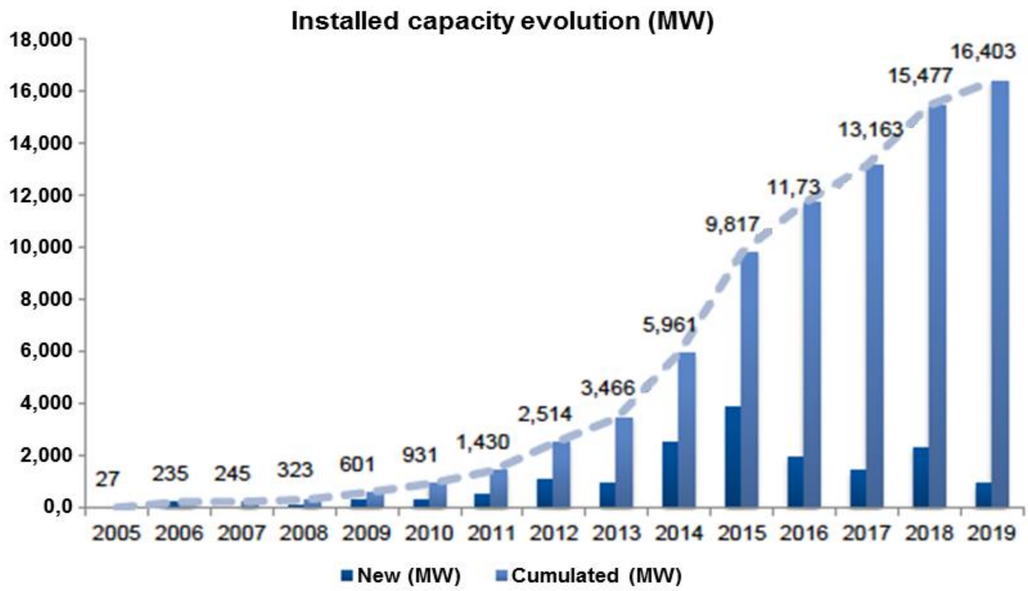

Figure 2.6 – Wind power installed capacity evolution in Brazil (with prospection).

Moreover, studies from ABEEólica indicate a total potential of approximately 300 GW in Brazil [45]. The prospected new investments in wind power should maintain Brazil among the 10 first nations in the rank of wind power installed capacity. Currently leading the rank, China has incredibly added more than 23GW of wind installed capacity to its grid only in 2014, meeting almost 115 GW of total installed capacity, as disposed next, in Figure 2.7.

Top 12 countries by total wind installations

Position 2013	Country/Region	Total capacity end 2014 ** [MW]	Added capacity 2014 *** [MW]	Growth rate 2014 [%]
1	China	114'763	23'350,0	25,7
2	USA	65'879	4'854,0	7,8
3	Germany	40'468	5'808,0	16,8
4	Spain	22'987	27,5	0,1
5	India	22'465	2'315,1	11,5
6	United Kingdom	11'998	1'467,0	13,9
7	Canada	9'694	1'871,0	25,9
8	France	9'296	1'042,0	12,6
9	Italy	8'663	107,5	1,3
10	Brazil	6'182	2'783,0	81,9
11	Sweden	5'425	1'050,0	21,4
12	Denmark *	4'850	78,0	1,6
	Rest of the World	47'300	7'000 (estimated)	16,0
	Total	**370'000**	**51'753**	**16,2**

* by november 2014
** Includes all installed wind capacity, connected and not-connected to the grid.
*** Includes the net capacity addded during the year 2014.

Figure 2.7 – Top 12 worldwide wind energy rank.

In a highly hydro-generation based country, case of Brazil, the penetration of a different source like wind can positively contribute to the diversification of the electrical matrix, due to its great unexplored potential and complementary pattern of its generation profile with the current generation profile of Brazil's electrical matrix. Not differently from small hydros and biomass plants, in Brazil the wind power production is also driven by strong seasonal pattern. In Figure 2.8, it is shown the generation profile of a typical wind power plant of the northeastern Brazil. Additionally, jointly analyzing Figure 2.3, Figure 2.4 and Figure 2.8 it is possible to note the complementary generation profiles among the sources, where the peak of wind power production coincides with the dry period on hydro generation and with the sugar cane harvest period. This complementarity characteristic of RES will be discussed in the next chapter.

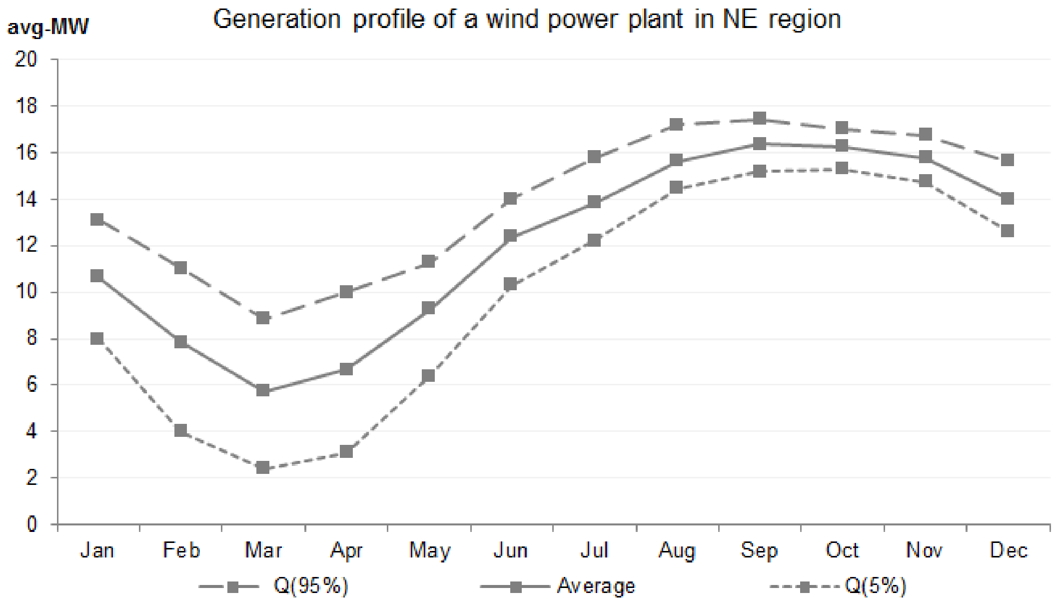

Figure 2.8 – Generation profile of a typical wind power plant.

3
Market setup

Since the beginning of the past decade, investments in renewables were mainly promoted in Brazil through the Regulated Trade Environment, with less appearance of RES in the Free Trade Environment. Those two environments were created within the Brazilian power sector reformulation of 2004, guided by the system regulator, ANEEL [2] [46]. This reformulation was necessary to correct structural problems that induced the energy crisis of 2001 and the consequent well succeeded volunteered energy-rationing campaign within the population. Among other measures, the regulatory plan of 2004 established two important firms for the power sector: (i) the Energetic Research Company (EPE in Portuguese), responsible for the long-term expansion planning of the sector and (ii) the Electrical Energy Commercialization Chamber (CCEE in Portuguese), responsible for operating the two trade environments, the RTE and the FTE (both created in the reformulation as well, as previously mentioned). The main goal of the reformulation was to ensure the power sector expansion in a safe and organized way, at the lower cost as possible, via incentives to attract private investments [47].

In this context, some actions were taken to enforce the safety of supply expansion. One of the most important was the obligation that supply contracts should be physically backed on energy generation, i.e., an agent can only sell a supply contract if possess enough energy rights to cover the amount sold. This was done through the creation of firm energy certificates (FEC) issued by the system regulator and assigned to each generator [48]. This value is calculated via long-term generation average of the units and is periodically revised to protect the system against underperformances. Moreover, FEC are defined in average-MW, or MWh/year, and defines the total amount of energy rights of generators,

[2] The Brazilian power system regulatory agency, ANEEL, was created in 1998 when the Brazilian government promoted a reformulation of the power sector in order to increase the participation of private investors.

working as an upper bound for the total amount that can be sold through contracts by generators.

Another measure taken to increase the system reliability was the creation of the long-term supply contracts. These kinds of contracts promote better conditions for new enterprises in the generation sector, since they provide better signals of future energy prices for investors, who are consequently beneficiated with better credit conditions. In general, these types of contracts have been practiced in the RTE, which is the environment operated by energy auctions controlled by CCEE. It is through the RTE that the distribution companies (DisCos) contract all the energy they need to supply their consumers (this is another example that any demand has to be 100% backed on supply contracts). In this setting, this environment has been fostering new investments in generation in Brazil.

On the other hand, the FTE is the one where GenCos and ETCs are free to sign bilateral contracts with special consumers [3], which in turns have the alternative of contracting energy from companies other than its local DisCo. Additionally, the contracts in the FTE frequently expose the agents to the well-known price-and-quantity risk. In spite of that, the inherent risks of the FTE have positive counterparts. For example, (i) generators get higher prices for the energy sold and (ii) for special consumers, energy bought from renewable generators grant discounts in energy transmission tariffs when compared with the ones charged from DisCos (this is the so called renewable energy incentive).

Another important aspect of the Brazilian power sector is that, differently from many other countries and markets, the system dispatch is done on a centralized way. The National System Operator (ONS, in Portuguese) is responsible for minimizing the final energy cost for consumers while operating the system. Since 75% of the Brazilian installed capacity is originated from large hydros [49], the system operator is capable of managing the load and contingencies on a weekly timescale basis, but it also faces strong uncertainties during the periods within a year, due to the intrinsic seasonal profile of inflows. Such uncertainty has direct influence on the total storage capacity of the system, addressing strong variation on the energy spot prices that have strong negative correlation with the total system storage. This is due to high storage levels favors

[3] The FTE special consumers are the ones that satisfy certain conditions on minimal demand and specific supply tension.

the system capability of attending the load at low costs (through hydro usage, which is much cheaper than thermal generation). Under this environment, the spot price, which is an outcome of the dispatch model and reflects the cost of the most expensive unit under dispatch, can increase rapidly in periods of strong storage drops. For the sake of clarification, Figure 3.1 denotes the (strong) negative correlation between the total system storage and the spot price (the graph was built with real historical data of the total water storage and energy spot prices for the Southeast/Center-west subsystem of Brazil).

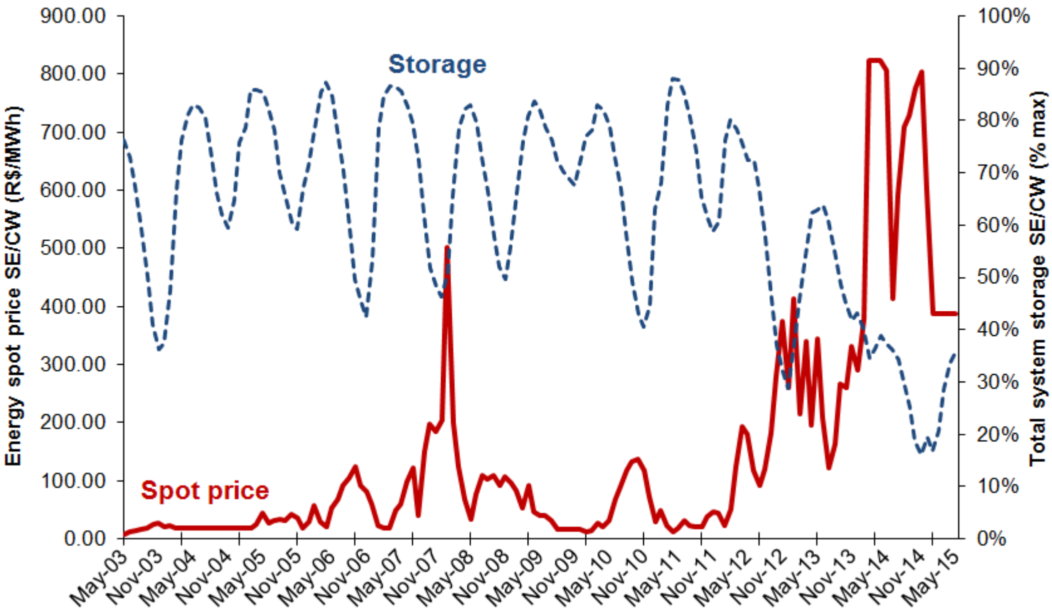

Figure 3.1 – The negative correlation between total storage and energy spot prices.

Strong uncertainty pattern and volatile behavior are also characteristics of RES generation profiles, not only because of their dependence on the availability of its natural resources (inflow, wind and biomass waste), but also because of the lack of any storage capacity. However, due to this last aspect, RES are considered must run units within the centralized dispatch model, i.e., differently from large hydros (which are capable of regulating their energy generation along time), RES are considered non-dispatchable units, injecting in the grid every single watthour that their resources can provide. This represents another challenge to be faced by the system regulator, since the actual scenario is of growth in the penetration of RES into the Brazilian grid, intensifying the volatility of the system.

It is also important to highlight that the two available environments of the Brazilian power sector are, in a certain manner, mutually exclusives. Nonetheless, in another sense, they are not. Summing up: investors can freely allocate portions of the FEC of their generation assets on both environments, as soon as the sum of energy credits sold throughout the two environments don't exceed units' total FEC. At the present work, however, it will be considered only opportunities in the FTE, as previously pointed out.

Granting all this, apart of the environment where the energy is traded, RES face many risks when signing annual energy supply contracts with consumers. The natural uncertain generation pattern of RES, allied to the intrinsic volatile spot prices of the Brazilian power market let traders and generators exposed to the so called price and quantity risk, or volume and price risk. This risk is intrinsically linked to the fact that agents are obligated to clear in the spot market the differences between the amount of energy produced and the contracted amount (or difference between the amount of energy rights and energy obligations, respectively). Such risk is exemplified in Figure 3.2, when a determined agent has a fixed monthly obligation within a bilateral contract during a given year.

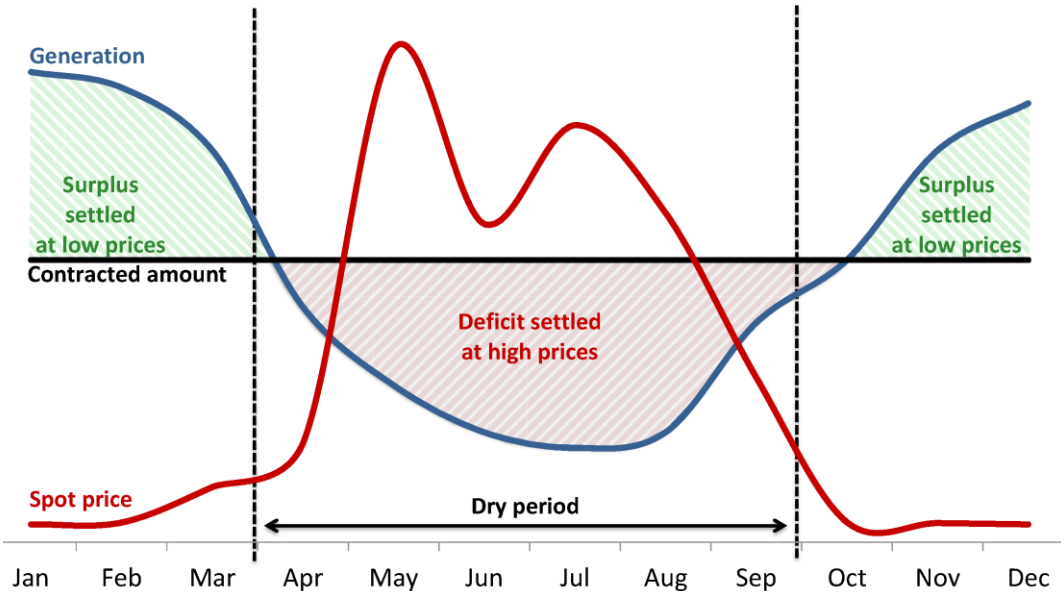

Figure 3.2 – Exemplification of price and quantity risk.

To deal with such risks, agents make use of modeling techniques capable of faithfully represent the variables involved in the power sector. The existing uncertainties in energy generation and spot prices are usually modeled through stochastic processes [50] that carry out historical information of the data series and, in some cases, can also carry information about the existing correlation among different processes. For example, [32] has modeled, through stochastic processes, the spatial and time correlated behavior of the resources of a small hydro (inflow) and a wind power plant (wind), also incorporating their dependence with the spot price, in order to produce realistic scenarios of realization of these resources.

Although inflow, wind and biomass present very seasonal availability profile, which imposes many risks to whomever wants to trade energy from such sources, they have a very important aspect: they present a strong complementarity among each other. Figure 3.3 shows the complementarity in the generation profile of the three main RES of the Brazilian power sector.

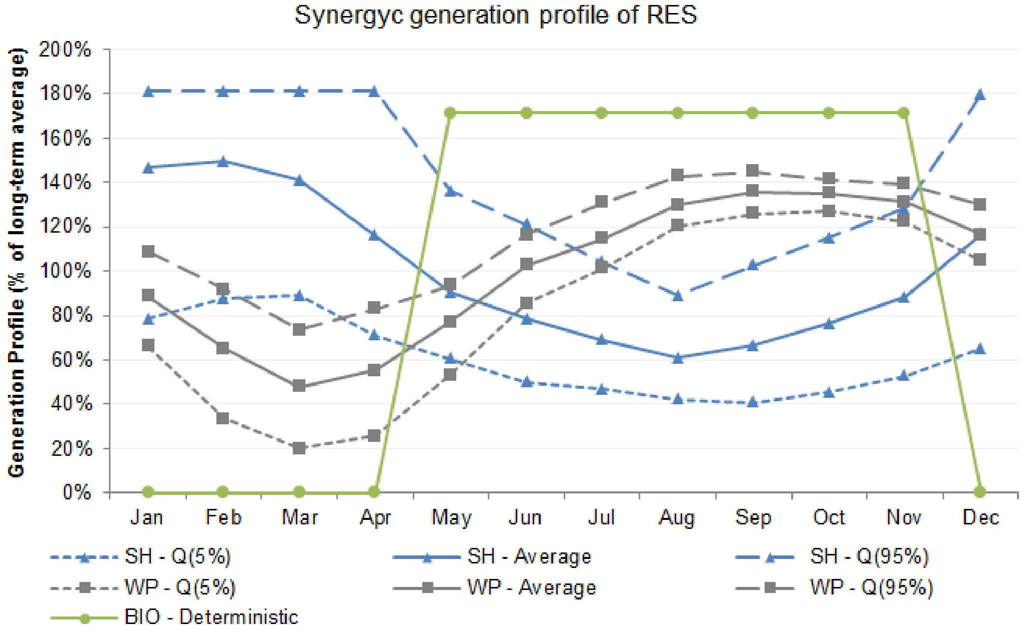

Figure 3.3 – Generation profile in % of the FEC (long-term average) for the three main renewable sources present in Brazil.

This aspect has been object of study by many authors, which came up to expectations when building portfolio-trading strategies to mitigate the price and quantity risk as in [30], [31] and [32], for instance. Besides, the trading scheme presented by [19] (an improved version of the idea presented in [13]), which also takes advantage on the complementary profiles of RES, serves as basis for the trading model at the present work as well. In such scheme an independent ETC is responsible for the risk management of the joint financial operation of the RES in the portfolio.

Differently from other schemes, like the one presented in [30], here the centralizing agent is not the only risk taker, since the risk is shared among all participants, which are in fact shareholders of the cooperative risk-mitigation pool. Moreover, the ETC models the portfolio's price and quantity risk to devise the joint selling strategy where the outcome is the defined optimal energy amount to be sold in a contract with a consumer. This optimally defined energy amount, together with the equivalent generation profile that is transferred to the ETC (as energy rights) are key elements to mitigate the price and quantity risk. The uncertainties involved in the joint selling strategy will be discussed in the next section.

3.1 Uncertainty characterization

The correct pricing of the joint trading strategy is crucial for the risk analysis of energy portfolios. One of the main challenges of such task is the appropriate mapping of the inherent interrelationship between the energy generation and spot prices. These are the two risk factors considered at the present work:

i. $\tilde{\pi}_t$, a random variable that represents the spot price at a given period t in $/MWh and;
ii. $\tilde{G}_{i,t}$, a random variable that expresses the amount of electrical energy produced by the generation unit i at period t, in MWh.

Throughout this work, random variables are assumed to be discrete following stochastic programming standards [51]. Therefore, both uncertainty factors $(\tilde{\pi}_t, \tilde{G}_{i,t})$ can be characterized by means of a set of possible realization scenarios and respective probabilities $\{(\pi_{t,s}, G_{i,t,s}), p_s\}_{s \in S}$, where S is the set of indexes for scenarios.

In this setting, as the solution approach is of stochastic nature, the simulation of renewable generation and spot price scenarios made through a multivariate statistical model is of crucial importance. This is due to the essence of the model that relies in the existing synergy of RES. For this reason, at the present work the data simulation process is done according to [33] and includes: i) the spatial and time (seasonal) dependence between the generation units and ii) the correlation between the simulated resources and the spot price scenarios. Moreover, the future energy spot prices scenarios come from the Newave program

[52] and [53], used in the centralized dispatch of the National System Operator (ONS).

Accordingly, following the procedure described in [34] a multivariate VARx model (**V**ector **A**uto **R**egressive with e**X**ternal variables) was used to produce the input data for the uncertainty factors $(\tilde{\pi}_t, \tilde{G}_{i,t})$. Such model is capable of properly simulating scenarios of renewable resources in fine-tune with the spot price scenarios as it uses external (or explanatory) information from the system's operative model. These explanatory variables are the simulated data of inflow from the Southeast, South, Northeast and North subsystems of Brazilian power sector. Such data is produced by a statistical module called PAR-p (**P**eriodic **A**uto **R**egressive model of order **p**), part of the Newave.

Additionally, it is also very important to clarify that synthetic series of inflow produced by the PAR-p model only consider data from medium to large hydro units. As renewables (such as small hydros, biomass, wind power plants and others) are much less representative in the system than the major plants, such units are considered in the model on an aggregate way and by means of the seasonal average production in each period of the operation horizon. This brings to RES the necessity of generating its own scenarios of energy generation, which should also save the relation with the scenarios of spot prices.

More specifically, besides PAR-p, Newave counts with a dispatch module that operates the electrical system in each one of the synthetic inflow scenarios. This is done via minimization of a measure of value over the total cost for operating the hydrothermal system in the whole time horizon of simulation. As outcome of the optimum dispatch policy, the model produces one spot price scenario for each one of the generated synthetic inflow scenarios. In this framework, the set of spot prices saves a one-to-one correspondence with the set of inflows. This same set of inflows is used by the VARx model to explain the spatial-and-time-correlated behavior of natural resources (wind and inflow) for the renewable units. As a consequence, its outcome (i.e., the set of renewable resources scenarios) is properly correlated with the energy spot prices.

In this framework, Figure 3.4 shows how the VARx model generates the scenarios of renewable resources correlated with the spot price scenarios.

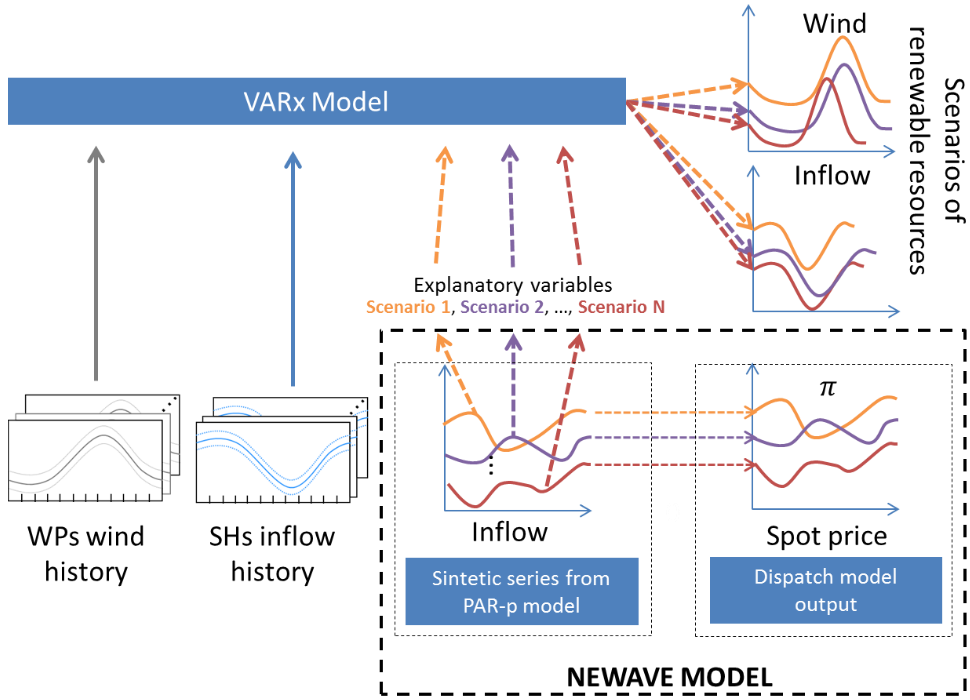

Figure 3.4 – The relation stablished by the VARx model between the renewable resources historical series (input data), the dispatch model (external information) and the correlated renewable generation (output data) and spot prices scenarios.

More details of the statistical model and simulated energy generation scenarios are disposed in the Appendix of the present document. However, it is important to highlight that both uncertainty factors $\tilde{\pi}_t$ and $\tilde{G}_{i,t}$ are considered as input data and further explanation of the process for simulating the renewable energy scenarios is out of the scope of this work. Please refer for [32] to [35] in order to obtain more details from the model.

Finally, in the hypothesis of market-driven liberalized frameworks where the operation is outcome of the energy spot prices (contrarily from the Brazilian case) and, for example, spot price is given in a shorter time basis (such as hourly or even few minutes), the model presented in the present work would still be valid, as soon as the set of uncertainty factors ($\tilde{\pi}_t$ and $\tilde{G}_{i,t}$) recovers all critical aspects from such markets.

3.2
Revenue of a RES pool

At the present work, it is considered only one type of contract: the bilateral quantity contract, also known as contract for differences, or forward contracts. This type of contract is arranged between two agents: (i) an agent that sells energy rights, e.g., a GenCo or an ETC, and (ii) an agent that buys energy rights, e.g., a liberalized consumer or also an ETC. In Brazil, like many other markets, contracts establish the obligation, from the part of the seller, to deliver the exact amount of energy contracted per period. However, this obligation is only a financial instrument, without the need of the physical energy delivery, since all the energy produced by generators is injected onto grid. So, when an agent sells the contract, it assumes an energy duty with the system. On the other side, the buyer receives the same amount in energy rights. These amounts of traded energy are then cleared at spot market. For the sake of exemplification, in the case of a GenCo that has sold Q MWh at a fixed price P \$/MWh for example, the difference between its amount of energy rights (i.e. amount of energy generated) and its amount of energy duty (Q) in a given period is cleared at the spot market. If such difference is positive, this operation will bring financial gains, on the contrary, losses. In this setting, the (stochastic) net revenue $\tilde{R}_t$ of the generator at a given period t is denoted by

$$\tilde{R}_t(Q) = P \cdot Q + \tilde{G}_t \cdot \tilde{\pi}_t - Q \cdot \tilde{\pi}_t, \tag{3.1}$$

where: (i) the first term of expression (3.1) defines the fixed payment received by the trade of the energy rights, which is part of the financial instrument of the bilateral contract and does not counts for the spot market clearance; (ii) the second term defines the clearance, at the spot price $\tilde{\pi}_t$, of the energy rights obtained with the amount of energy generated at period t and finally; (iii) the last term in expression (3.1) defines the clearance of the energy duty with the system (Q) at the spot market. For further details on bilateral contracts, please refer to [30], [31], [32] and [54].

In a more general sense, and introducing this concept to a portfolio framework, expression (3.1) can be reorganized in order to reflect the (stochastic) net revenue $\tilde{R}_t$ of a pool of RES that recovers the sale of Q MWh on a financial bilateral quantity contract at a fixed payoff at price P (in \$/MWh):

$$\tilde{R}_t^{POOL}(Q) = (P - \tilde{\pi}_t) \cdot Q + \sum_{i \in I} \tilde{G}_{i,t} \cdot \left(\tilde{\pi}_t - C_{i,t}^U\right), \ \forall \, t \in \, T. \qquad (3.2)$$

Where $C_{i,t}^U$ is introduced to denote the unitary cost of energy production of unit i in period t in \$/MWh, I denotes the set of generators in the pool and T denotes the set of periods in the horizon of the contract. In this new scheme, first term of expression (3.2) denotes the financial result of the (fixed) payoff against the spot market. The second term is the financial result due to the equivalent aggregated generation of the pool in the spot market. Moreover, as depicted in Figure 3.5, an optimal selling strategy Q^{*POOL} of the pool should maximize a selected risk measure of value of the total stochastic cash flow given by (3.2). This is done by properly balancing the trade-off between the deterministic fixed payment part of the revenue of the contract against the risks associated with the spot market. A proper risk-adjusted measure for the future stochastic cash flow will be discussed further in the text.

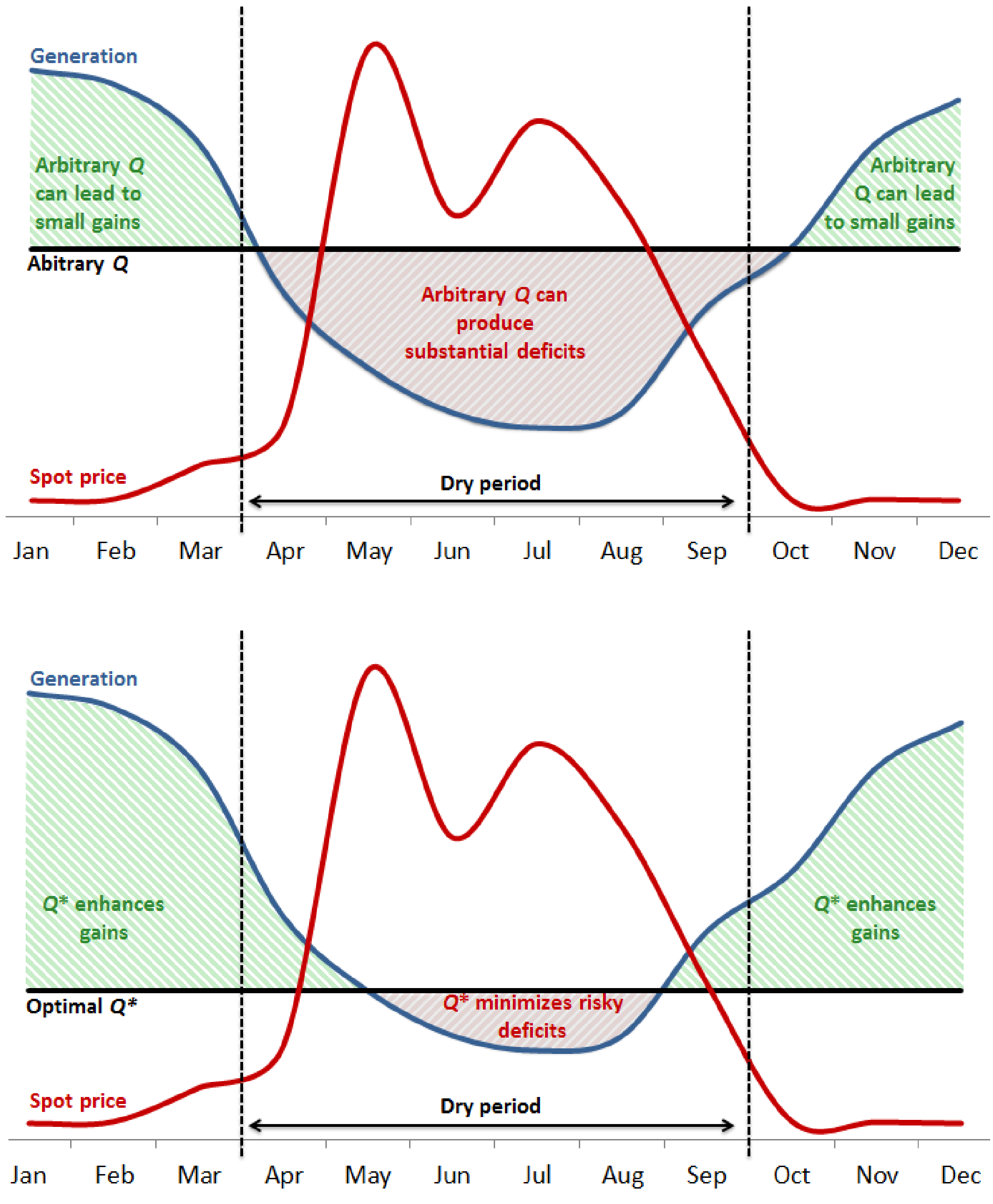

Figure 3.5 – Exemplification of the contracted amount Q optimization benefits.

Nevertheless, such amount Q is also subject to the regulatory limit of the total FEC of the pool: $Q^{POOL} \leq \sum_{i \in I} FEC_i$, where FEC_i is the total amount of FEC issued by the regulator to generator i in avg-MW[4] (see [30], [31] and [32] for further details). Although this is not an aspect discussed in the present work, it is also important to note that despite of the obligation on energy delivery be placed over seller agents, consumers must also settle differences between the contracted

[4] Firm Energy Certificates (FEC) are usually issued in terms of avg-MW (MWh/year), based on the unit's total long term average production per year.

amount and the energy effectively consumed. Nonetheless, usually the seller, legally representing the buyer, is responsible for the clearance operation.

4
RES pool under a cooperative game framework

In general, games theory models mutual and independent behaviors of agents that interact among themselves. Such interaction can assume a cooperative character or even a conflicted character. These so called agents can be people, companies, institutions, countries, coalitions, etc., and are usually called players under this framework. The games theory is based on the idea that the best decision for a given agent should always be taken considering the possible decisions the other players in the market can take. Thus, under this framework, the optimal decision for such a player is conditional upon the optimal decisions of others.

Games theory had its upsurge after John Nash[5] published his articles [55], [56], [57] and [58] between 1950 and 1953, initiating the modern times of this area, with the introduction of the concepts of Nash equilibrium in non-cooperative games and also modeling a solution to cooperative games in [57] and [58]. The Nash equilibrium concept produced the distinction between cooperative and non-cooperative games. The main difference is that in cooperative games, agreements can be forced (through contracts or decision on tribunals, for example), differently from non-cooperative games where only the equilibrium results are self-sustainable (i.e., stable).

Moreover, in cooperative games, players interact on a coordinate way in order to obtain better benefits than the ones that could be reached in case of non-cooperation. Once the goal is reached, the direct challenge is placed on the way such benefits are distributed among players. As one of the assertions of games theory, in a cooperative game it is necessary to analyze the optimum strategies of each one of the possible coalitions of the game. Those can put in risk the stability of the grand coalition, producing the desertion by the part of some players from the pool in order to form smaller coalitions, even at the expense of others.

[5] It is left here a respectful recognition for the contributions of John Forbes Nash, Jr, in Games Theory, a field of study which is vastly explored in the present work. Nash died tragically along with his wife in a car accident on Saturday, 23th of March, 2015, at the time of the production of this work.

In this context, cooperative games theory appears as an interesting area of study for allocation methods which are capable of producing results where the equilibrium among players can be reached. For example, cooperative games commonly use axioms and methods that produce a result considered fair by all participants. In this framework, the optimum Pareto is searched. The Pareto efficiency was developed by Vilfredo Pareto between centuries XIX and XX (see [59] and [60]). It states that an economic allocation is efficient in Pareto's principle only if in case of loss on the utility of others, it is possible an increase in the utility of a given player.

Granting all this, the pool scheme proposed at the present work is naturally embedded on the cooperative games framework since its main idea is aggregate value to a joint selling strategy of a set of RES, in order to take advantage of the intrinsic synergy on the energy production profile of different sources. Such scheme should also share the reduced price-and-quantity risk among the players of the pool. Furthermore, the sharing strategy should be such that every single player or coalition of players stays satisfied with the benefits received, that is, the decision of the quotas allocation should take into account the necessity of attending all possible coalitions of players. Hence, the only path to success is reaching the so important stability of the pool, when no player has any economic incentive to leave the grand coalition or move to any other subcoalition. Summarizing, this set of aspects of the proposed pool justifies the cooperative games theory as the appropriate framework to treat the quotas allocation problem.

4.1
The game setup

In order to setup the cooperative game, it is relevant to formally introduce two important and vastly used concepts of cooperative games theory: (i) coalition and, (ii) characteristic function. First, given a set I of n players, i.e., $I = \{1, \ldots, n\}$, a coalition of players is formally defined as any combination of the elements (players) of I, reserving the name grand-coalition to the particular case of the coalition formed by all n players of the set I. At last, the term characteristic function was primarily introduced in 1947 by John Von Neumann and Oskar Morgenstern [61] to designate the function that calculates the value of the higher benefits members of a given coalition can exploit through a cooperative action. Already, a game is defined by the set I of n players and a characteristic function v, which assigns to each coalition of players a real number that represents the measure of the optimal selling strategy of its aggregated generation.

4.1.1 Notation

This work uses the following transformation (or isomorphism) throughout the text: for each set $C \subseteq I$, there exists a transformation $T: \wp\{I\} \to \mathbb{B}^n$, with domain on the powerset of I and image on the set of n-dimensional binary vectors $\mathbb{B}^n$ such that $T(C) = \boldsymbol{c}$. In this setting, hereinafter the coalitions of players are defined by a binary vector $\boldsymbol{c} \in \mathbb{B}^n$ with n entries, in which component c_i assumes value 1 (one) if player i belongs to the coalition and 0 (zero) otherwise.

Thus, the characteristic function $v: \mathbb{B}^n \to \mathbb{R}$ measures the value of a coalition $\boldsymbol{c} \in \mathbb{B}^n$. Still, the two elements, C and $\boldsymbol{c} = T(C)$, have a one-to-one correspondence or, in a more formal manner, are isomorphs. Yet, there is also the inverse transformation $T^{-1}: \mathbb{B}^n \to \wp\{I\}$ such that $T^{-1}(\boldsymbol{c}) = C$. In this context, the characteristic function $v: \wp\{I\} \to \mathbb{R}$, with domain on the powerset of I and image in the Real Numbers, is now represented by $v: \mathbb{B}^n \to \mathbb{R}$, having its domain shifted to the set of n-dimensional binary vectors (number of players). Note that in fact what is considered is $v(T^{-1}(\boldsymbol{c}))$, but for the sake of convenience on notation, from now on, it is simply used $v(\boldsymbol{c})$. In particular, $\boldsymbol{c}^{\{i\}}$ is the vector that represents the individual coalition where all elements assume value 0 (zero) except the one on the i-th position, which assumes value 1 (one). In other words, $\boldsymbol{c}^{\{i\}}$ is the coalition that represents the player i, individually. Also, vector $\mathbb{1}$, of dimension n, which all elements assume value 1 (one) represents the great coalition itself and $v(\mathbb{1})$ its respective characteristic function. In this same context, vector $\mathbb{0}$, also of dimension n, is the null vector (the origin) which has the value 0 (zero) associated with its respective characteristic function, $v(\mathbb{0})$. Finally, $\mathcal{C}$ is defined as the set of all possible subcoalitions of I, excluding the grand coalition and the empty coalition, that is, $\mathcal{C} = \mathbb{B}^n \backslash \{\mathbb{0}, \mathbb{1}\}$. Still, it is worth anticipating the set $\mathcal{C}$ as of great value in terms of notation convenience throughout the text.

4.1.2 Pool scheme

As a risk mitigation mechanism, the RES pool does not ensure a prior fixed (deterministic) payoff for its participants. What is really allocated to each participant i is a percentage x_i of the future stochastic net revenue of the optimized joint trading strategy of the pool in both contract and spot market. Both the selling and the quota allocation strategies of the grand coalition are, hence, optimized by the pool manager, e.g., an independent ETC, following the common-agreed pool standards. In this setting, the scheme proposed in this work belongs to the class of stochastic cooperative games [36]. Finally, the revenue of player i in the pool is defined as follows:

$$\tilde{R}_{i,t}^{pool}(Q^{*pool}) = x_i \tilde{R}_t^{pool}(Q^{*pool}), \ \forall t \in T. \quad (4.1)$$

For the sake of clarification, Figure 4.1 exemplifies the payment flow of the pool.

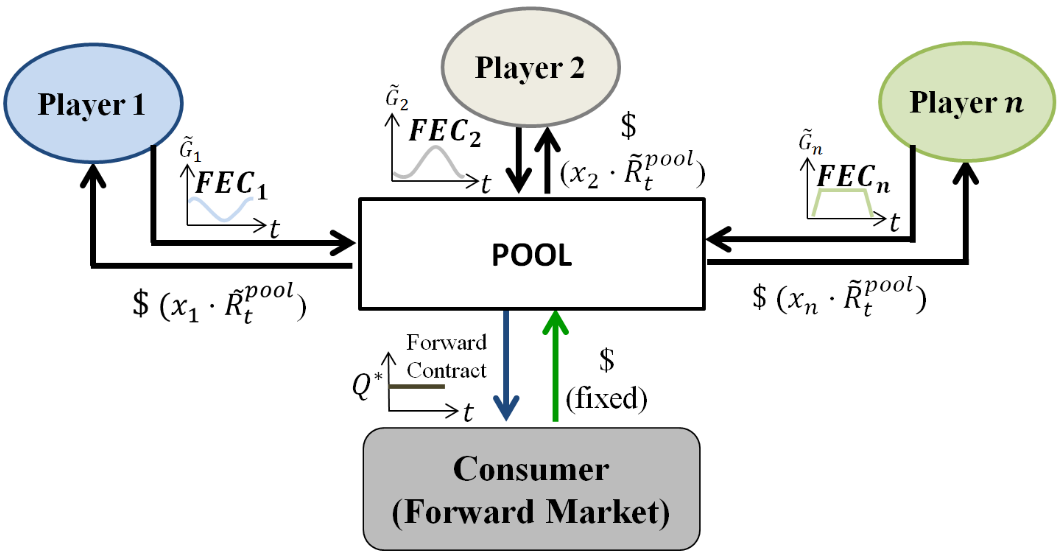

Figure 4.1 – Exemplification of the energy and payment flows of the proposed cooperative RES trading mechanism.

4.1.3 The value function

The definition of a characteristic function in the context of a stochastic game requires a measure of value, or certainty equivalent, to assess the collective value of the stochastic outcome obtained with the optimal selling strategy for a given coalition [18]. Measures of risk must recover the value of the future cash flow of a given asset and transcribe it in a single number. Additionally, measures of value have the goal of quantifying the exposure of the asset to risks and limit such risks to acceptable values. They compute the risk that a given position provides to agents, following the most different criteria and should also satisfy the agents in terms of comprehension, complexity and objectivity necessities.

Additionally, it is worthwhile the use of measures of risk which exhibits particular characteristics that provide decision takers with tools that support operations involving risk control in decision processes. For example, one desirable aspect of the characteristic function in cooperative games is the superadditivity property, formally defined as follows. Let C^1 and C^2 be two coalitions of players (recall that $C^1 = T(\boldsymbol{c}^1)$ and $C^2 = T(\boldsymbol{c}^2)$). So, a given measure of value v is superadditive if the inequality on expression (4.2) holds.

$$v(C^1 \cup C^2) \geq v(C^1) + v(C^2), \qquad \forall\, C^1, C^2 \subseteq I; C_1 \cap C_2 = \emptyset. \qquad (4.2)$$

Thus, in the cooperative games framework, superadditivity determines that the associated benefit of any coalition must be always greater or equal to the sum of the associated benefits of any partition of subcoalitions. This aspect is important because, since superadditivity must be satisfied for any C_1 and C_2, the property thus guarantees that the cooperation explores on a benefic way the available synergy among players and always generates value for the pool (in terms of increase on the pool's total welfare).

Hence, it is used at the present work the risk averse α-left-tail conditional value at risk (CVaR) to model the pool's risk profile. Roughly speaking, the CVaR [62] applied to measure the value of stochastic incomes (net revenues) is defined as the average of the $(1-\alpha)100\%$ worst-case scenarios, i.e., those below the $(1-\alpha)$ quantile, or α Value at Risk (VaR) [63]. In this setting, α is a confidence level demanded by a risk-averse decision maker, generally ranging between 0.95 and 0.99. For purpose of comparison, Figure 4.2 illustrates the α-VaR and α-CVaR for a general continuous probability density function f.

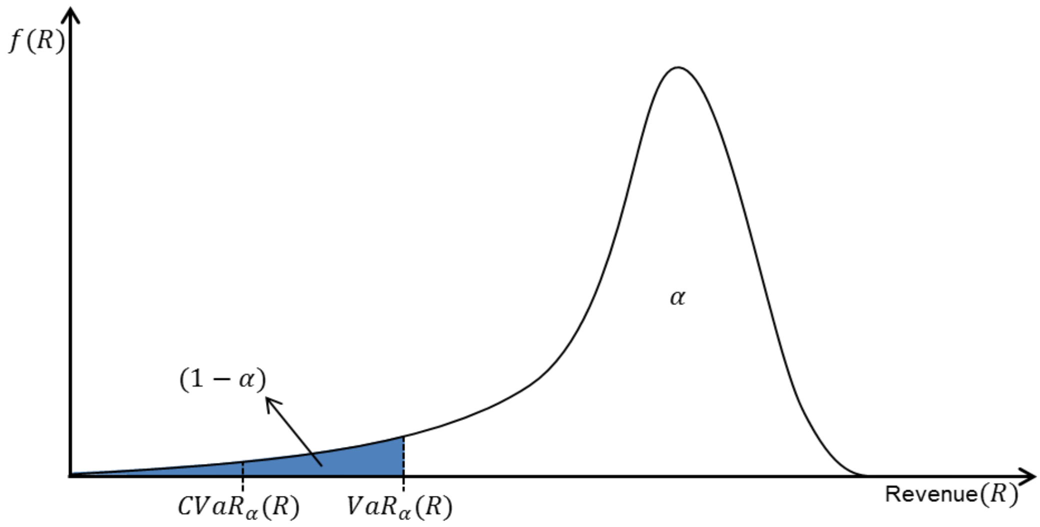

Figure 4.2 – CVaR of a general revenue probability mass function.

Moreover, the CVaR presents all desirable properties of a coherent measure of risk: it is monotone, homogeneous, superadditive and more (see [64] and [65] for further details). Besides that, the CVaR can be written as a maximization problem with linear constraints and be easily embedded in portfolio optimization problems [66], where what agents want is to maximize its objective function, or its certainty equivalent.

Taking all this in consideration, the adopted certainty equivalent (or characteristic function) of the present work, assessed in a given point $\boldsymbol{c} \in \mathcal{C}$, is a convex combination between this risk averse measure and the expected value, and defined as follows:

$$v(\boldsymbol{c}) = \max_{Q \geq 0} \left\{ \rho_{\alpha,\lambda} \left(\sum_{t \in T} \frac{\tilde{R}_t(Q, \boldsymbol{c})}{(1+J)^t} \right) \middle| \; Q \leq \sum_{i \in I} FEC_i c_i \right\}. \tag{4.3}$$

Where,

$$\tilde{R}_t(Q, \boldsymbol{c}) = (P - \tilde{\pi}_t) h_t Q + \sum_{i \in I} \tilde{G}_{i,t} \left(\tilde{\pi}_t - C_{i,t}^U \right) c_i \tag{4.4}$$

is the stochastic net revenue in both contract and spot markets considering the total energy produced by the set of generators in the coalition and $\rho_{\alpha,\lambda}$ is the adopted risk measure, formally defined as:

$$\rho_{\alpha,\lambda}(\cdot) = \lambda \cdot CVaR_\alpha(\cdot) + (1 - \lambda) \cdot \mathbb{E}(\cdot). \tag{4.5}$$

In (4.3), J is the risk-free opportunity cost of money between two periods (in percentage per period), and FEC_i is the amount of FEC (in average-MW) of player i. The right-hand-side of expression (4.3) assesses the value of the optimal selling strategy of the coalition by choosing, within the total amount of FEC in the coalition, the energy amount to be sold in contract that maximizes the net present value of the stochastic cash flow defined in (4.4). In (4.5), $\mathbb{E}(\cdot)$ accounts for the expected value operator and α and λ are the risk-aversion parameters. Expression (4.3) is then a two-stage stochastic optimization problem and can be assessed by means of the following deterministic equivalent linear program (LP):

$$v(\boldsymbol{c}) = \max_{Q,\Delta_s,z} \lambda\left(z - \sum_{s\in S} \frac{p_s\,\Delta_s}{(1-\alpha)}\right) + (1-\lambda)\sum_{s\in S} p_s\left(\sum_{t\in T} \frac{R_{t,s}(Q,\boldsymbol{c})}{(1+J)^t}\right) \quad (4.6)$$

subject to:

$$\Delta_s \geq z - \sum_{t\in T} \frac{R_{ts}(Q,\boldsymbol{c})}{(1+J)^t}, \qquad \forall\, s \in S \quad (4.7)$$

$$\Delta_s \geq 0, \qquad \forall\, s \in S \quad (4.8)$$

$$0 \leq Q \leq \sum_{i\in I} FEC_i c_i. \quad (4.9)$$

In this model, expressions (4.6)-(4.8) account for the CVaR, following the findings of [66] and (4.9) states the regulatory constraint on the amount of contracted energy, described earlier at the end of section 3.2.

It is worth mentioning that the value of the optimal selling strategy of the pool (grand coalition) is a particular case of (4.3) and can be found by calculating $v(\mathbb{1})$, i.e., the value of the characteristic function assessed at the point $\boldsymbol{c} = \mathbb{1} = [1, \ldots, 1]^T$. In this case, expression (4.4) precisely meets the revenue of the pool, presented in (3.1) (and (3.2)), and the optimized amount of contract obtained in (4.3) meets the optimal selling strategy of the grand coalition, Q^{*POOL}.

4.2 The core is non-empty

Mathematically, the core of a cooperative game can be defined as the set of quota allocation vectors $\boldsymbol{x}$ such that the value allocated to every possible coalition $\boldsymbol{c} \in \mathcal{C}$ in the pool, given by $v(\mathbb{1})\sum_{i\in I} x_i$ or, in vector notation,

$v(\mathbb{1})(\boldsymbol{c}^T \cdot \boldsymbol{x})$, is greater than or equal to its independent result $v(\boldsymbol{c})$. In this setting, the formal definition of the core is given as follows:

$$Core(v) \coloneqq \left\{ \boldsymbol{x} \in [0,1]^n \,\middle|\, \begin{array}{c} \mathbb{1}^T \boldsymbol{x} = 1 \\ v(\mathbb{1})(\boldsymbol{c}^T \cdot \boldsymbol{x}) \geq v(\boldsymbol{c}),\ \forall\, \boldsymbol{c} \in \mathcal{C} \end{array} \right\}. \tag{4.10}$$

Inside the context of the core, it might be important to mention that the superadditivity property of the CVaR (and consequently, of the adopted certainty equivalent) does not guarantee the existence of the core in cooperative games, as disposed in [67]. In the next paragraphs it is presented the proof of non-emptiness of the core of the proposed cooperative game.

The proof is based on the concept of a balanced collection of the coalitions in a cooperative game. The characteristic function $v(\boldsymbol{c})$ of a cooperative game which is defined as the optimal solution of a linear programming problem where the entire right-hand-side vector (from the set of constraints) is composed of a linear transformation of $\boldsymbol{c}$ is said to be concave and positive homogeneous. This is the case of the present adopted characteristic function and the mutual existence of these two properties guarantee the existence of a balanced collection of the coalitions (see [15]). Therefore, the Bondareva-Shapley Theorem, which was independently proven by Olga Bondareva in 1963 [68] and by Loyd Shapley in (1967) [69] can be applied to prove the existence of the core.

Likewise and more formally, the core of a cooperative game is nonempty if and only if there is at least one allocation vector of quotas that provides all coalitions in $\mathcal{C}$ with a surplus value when participating in the grand coalition (pool). Therefore, the core is nonempty if and only if $f^* \leq v(\mathbb{1})$, where f^* is defined as follows:

$$f^* = \min_{\boldsymbol{x}} v(\mathbb{1})(\mathbb{1}^T \boldsymbol{x}) \tag{4.11}$$

subject to:

$$v(\mathbb{1})(\boldsymbol{c}^T \boldsymbol{x}) \geq v(\boldsymbol{c}) \qquad : \mu_c, \qquad \forall\, \boldsymbol{c} \in \mathcal{C}. \tag{4.12}$$

Note that the absence of the constraint $\mathbb{1}^T \boldsymbol{x} = 1$ in formulation (4.11)-(4.12) allows the search of the minimum amount of value f^* necessary to provide all possible coalitions with a surplus when shared among players through an optimal solution $\boldsymbol{x}^*$. Thus, if f^*, through the quota allocation vector $\boldsymbol{x}^*$ such that $\mathbb{1}^T \boldsymbol{x}^* < 1$, is strictly lower than $v(\mathbb{1})$, one can share the excess among players in many ways in order to guarantee gains for individual players and, consequently, it is guaranteed the core is non-empty. Additionally, note also that in this case the excess is given by $v(\mathbb{1})(1 - \mathbb{1}^T \boldsymbol{x}^*) > 0$.

So, according to the strong duality theorem, the optimal solution of problem (4.11)-(4.12) equals the optimal solution of its dual problem, which is given by:

$$f^* = \max_{\mu} \sum_{\boldsymbol{c} \in \mathcal{C}} \mu_{\boldsymbol{c}} v(\boldsymbol{c}) \tag{4.13}$$

subject to:

$$\sum_{\boldsymbol{c} \in \mathcal{C}} \boldsymbol{c}^T \mu_{\boldsymbol{c}} = \mathbb{1}^T \tag{4.14}$$

$$\mu_{\boldsymbol{c}} \geq 0, \qquad \forall\, \boldsymbol{c} \in \mathcal{C}, \tag{4.15}$$

where $\mu_{\boldsymbol{c}}$ are the dual variables of the set of constraints (4.12). Hence, the core is nonempty if and only if

$$f^* = \sum_{\boldsymbol{c} \in \mathcal{C}} \mu_c^* v(\boldsymbol{c}) \leq v(\mathbb{1}). \tag{4.16}$$

For instance, this derivation follows the aforementioned Bondareva-Shapley Theorem. Furthermore, in (4.16), $\{\mu_c^*\}_{\boldsymbol{c} \in \mathcal{C}}$ is the optimal solution of problem (4.13)-(4.15).

According to (4.6)-(4.9), $v(\boldsymbol{c})$ is a convex optimization problem whose right-hand-side (RHS) is composed by a linear transformation of $\boldsymbol{c}$. Then, from linear programming theory, the following properties of function v are verified (for the sake of brevity the proofs will be omitted):

i. **$\boldsymbol{v}$ is concave**: $\sum_{\boldsymbol{c} \in \mathcal{C}} a_{\boldsymbol{c}} v(\boldsymbol{c}) \leq v(\sum_{\boldsymbol{c} \in \mathcal{C}} a_{\boldsymbol{c}} \boldsymbol{c})$, if $a_{\boldsymbol{c}} \geq 0, \forall\, \boldsymbol{c} \in \mathcal{C}$ and $\sum_{\boldsymbol{c} \in \mathcal{C}} a_{\boldsymbol{c}} = 1$;

ii. **$\boldsymbol{v}$ is positive homogeneous:** $v(b \cdot \boldsymbol{c}) = b \cdot v(\boldsymbol{c})$, for all $b > 0$.

Thus, according to properties (i) and (ii), which guarantees a balanced collection of coalitions, by setting $\left\{a_{\boldsymbol{c}} = \frac{\mu_c^*}{\sum_{\boldsymbol{c} \in \mathcal{C}} \mu_c^*}\right\}_{\boldsymbol{c} \in \mathcal{C}}$, the following inequality holds true:

$$\sum_{\boldsymbol{c} \in \mathcal{C}} \mu_{\boldsymbol{c}}^* v(\boldsymbol{c}) \leq v\left(\sum_{\boldsymbol{c} \in \mathcal{C}} \mu_{\boldsymbol{c}}^* \boldsymbol{c}\right). \tag{4.17}$$

Finally, the proof is complete by replacing the RHS of (4.17) by $v(\mathbb{1})$, which can be justified by the dual constraint (4.14), and thus meeting the existence condition (4.16).

5
Allocation methods

Once proved the joint trading strategy always brings financial benefits in terms of risk mitigation ([13],[19]), the subsequent challenge is how to share the quotas of the pool among its participants in order to secure their financial benefits and the consequent stability of the pool. As previously mentioned, this need relies in the problem of finding allocation quotas that belong to the core of the game, since not all allocation methods has the property of generating solutions of this kind. For the sake of motivation, next it is presented a small example for the quota allocation problem.

5.1
A brief motivation

In a simple example, take three generator units with different energy production profiles that trade their energy in the FTE. For instance, the three generators are each one of the RES disposed in Chapter 2. Suppose risk-averse agents and that the metric used to calculate the future financial cash flow of their assets is the one defined by (4.3), applied to various scenarios of future stochastic income (under uncertainty), under risk-averse parameters $\lambda = 90\%$ and $\alpha = 95\%$. Furthermore, the referenced metric is also applied to medium-term contracts (e.g. 12 months).

For the sake of comparison, in this example all agents maximize their respective certainty equivalent to obtain optimum energy trading revenues. Still, the trading strategy is done via standard sales contract with a fixed price of 100 \$/MWh. Thus, Table 5.1 disposes the values agents quantify their respective businesses, as well as the joint selling strategy of the aggregated energy of all possible combinations from the three sources, in order to help the investigation of the cooperation among the three profiles.

Table 5.1 – Comparison of the future stochastic revenues of three different renewable units that trade their energy in the FTE individually and through cooperation.

Unit (or coalition) ($\boldsymbol{c}$)	FEC (avg-MW)	Contracting level (% of FEC)	Revenue in \$ ($v(\boldsymbol{c})$)
Small hydro (SH)	5	59%	2,447.00
Biomass (Bio)	1	79%	594.99
Wind Power (WP)	1	84%	683.00
SH – Bio	6	65%	3,411.03
SH – WP	6	72%	3,509.97
Bio – WP	2	84%	1,378.93
SH – Bio - WP	7	75%	4,467.94

The first column of Table 5.1 denotes the FEC of each player, and the respective aggregated FEC of each possible coalition of players, as a sum of the FEC of its participants. The third column expresses the optimum quantity, regarding to the FEC, that each coalition of units sells through the standard sales contract, leaving the difference to be settled at the spot market, using the same logic of Figure 3.5 (regarding to optimal contracting level Q^*). At last, the forth column disposes the value of the optimum trading strategy of coalitions' energy.

Analyzing the results, it can be noticed the cooperation would result in a future stochastic revenue (R^{coop}) greater then the sum of the individual ones. Therefore, the generated synergic gain should be shared among units. The first and intuitive guess is to divide the benefit in the same rate as the unit's FEC represent from the total, which is theoretically equivalent to the amount of energy that each player contributed with. The results of this sharing scheme are disposed in Table 5.2. The first three lines of second column denote the quotas sharing for each individual player and the subsequent lines denote the quota of each coalition, which is equal to the sum of the quotas of individual players that participate in each coalition. Thus, following expression (4.1), the total monetary value allocated to a given coalition is equal to its quotas times the total value of the grand coalition (disposed in last cell of Table 5.1). Finally, the last row denotes the difference between the value a given coalition receives in the cooperation and its value if trading its energy by itself.

Table 5.2 – Allocation of the benefits in a possible cooperation proportionally to the respective FEC of each player in relation to the cooperation's total FEC.

Unit (or coalition) ($\boldsymbol{c}$)	Allocation ($\boldsymbol{c}^T\boldsymbol{x}$)	Original Revenue in \$ ($v(\boldsymbol{c})$)	Revenue in the cooperation in \$ ($R^{coop} \cdot \boldsymbol{c}^T\boldsymbol{x}$)	Absolute gains in \$ ($R^{coop} \cdot \boldsymbol{c}^T\boldsymbol{x} - \boldsymbol{v}(\boldsymbol{c})$)
Small hydro (SH)	71.43%	2,447.00	3,191.45	744.45
Biomass (Bio)	14.29%	594.99	638.47	43.48
Wind Power (WP)	14.29%	683.00	638.47	- 44.53
SH – Bio	85.72%	3,411.03	3,829.92	418.89
SH – WP	85.72%	3,509.97	3,829.92	319.95
Bio – WP	28.58%	1,378.93	1,276.94	- 101.99

The results show a curious aspect of cooperative games: in spite of the cooperation generates value for the units' energy trading, depending on the way the allocation is made, some units can obtain great advantages of the cooperation, case of the small hydro, whereas others can even have losses, case of the wind power plant. Under this condition, the cooperation is not stable, since the wind power plant would rather trade its energy individually in the market. Thus, under this framework, it is important to share the quotas on a smart way, with the main objective of keeping the cooperation stable. The next sections present different types of methods for sharing the quotas of a cooperative game.

5.2 FEC-proportional

This allocation method is built as the one in the above example of section 5.1 and does not require any games theory technique since the combination of players does not disturb the marginal contribution of each player in the coalition. Mathematically, the bellow-disposed expression denotes the formal definition of this allocation method.

$$x_i^{FEC} = v(\mathbb{1}) \frac{FEC_i}{\sum_{i \in I} FEC_i}. \quad (5.1)$$

Please note that, differently from the other methods (that will be presented next) the sharing represented by (5.1) is independent of coalitional synergic effects. Therefore, an agent i is remunerated only by its individual energy contribution, in terms of FEC, for the pool. Additionally, this allocation method does not guarantee a solution inside the core of the cooperative game, even if the core is non-empty, as already shown.

5.3
Shapley value

The Shapley value was introduced by Lloyd Shapley in 1953 [70] and is a solution method for quota allocation problems in cooperative games theory. In this method, the distribution of the total benefits generated by the grand coalition among all players is calculated at once, through the determination of the respective contribution of each player to the coalition. Thus, the Shapley value of each player i can be retrieved through the average of its marginal contribution in all possible (sub)coalitions that such player can participate. For the quota allocation problem of a RES pool, the Shapley value associated with a given player i is given as

$$x_i^{SV} = \frac{\left[\frac{1}{n!}\left(\sum_{(\boldsymbol{c}\in\mathcal{C})|c_i=0}[(\mathbb{1}^T\boldsymbol{c})!\,(n-(\mathbb{1}^T\boldsymbol{c})-1)!]\cdot\left[v\left(\boldsymbol{c}+\boldsymbol{c}^{\{i\}}\right)-v(\boldsymbol{c})\right]\right)\right]}{v(\mathbb{1})}, \quad (5.2)$$

where x_i^{SV} is the quota associated to player i, n is the number of players in the grand coalition, and $(\mathbb{1}^T\boldsymbol{c})$ accounts for the number of players in coalition. Additionally, $\boldsymbol{c}^{\{i\}}$ is the aforementioned vector full of zeros with input 1 at position i that represents the coalition of player i, individually. Thereby, $\boldsymbol{c}+\boldsymbol{c}^{\{i\}}$ represents the coalition $\boldsymbol{c}$ with the inclusion of player i. Hence, $v(\boldsymbol{c}+\boldsymbol{c}^{\{i\}})$ is the characteristic function associated to coalition $\boldsymbol{c}$ including agent i. Note that in this case, $\boldsymbol{c}$ denotes a vector which has value 0 at position i and vector $\boldsymbol{c}^{\{i\}}$ is the null vector, except by value 1 in position i. Thus, the sum $\boldsymbol{c}+\boldsymbol{c}^{\{i\}}$ will obligatorily place the value 1 at position i of vector $\boldsymbol{c}$. Finally, $v(\boldsymbol{c})$ the characteristic function associated to coalition $\boldsymbol{c}$ only (without agent i).

Although the method is constructed on an intuitive way, it unfortunately presents two drawbacks. The first is the increase on its complexity while the number of players in the coalition grows, since it is built under a combinatory basis. The second drawback is the lack of isonomy, where large players can obtain, on a disproportional way, greater benefits in a given coalition. This effect occurs because, differently than the smaller ones, large players are less sensible in the input order in the analytical expression used to calculate the allocation.

5.4 Marginal Benefits

The allocation method by Marginal Benefits is based on the idea that a fair solution for quotas allocation in a cooperative game is achieved when the benefit regarding to a given player i is proportional to the marginal contribution of such player to the coalition's total benefit. In other words, the quota sharing is done according to the marginal increase player i grant to the benefit $v(\boldsymbol{c})$ of a given coalition $\boldsymbol{c}$ (e.g., the grand coalition $\boldsymbol{c} = \mathbb{1}$), which is promoted by the contribution of the vector of resources $\boldsymbol{b}_i$. So, the monetary benefit that player i receives in the particular case of the grand coalition is given by $\phi_i = \left.\frac{\partial v(\boldsymbol{c})}{\partial c_i}\right|_{\boldsymbol{c}=\mathbb{1}}$, which is the marginal increment of the benefit function assessed in $\boldsymbol{c} = \mathbb{1}$, induced by the component c_i (regarding player i) of the total vector of resources $\boldsymbol{c}$ of the grand coalition. For instance, the resources a given player i brings to a given coalition are: (i) FEC, which can be translated as contracting capacity and; (ii) the spot price revenue under the measure of value given by (4.5).

Furthermore, since the benefit function of the RES pool is obtained through an LP (Linear Programming problem) (model (4.6)-(4.9)), the marginal benefit can be directly retrieved from the value of the dual variables associated with the constraints of the resources of each player. Such variables carry the information on how the objective function, i.e., total benefit of the pool, varies with the marginal increment of resource associated to such constraint (see [17] for further details).

Thus, in order to devise the Marginal Benefits allocation, first take the generalization of characteristic function disposed in (4.3), as follows:

$$v(\boldsymbol{c}) = \max_{\boldsymbol{x}}\{\boldsymbol{a}^T\boldsymbol{x}\} + \sum_{i\in I} c_i q_i\,. \tag{5.3}$$

subject to:

$$\boldsymbol{A}\boldsymbol{x} \le \sum_{i\in I} c_i \boldsymbol{b}_i \qquad :\boldsymbol{\gamma} \tag{5.4}$$

$$x_i \ge 0, \qquad \forall i \in I, \tag{5.5}$$

where $\boldsymbol{x}$ is the vector of decision variables for the generalized problem and $\boldsymbol{\gamma}$ is the vector of dual variables associated to problem's set of constraints, (5.4). From the duality theory, the derivative of the objective function $v(\boldsymbol{c})$ with respect to the resource $\boldsymbol{b}_i$ grant by player i in the coalition is given by:

$$\frac{\partial v}{\partial c_i}(\boldsymbol{c}) = \boldsymbol{\gamma}^*, \tag{5.6}$$

where $\boldsymbol{\gamma}^*$ is the vector of dual variables in the optimal solution. Hence, the monetary amount received by player i is given by

$$\phi_i = \boldsymbol{\gamma}^{*T}\boldsymbol{b}_i + q_i\,, \tag{5.7}$$

where q_i denotes the derivation of the second part of the RHS of expression (5.3) in relation to the component c_i of the pool's total resource.

Now, recall the LP model that defines the characteristic function in (4.6)-(4.9), which can be properly rearranged in order to separate the resources terms from the decision variables terms, in ways of model (5.3)-(5.5), as follows.

$$\begin{aligned} v(\boldsymbol{c}) = \max_{Q,z,\Delta_s} \Bigg\{ & \lambda\left(z - \sum_s \frac{p_s \Delta_s}{1-\alpha}\right) \\ & + (1-\lambda)\left(\sum_{t,s} p_s\left(P - \pi_{t,s}\right)Qh_t\right)\Bigg\} \\ & + \sum_{i\in I} c_i \sum_{t,s}(1-\lambda)p_s\pi_{t,s}G_{i,t,s}h_t \end{aligned} \tag{5.8}$$

subject to:

$$z - \Delta_s - \sum_t \left(P - \pi_{t,s}\right)Qh_t \le \sum_{i,t} c_i\pi_{t,s}G_{i,t,s}h_t \qquad :\gamma_s, \forall s \in S \tag{5.9}$$

$$Q \le \sum_i FEC_i c_i \qquad :\beta \tag{5.10}$$

$$Q \ge 0 \tag{5.11}$$

$$\Delta_s \ge 0, \qquad \forall s \in S, \tag{5.12}$$

where γ_s and β are the dual variables associated to constraints (5.9) and (5.13), which are reformulated versions of (4.7) and (4.9), respectively. The segregation of the players' resources onto the RHS of constraints (5.9)-(5.10), like in (5.4), makes the derivation of the benefit function much easier. Furthermore, since the last term of the objective function (5.8) is not part of the maximization operator, the dual variables incorporate no information of such component, which in turns have to be derived separately. In this setting, this pair of derivations is done following (5.7).

Finally, the derivation of $v(\boldsymbol{c})$ in relation to a given component c_i is denoted by:

$$\phi_i = \frac{\partial v}{\partial c_i}(\boldsymbol{c})\bigg|_{\boldsymbol{c}=\mathbb{1}} = \sum_{t,s} \gamma_s^* \pi_{t,s} G_{i,t,s} h_t + \beta^* FEC_i + (1-\lambda) \sum_{t,s} p_s \pi_{t,s} G_{i,t,s} h_t, \quad (5.13)$$

where the first two terms are the derivatives, with respect to c_i, of the RHS (resources) of constraints (5.9) and (5.10), respectively and the last term is the derivative, also with respect to c_i, of the constant (last) term of the objective function (5.8), following (5.7), accordingly. For instance, the resources a given player i brings to a given coalition are: (i) FEC, which can be translated as contracting capacity, associated to the dual variable β; (ii) the spot price revenue under the measure of value given by (4.5), where γ_s is related to for the CVaR and q_i regards to the expected value. In this setting, the Marginal Benefits method allocates to each player the respective portion of the objective function of the corresponding dual problem, which is equal to the primal problem in the optimal solution. Thus, it recovers the total value of $\boldsymbol{v}(\mathbb{1})$.

Additionally, it is important to note that in (5.13) ϕ_i is the monetary benefit value (in $) produced by the resources with which player i contributes to the pool. Hence, in order to obtain the value of the quota itself, in percentage of the total value of the grand coalition, as done in the other methods, it is necessary to divide such number by the total value of the pool.

Finally, the allocation of a given player i in the pool, through the Marginal Benefits method, is given by:

$$x_i^{MB} = \frac{\phi_i}{v(\mathbb{1})}. \quad (5.14)$$

5.5 Nucleolus

The goal of the renewable pool is to find the optimum-selling strategy for its aggregated energy generation profile and guarantee its own stability by distributing benefits for its participants. Therefore, the pool manager needs to share the participants' quotas in such a way that the value of the quotas allocated to any coalition of players exceeds the value of its optimal selling strategy, outside the pool, for a given contract price opportunity, P ($/MWh), and risk-profile, α and λ. In other words, the quota allocation must be such that the difference

$v(\mathbb{1})(\boldsymbol{c}^T \cdot \boldsymbol{x}) - v(\boldsymbol{c})$ holds for all $\boldsymbol{c} \in \mathcal{C}$. Thereby, such expression represents the monetary gain (in \$) that a given coalition $\boldsymbol{c}$ has when participating in the pool.

An allocation with these properties belongs to the core of the cooperative game. This work studies the Nucleolus allocation method, which is known to be computationally intensive, for a large number of players. The main goal of the Nucleolus method is to share the quotas such that the monetary worst-case gain among all coalitions is maximized. Hence, by construction, if the core of the cooperative game is non-empty, the solution produced by the Nucleolus lays inside the core. It is worth mentioning that, in cooperative games theory, the word *Nucleolus* is the given name for the point (or solution) that maximizes the worst-case gain among all players. Therefore, the name of the method coincides with the optimum value it produces as output.

Moreover, the gain of a given coalition $\boldsymbol{c} \in \mathcal{C}$ in the pool can be retrieved by computing the difference between the revenue, in terms of $\rho_{\alpha,\lambda}$, of the cash flow obtained with the totality of the quotas allocated for such coalition and the cash flow it would obtain with the respective optimal selling strategy outside the pool. Thus, under a given shared quotas, defined by the vector $\boldsymbol{x} = [x_1, \dots, x_n]^T$, the gain of a given coalition $\boldsymbol{c}$ in the pool is defined by:

$$g(\boldsymbol{c}, \boldsymbol{x}) = \rho_{\alpha,\lambda}\left(\sum_{t \in T} \frac{\tilde{R}_t^{POOL}(Q^{*POOL})}{(1+J)^t}(\boldsymbol{c}^T \cdot \boldsymbol{x})\right) - v(\boldsymbol{c}), \tag{5.15}$$

where $\boldsymbol{c}^T \cdot \boldsymbol{x} = \sum_{i \in I} c_i \cdot x_i$ is the inner product that computes the sum of quotas allocated to coalition $\boldsymbol{c}$.

Since the CVaR is a coherent and positive-homogeneous measure of risk (as previously mentioned), so the product $(\boldsymbol{c}^T \cdot \boldsymbol{x})$ can be extracted from the measure $\rho_{\alpha,\lambda}$ and because, according to expression (4.3), the characteristic function of the grand coalition can be assessed *a priori* as $v(\mathbb{1}) = \rho_{\alpha,\lambda}\left(\sum_{t \in T} \frac{\tilde{R}_t^{POOL}(Q^{*POOL})}{(1+J)^t}\right)$, expression (5.15) can be rewritten as:

$$g(\boldsymbol{c}, \boldsymbol{x}) = v(\mathbb{1})(\boldsymbol{c}^T \cdot \boldsymbol{x}) - v(\boldsymbol{c}). \tag{5.16}$$

In this context, the quota allocation vector $\boldsymbol{x}$ for the Nucleolus allocation method for the proposed RES pool can be retrieved via the following mathematical programming problem:

$$\delta^* = \max_{\delta, \boldsymbol{x}} \delta \tag{5.17}$$

subject to:

$$\delta \leq v(\mathbb{1})(\boldsymbol{c}^T \cdot \boldsymbol{x}) - v(\boldsymbol{c}), \qquad \forall \boldsymbol{c} \in \mathcal{C} \tag{5.18}$$

$$\mathbb{1}^T \cdot \boldsymbol{x} = 1 \tag{5.19}$$

$$x_i \geq 0, \qquad \forall i \in I. \tag{5.20}$$

In (5.17), the auxiliary variable δ is maximized. Constraints (5.18) ensure the gains of all coalitions $\boldsymbol{c} \in \mathcal{C}$, except the grand coalition, and are responsible for making δ to push the worst-case gain coalition to the higher value as possible and constraint (5.19) guarantees the quotas are shared on its totality. Moreover, note that in the optimum point (solution), $(\delta^*, \boldsymbol{x}^*)$, $\delta^* = min_{\boldsymbol{c}\in\mathcal{C}}\{g(\boldsymbol{c}, \boldsymbol{x}^*)\}$.

The drawback of (5.17)-(5.19) is its famous computational burden, since the number of constraints grows exponentially while the number n of players in the pool increases, due to the combinatory nature of the coalitions. The number of constraints in (5.17)-(5.19) relies on the cardinality of the set of (sub)coalitions $\mathcal{C}$, which is $2^n - 1$. As an example, if $n = 30$, one would need to assess the characteristic function $v(\boldsymbol{c})$ more than one billion of times to obtain the right-hand-side (RHS) vector of the problem. The complexity of this assessment is enforced by the fact that, in the present problem, the RHS vector is defined as a two-stage stochastic optimization portfolio problem, i.e., the computation of $v(\boldsymbol{c})$. Thus, in the next chapter, an efficient methodology is derived to solve problem (5.17)-(5.19) without needing to explicitly impose the full set of gain constraints (5.18).

5.6 Proportional Nucleolus

Since the Nucleolus method is based on the maximization of the worst-case gain in terms of monetary value, in cases where a coalition is formed with players of different size, the solution can produce some distortion in terms of the relative gain among coalitions. For the sake of simplicity, imagine a simple game of three players where the quota-sharing is given by the Nucleolus method, as follows.

Table 5.3 – Quotas sharing via the Nucleolus allocation method for a game with players disproportionally sized.

coalition ($\boldsymbol{c}$)	FEC (avg-MW)	Revenue in $\$10^3$ ($v(\boldsymbol{c}^{\{i\}})$)	Allocation (x_i)	Absolute Gain in (\$) ($v(\mathbb{1}) - v(\boldsymbol{c}^{\{i\}})$)	Proportional Gain $\left(\frac{v(\mathbb{1}) - v(\boldsymbol{c}^{\{i\}})}{v(\boldsymbol{c}^{\{i\}})}\right)$
SH	5	2,447.00	63.27%	379.86	15.52%
Bio	1	594.99	17.38%	181.54	30.51%
WP	1	683.00	19.35%	181.54	26.58%
SH – Bio	6	3,411.03	80.65%	192.37	5.64%
SH – WP	6	3,509.97	82.62%	181.45	5.17%
Bio – WP	2	1,378.93	36.73%	262.15	19.01%
SH – Bio - WP	7	4,467.94	w-c gain:	181.45	5.17%

Although the (monetary) absolute worst-case gain was maximized, resulting in a gain of $\$\,379.86$ thousand for the small hydro unit which is at principle a very good gain, the unit could claim better benefits, once the two smaller units, biomass and wind power, had not so smaller monetary gains but contributed only with 1 avg-MW of FEC each for the cooperation, while the small-hydro, contributed with much more energy. This leaves a sensation that the smaller units are "surfing the wave" of the small hydro. Additionally, the percentage gains, disposed in the last column of Table 5.3 shows a smaller gain for the bigger unit, besides pointing a considerable difference between the smaller ones, while both contributed with the same amount of FEC.

The Proportional Nucleolus reduces such distortion since its main goal is to maximize the worst-case gain proportionally (or relatively) to the value coalitions receive by themselves. Nevertheless, the gain function the Proportional Nucleolus maximizes is $\frac{v(\mathbb{1})-v(\boldsymbol{c})}{v(\boldsymbol{c})}$, which should also hold for all $\boldsymbol{c} \in \mathcal{C}$ (as stated in the Nucleolus gain function, in previous section) and which is also the object in focus of the last column in Table 5.3. For comparison purposes, Table 5.4, disposed bellow, denotes the results of the quota allocation via the Proportional Nucleolus method for the same game of Table 5.3.

Table 5.4 – Contrast of the quotas sharing via the Proportional Nucleolus allocation method for the game of Table 5.3.

coalition ($\boldsymbol{c}$)	FEC (avg-MW)	Revenue in $\$10^3$ ($v(\boldsymbol{c}^{\{i\}})$)	Allocation (x_i)	Absolute Gain in (\$) ($v(\mathbb{1}) - v(\boldsymbol{c}^{\{i\}})$)	Proportional Gain $\left(\frac{v(\mathbb{1}) - v(\boldsymbol{c}^{\{i\}})}{v(\boldsymbol{c}^{\{i\}})}\right)$
SH	5	2,447.00	66.77%	536.24	21.91%
Bio	1	594.99	15.42%	93.97	15.79%
WP	1	683.00	17.81%	112.74	16.51%
SH – Bio	6	3,411.03	82.19%	261.17	7.66%
SH – WP	6	3,509.97	84.58%	269.02	7.66%
Bio – WP	2	1,378.93	33.23%	105.77	7.67%
SH – Bio - WP	7	4,467.94	w-c gain:	93.97	7.66%

In this second case, admitting that the absolute worst-case gain dropped from \$ 181.45 thousand on the previous example by half, it is undeniable the proportional benefit of 21.91% for the small hydro is more reasonable. This would also keep away the risk of the bigger player (small hydro) quit the cooperation, which would be a bad deal for the cooperation and, first and foremost, for the smaller players. Still, note that the difference between the gains of the small players was decreased. Finally, the proportional worst-case gains among all coalitions increased from 5.17% to 7.66%.

In the formal (and further) definition of the Proportional Nucleolus method, it might be worth make use of the Nucleolus notation previously presented, in order to save time, since both methods are really similar. Therefore, the proportional gain function for a given coalition $\boldsymbol{c}$ and quota allocation vector $\boldsymbol{x}$ is given by:

$$g^p(\boldsymbol{c}, \boldsymbol{x}) = \frac{v(\mathbb{1})(\boldsymbol{c}^T \cdot \boldsymbol{x}) - v(\boldsymbol{c})}{v(\boldsymbol{c})}. \tag{5.21}$$

Furthermore, in this setting the Proportional Nucleolus allocation method to share the quotas of the proposed RES pool can be retrieved via the following mathematical programming problem.

$$\delta^{p*} = \max_{\delta^p, \boldsymbol{x}} \delta^p \tag{5.22}$$

subject to:

$$\delta^p \leq \frac{v(\mathbb{1})(\boldsymbol{c}^T \cdot \boldsymbol{x}) - v(\boldsymbol{c})}{v(\boldsymbol{c})}, \qquad \forall \boldsymbol{c} \in \mathcal{C} \tag{5.23}$$

$$\mathbb{1}^T \cdot \boldsymbol{x} = 1 \tag{5.24}$$

$$x_i \geq 0, \qquad \forall i \in I. \tag{5.25}$$

In (5.22), the auxiliary variable δ^p is maximized. Constraints (5.23) ensure the proportional gains of all coalitions $\boldsymbol{c} \in \mathcal{C}$, except the grand coalition, and are responsible for making δ^p to push the proportional worst-case gain coalition to the higher value as possible and constraint (5.24) guarantees the quotas are shared on its totality. Moreover, note that in the optimum point (solution), $(\delta^{p^*}, \boldsymbol{x}^*)$, $\delta^{p^*} = min_{\boldsymbol{c}\in\mathcal{C}}\{g^p(\boldsymbol{c}, \boldsymbol{x}^*)\}$.

The Proportional Nucleolus method has the same computational drawback of his predecessor. Additionally, it brings one more difficulty when the devised efficient methodology that bypasses such computational issue is applied to it. Fortunately, this second drawback will also be appropriately solved in the sequel.

5.7 The pros and cons of studied allocation methods

The fact that the core of the proposed cooperative game is non-empty provides a very good signal for the stability of the pool, since some of the methods presented above always generate solutions inside the core, whenever it exists (which is thus the case of the present work). Therefore, with such solutions, no player has incentives to leave the pool, once benefits are guaranteed for all possible coalitions of players.

On a slightly different manner, games theory also discusses the concept of fairness of the many different solution methods. Despite a given solution for a cooperative game could bring substantial gains for the cooperation, this does not grant fairness, necessarily. For instance, if a given solution allocates great part of the global gains for a single player and little gains for the remaining players, such solution would be said to bring stability for the cooperation, although not being fair at all. Contradictorily, in such situation the absence of fairness could even break the stability of the cooperation, forcing some players to quit the pool.

In this context, researchers in cooperative games evaluate the level of fairness from solution methods on different ways. Moreover, it is also important to state that the concept of fairness is subjective and very personal. It depends on the characteristics of each case or application. For example, a very common way to evaluate the fairness of a given solution is via the marginal contributions of each player to the final results of the grand coalition, which is the main concept behind the Shapley value and Marginal Benefits methods. However, as stated before, Shapley value method is not scalable and case studies presented in Chapter 7 will show that the Marginal Benefits method can produce solutions with null worst-case gains, i.e., the method is not as stable as the Nucleolus, for example. As pointed before, another intuitive way of generating a fair solution would be proportionally by terms of the size of each player in the cooperation, via the rational “what you give is what you get”. However, the motivation Section 5.1 of the present Chapter has already shown that this kind of solution does not bring stability for the cooperation in certain cases.

On the other hand, another common argument to produce fair solutions is based on the maximization of the worst-case gain, such as the Nucleolus methods do. Summarizing, it is important to note that there is no unanimity about which is the best type of allocation. Each of the different aforementioned methods has their pros and cons, which shall be further pointed out throughout the text. So, while choosing a quota sharing methodology, it is necessary to take into account the appropriate balance between the stability and the fairness each solution method can provide.

Particularly, the nucleolus allocation methods are associated to the concept of strong fairness, as well as strong stability: while maximizing the worst-case gain among all possible coalitions of players, the methods are maximizing a measure of stability. For this reason the present work focus on nucleolus methods. Additionally, after solving the Nucleolus problems, the proposed methodology runs a post-optimization algorithm to maximize individual worst-case gains, in order to improve even more the incentives for individual agents. Moreover, it is intuitive that players would pursue greater individual gains relatively to the best it could be obtained outside the cooperation, which motivates the present work to focus on the Proportional Nucleolus method. Finally, next chapter presents the methodology that overcomes the computational burden of the nucleolus methods, which is responsible for placing these methods a little ahead from the other ones.

6
Allocation methods by Benders decomposition

As previously mentioned, in order to work-around the combinatory explosion problem on the number of constraints, an efficient methodology is devised to solve (5.22)-(5.24) without needing to explicitly build the full set of constraints. Hence, problem (5.22)-(5.24) can be rewritten as the following nonlinear optimization problem:

$$\delta^* = \max_{\delta, \boldsymbol{x}} g^*(\boldsymbol{x}) \tag{6.1}$$

subject to:

$$\mathbb{1}^T \cdot \boldsymbol{x} = 1 \tag{6.2}$$

$$x_i \geq 0, \qquad \forall i \in I. \tag{6.3}$$

Where, $g^*(\boldsymbol{x}) = min_{\boldsymbol{c} \in \mathcal{C}}\, g(\boldsymbol{c}, \boldsymbol{x})$ represents the optimal value of δ for each feasible vector $\boldsymbol{x}$ in (5.22)-(5.24) which is the worst-case gain, among all coalitions for a given allocation vector $\boldsymbol{x}$. Thus, according to (5.21), the worst-case gain function can be defined as the following expression:

$$g^*(\boldsymbol{x}) = \min_{\boldsymbol{c} \in \mathcal{C}} \{ v(\mathbb{1})(\boldsymbol{c}^T \cdot \boldsymbol{x}) - v(\boldsymbol{c}) \}. \tag{6.4}$$

Moreover, according to (4.3), expression (6.4) can be expanded to:

$$g^*(\boldsymbol{x}) = \min_{\boldsymbol{c} \in \mathcal{C}} \left\{ v(\mathbb{1})(\boldsymbol{c}^T \cdot \boldsymbol{x}) - \max_{0 \leq Q \leq \sum_{i \in I} FEC_i c_i} \rho_{\alpha,\lambda} \left(\sum_{t \in T} \frac{\tilde{R}_t(Q, \boldsymbol{c})}{(1+J)^t} \right) \right\}. \tag{6.5}$$

Once that for a general function f, the equality $max(f) = -min(-f)$ always holds, expression (6.5) can be expressed as:

$$g^*(\boldsymbol{x}) = \min_{\boldsymbol{c} \in \mathcal{C}} \left\{ v(\mathbb{1})(\boldsymbol{c}^T \cdot \boldsymbol{x}) + \min_{0 \leq Q \leq \sum_{i \in I} FEC_i c_i} -\rho_{\alpha,\lambda} \left(\sum_{t \in T} \frac{\tilde{R}_t(Q, \boldsymbol{c})}{(1+J)^t} \right) \right\}. \tag{6.6}$$

Furthermore, since vector $\boldsymbol{c}$ is a parameter for the $max_Q\{\cdot\}$ operator of the characteristic function (in expression (4.3)), the joint minimization of variables $\boldsymbol{c}$ and Q can be properly done, as disposed in the following two steps:

$$g^*(\boldsymbol{x}) = \min_{\boldsymbol{c}\in\mathcal{C}}\left\{\min_{0\le Q\le\sum_{i\in I}FEC_ic_i}\left[v(\mathbb{1})(\boldsymbol{c}^T\cdot\boldsymbol{x}) - \rho_{\alpha,\lambda}\left(\sum_{t\in T}\frac{\tilde{R}_t(Q,\boldsymbol{c})}{(1+J)^t}\right)\right]\right\} \quad (6.7)$$

and, finally,

$$g^*(\boldsymbol{x}) = \min_{\substack{\boldsymbol{c}\in\mathcal{C}\\ 0\le Q\le\sum_{i\in I}FEC_ic_i}}\left\{v(\mathbb{1})(\boldsymbol{c}^T\cdot\boldsymbol{x}) - \rho_{\alpha,\lambda}\left(\sum_{t\in T}\frac{\tilde{R}_t(Q,\boldsymbol{c})}{(1+J)^t}\right)\right\}, \quad (6.8)$$

which is equivalent to (6.4), representing the worst-case gain among all possible coalitions of the cooperative game, for a given allocation vector $\boldsymbol{x}$. Additionally, in (6.4), $g^*(\boldsymbol{x})$ is defined as a pointwise minimum within a family of affine functions in $\boldsymbol{x}$, inside a set indexed by $\boldsymbol{c}\in\mathcal{C}$, $\{v(\mathbb{1})(\boldsymbol{c}^T\cdot\boldsymbol{x}) - v(\boldsymbol{c})\}_{\boldsymbol{c}\in\mathcal{C}}$. Hence, $g^*(\boldsymbol{x})$ is a concave function of $\boldsymbol{x}$ (see chapter 3.2.3 of [41]). In such a framework, the MILP (6.1)-(6.3) is suitable for the Benders decomposition approach.

6.1
MILP formulation for the Nucleolus worst-case gain function

The challenge of computing $g^*(\boldsymbol{x})$ is the need to evaluate expression (5.16) for all possible coalitions of players. This would demand the assessment of $2^n - 2$ two-stage stochastic programs to find $v(\boldsymbol{c})$ for all $\boldsymbol{c}\in\mathcal{C}$. However, $g^*(\boldsymbol{x})$ can be written as a MILP that finds $\boldsymbol{c}^*$ for each $\boldsymbol{x}$ and is solved efficiently by Branch-and-Cut algorithms [71], thus avoiding the need to explicitly explore the whole set of coalitions. According to the definition of $v(\boldsymbol{c})$ in (4.3), the worst-case gain function can be assessed by the following program:

$$g^*(\boldsymbol{x}) = \min_{\boldsymbol{c},Q}\ v(\mathbb{1})(\boldsymbol{c}^T\cdot\boldsymbol{x}) - \rho_{\alpha,\lambda}\left(\sum_{t\in T}\frac{\tilde{R}_t(Q,\boldsymbol{c})}{(1+J)^t}\right) \quad (6.9)$$

subject to:

$$0 \le Q \le \sum_{i\in N} FEC_ic_i \quad (6.10)$$

$$1 \le \mathbb{1}^T\cdot\boldsymbol{c} \le n-1 \quad (6.11)$$

$$c_i \in \{0,1\}, \qquad \forall i\in I. \quad (6.12)$$

In (6.9)-(6.12), the objective function minimizes expression (5.16) to assess the worst-case function (6.4). Expression (6.10) is directly inferred from (4.3), and expression (6.11) excludes the null and the grand coalitions from the feasible set according to (6.4). Finally, (6.12) imposes the binary nature to the decision vector

$\boldsymbol{c}$. The problem (6.9)-(6.12) can be solved by its deterministic equivalent formulation, which can be obtained by replacing the second term of (6.9) with expressions (4.6)-(4.8). This formulation leads to a MILP that can be solved by commercial solvers such as IVE Xpress [72].

It is worth to highlight that the problem (6.9)-(6.12) is a separation problem that discovers the worst-case constraint violation for a given quota allocation vector. To do that, it co-optimizes the vector $\boldsymbol{c}$ and quantity Q to select, in the set of constraints defined in (5.18), the constraint with the lowest right-hand-side (a similar idea is proposed in [38]). Then, the current vector of quotas is said to belong to the core of the game if and only if the result of this minimization is non-negative. Under this framework, problem (6.9)-(6.12) can be also understood as an "oracle" that determines if a given vector of allocated quotas belongs to the core independently of the allocation method used to find it. This separation problem will be used as a cut generator in the developed decomposition framework of the next section.

6.2 Benders decomposition algorithm

Pursuing a more efficient solution in terms of computational effort, Benders decomposition technique [73] prompted as an alternative: the main idea consists in splitting the problem (6.1)-(6.3), which has high computational complexity, into two problems of low computational cost. Benders decomposition was developed to solve problems of elevated computation cost and is quite common in large-scale stochastic programming problems applications. The two problems that compose the technique are so-called primary and secondary problems or problems of first and second levels, respectively. Still, the primary problem is also commonly known as master problem.

The intuition behind the method is to approximate a convex function with the first-order Taylor expansion through hyperplanes, known as Benders cuts. Under this framework, the problem is solved by way of an iterative process, with the addition of new cuts in each step, until a certain tolerance level over the value of the objective function is achieved as well as the consequent convergence (termination) of the algorithm. In the worst case analysis, the first level completely recovers the feasible region of the original problem, by means of the cuts generated by the secondary level, which is the one that assesses the true function under inspection. However, in most cases, few iterations are necessary.

At the proposed method, a master problem is solved to determine an upper bound, $UB_{(k)}$, for δ* and an optimal solution $\boldsymbol{x}_{(k)}$ in each iteration k. This allocation $\boldsymbol{x}_{(k)}$ is a trial solution for the Nucleolus problem (6.1)-(6.3). Therefore, the true function $g^*(\boldsymbol{x})$ can be evaluated in $\boldsymbol{x}_{(k)}$, by (6.9)-(6.12) to determine a lower bound, $LB_{(k)}$, for δ^* and to obtain a new cut for the first level. The master problem of each iteration k optimizes an outer approximation for $g^*(\boldsymbol{x})$ built as the pointwise minimum of the Benders cuts, which are affine approximations of the function around $\boldsymbol{x}_{(k)}$. This problem can be written as the following LP:

$$UB_{(k)} = \max_{\delta,\boldsymbol{x}} \delta \tag{6.13}$$

subject to:

$$\delta \leq g^*\left(\boldsymbol{x}_{(j)}\right) + \partial g^*\left(\boldsymbol{x}_{(j)}\right)^T\left(\boldsymbol{x} - \boldsymbol{x}_{(j)}\right), \qquad \forall j = 1, \dots, k-1 \tag{6.14}$$

$$\mathbb{1}^T \cdot \boldsymbol{x} = 1 \tag{6.15}$$

$$x_i \geq 0, \qquad \forall i \in I. \tag{6.16}$$

Where the objective function (6.13) maximizes an auxiliary variable δ that, in the optimal solution, meets the value of the lowest cut considered in the problem. The set of constraints (6.14) represent the Benders cuts, in which $\partial g^*\left(\boldsymbol{x}_{(j)}\right)^T$ is the subgradient of g^* when $\boldsymbol{x} = \boldsymbol{x}_{(j)}$. And according to (6.4):

$$\partial g^*\left(\boldsymbol{x}_{(j)}\right)^T = v(\mathbb{1})\boldsymbol{c}_{(k)}^T. \tag{6.17}$$

The following algorithm summarizes the Benders Decomposition approach to solve problem (6.1)-(6.3).

Benders decomposition algorithm

1: **Initialization:**
2: $k \leftarrow 1$ and $\boldsymbol{x}_{(k)} = \mathbb{1}n^{-1}$;
3: $UB_{(k)} \leftarrow +\infty$ and $LB_{(k)} \leftarrow g^*\left(\boldsymbol{x}_{(k)}\right)$.
4: **While** $UB_{(k)} - LB_{(k)} > \varepsilon$ **do:**
5: $k \leftarrow k + 1$;
6: Solve the master problem (6.13)-(6.16) and store $UB_{(k)}$ and $\boldsymbol{x}_{(k)}$;
7: Solve the MILP (6.9)-(6.12) for $\boldsymbol{x}_{(k)}$ and store $g^*(\boldsymbol{x}_{(k)})$ and $\partial g^*\left(\boldsymbol{x}_{(k)}\right)^T$;
8: $LB_{(k)} \leftarrow g^*(\boldsymbol{x}_{(k)})$.
9: **Repeat do**

Because g^* is concave, the algorithm finitely converges to an ε-near optimal (global) solution $\boldsymbol{x}_{(k^*)}$, i.e., $\left|g^*\left(\boldsymbol{x}_{(k^*)}\right) - \delta^*\right| \le \varepsilon$. In the case study section, the performance of the Benders algorithm is compared with the classical full coalition-dependent formulation (5.22)-(5.24).

It is important to note that the Benders cuts expression, (6.14), precisely recovers expression (5.23) for each coalition found in step 7 of the algorithm. Therefore, in the specific case of this application, the proposed Benders algorithm, devised as an outer (dual) representation of the worst-case gain function, can also be understood as a primal algorithm. In this setting, the algorithm finds a subset of the most-inner constraints of (5.23) necessary to allow the master problem, which is a relaxed version of the full problem, to obtain the optimal solution of (5.22)-(5.24). This set of constraints is also known as umbrella constraints [40]. Next, it will be illustrated the algorithm's iterative process.

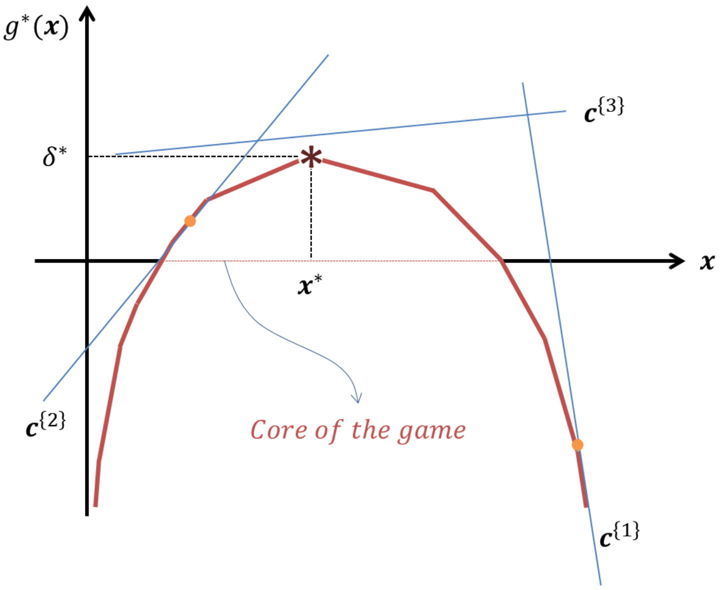

Figure 6.1 – Iteration 1 of the illustrative example of the Nucleolus allocation algorithm via Benders decomposition.

The illustrative example of Figure 6.1 represents the approximation of the worst-case gain function $g^*(\boldsymbol{x})$ (vertical axis), which is function of the allocation vector $\boldsymbol{x}$ (horizontal axis). What the algorithm does is approximate the worst-case function through support planes, represented in the figure by the linear by parts concave curve, until finding the point $\boldsymbol{x}^*$ of Figure 6.1, that maximizes the function $g^*(\boldsymbol{x})$ (note that $g^*(\boldsymbol{x}^*) = \delta^*$). The dashed line in the horizontal axis represents the range of allocations that defines the core of the cooperative game, since it is the rage where the worst-case gain function assumes positive values. Additionally, the figure denotes an example where the problem's initial formulation that contains the constraints of individual gain. Here only the cuts for players $\boldsymbol{c}^{\{1\}}$, $\boldsymbol{c}^{\{2\}}$ and $\boldsymbol{c}^{\{3\}}$ are represented, but actually there is one cut for each player in the cooperative game. Nonetheless, these initial cuts will be employed on an improved version of the algorithm, further in the text. The next steps of the algorithm are depicted in the sequel.

At initialization, first, the iteration counter (j) is set to 1 and a confidence level ε is defined. Next, the first trial solution $\boldsymbol{x}^{(1)}$ is found, together with the first upper bound, $UB^{(1)}$, for the problem. Both are represented in Figure 6.2. Note that the point that intersects cuts $\boldsymbol{c}^{\{1\}}$ and $\boldsymbol{c}^{\{3\}}$ defines $UB^{(1)}$, since it maximizes $g^*(\boldsymbol{x})$, given cuts $\boldsymbol{c}^{\{1\}}$, $\boldsymbol{c}^{\{2\}}$ and $\boldsymbol{c}^{\{3\}}$.

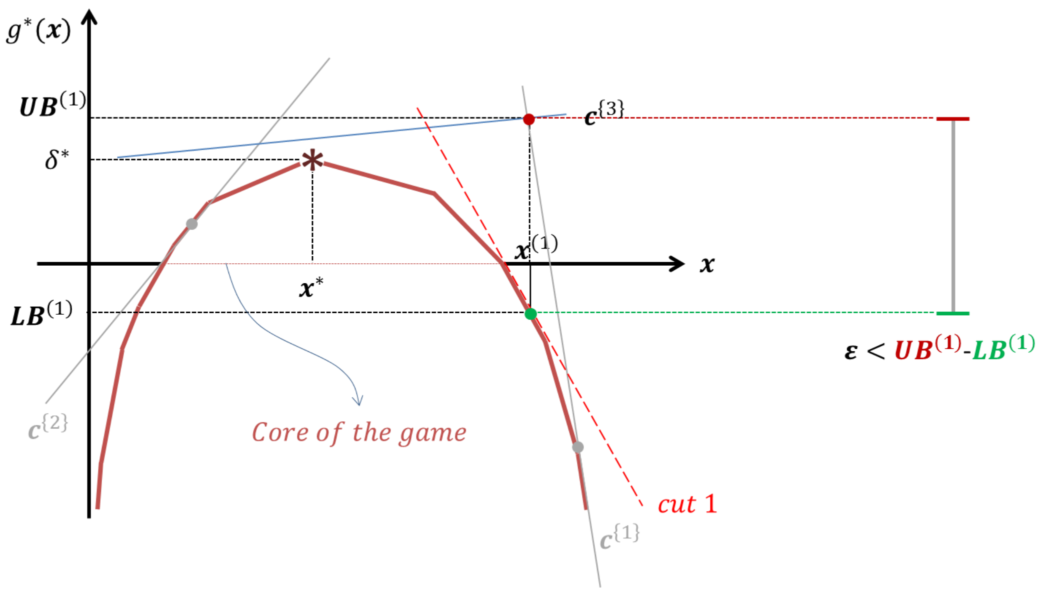

Figure 6.2 – Iteration 2 of the illustrative example of the Nucleolus allocation algorithm via Benders decomposition.

Now equipped with $\boldsymbol{x}^{(1)}$, the algorithm adds the cut 1 to the problem's formulation, since the worst-case coalition $\boldsymbol{c}^{\{1\}}$ was found. At this moment, the first lower bound for the problem is also obtained, when $g^*\left(\boldsymbol{x}^{(1)}\right)$ defines $LB^{(1)}$, since the assessment of the real worst-case gain function is done at the point $\boldsymbol{x}^{(1)}$. As the gap between the upper and lower bounds is still not satisfactory, i.e.

$\varepsilon < UB^{(1)} - LB^{(1)}$, the iteration counter is incremented in one unit and the algorithm follows to step 3.

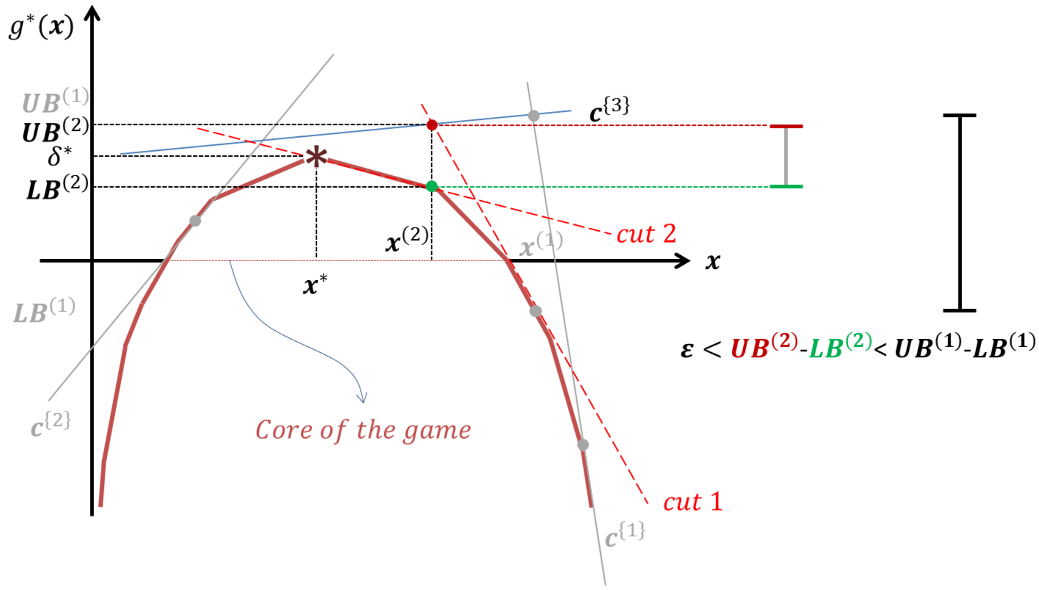

Figure 6.3 – Iteration 3 of the illustrative example of the Nucleolus allocation algorithm via Benders decomposition.

A new trial solution, $\boldsymbol{x}^{(2)}$, is found solving once more the problem defined in (6.13)-(6.16), that now contains the cut 1, added at the previews step. A new upper bound, $UB^{(2)}$, is also retrieved. With the new trial allocation vector $\boldsymbol{x}^{(2)}$ in hands, the algorithm finds the new worst-case coalition $\boldsymbol{c}^{(2)}$ and the respective lower bound of iteration 2, $LB^{(2)}$. Such procedures are disposed in Figure 6.3. Note that the new gap $(UB^{(2)} - LB^{(2)})$ is smaller than the first one but it is, also, bigger then ε and therefore, not yet satisfactory $(\varepsilon < UB^{(2)} - LB^{(2)} < UB^{(1)} - LB^{(1)})$. Therefore, the second cut is added, the iteration counter incremented and the procedure is continued. Finally, it can be noted that by repeating the procedure once more, only, with the addition of the next cut, the illustrative algorithm converges to the optimum solution, reaching the pair $(\delta^*, \boldsymbol{x}^*)$.

Hence, expression (6.14) represents each one of the generated cuts in the problem's original polyhedron (expression (5.23)) during the iterative process, significantly reducing the feasible space, since the problem's feasible region frontier is approximated through hyperplanes. The main objective is to find and allocation vector $\boldsymbol{x}$ such that all subcoalition receives from the pool a greater value then it would individually obtain. Instead of worrying about with all possible coalitions, which is already known to be an intractable problem, what the algorithm pursues is to approximate, linearly by parts, via the function $g^*(\boldsymbol{x}) = \min_{c \in \mathcal{C}} g(\boldsymbol{c}, \boldsymbol{x})$, the worst-case gain function, given an allocation $\boldsymbol{x}$.

Under this framework, the methodologies presented in (6.14) and (5.22)-(5.24) are equivalent. To demonstrate such aspect, it is sufficient the substitution of (6.17) in (6.14) that, for a given cut in a given iteration k, leads towards the following expression:

$$\delta \leq g^*\left(\boldsymbol{x}^{(k)}\right) + v(\mathbb{1})\left(\boldsymbol{c}^{*T} \cdot \left(\boldsymbol{x} - \boldsymbol{x}^{(k)}\right)\right). \tag{6.18}$$

Then, since $g^*\left(\boldsymbol{x}^{(k)}\right) = min_{c \in \mathcal{C}}\, g\left(\boldsymbol{c}, \boldsymbol{x}^{(k)}\right) = v(\mathbb{1})\left(\boldsymbol{c}^{*T} \cdot \boldsymbol{x}^{(k)}\right) - v(\boldsymbol{c}^*)$, the following expression holds.

$$\delta \leq v(\mathbb{1})\left(\boldsymbol{c}^{*T} \cdot \boldsymbol{x}^{(k)}\right) - v(\boldsymbol{c}^*) + v(\mathbb{1})\left(\boldsymbol{c}^{*T} \cdot \boldsymbol{x}\right) - v(\mathbb{1})\left(\boldsymbol{c}^{*T} \cdot \boldsymbol{x}^{(k)}\right), \tag{6.19}$$

which finally yields to:

$$\delta \leq v(\mathbb{1})\left(\boldsymbol{c}^{*T} \cdot \boldsymbol{x}\right) - v(\boldsymbol{c}^*). \tag{6.20}$$

In this setting, the problem constituted by expressions (6.13)-(6.16) can have the expression (6.14) substituted by (6.20). Thus, it is built in the same way of problem (5.22)-(5.24).

The presented methodology saves computational effort since the second level does not need to visit all possible coalitions of players, focusing only in the ones that have the lower gains. In this context, a reasoning gain in terms of computing time is expected in the RES pool quotas sharing problem. Case studies will be further presented in the text to corroborate with such expectation and also will all theory and concepts presented throughout the work. The most relevant result is the comparison table between the quotas sharing of the RES pool via the Full Coalition Dependent Nucleolus methods and the Benders decomposition approaches.

6.3 MILP formulation for the Proportional Nucleolus worst-case gain function

As previously mentioned, besides the computational issue, which can be properly solved through the model disposed in the previews section, the

Proportional Nucleolus method has one more drawback: when the just described Benders decomposition method is applied to it, the problem becomes non-linear. That is due to the method's original formulation, which aims to maximize the proportional gain of the coalitions, $\frac{v(\mathbb{1})(\boldsymbol{c}^{\boldsymbol{T}}\cdot\boldsymbol{x})-v(\boldsymbol{c})}{v(\boldsymbol{c})}$, where the vector of coalitions $\boldsymbol{c}$ in this form of the problem is a decision variable. Fortunately, the formulation can be properly linearized through fractional programming technique, found in a general form in section 4.3.2 of [41]. However, the present application follows the findings of [42], which differently from [41] presents a version of the technique where (besides the linear variables) the integer variables are strictly binary, case of the present problem. The main idea behind the technique is to introduce new variables that yield to a non-linear optimization problem containing only products of decision variables (instead of division of variables) that can, so, be linearized via the big- M technique. The non-linear formulation for the Proportional Nucleolus allocation method and the respective linearized solution problem are disposed in the sequel.

Hence, the fractional programming technique presented in [42] will be depicted, but considering only the variables involved in the particular case of the present work. That is due to the fact that, in the referred model, linear and binary variables are present at both numerator and denominator of the function object of linearization. In the case of the present work, the integer variables are also binary, which allows the application of the solution presented in [42]. However, the Proportional Nucleolus presents only binary variables at the numerator and only linear variables in the denominator. Therefore and reinforcing, please do refer to [41] for a more general model and further information.

Thus, first take the general non-linear problem:

$$f = \min_{x,y} \frac{\boldsymbol{a}^T\boldsymbol{y}}{\boldsymbol{b}^T\boldsymbol{x}} \tag{6.21}$$

subject to:

$$\boldsymbol{q}^T\boldsymbol{x} + \boldsymbol{r}^T\boldsymbol{y} = 0 \tag{6.22}$$

$$\boldsymbol{x} \geq \mathbb{0}, \boldsymbol{y} \in \{0,1\}^n, \tag{6.23}$$

where $\boldsymbol{b}^T\boldsymbol{x}$ must be strictly positive, since it is on the denominator of (6.21). Then, the variables $u = \frac{1}{\boldsymbol{b}^T\boldsymbol{x}}$ and $\boldsymbol{x}^u = \frac{\boldsymbol{x}}{\boldsymbol{b}^T\boldsymbol{x}} = \boldsymbol{x}\cdot u$ are introduced. Please note that the superscript 'u' in $\boldsymbol{x}^u$ is only for notation purposes and does not denote an exponent. Next, $\frac{1}{\boldsymbol{b}^T\boldsymbol{x}}$ is replaced by u in the objective function and since u is also strictly positive, (6.22) can be multiplied by u without further problems. These steps yield to the following formulation.

$$f = \min_{x,y,u} \ \boldsymbol{a}^T(\boldsymbol{y} \cdot u) \tag{6.24}$$

subject to:

$$\boldsymbol{q}^T(\boldsymbol{x} \cdot u) + \boldsymbol{r}^T(\boldsymbol{y} \cdot u) = 0 \tag{6.25}$$
$$\boldsymbol{b}^T(\boldsymbol{x} \cdot u) = 1 \tag{6.26}$$
$$\boldsymbol{x} \geq \mathbb{0}, u \geq 0, \boldsymbol{y} \in \{0,1\}^n, \tag{6.27}$$

where (6.26) defines variable u. Then, the substitution of variables $\boldsymbol{x}^u$ is properly done, together with the replacement of the product $\boldsymbol{y} \cdot u$ by the new variable $\boldsymbol{w} = \boldsymbol{y} \cdot u$, leading to the final problem:

$$f = \min_{x,y,w,u} \ \boldsymbol{a}^T\boldsymbol{w} \tag{6.28}$$

subject to:

$$\boldsymbol{q}^T\boldsymbol{x}^u + \boldsymbol{r}^T\boldsymbol{w} = 0 \tag{6.29}$$
$$\boldsymbol{b}^T\boldsymbol{x}^u = 1 \tag{6.30}$$
$$\boldsymbol{w} \leq u \cdot \mathbb{1} \tag{6.31}$$
$$\boldsymbol{w} \leq M \cdot \boldsymbol{y} \tag{6.32}$$
$$\boldsymbol{w} \geq u \cdot \mathbb{1} - M(\mathbb{1} - \boldsymbol{y}) \tag{6.33}$$
$$\boldsymbol{x} \geq \mathbb{0}, \boldsymbol{w} \geq \mathbb{0}, u \geq 0, \boldsymbol{y} \in \{0,1\}^n, \tag{6.34}$$

which is the final linear and equivalent formulation for the original problem, where M is a sufficiently large number and (6.31)-(6.33) are the so-called big-M constraints, responsible for making $\boldsymbol{w}$ properly recover the dynamic of the product $\boldsymbol{y} \cdot u$. In this setting, when a given entry of $\boldsymbol{y}$ assumes value 1, the product should assume the value u. This is guaranteed by (6.31)-(6.33), since the sufficiently large M leaves (6.32) unrestricted and the pair {(6.31), (6.33)} forces the relation $u \cdot \mathbb{1} \leq \boldsymbol{w} \leq u \cdot \mathbb{1}$, i.e. $\boldsymbol{w} = u \cdot \mathbb{1}$. On the other hand, when a given entry of $\boldsymbol{y}$ assumes value 0, the product should also assume the value 0. Again, this is also guaranteed by (6.31)-(6.33), since the sufficiently large number M and the pair {(6.32),(6.33)} force the relation $(u - M) \cdot \mathbb{1} \leq \mathbb{0} \leq \boldsymbol{w} \leq \mathbb{0}$, i.e. $\boldsymbol{w} = \mathbb{0}$, and $\boldsymbol{w}$ is non-negative. Under this framework, the choice of a proper value for M is crucial for the correct functioning of the formulation, in computational terms, due to numerical-precision issues. This choice will be discussed further in the text. Hereinafter in the chapter the MILP formulation for the Proportional Nucleolus will be presented, culminating with the fractional programing technique applied to the specific case of this work.

For convenience purposes, since the Proportional Nucleolus method is very similar to its precursor method and the MILP formulations for both methods are almost identical, once again, the proceeding to devise the formulation for the MILP will be skipped. In this context, please refer to section 6.1 for the full development. Hence, the worst-case gain function for the Proportional Nucleolus can be assessed by the following program:

$$g^{p^*}(\boldsymbol{x}) = \min_{\boldsymbol{c},Q} \frac{v(\mathbb{1})(\boldsymbol{c}^T \cdot \boldsymbol{x}) - v(\boldsymbol{c})}{v(\boldsymbol{c})} \tag{6.35}$$

subject to:

$$0 \leq Q \leq \sum_{i \in N} FEC_i c_i \tag{6.36}$$

$$1 \leq \mathbb{1}^T \cdot \boldsymbol{c} \leq n - 1 \tag{6.37}$$

$$c_i \in \{0,1\}, \qquad \forall i \in I. \tag{6.38}$$

Accordingly, the problem that determines the upper bound $UB_{(k)}$ for the Proportional Nucleolus at a given iteration k of the Benders decomposition algorithm is stated as:

$$UB_{(k)} = \max_{\delta,\boldsymbol{x}} \delta \tag{6.39}$$

subject to:

$$\delta \leq g^{p^*}(\boldsymbol{x}_{(j)}) + \partial g^{p^*}(\boldsymbol{x}_{(j)})^T (\boldsymbol{x} - \boldsymbol{x}_{(j)}), \ \ \forall j = 1, \dots, k-1 \tag{6.40}$$

$$\mathbb{1}^T \cdot \boldsymbol{x} = 1 \tag{6.41}$$

$$x_i \geq 0, \qquad \forall i \in I. \tag{6.42}$$

Moreover, the set of constraints (6.40) represent the Benders cuts for the Proportional Nucleolus, in which $\partial g^{p^*}(\boldsymbol{x}_{(j)})^T$ is the subgradient of g^{p^*} when $\boldsymbol{x} = \boldsymbol{x}_{(j)}$. And according to (5.21):

$$\partial g^{p^*}(\boldsymbol{x}_{(j)})^T = \frac{v(\mathbb{1})}{v(\boldsymbol{c})} \boldsymbol{c}_{(k)}^T. \tag{6.43}$$

Under this structure, since the vector of coalitions $\boldsymbol{c}$ is a decision variable and due to the characteristic function $v(\boldsymbol{c})$ placed at the denominator of the objective function expression that involves the proportional gains, problem (6.35)-(6.38) turns to be non-linear. As a consequence, it cannot be solved by MILP solvers. Nevertheless, it is good to highlight that, although the cut constraints (6.40) in the method also involves a proportional gain expression, it does not represent a problem for the method. The reason is that the problem that determines the upper bound does not see this non-linearity, once the decision variable under optimization is only the trial quota allocation vector $\boldsymbol{x}$, with $\boldsymbol{c}$ simply playing the role of a parameter.

The fractional programming technique is then applied to solve the non-linearity of (6.35)-(6.38). The starting point is the Proportional Nucleolus worst-case gain function:

$$g^{p^*}(\boldsymbol{x}) = \min_{\boldsymbol{c}\in\mathcal{C}} \left\{ \frac{v(\mathbb{1})(\boldsymbol{x}^T \cdot \boldsymbol{c})}{v(\boldsymbol{c})} \right\}, \tag{6.44}$$

which is a non-linear minimization problem on the binary variable $\boldsymbol{c}$. Replacing $v(\boldsymbol{c})$ in (6.44) by the expression of the characteristic function, the following equivalent form of the worst-case gain function is:

$$g^{p^*}(\boldsymbol{x}) = \min_{\boldsymbol{c}\in\mathcal{C}} \left\{ \frac{v(\mathbb{1})(\boldsymbol{x}^T \cdot \boldsymbol{c})}{\max_{Q} \rho\left(\tilde{R}(Q,\boldsymbol{c})\right)} \right\}, \tag{6.45}$$

which is a different case of the ones studied in [41] and [42] due to the maximization problem in the denominator of the objective function of the outer minimization. Fortunately, since the maximization of the risk measure $\rho\{\tilde{R}(Q,\boldsymbol{c})\}$ in the denominator naturally minimizes $\left\{\frac{v(\mathbb{1})(\boldsymbol{x}^T\cdot\boldsymbol{c})}{max_Q\, \rho(\tilde{R}(Q,\boldsymbol{c}))}\right\}$, the following joint minimization:

$$g^{p^*}(\boldsymbol{x}) = \min_{\substack{\boldsymbol{c}\in\mathcal{C} \\ 0\le Q\le \sum_{i\in I} FEC_i c_i}} \left\{ \frac{v(\mathbb{1})(\boldsymbol{x}^T \cdot \boldsymbol{c})}{\rho\left(\tilde{R}(Q,\boldsymbol{c})\right)} \right\}, \tag{6.46}$$

is valid. Recall that the denominator should be strictly positive and this is the case of the present problem. This will be demonstrated through the analysis of the behavior of the characteristic function of the game where $\tilde{R}_{t,s}(Q,\boldsymbol{c})$ was expanded via (3.2):

$$v(\boldsymbol{c}) = \max_{Q} \left\{ \rho_{\alpha,\lambda} \left(\sum_{t\in T} \frac{(P - \tilde{\pi}_t) h_t Q + \sum_{i\in I} \tilde{G}_{i,t} \left(\tilde{\pi}_t - C^U_{i,t}\right) c_i}{(1+J)^t} \right) \right\} \tag{6.47}$$

subject to:

$$0 \le Q \le \sum_{i\in I} FEC_i \cdot c_i. \tag{6.48}$$

In (6.47)-(6.48), note that: (i) the second part of the revenue expression is always positive, since $\tilde{G}_{i,t} \ge 0 \; \forall i \in I, \forall t \in T$, $\tilde{\pi}_t \ge 0 \; \forall t \in T$ and the unit is only dispatched if its generation cost $C_{i,t}^{U}$ is lower than the spot price $\tilde{\pi}_t$ (system's marginal cost); (ii) since the objective function maximizes the measure $\rho_{\alpha,\lambda}$, whenever the first part of the (measure over the) revenue expression attempt to be negative, the $max_Q\{\cdot\}$ operator would annul Q, preventing a negative revenue. Thus, $v(\boldsymbol{c})$ is strictly positive.

The equivalent deterministic formulation for (6.46) is given by:

$$g^{p^*}(\boldsymbol{x}) = \min_{\boldsymbol{c},\Delta_s,z,Q} \frac{v(\mathbb{1})(\boldsymbol{x}^T \cdot \boldsymbol{c})}{\lambda\left(z - \sum_{s\in S} \frac{p_s}{(1-\alpha)} \Delta_s\right) + (1-\lambda) \sum_{s\in S} p_s \left(\sum_{t\in T} \frac{R_{t,s}(Q,\boldsymbol{c})}{(1+J)^t}\right)} \quad (6.49)$$

subject to:

$$0 \le \Delta_s \ge z - \sum_{t\in T} \frac{R_{t,s}(Q,\boldsymbol{c})}{(1+J)^t}, \qquad \forall\, s \in S \quad (6.50)$$

$$0 \le Q \le \sum_{i\in I} FEC_i \cdot c_i \quad (6.51)$$

$$1 \le \mathbb{1}^T \cdot \boldsymbol{c} \le n-1 \quad (6.52)$$

$$c_i \in \{0,1\}, \qquad \forall i \in I. \quad (6.53)$$

It is worth mentioning that such problem has a very specific construction and could fortunately be put in the form of (6.21)-(6.23) and so be solved through the solution presented in [42]. Finally, the particular function of the present work is, thus, now suitable for fractional programming.

Hence, the following variables are introduced:

$$u = \frac{1}{v(\boldsymbol{c})} = \frac{1}{\lambda\left(z - \sum_{s\in S} \frac{p_s}{(1-\alpha)} \Delta_s\right) + (1-\lambda) \sum_{s\in S} p_s \left(\sum_{t\in T} \frac{R_{t,s}(Q,\boldsymbol{c})}{(1+J)^t}\right)}, \quad (6.54)$$

$$z^u = \frac{z}{\lambda\left(z - \sum_{s\in S} \frac{p_s}{(1-\alpha)} \Delta_s\right) + (1-\lambda) \sum_{s\in S} p_s \left(\sum_{t\in T} \frac{R_{t,s}(Q,\boldsymbol{c})}{(1+J)^t}\right)}, \quad (6.55)$$

$$\Delta_s^u = \frac{\Delta_s}{\lambda\left(z - \sum_{s\in S} \frac{p_s}{(1-\alpha)} \Delta_s\right) + (1-\lambda) \sum_{s\in S} p_s \left(\sum_{t\in T} \frac{R_{t,s}(Q,\boldsymbol{c})}{(1+J)^t}\right)}, \quad \forall s \in S \quad (6.56)$$

$$Q^u = \frac{Q}{\lambda\left(z - \sum_{s\in S} \frac{p_s}{(1-\alpha)} \Delta_s\right) + (1-\lambda) \sum_{s\in S} p_s \left(\sum_{t\in T} \frac{R_{t,s}(Q,\boldsymbol{c})}{(1+J)^t}\right)}. \quad (6.57)$$

The next steps are then: (i) replace the denominator of the objective function (6.49) by $\frac{1}{u}$ and; (ii) multiply both sides of constraints (6.50)-(6.52) by u. These two steps are valid in this case, since u is strictly positive, so the formulation remains unsullied. This leads to the following formulation where, again, $R_{t,s}(Q,\boldsymbol{c})$ was expanded via (3.2):

$$g^{p^*}(\boldsymbol{x}) = \min_{\boldsymbol{c},\Delta_s,z,Q,u} v(\mathbb{1}) \sum_{i \in I} x_i\,(c_i\,u) \tag{6.58}$$

subject to:

$$\Delta_s\,u \geq z\,u - \sum_{t \in T} \frac{(P - \pi_{t,s}) h_t Q\,u + \sum_{i \in I} G_{i,t,s}(\pi_{t,s} - C_{i,t}^U) c_i\,u}{(1+J)^t}, \forall\, s \in S \tag{6.59}$$

$$Q\,u \leq \sum_{i \in I} FEC_i\,c_i\,u \tag{6.60}$$

$$1\,u \leq c_i\,u \leq (n-1)\,u, \forall i \in I \tag{6.61}$$

$$\lambda\left(z\,u - \sum_{s \in S} \frac{p_s}{(1-\alpha)} \Delta_s\,u\right) + (1-\lambda) \sum_{s \in S} p_s \left(\sum_{t \in T} \frac{(P - \pi_{t,s}) h_t Q\,u + \sum_{i \in I} G_{i,t,s}(\pi_{t,s} - C_{i,t}^U) c_i\,u}{(1+J)^t}\right) = 1 \tag{6.62}$$

$$u \geq 0 \tag{6.63}$$

$$c_i \in \{0,1\}, \qquad \forall i \in I \tag{6.64}$$

$$\Delta_s \geq 0, \qquad \forall s \in S, \tag{6.65}$$

where the new expression (6.62) imposes the definition of u through the relation (6.54). Furthermore, the resulting formulation is still non-linear, since the objective function had the division of decision variables replaced by a product of decision variables. However, this new formulation can be linearized, through the transformation of the product $(c_i\,u)$ into the new variable w_i. Hence, replacing $(c_i\,u)$ by w_i in the formulation (6.58)-(6.62), the following problem is obtained.

$$g^{p^*}(\boldsymbol{x}) = \min_{\boldsymbol{c},\boldsymbol{w},\Delta_s^u,z^u,Q^u,u} v(\mathbb{1}) \sum_{i \in I} x_i\,w_i \tag{6.66}$$

subject to:

$$\Delta_s^u \geq z^u - \sum_{t \in T} \frac{(P - \pi_{t,s}) h_t Q^u + \sum_{i \in I} G_{i,t,s}(\pi_{t,s} - C_{i,t}^U) w_i}{(1+J)^t}, \quad \forall\, s \in S \tag{6.67}$$

$$Q\,u \leq \sum_{i \in I} FEC_i\,w_i \tag{6.68}$$

$$u \leq w_i \leq (n-1) \cdot u, \forall i \in I \tag{6.69}$$

$$\lambda\left(z^u - \sum_{s \in S} \frac{p_s}{(1-\alpha)} \Delta_s^u\right) + (1-\lambda) \cdot \sum_{s \in S} p_s \left(\sum_{t \in T} \frac{(P - \pi_{t,s}) h_t Q^u + \sum_{i \in I} G_{i,t,s}(\pi_{t,s} - C_{i,t}^U) w_i}{(1+J)^t}\right) = 1 \tag{6.70}$$

$$w_i \leq u \qquad \forall i \in I \tag{6.71}$$

$$w_i \leq M\,c_i \qquad \forall i \in I \tag{6.72}$$

$$w_i \leq u - M(1 - c_i) \quad \forall i \in I \tag{6.73}$$

$$Q \geq 0, u \geq 0 \tag{6.74}$$

$$w_i \geq 0, c_i \in \{0,1\}, \qquad \forall i \in I \tag{6.75}$$

$$\Delta_s \geq 0, \qquad \forall s \in S. \tag{6.76}$$

Under this framework, the Proportional Nucleolus method is now suitable for the Benders decomposition approach presented in section 6.2. The bellow-disposed algorithm summarizes the Benders Decomposition approach to solve problem:

$$\delta^* = \max_{\delta, x} g^{p*}(\boldsymbol{x}) \tag{6.77}$$

subject to:

$$\mathbb{1}^T \cdot \boldsymbol{x} = 1 \tag{6.78}$$

$$x_i \geq 0, \qquad \forall i \in I, \tag{6.79}$$

where, $g^{p*}(\boldsymbol{x}) = min_{\boldsymbol{c} \in \mathcal{C}}\, g^p(\boldsymbol{c}, \boldsymbol{x})$ represents the optimal value of δ for each feasible vector $\boldsymbol{x}$ in (5.22)-(5.24) which is the worst-case gain, among all coalitions for a given allocation vector $\boldsymbol{x}$.

Benders decomposition algorithm

1: **Initialization:**
2: $k \leftarrow 1$ and $\boldsymbol{x}_{(k)} = \mathbb{1}n^{-1}$;
3: $UB_{(k)} \leftarrow +\infty$ and $LB_{(k)} \leftarrow g^{p*}(\boldsymbol{x}_{(k)})$.
4: **While** $UB_{(k)} - LB_{(k)} > \varepsilon$ **do:**
5: $k \leftarrow k + 1$;
6: Solve the master problem (6.39)-(6.41) and store $UB_{(k)}$ and $\boldsymbol{x}_{(k)}$;
7: Solve the MILP (6.66)-(6.73) for $\boldsymbol{x}_{(k)}$ and store $g^{p*}(\boldsymbol{x}_{(k)})$ and $\partial g^{p*}(\boldsymbol{x}_{(k)})^T$;
8: $LB_{(k)} \leftarrow g^{p*}(\boldsymbol{x}_{(k)})$.
9: **Repeat do**

Because g^{p*} is concave, the algorithm finitely converges to an ε-near optimal (global) solution $\boldsymbol{x}_{(k^*)}$, i.e., $\left|g^{p*}(\boldsymbol{x}_{(k^*)}) - \delta^*\right| \leq \varepsilon$. In the case study section, the performance of the Benders algorithm is compared with the classical full coalition-dependent formulation (5.22)-(5.24).

Finally, it is important to note that, for the present problem, the choice of a proper value for M is crucial for the well-functioning of the algorithm. For instance, the value of M should be as tight as possible, i.e., the upper bound of variable u. More precisely, it can be noted from (6.54) that M should assume the lowest value of $v(\boldsymbol{c})$ for all $\boldsymbol{c} \in \mathcal{C}$ which, in a first moment, does not seams an easy task. However, since the measure $v(\boldsymbol{c})$ counts with the superadditivity property, its lowest value comes from one of the individual coalitions (the lowest one, for instance). Fortunately, the characteristic function $v(\boldsymbol{c})$ is computed *a priori* of the resolution of the present method (this will be clarified in the beginning of section 7.4). Hence, the determination of M is done with no further problems.

6.4
The post-optimization algorithm

During the developments of the present work it was noted that, for certain instances of the pool, the Nucleolus methods produce solutions where the gain of a given coalition exceeds by far the gains of the others. This did not happen for all cases, but still represents a weak point of the methods. With this in mind, a post-optimization algorithm was devised on an attempt to minimize such drawback. This happens because the approach for the Nucleolus methods at the present work has the characteristic of producing the so-called degenerated solutions. This kind of solution is common in Linear Programming problems, where optimality can be reached with not only a unique solution, but within a set of possible realizations of the decision variables. For instance, in the case of the Nucleolus methods, there might be a set of optimal allocation vectors that, besides maximizing the worst-case gain, it distributes the pool's total excess among players in different ways. This is due to their single target of maximization of the worst-case gain among all possible coalitions. Thus, these methods are blind when concerning the gains of non-binding players, i.e., players that are not within the group of worst-case gains coalitions.

In this context, after a Nucleolus (or Proportional Nucleolus) solution is found, one could try to equalize the gains among individual players respecting, of course, the just obtained worst-case gain. This should be done through a round of $n-1$ optimizations, where n is the number of players. The round starts with the attempt of optimizing the worst-case gain within the n individual players. Once this goal is reached, the gains of such player is ensured by an added new constraint and the process is repeated for the remaining $n-1$ players, ending-up with the determination of the maximum gains of the last two players, that are always obtained at once. These rounds are said to be lexicographical optimization rounds, since the coalitions are satisfied from the ones with the lower gains to the ones with higher gains, mandatorily in this order. This process is also expected in the classic-fashioned Nucleolus methods, and leads to a more fair (and stable) solution.

Prior to the presentation of the devised algorithm, for the sake of exemplification and to motivate the reader, three examples were produced in which a RES pool is again composed by three players: a small hydro (SH); a biomass plant (Bio) and a wind power plant (WP). Moreover, in such examples, two of those three players have a FEC of 50 avg-MW and the other one has a FEC of 1 avg-MW. The results are presented in Table 6.1, where the first column shows the FEC of each player, which are themselves disposed in the second column and their respective characteristic function values denoted in the third column. Additionally, the next two pairs of columns present, for the Nucleolus and Proportional Nucleolus methods, respectively, the results obtained with the original solutions followed by the solutions obtained after the post-optimization algorithm was employed, accordingly.

Table 6.1 – Comparison of individual gains before and after the post-optimization algorithm for a pool with 3 players in cases where players have different FEC.

FEC (avg-MW)	$\boldsymbol{c}$	$v(\boldsymbol{c})$ ($\$10^3$)	Abs. Nuc. gains		Prop. Nuc. gains	
			Original	Post	Original	Post
50	$\boldsymbol{c}_{SH}$	24,470.03	11,606.72	5,915.51	2.17%	22.10%
50	$\boldsymbol{c}_{Bio}$	29,749.39	224.31	5,915.51	38.50%	22.10%
1	$\boldsymbol{c}_{WP}$	683.00	155.86	155.86	0.47%	0.47%
50	$\boldsymbol{c}_{SH}$	24,470.03	9,663.58	4,940.23	2.09%	17.11%
1	$\boldsymbol{c}_{Bio}$	594.99	150.12	150.12	0.44%	0.44%
50	$\boldsymbol{c}_{WP}$	34,150.14	216.88	4,940.23	27.87%	17.11%
1	$\boldsymbol{c}_{SH}$	489.40	225.94	225.94	0.65%	0.65%
50	$\boldsymbol{c}_{Bio}$	29,749.39	4,906.50	2,636.39	2.21%	8.60%
50	$\boldsymbol{c}_{WP}$	34,150.14	366.28	2,636.39	14.17%	8.60%

Note that both the absolute and proportional gains of the big players (FEC = 50 avg-MW) were equalized in all cases, after the post-optimization algorithm was employed. It is also worth mentioning that the gains of the small player (FEC = 1 avg-MW) could not be improved, since that, for these examples, they constitute the binding coalition in terms of worst-case gains. Next, the proceeding of the post-optimization algorithm for the equalization of individual gains is presented. Additionally, for the sake of brevity, the algorithm will be presented using the Nucleolus method as reference. It is worth to highlight the particular case of the algorithm for the Proportional Nucleolus method is very similar to this one, with no further complications.

The algorithm is initialized with the computation of the k worst-case gain coalitions from the solution of the Nucleolus (or Proportional Nucleolus) method as well as the optimal solution value δ^*. Next, the following linear programing problem is solved.

$$f = \max_{x,\delta^{post}} \delta^{post} \tag{6.80}$$

subject to:

$$\delta^{post} \leq v(\mathbb{1})x_i - v\left(\boldsymbol{c}^{\{i\}}\right), \qquad \forall i \in I \tag{6.81}$$

$$\delta^* \leq v(\mathbb{1})\left(\boldsymbol{c}_{(j)}^T \boldsymbol{x}\right) - v\left(\boldsymbol{c}_{(j)}\right), \qquad \forall j = 1, \dots, k \tag{6.82}$$

$$\mathbb{1}^T \cdot \boldsymbol{x} = 1 \tag{6.83}$$

$$x_i \geq 0, \qquad \forall i \in I. \tag{6.84}$$

In (6.80) an auxiliary variable δ^{post} maximizes the players' individual gains, supported by the set of constraints (6.81). Furthermore, the set of constraints (6.82) accounts for the computed worst-case gain coalitions of the original solution of the Nucleolus that composes the well-known set of umbrella constraints. Finally, constraint (6.83) ensures the complete share of the excess of the pool. On a given iteration k^{post} of the algorithm, a trial solution $\boldsymbol{x}_{(k^{post})}$ is obtained and the useful "oracle" problem (6.9)-(6.12), i.e., the secondary problem of the prior-proposed Benders decomposition algorithm, is used to determine: (i) whether the value of the worst-case gain coalition $g^*\left(\boldsymbol{x}_{(k^{post})}\right)$ for the given trial solution equals δ^* and; (ii) the worst-case gain coalition $\boldsymbol{c}_{(k^{post})}$ itself.

If the new worst-case gain value $g^*\left(\boldsymbol{x}_{(k^{post})}\right)$ equals δ^*, the satisfied individual constraints counter k^{ind} is incremented, the current tighter individual player i^{ind} gain was maximized to the respective value of δ^{post} and its associated constraint in the set (6.81) must be substituted by:

$$\delta_{(k^{post})}^{post} \leq v(\mathbb{1})x_{i^{ind}} - v(\boldsymbol{c}^{\{i^{ind}\}}). \tag{6.85}$$

Otherwise, if $g^*\left(\boldsymbol{x}_{(k^{post})}\right)$ violates (is greater than) δ^*, a new constraint has to be added in the original set of constraints (6.82), as follows:

$$\delta^* \leq v(\mathbb{1})\left(\boldsymbol{c}_{(k^{post})}^T \boldsymbol{x}\right) - v(\boldsymbol{c}_{(k^{post})}). \tag{6.86}$$

After this verification, the iteration counter is increased and the process loops until all individual gains are maximized. It is also important to note that the individual player i^{ind} satisfied in a given iteration of the algorithm is the one that has the higher value of the dual variables associated with the set of constraints (6.81).

The following algorithm summarizes the approach for the individual gains equalization.

Post-optimization algorithm for individual gains equalization

Initialization:
  Compute the set of umbrella constraints and δ^* for the Nucleolus method;
  $k^{post} \leftarrow 1$;
  $k^{ind} \leftarrow 0$.
While $k^{ind} < n$ **do:**
  Solve problem (6.80)- (6.83) and store $\boldsymbol{x}_{(k^{post})}$;
  Solve the MILP (6.9)-(6.12) for $\boldsymbol{x}_{(k^{post})}$ and store $g^*(\boldsymbol{x}_{(k^{post})})$;
  If $g^*(\boldsymbol{x}_{(k^{post})}) = \delta^*$ **then**
    $k^{ind} \leftarrow k^{ind} + 1$;
    Determine the individual player i^{ind} by the higher dual variable of (6.81);
    Replace $\delta^{post} \leq v(\mathbb{1})x_{i^{ind}} - v(\boldsymbol{c}^{\{i^{ind}\}})$ in (6.81) by $\delta^{post}_{(k^{post})} \leq v(\mathbb{1})x_{i^{ind}} - v(\boldsymbol{c}^{\{i^{ind}\}})$;
  Else
    Add $\delta^* \leq v(\mathbb{1})(\boldsymbol{c}_{(k^{post})}{}^T\boldsymbol{x}) - v(\boldsymbol{c}_{(k^{post})})$ to (6.82);
  End-if
  $k^{post} \leftarrow k^{post} + 1$.
Repeat do

For the sake of clarification, a case study with 3 players of size equal to 1 avg-MW is presented to show the path the algorithm takes during execution. Firstly, the original and post-optimization results for the small pool are presented in Table 6.2. Next, the iterations of the algorithm are disposed in Table 6.3.

Table 6.2 – Comparison of coalition gains before and after the post-optimization algorithm for a pool with 3 players in a case where players have the same FEC of 1 avg-MW.

FEC (avg-MW)	$\boldsymbol{c}$	$v(\boldsymbol{c})$ (\$10³)	Abs. Nuc. gains Original	Abs. Nuc. gains Post	Prop. Nuc. gains Original	Prop. Nuc. gains Post
1	$\boldsymbol{c}_{SH}$	489.40	218.53	194.61	42.68%	42.68%
1	$\boldsymbol{c}_{Bio}$	594.99	100.94	124.86	19.50%	19.50%
1	$\boldsymbol{c}_{WP}$	683.00	85.96	85.96	11.79%	11.79%
2	$\boldsymbol{c}_{SH,Bio}$	1,317.89	85.96	85.96	6.93%	6.93%
2	$\boldsymbol{c}_{SH,WP}$	1,367.01	109.88	85.96	6.93%	6.93%
2	$\boldsymbol{c}_{Bio,WP}$	1,378.93	85.96	109.88	6.93%	6.93%
3	$\boldsymbol{c}_{SH,Bio,WP}$	2,172.81	-	-	-	-
Worst case gain (\$10³ | %):			85.96	85.96	6.93%	6.93%

Table 6.3 – Case study example for the post-optimization algorithm results for the Nucleolus method.

Post-optimization algorithm iterations for the Nucleolus (k^{post})				
$\boldsymbol{x}_{(k^{post})}$ | $\boldsymbol{c}_{(k^{post})}$	Original	Step 1	Step 2	Step 3
$\boldsymbol{x}_{SH}$ | $\boldsymbol{c}_{SH}^{wc}$	32.58% | 1	32.58% | 1	29,88% | 1	31.48% | 1
$\boldsymbol{x}_{BIO}$ | $\boldsymbol{c}_{Bio}^{wc}$	32.03% | 1	32.03% | 1	34,73% | 0	33.13% | 0
$\boldsymbol{x}_{WP}$ | $\boldsymbol{c}_{WP}^{wc}$	35.39% | 0	35.39% | 0	35,39% | 1	35.39% | 1
$LB_{(k^{post})}$ (\$ 10^3):	85.96	85.96	51.08	85.96
Found $\boldsymbol{c}^{\{i^{ind}\}}$:	-	$\boldsymbol{c}_{WP}$	$\boldsymbol{c}_{SH}, \boldsymbol{c}_{Bio}, \boldsymbol{c}_{WP}$	$\boldsymbol{c}_{SH}$ | $\boldsymbol{c}_{Bio}$
$\boldsymbol{c}^{\{i^{ind}\}}$ gain (\$ 10^3):	-	85.96	159.73	194.61 | 124.86

In the original solution, the worst-case gain of $\$85.96$ thousand for the coalition $c_0 = [1\ 1\ 0]$ was found. Then, in the first iteration of the post-optimization algorithm, the same gain of $\$85.96$ thousand was assigned to the wind power plant and the same worst-case gain coalition was obtained with this solution. Step 2 then attempted to equalize the gain of both small hydro and biomass to a value of $\$159.73$ thousand. Note that 159.73 x 2 (SH and Bio gains) + 85.96 (WP gains) equals the total excess of $\$405.42$ thousand for the pool. However, the solution would promote a worst-case gain of $\$51.08$ thousand for coalition $c_2 = [1\ 0\ 1]$, which did not belonged to the original set of umbrella constraints yet. Thus, since the given solution violates the original worst-case gain of $\$85.96$ thousand the constraint for the gains of coalition c_2 is added to the problem. The next step takes charge of satisfying such gains of coalition c_2 and produces a solution that assigns gains of $\$194.61$ thousand for the small hydro and $\$124.86$ thousand for the biomass plant.

Finally, it can be noted that the algorithm had no effect over the Proportional Nucleolus solution, which indicates that such outcome was already equalized since the original solution.

7
Case studies

In order to demonstrate the benefits of the proposed model three computational case studies are presented. The first experiment is of qualitative manner, where a small pool composed by three generators, being one small hydro, one biomass plant and one wind power plant is explored to show the differences among the solutions provided by the different proposed allocation methods. The following case study is responsible for depicting the Benders algorithm behavior for the Nucleolus methods for the same 3-players pool with the objective of giving primary indications of the benefits the proposed method supply. The final experiment runs the allocation methods for different instances of the pool, gradually increasing the number n of players and, consequently, its computational complexity, in order to stress the methods. Notwithstanding, case studies were executed in a computer with Intel® Core™ i7-3960X CPU @3.30GHz and 64Gb of RAM memory. The linear programming problems (LPs) and mixed-integer linear programming problems (MILPs) were implemented in Mosel language and made use of the FICO™ Xpress solver [72].

Furthermore, the experiments were made with realistic data from the Brazilian electrical system. The energy generation scenarios of 23 small hydros and 17 wind power units were produced simultaneously via the VARx statistical model introduced in Section 3.1, while 10 biomass units were considered as deterministic. Furthermore, the biomass generation scenarios contain small disturbances between the units without compromising, however, the typical pattern of this kind of source, which was previously depicted in Figure 2.4 of section 2.2.

It is also important to state the following data set is common to all experiments:

i. the time horizon is 12 months, from Jan/2020-Dec/2020;
ii. the energy selling is always done through a flat quantity contract at fixed price $P = 100$ \$/MWh;
iii. the simulated data of energy generation and spot prices contains 200 scenarios within the time horizon;
iv. it is considered the certainty equivalent disposed in model (4.3)-(4.5) with risk aversion parameters $\lambda = 90\%$ and $\alpha = 95\%$;
v. the discount rate considered is $J = 0\%$ per month;
vi. energy losses are considered as 3%;
vii. the FEC of each unit is always 1 avg-MW and, consequently, in each instance of n players of the case studies presented in this work, the pool has a total FEC of n avg-MW;
viii. the convergence gap level ε is considered equal to 10 \$/MWh for the objective functions of the allocation methods via Benders algorithm.

7.1 Isonomic 3-players RES pool case study – comparison of allocation methods

In this section it is presented a case study with real data for the proposed RES pool composed by a small hydro (SH), a biomass (Bio) plant and a wind power plant (WP). The 5 methods presented in Chapter 5 were used to share the quotas and the respective results are disposed in Table 7.1. In order to promote a qualitative comparison of the behavior of each one of the methods on an isonomic pool, in this case study the players have the same size, equal to 1 avg-MW.

Table 7.1 – Quota allocation results for each of the methods presented for a RES pool formed by 3 players.

$\boldsymbol{x}$	FEC proportional	Shapley value	Marginal Benefits	Nucleolus	Proportional Nucleolus
$\boldsymbol{x}_{SH}$	33.33%	30.48%	30.95%	31.48%	32.14%
$\boldsymbol{x}_{Bio}$	33.33%	33.18%	31.80%	33.13%	32.72%
$\boldsymbol{x}_{WP}$	33.33%	36.34%	37.25%	35.39%	35.14%

Firstly, it can be noted that the allocation results from the different methods do not differ too much from the ones presented by the FEC proportional method, which is the one that distribute the benefits equally among players, since they have the same size (FEC). However, on the qualitative side, such a small variation produces crucial differences in the results. This will be further clarified in the sequel. The first column in Table 7.2 and Table 7.3 dispose each of the possible 7 coalitions formed by a combination of the three players. The optimal trading quantities Q^* (as a percentage of the total FEC in a coalition) are disposed in the second column with the respective average of its individual players' contracted amount denoted in column 3. Next, the certainty equivalents $v(\boldsymbol{c})$ of each coalition are presented. Moreover, Table 7.2 disposes the coalition's correspondent absolute gains (in \$), with the proportional gains (%) being disposed in Table 7.3. Finally, both kinds of gains were obtained via the solution of each one of the studied methods.

Table 7.2 – Absolute gains for a RES pool formed by 3 players of equal FEC (1 avg-MW).

$\boldsymbol{c}$	Q^* (%FEC)	$\frac{\sum_{i\in I} c_i Q_i^*}{\sum_{i\in I} c_i}$	$v(\boldsymbol{c})$ (10^3\$)	Absolute gain of coalition $\boldsymbol{c}$ (10^3\$): $v(\mathbb{1})(\boldsymbol{x}^T\boldsymbol{c}) - v(\boldsymbol{c})$				
				FEC prop	SV	MB	Nuc.	Prop. Nuc.
$\boldsymbol{c}_{SH}$	58.55	58.55	489.40	234.87	172.85	183.17	194.61	208.87
$\boldsymbol{c}_{Bio}$	79.23	79.23	594.99	129.28	126.01	95.87	124.86	116.02
$\boldsymbol{c}_{WP}$	83.59	83.59	683.00	41.27	106.56	126.38	85.96	80.53
$\boldsymbol{c}_{SH,Bio}$	74.65	68.99	1,317.89	130.65	65.36	45.54	85.96	91.39
$\boldsymbol{c}_{SH,WP}$	76.12	71.07	1,367.01	81.53	84.80	114.94	85.96	94.79
$\boldsymbol{c}_{Bio,WP}$	84.21	81.41	1,378.93	69.62	131.64	121.32	109.88	95.62
$\boldsymbol{c}_{SH,Bio,WP}$	84.11	73.79	2,172.81	-	-	-	-	-
Absolute worst-gain (\$):				41.27	65.36	45.54	85.96	80.53

Table 7.3 – Proportional gains for a RES pool formed by 3 players of equal FEC (1 avg-MW).

$\boldsymbol{c}$	Q^* (%FEC)	$\frac{\sum_{i\in I} c_i Q_i^*}{\sum_{i\in I} c_i}$	$v(\boldsymbol{c})$ (10^3\$)	Proportional gain of coalition $\boldsymbol{c}$ (%): $\frac{v(\mathbb{1})(\boldsymbol{x}^T\boldsymbol{c}) - v(\boldsymbol{c})}{v(\boldsymbol{c})}$				
				FEC prop	SV	MB	Nuc.	Prop. Nuc.
$\boldsymbol{c}_{SH}$	58.55	58.55	489.40	47.99%	35.32%	37.43%	39.76%	42.68%
$\boldsymbol{c}_{Bio}$	79.23	79.23	594.99	21.73%	21.18%	16.11%	20.98%	19.50%
$\boldsymbol{c}_{WP}$	83.59	83.59	683.00	6.04%	15.60%	18.50%	12.59%	11.79%
$\boldsymbol{c}_{SH,Bio}$	74.65	68.99	1,317.89	9.91%	4.96%	3.46%	6.52%	6.93%
$\boldsymbol{c}_{SH,WP}$	76.12	71.07	1,367.01	5.96%	6.20%	8.41%	6.29%	6.93%
$\boldsymbol{c}_{Bio,WP}$	84.21	81.41	1,378.93	5.05%	9.55%	8.80%	7.97%	6.93%
$\boldsymbol{c}_{SH,Bio,WP}$	84.11	73.79	2,172.81	-	-	-	-	-
Proportional worst-gain (%):				5.05%	4.96%	3.46%	6.29%	6.93%

The FEC (energy) proportional method provides an intuitive solution for the quota sharing via the rationale “what you give is what you get”. However, in terms of absolute gains, this method provides the least stable solution, since the absolute worst-case gain is the smaller one among the methods. On the proportional side, it goes better than the Shapley Value and Marginal Benefits methods. Furthermore, although the FEC proportional method produced a solution that belongs to the core of the cooperative game, this is not always guaranteed. It can also be noted that, for this case study, both Nucleolus methods produced the best solutions in terms of absolute and proportional gains, which shows a good qualitative behavior for these methods on isonomic pools. Additionally, by analyzing the generation profiles of the different sources in Figure 3.3, it can be noted that the biomass plant has a profile more similar to the wind power plant, and both present good seasonal complementarity with the small hydro plant. Hence, both biomass and wind power plants compete for the perfect matching with the small hydro, which benefits from this complementarity and present the higher values in terms of both absolute and proportional gains for all sharing methodologies. Moreover, it can be seen that, besides the wind power has the highest payoff among the three players (individual coalitions), it receives the lowest quota allocation and gains, when compared with the other two players, mainly because its synergy with the complementary player, small hydro, is lower than the synergy between the biomass and the small hydro plant. Finally, the complementary synergy effect is enforced by the fact that coalitions $\boldsymbol{c}_{SH,Bio}$, $\boldsymbol{c}_{SH,WP}$ and $\boldsymbol{c}_{Bio,WP}$ exhibits more aggressive trading strategies (higher values of Q^* in a percentage of the total FEC of the coalition) when compared to the average of the contracted amount of the individual players that compose such coalitions, disposed in the third column of the tables.

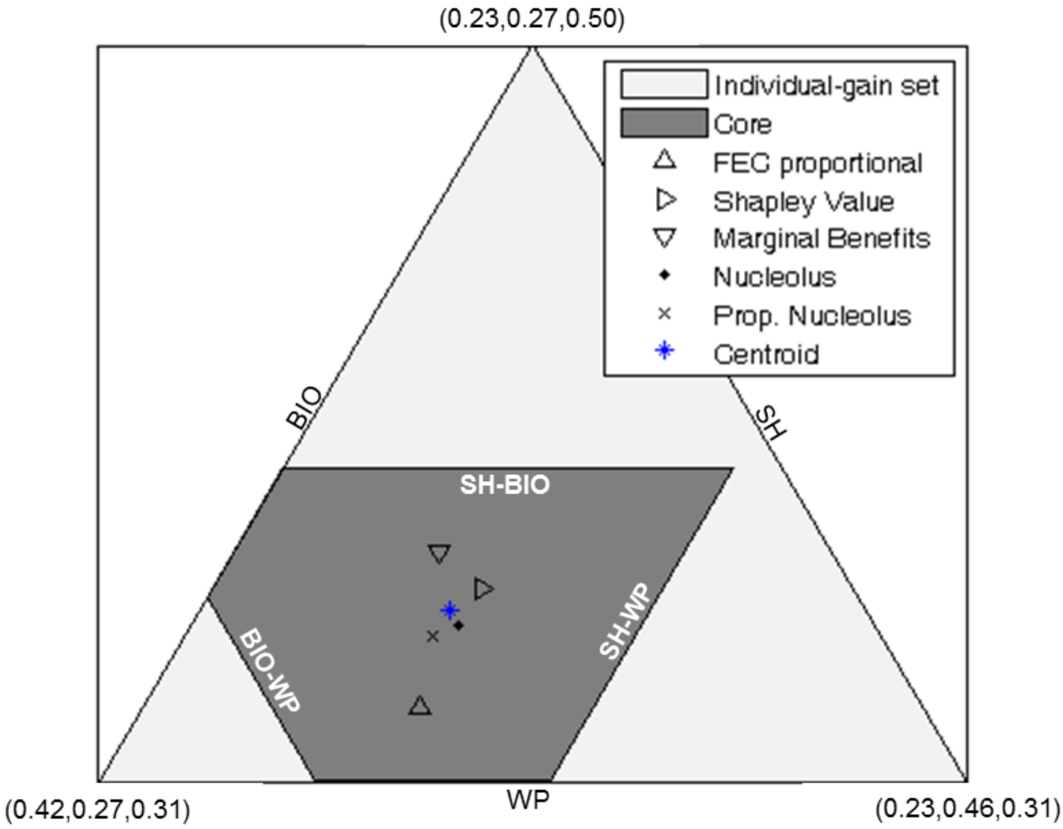

Figure 7.1 – Core of the cooperative game and respective quota allocation by the different presented methods.

In Figure 7.1, the core of the cooperative game and the set of allocations is represented on the sum-one plane projection, in two dimensions, developed by [74]. The outer triangle defines the frontier of the set of quota allocations solutions that ensure the players' individual gains. Still, the core of the cooperative game is defined by the inner dark polygon, where the gains of all possible coalitions are ensured. In this context, the Nucleolus methods pursue points inside the core's polygon, which goes in accordance with its objective. Furthermore, these methods tend to push the solution as far as possible from the coalitions' gain frontiers and that is why their solutions are the closest ones from the polygon's centroid. In addition, it might be highlighted that even though the Shapley Value and Marginal Benefits solutions lay inside the core of the game, it was not proved that this necessarily holds for all cases.

7.2 Non-isonomic 3-players RES pool case study – comparison of allocation methods

In order to promote the investigation of methods' behavior in non-isonomic pools, it was done a parametrization on the units' size for a pool with the same three players of earlier case studies. For this purpose, the size of each player was increased in separate, to produce instances where one player is bigger than the other two (in terms of FEC). As the cases are many, only the results for individual players are disposed. Thus, the absolute and proportional gains for the individual players are disposed in Table 7.1 and Table 7.2, respectively, and in which the first column shows the size of each player in each pool (lines). It is worth mentioning that: (i) except for the FEC proportional solutions with negative gains, all solutions lied in the core of the cooperative game and; (ii) the first line represents the same pool for the previews case study (section 7.1), used as the reference pool where all players have the size of 1avg-MW.

Table 7.4 – Absolute gains ($\$10^3$) for a RES pool formed by 3 players of different FEC.

FEC (avg-MW)			FEC Proportional			Shapley Value			Marginal Benefits			Nucleolus			Proportional Nucleolus		
SH	BIO	WP	SH	BIO	WP	SH	BIO	WP	SH	BIO	WP	SH	BIO	WP	SH	BIO	WP
1	1	1	235	129	41	173	126	107	183	96	126	194	125	86	209	116	81
3	1	1	575	86	-2	301	187	172	173	276	210	357	151	151	442	121	97
10	1	1	907	-15	-103	358	215	216	16	395	379	386	202	202	615	84	91
20	1	1	1002	-55	-144	365	219	219	24	395	385	393	205	205	659	70	74
50	1	1	1084	-84	-172	378	222	228	34	415	380	413	208	208	708	54	67
1	3	1	205	300	12	197	194	126	313	58	147	236	188	94	201	256	61
1	10	1	153	475	-41	218	233	137	329	58	201	263	224	100	195	360	33
1	20	1	134	571	-59	246	261	138	329	116	201	319	224	102	205	430	10
1	50	1	118	641	-75	262	277	145	460	0	224	347	224	112	199	480	5
1	1	3	237	131	129	205	132	159	283	163	52	251	105	141	222	74	200
1	1	10	217	111	229	225	138	194	382	158	16	278	105	173	218	44	295
1	1	20	206	101	257	223	141	201	366	185	13	270	105	189	207	42	316
1	1	50	199	94	290	222	147	214	366	185	31	258	108	217	195	45	343

Table 7.5 – Proportional gains (%) for a RES pool formed by 3 players of different FEC.

FEC (avg-MW)			FEC Proportional			Shapley Value			Marginal Benefits			Nucleolus			Proportional Nucleolus		
SH	BIO	WP	SH	BIO	WP	SH	BIO	WP	SH	BIO	WP	SH	BIO	WP	SH	BIO	WP
1	1	1	48	22	6	35	21	16	37	16	19	40	21	13	43	20	12
3	1	1	39	14	0	20	31	25	12	46	31	24	25	22	30	20	14
10	1	1	19	-2	-15	7	36	32	0	66	55	8	34	30	13	14	13
20	1	1	10	-9	-21	4	37	32	0	66	56	4	34	30	7	12	11
50	1	1	4	-14	-25	2	37	33	0	70	56	2	35	30	3	9	10
1	3	1	42	17	2	40	11	18	64	3	21	48	11	14	41	14	9
1	10	1	31	8	-6	45	4	20	67	1	29	54	4	15	40	6	5
1	20	1	27	5	-9	50	2	20	67	1	29	65	2	15	42	4	2
1	50	1	24	2	-11	54	1	21	94	0	33	71	1	16	41	2	1
1	1	3	48	22	6	42	22	8	58	27	3	51	18	7	45	13	10
1	1	10	44	19	3	46	23	3	78	27	0	57	18	3	45	7	4
1	1	20	42	17	2	46	24	1	75	31	0	55	18	1	42	7	2
1	1	50	41	16	1	45	25	1	75	31	0	53	18	1	40	8	1

Firstly, it is good to note that the FEC proportional method promoted instable solutions for some instances, where the gains became negative for the biomass and wind power units, which compete for the small hydro synergic-complementary effects. Thus, these solutions lie outside the core of the cooperative game. Secondly, it can be noted that the Marginal benefits method produced solutions with gain 0 (zero) for some cases, which represents quite unstable solutions for the pool.

For a more visual inspection, the results are condensed in Figure 7.2, in the next page. The radar graphs are separated by player and display its gains in each instance of the pool, for each one of the studied methods. The absolute gains graphs are disposed in the left side, while the proportional gains ones are disposed in the right side. Please note that the axis legends that represent the gains are placed in vertical just as a reference. Actually the gains increase from the center of the graphs (lower gains) to the border (higher gains), regardless if goes up, down, left or right, for example. Yet, the superior vertical line denotes the reference pool, where all players have size 1 (avg-MW). Furthermore, the cases were placed in the same order of Table 7.1 and Table 7.2, with the objective of condensing the cases were the small hydro increases firstly, followed by the biomass unit and the wind power at last. Finally, the instances are denoted by the radial lines, with the size of each player placed in the order $\{SH, BIO, WP\}$.

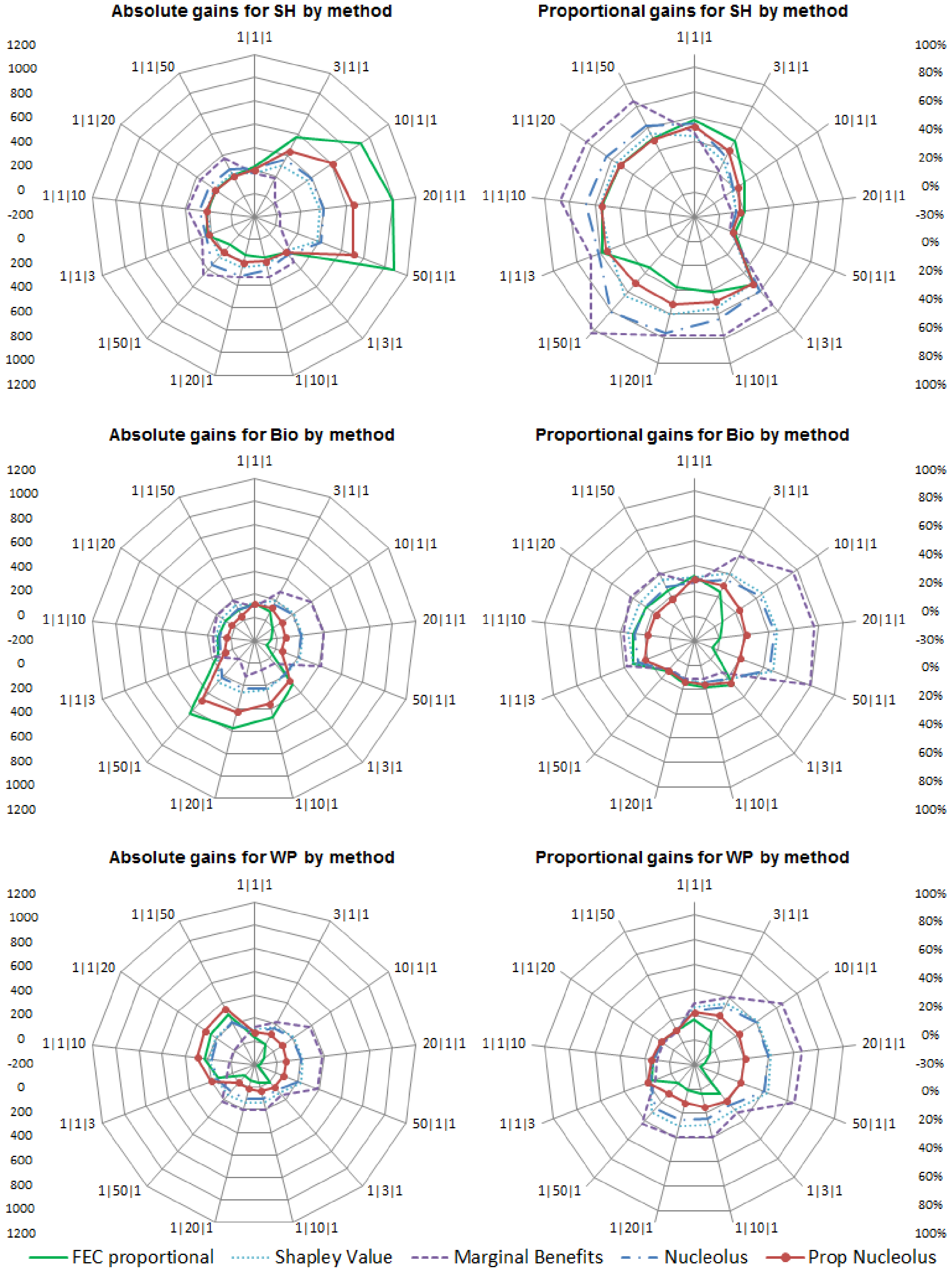

Figure 7.2 – Distribution of absolute and proportional gains of each source, by method, for different case studies, when the units' FEC are modified.

Finally, it is worth mentioning that these graphs highlight the cases where some player receives a higher allocation from a given method than the others, as seen next.

The first conclusion is that the graphs allowed the capture of the curious effect of, while the size of the small hydro gradually increases, so do its absolute gains. However, its proportional gains go on the other way. This shows that the (marginal) increments on its absolute gains do not consort with the (large) increment on its FEC, which makes total sense. The same happen for the other players, but in a smaller scale, since the small hydro is the one that most benefit from the coalitional synergic effects. In this context, the Proportional Nucleolus always tries to equalize the (proportional) gains of the smaller players, presenting a more stable result. Note that its solution has a smaller variance then the other methods, which can be noticed via the lines on the graphs that lie almost in a "constant" level of gains, i.e., the lines present a "almost-constant" radius (for the small players areas). Contrarily, the other methods produce solutions that keep "jumping" from a gain level to another, more frequently, especially the FEC proportional and the Marginal Benefits methods. So, besides the Shapley Value and the absolute Nucleolus solutions presented a behavior that varies between the smooth Proportional Nucleolus and the two volatile ones, they also produced solutions that are similar to each other.

Finally, one more effect can be highlighted: the Proportional Nucleolus produces higher absolute gains than the Absolute Nucleolus and vice-versa. This is caused by the fact that the given methods maximize its own respective rationale for all possible coalitions, which leads to a smaller surplus for individual gains than the other one (please recall that the graphs only display the gains for coalitions of individual players, but respect the gains of all possible coalitions). This justifies the higher proportional gains for the Absolute Nucleolus, Shapley Value and Marginal Benefits methods (on the left-sided graphs).

Next, a final case study is presented, where two of the three players are much bigger in FEC then the other one, also with the purpose of studying the behavior of the proposed methods for the underlying non-isonomic pools.

Table 7.6 – Comparison of individual absolute gains in very non-isonomic pools.

FEC (avg-MW)	c	$v(c)$ (10^3)	Absolute gains (10^3)				
			FEC prop.	Shapley	MB	Nuc.	Prop. Nuc.
50	c_{SH}	24,470.03	8,643.49	5,935.61	7,557.29	5,915.51	5,408.42
50	c_{Bio}	29,749.39	3,364.13	5,841.10	4,117.87	5,915.51	6,575.27
1	c_{WP}	683.00	(20.73)	210.18	311.73	155.86	3.20
50	c_{SH}	24,470.03	9,810.04	4,959.96	6,418.67	4,940.23	4,186.02
1	c_{Bio}	594.99	90.61	203.23	300.23	150.12	2.59
50	c_{WP}	34,150.14	129.93	4,867.39	3,311.68	4,940.23	5,841.97
1	c_{SH}	489.40	202.56	288.27	451.87	225.94	3.18
50	c_{Bio}	29,749.39	4,848.46	2,628.55	3,209.61	2,636.39	2,558.53
50	c_{WP}	34,150.14	447.70	2,581.90	1,837.23	2,636.39	2,937.00

Table 7.7 – Comparison of individual proportional gains in very non-isonomic pools.

FEC (avg-MW)	c	$v(c)$ (10^3)	Proportional gains (%)				
			FEC prop.	Shapley	MB	Nuc.	Prop. Nuc.
50	c_{SH}	24,470.03	35.32%	24.26%	30.88%	24.17%	22.10%
50	c_{Bio}	29,749.39	11.31%	19.63%	13.84%	19.88%	22.10%
1	c_{WP}	683.00	-3.04%	30.77%	45.64%	22.82%	0.47%
50	c_{SH}	24,470.03	40.09%	20.27%	26.23%	20.19%	17.11%
1	c_{Bio}	594.99	15.23%	34.16%	50.46%	25.23%	0.44%
50	c_{WP}	34,150.14	0.38%	14.25%	9.70%	14.47%	17.11%
1	c_{SH}	489.40	41.39%	58.90%	92.33%	46.17%	0.65%
50	c_{Bio}	29,749.39	16.30%	8.84%	10.79%	8.86%	8.60%
50	c_{WP}	34,150.14	1.31%	7.56%	5.38%	7.72%	8.60%

The first relevant aspect of the case study is the big players' gains equalization for the Nucleolus methods, in comparison to solutions produced by the other methods, which is due to the algorithm presented in section 6.4. The second aspect is that specifically, the Proportional Nucleolus solutions allocated really small gains to the small player in all cases, which can be seen as a method's week point. However, this is again due to the fact that, since the method maximizes the worst-case gains among all coalitions, the distribution of the pool's total excess among the individual players gets tied. In this framework, this represents the contradictory balance of the pool's stability, which has the strategy of defending itself from the most threatening coalitions but can also, sometimes, lead to solutions where a given player would have small individual gains, which would also bring instability for the pool. Finally, it can be mentioned that once more the FEC Proportional method produces solutions outside the core of the cooperative game.

7.3 Benders algorithm exemplification for a 3-players RES pool case study

The main goal of this experiment is to enforce the convergence between the Benders algorithms and the Nucleolus methods that enumerates all gain constraints from all coalitions, called here Full Coalition Dependent method (FCD), of models (5.17)-(5.19) and (5.22)-(5.24). Secondly, the experiment presents some primary evidences of the algorithms' efficiency, which will be of great value on the large-scale instances of the next (and last) case study. In this context, the cutting plane algorithms were executed to the same data-set of the isonomic pool composed by the three RES plants of experiment 7.1. The results of the algorithm iterations are presented in Table 7.8 and Table 7.9 for the Nucleolus and Proportional Nucleolus, respectively, until the optimum solution are found. Moreover, such tables dispose the quotas share, the worst-case gain coalition and the upper and lower bound for each iteration. Finally, it is worth mentioning that the solutions disposed bellow constitute the original Nucleolus methods solutions, and do not include the post-optimization algorithm solution. Thus, the final worst-case gain results of Table 7.6 and Table 7.7 should be compared with the original worst-case gain solutions of Table 6.2, of section 6.4.

Table 7.8 – Benders algorithm results for the Nucleolus method.

		Algorithm iterations for Nucleolus (k)		
$\boldsymbol{x}_{(k)}\|\boldsymbol{c}_{(k)}$	FCD	1	2	3
$\boldsymbol{x}_{SH}\|\boldsymbol{c}_{SH}$	32.58%	28.74% | 1	33.27% | 0	32.58% | 1
$\boldsymbol{x}_{BIO}\|\boldsymbol{c}_{BIO}$	32.03%	33.60% | 1	31.34% | 1	32.03% | 1
$\boldsymbol{x}_{WP}\|\boldsymbol{c}_{WP}$	35.39%	37.65% | 0	35.39% | 1	35.39% | 0
$UB_{(k)}$	-	$ 135,140.91	$ 85,959.79	$ 85,959.79
$LB_{(k)}$	-	$ 36,778.67	$ 70,982.75	$ 85,959.79

Table 7.9 – Benders algorithm results for the Proportional Nucleolus method.

		Algorithm iterations for Proportional Nucleolus (k)			
$\boldsymbol{x}_{(k)}\|\boldsymbol{c}_{(k)}$	FCD	1	2	3	4
$\boldsymbol{x}_{SH}\|\boldsymbol{c}_{SH}$	32.14%	27.69% | 1	36.13% | 0	31.08% | 1	32.14% | 1
$\boldsymbol{x}_{Bio}\|\boldsymbol{c}_{Bio}$	32.72%	33.66% | 1	29.74% | 1	34.78% | 0	32.72% | 0
$\boldsymbol{x}_{WP}\|\boldsymbol{c}_{WP}$	35.14%	38.64% | 0	34.13% | 1	34.13% | 1	35.14% | 1
$UB_{(k)}$	-	22.94%	8.59%	8.59%	6.93%
$LB_{(k)}$	-	1.16%	0.64%	3.66%	6.93%

In the each iteration, the master problem provides trial solutions with partial information of the worst-case gain constraints, while the oracle (MILP approach), identify if such solutions lie within the core and feed the master with new information (by means of the Benders cuts) to approximate the FCD problem. Finally, the algorithm converges providing the same solution found by the FCD problem.

At last, it is worth mentioning that only a part of the possible 7 constraints were necessary to allow the BA to find optimal solutions. Although this saving is not representative, which is due to the reduced size of these instances, this is a salient feature of the method and very valuable in large instances. In the next case study, it is shown that the proposed BA considerably outperforms the FCD problem in medium and large-scale instances.

Next, it is presented in Figure 7.3 the path the algorithm takes while pursuing the optimal solution until convergence for the game of sum-one projection plane, for the (Absolute) Nucleolus method.

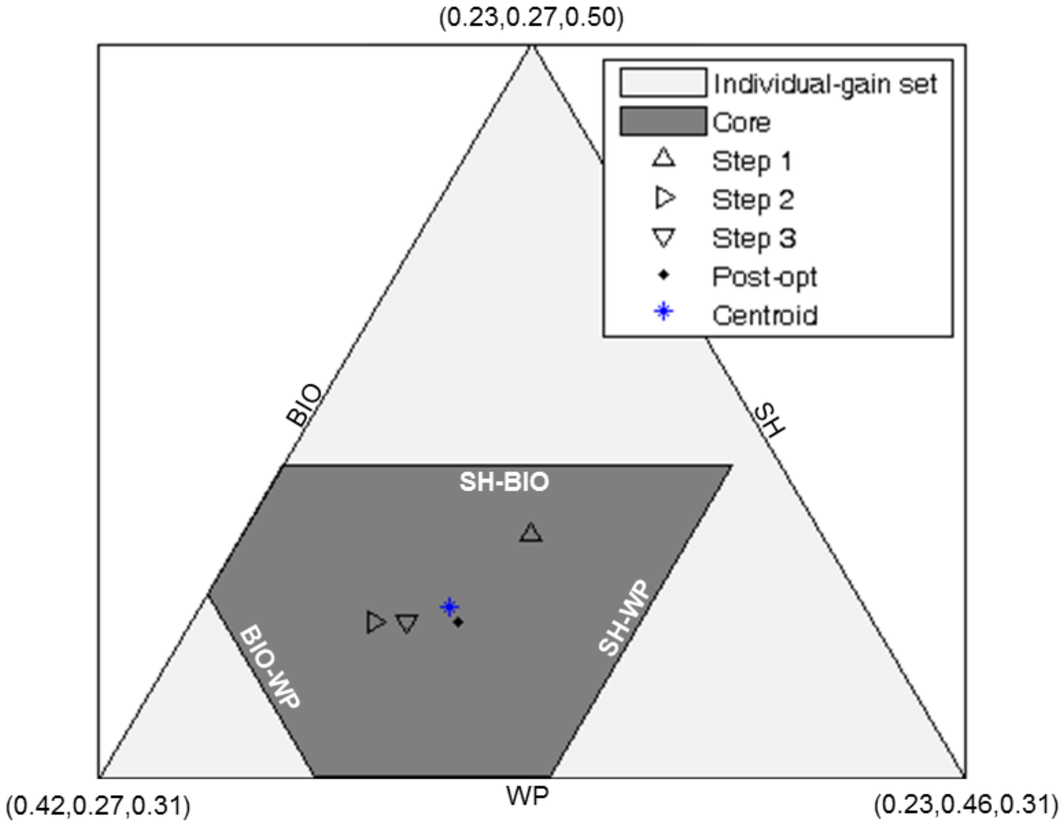

Figure 7.3 – Core of the cooperative game and respective quota allocation by the Benders algorithm for the Nucleolus method.

Note that the trial solution of the first step already presented a positive worst-case gain (last row of Table 7.6). Thereby, this solution lied inside the core of the cooperative game. Additionally, the first cut is added for coalition $\boldsymbol{c}_{SH,Bio}$. Thus the second step takes charge of repelling the solution from the SH-WP gains frontier. Next, the same happens for coalition $\boldsymbol{c}_{Bio,WP}$ and the following point is placed a little bit way from the BIO-WP frontier. Finally, the post-optimization algorithm of section 6.4 pushes the solution as close as possible to the centroid of the inner-polygon (core region).

7.4
50-players RES pool case study – stress analysis

In this experiment, it is compared the computational execution time of three models: (i) the Full Coalition Dependent (FCD) model that calculates the quotas sharing through the Nucleolus and Proportional Nucleolus methods, by the enumeration of all possible coalitions of players; (ii) the same methods calculated via the Benders algorithm and; (iii) the Marginal Benefits method. To do so, the methods were tested in different instances of the problem, with the unitary increase of the number n of players in the pool until the execution time for the first method was considered unacceptable. Furthermore, this case study aims a qualitative analysis of the methods' performance, since it is known the complexity of the cooperative game grows exponentially on the number of players n. For instance, the number of constraints in the FCD model is ruled by the number of possible coalition that can be formed with n players which is, precisely, $2^n - 1$. Thereby, on a realistic case of a pool formed by 30 generators, one would have to assess the two-stage stochastic problem that evaluates the coalition's characteristic function more than one billion times. Moreover, since the formulation of the Shapley Value method also imposes such assessment, this method is omitted from this case study, since its computational time would be practically the same of the FCD method.

In addition, in this experiment an improved algorithm is used. The following modifications are employed to enhance the algorithms performance: (i) expression (6.14) is included, for all the individual coalitions into the master problem (6.13)-(6.16) and; (ii) step 3 of the Benders algorithm is replaced by a copy of steps 6-8. This provides the algorithm with trial solutions that satisfies individual gains since the first iteration. Moreover, although the addition of such constraints does not conceptually change the solution of the algorithm, the consideration of such tighter space helps in stabilizing the algorithm convergence.

Finally, in Table 7.10: column 1 exhibits the number of players in the pool for each iteration; column 2 shows the growth of the number of constraints in problem (5.17)-(5.19), i.e. the number of two-stage stochastic programming problems needed to solve the Nucleolus method in the FCD formulation; column 3 shows the computing time of the solutions found by the FCD problem; Columns 4-6 display the value of the absolute and proportional worst-case gains of the solutions given by the Marginal Benefits method and its execution time and; finally, the other columns disposes the worst-case gain (wc-gain), the computing time and the number of cuts for the Nucleolus problems.

Table 7.10 – Comparison among the FCD formulation, Marginal Benefits and Benders algorithm for both Nucleolus methods on the solution of large instances of the problem

n	$2^n - 1$	FCD Time (s)	Marginal Benefits			Nucleolus			Proportional Nucleolus		
			wc-gain ($10³)	wc-gain (%)	Time (sec.)	wc-gain ($10³)	BA Time (s)	BA Cuts	wc-gain (%)	BA Time (s)	BA Cuts
3	7	<1	857.35	3.24%	<1	1,397.42	<1	4	6.06%	<1	4
4	15	<1	396.92	0.97%	<1	1,240.98	<1	5	3.43%	<1	7
5	31	<1	169.40	0.32%	<1	648.99	<1	7	1.41%	<1	11
6	63	1	20.32	0.03%	<1	406.97	1	12	0.91%	1	13
7	127	1	29.23	0.07%	<1	96.74	1	17	0.21%	1	18
8	255	3	112.93	0.12%	<1	208.20	1	20	0.33%	2	25
9	511	6	42.17	0.05%	<1	166.75	2	25	0.28%	2	31
10	1,023	14	0.00	0.00%	<1	107.82	2	33	0.14%	3	42
11	2,047	30	0.00	0.00%	<1	28.75	3	46	0.03%	5	61
12	4,095	64	0.00	0.00%	<1	29.24	4	51	0.03%	5	59
13	8,191	134	0.00	0.00%	<1	7.46	4	48	0.01%	7	74
14	1.6E04	294	0.00	0.00%	<1	0.33	6	73	0.00%	8	76
15	3.3E04	644	0.00	0.00%	<1	0.48	8	87	0.00%	8	83
16	6.6E04	1362	0.00	0.00%	<1	0.00	9	92	0.00%	10	93
17	1.3E05	3051	0.00	0.00%	<1	0.00	6	63	0.00%	9	81
18	2.6E05	7117	0.00	0.00%	<1	0.00	7	68	0.00%	11	91
19	5.2E05	-	0.00	0.00%	<1	0.00	6	63	0.00%	11	94
20	1.0E06	-	0.00	0.00%	<1	0.00	18	121	0.00%	20	142
25	3.4E07	-	0.00	0.00%	<1	0.42	44	208	0.00%	52	233
30	1.1E09	-	0.00	0.00%	<1	27.04	69	368	0.01%	119	534
35	3.4E10	-	0.00	0.00%	<1	2.65	103	405	0.00%	177	609
40	1.1E12	-	0.00	0.00%	1	0.00	279	481	0.07%	232	551
45	3.5E13	-	0.00	0.00%	1	31.29	368	739	0.01%	756	974
50	1.1E15	-	0.00	0.00%	1	17.29	312	613	0.03%	896	1010

It is important to highlight, first, that the optimality gap was closed (zero) for all reported solutions for the improved Benders algorithm for both Nucleolus methods for all instances. Moreover, in the Benders algorithm the secondary problem is responsible for the majority of the computational time needed to find the optimal solution. Now analyzing the results, as expected, the FCD methodology is quickly beaten by the other three methods, which present expressive results in terms of computational time when the number of players increase, principally the Marginal Benefits method. For small instances, with up to 7 players, the FCD model provides low computing time. However, for instances with more than 9 players, the computing time demanded to solve the FCD problem is twofold greater than the time needed for the BA-based methods to achieve convergence. When the instance reached 18 players, the FCD was interrupted after more than 2 hours of execution, without reaching the solution and was then considered intractable, since the Benders approach for the Nucleolus and Proportional Nucleolus solved the 50-players instance within 312 (5 minutes) and 896 seconds (15 minutes) of execution time, respectively. The difference in the execution time between the Nucleolus and the proportional Nucleolus method is due to the “spare” sets of constraints (6.69)-(6.73), needed for the linearization of the nucleolus method, as they enhance the complexity of the solution methodology.

A special attention might be given for the Marginal Benefits method, which solves the problem very quickly, since it is needed only the solution of one single LP (the characteristic function of the grand coalition, $v(\mathbb{1})$) to obtain such solution. However, as can be seen in the results, the method does not produce a solution of the quality of the Nucleolus methods, in the sense that the worst case gains for this method is always smaller than the ones produced by the Nucleolus methods. For instance, the 10-players pool is the first where a really small unit enters the pool. At this arrangement, the marginal Benefits method starts to produce worst-case null-gain solutions. For transparency purposes, the table with the size (in terms of FEC) of each player for the proposed method is disposed in Table A, on the Appendix.

In this context, it can be concluded from the Nucleolus methods that: (i) the method produces solutions that are the most adequate for the proposed RES pool model, since it provides better signals for players, in terms of stability and; (ii) the computational burden of the FCD formulation, which makes the method inappropriate for large-scale problems, was successfully solved through the Benders algorithm approach. Finally, it is important to mention that the Benders algorithm approach successfully reached all expectations, since it solved the intractability of the Nucleolus problems for the quota allocation of the proposed RES pool, allowing a consistent analysis of a pool with 50 players in about 15 minutes for the proportional Nucleolus.

Finally, in Figure 7.4, the number of cuts, or number of iterations, of the improved BA approximately exhibits a polynomial growth within the set of tested instances, whereas the full set of constraints used in the FCD problem is exponential (see the first two columns of Table 7.10). This provides a tractable algorithm even in the case of large-scale instances.

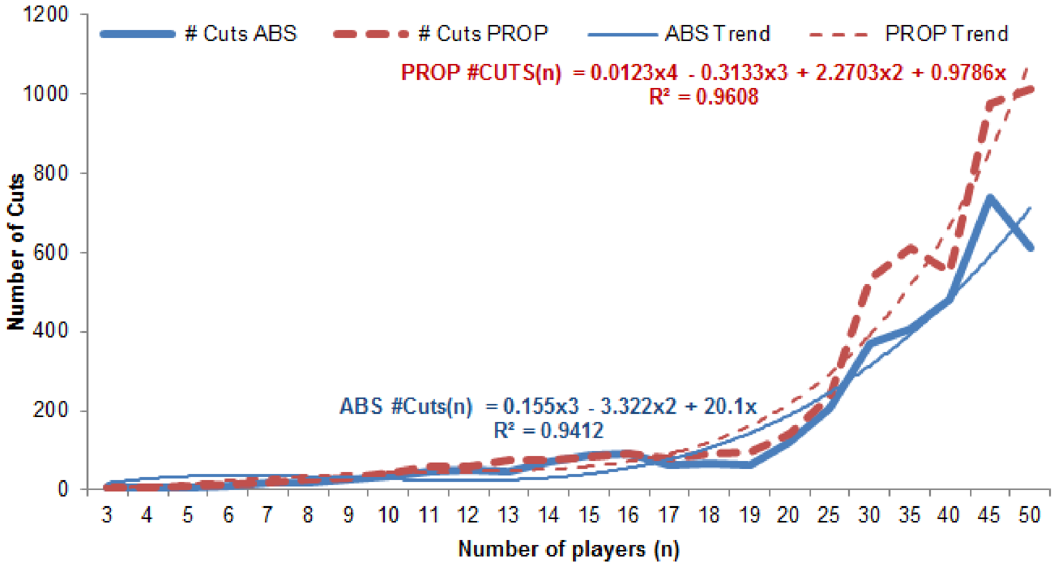

Figure 7.4 – Number of cuts as a function of the number of players in the pool. The thin line represents a polynomial trend (with equation and R^2 index displayed in the graph) within the observed data.

8
Final remarks

This work compared different quota allocation methods for a stochastic cooperative portfolio of renewable energy. The compared methods were: FEC-proportional, Shapley Value, Marginal Benefits, Nucleolus and Proportional Nucleolus. The work has proved the core of the proposed game to be non-empty. Further and more important, the work presented an efficient methodology to find the Nucleolus and Proportional Nucleolus shares of quotas for the proposed large-scale RES pool. The proposed method is a MILP-based Benders decomposition applied to the full Nucleolus and the non-linear Proportional Nucleolus problems where the last one had a further difficulty that was overcame by means of Fractional Programming linearization technique. Finally, such method was capable of solving instances with realistic data of the Brazilian power sector that were, until then, intractable in the full problem formulation. In this context, this arises as the work's strongest contribution.

It was highlighted that regardless of the existing regulatory incentives for renewables, investors still remain highly exposed to risks with an inappropriate vision for the future cash flow of new investments. This is due to the characteristic intermittent and seasonal energy generation profile of such sources, which are intensified by the market's intrinsic uncertain and volatile spot prices. Moreover, the work proposed a model for trading renewable energy in the Free Trade Environment of the Brazilian power sector aiming to bring incentives to the penetration of the three main Brazilian renewable energy sources in such environment. Thus, RES can substantially reduce the inherent risks of the energy trade in the FTE, by means of the proposed model that can take advantage from the existing synergy among their complementary seasonal generation profiles. In this context the cooperative games framework, in tune with stochastic programming techniques as well as the correct risk-aversion measures has emerged as an appropriate approach, given the strong uncertainties that are typical of the energy trading in the FTE. Still, the use of realistic data of the Brazilian power sector, with spatial-and-time correlated scenarios of energy generation and spot prices had a very important role to capture the portfolio synergic effects of the proposed model.

8.1 Future research

Probably demanding low effort, the bellow-disposed topics arise as immediate promising investigations, which can bring solid improvements to the thesis' contents.

i. Investigate if the solutions of the Marginal Benefits method always lie inside the core of the cooperative game. Besides being computationally fast, the Marginal Benefits method can, in some cases, always lie inside the core of the cooperative game. For instance, [75] states that the method always produces solutions inside the core, when applied to a game modeled as a LP where the modification of the RHS of the constraints is the only necessary change to calculate the benefit of any subcoalition of players (which is the case of the present game). Still, results from section 7.4 strongly indicates that the Marginal Benefits method is always inside the core of the game, since no results from the different tested instances produced negative (worst-case) gains.

ii. Explore the equivalence between the Marginal Benefits and the Aumann-Shapley methods. One of the drawbacks of the Marginal Benefits method is that it can become inadequate in problems where the resources are discrete. Since the method is based in small incremental variations, it can become (too) sensible to small changes in the resources of agents, producing disproportional variations on the allocation of players that have their resources slightly modified. However, the proposed cooperative game of the present work is similar to the one studied in [75], which disposes aspects that lead to the equivalence between the Marginal Benefits and the Aumann-Shapley allocation methods. Thus, since the Aumann-Shapley method does not present the above mentioned sensibility issue, the Marginal Benefits method also does not, since they are equivalent. On the other hand, the expected property of always producing solutions that belongs to the core would be directly expanded to the Aumann-Shapley method.

As proposals for future work, the first suggestion is to study the concept of a dynamic framework where quotas are reallocated over time. The stability of the model under new conditions and under the realization of the uncertainties, as well as the volatility of the quotas are relevant themes of future research that represent the border line between academic and practical implementation of RES pools. Additionally, another important and direct extension of the proposed methodology relates to the consideration of non-convex characteristic functions, incorporating for example price maker generators and risk-constrained self-scheduling decisions [76]. In the latter case, such decisions could be considered in the methodology by incorporating the generation and the related technical constraints, e.g., as ramp and minimum up and down times, into model (4.6)-(4.9). This would allow for the consideration of physical contracts, since generation and the contract decisions would be part of the decision variables, and for the possibility of considering units with storage capacity to alleviate operational constraints and maximize incomes. Although the proposed methodology could be extended to incorporate such features and the Benders algorithm would still be valid, the existence proof might not be possible for the general case due to the non-convex nature of the characteristic function. However, in this case, the justification for the Nucleolus method would be strengthened due to its nature of pursuing the maximization of the worst-case allocation, thus, finding the most stable pool.

One more proposal for future research is to use the CVaR risk measure in all formulations as a constraint for the problems, instead of as part of the Certainty Equivalent (or objective function) throughout the work. This is a common practice in many applications of portfolio management, where authors maximize a more simple revenue expression, such as the expected value of the portfolio and use the CVaR as a hard constraint that forces a minimal gain for the portfolio. However, for the models presented at the present work the correct functioning of developments that make use of properties like superadditivity, positive-homogeneity such as Benders decomposition techniques and fractional programming for example are not guaranteed and would demand further inspection.

Finally, a new Nucleolus-based method can be properly developed. What is pursued by the presented Nucleolus methods in the proposed cooperative game, in fact, is to defend the grand coalition from the most threatening subcoalitions and promote, indeed, an environment where those do not exist. However, in theory, once a solution inside the core of the cooperative game is reached, this is automatically guaranteed. In this context, a new variation of the Nucleolus methods can be devised in order to satisfy coalitions that belongs to the set of umbrella constraints with a specific small fixed non-negative gains (for example, zero?), instead of pushing such gains to the higher possible values. This would sound illogical in the first moment. However, in terms of stability, the high disparity of individual gains emerged in case-studies of section 7.2 exposed a weak point of the Proportional Nucleolus method. Thus, one can argue that this can be more dangerous for the grand coalition than a distribution of null benefits for the most threatening coalitions, since the solution already belongs to the core. Under this framework, such coalitions would then be placed in the edge of the indifference between being in the pool or not. Hence, any little monetary incentive would keep those coalitions in the pool. However, from a practical view-point, the underlying new Nucleolus variations methods could promote more flexibility for the post optimization algorithm of section 6.4 on its duty for the equalization of individual gains, thus enhancing the pool’s stability.

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Appendix

Size of the 50-players for the case study of section 7.4

Table A – Size of the n players of case study of section 7.4, in terms of Firm Energy Certificates (FEC, in average-MW).

n	Name of unit	FEC (avg-MW)	n	Name of unit	FEC (avg-MW)
1	SH1	12.13	26	BIO9	17.54
2	BIO1	17.54	27	WP9	17.65
3	WP1	20.76	28	SH10	7.34
4	SH2	17.52	29	BIO10	17.54
5	BIO2	17.54	30	WP10	207.53
6	WP2	15.43	31	SH11	11.63
7	SH3	20.23	32	WP11	12.37
8	BIO3	17.54	33	SH12	0.85
9	WP3	20.53	34	WP12	13.11
10	SH4	0.44	35	SH13	22.93
11	BIO4	17.54	36	WP13	17.71
12	WP4	11.33	37	SH14	9.63
13	SH5	7.90	38	WP14	10.23
14	BIO5	17.54	39	SH15	18.26
15	WP5	11.33	40	WP15	28.83
16	SH6	9.00	41	SH16	16.19
17	BIO6	17.54	42	WP16	20.74
18	WP6	11.33	43	SH17	17.06
19	SH7	7.98	44	WP17	18.31
20	BIO7	17.54	45	SH18	16.56
21	WP7	25.15	46	SH19	2.55
22	SH8	2.45	47	SH20	10.82
23	BIO8	17.54	48	SH21	4.42
24	WP8	7.93	49	SH22	13.56
25	SH9	3.77	50	SH23	16.36

The data set of simulated scenarios of renewable energy resources and the energy spot prices used in the computational experiments

This section presents graphs that expose the characteristics of the data produced by the VARx model used in the computational experiments. It is also important to highlight that the data used to simulate the wind power plants natural resource (wind) is given in terms of the capacity factor (% of maximum production) of the wind power plants while the data for hydro plants are given by the natural resource itself, inflow, in m^3/s. The raw data of the simulated scenarios of wind (capacity factor) and inflow (m^3/s) is then used to calculate the energy generation of each one of the wind and small hydro plants of the case studies, respecting the characteristics of the set of plants denoted in Table A. Additionally, as previously mentioned in Section 3.1, more details of the complete process for generating such data can be found in [32] to [35].

To illustrate the data related to the two simulated renewable resources, in Figure A and Figure B the historical and simulated data of wind farm WP1 and small hydro SH1 from Table A, respectively, are contrasted in a seasonal (monthly) basis. Notwithstanding, the historical data is depicted by the dots while the simulated scenarios are represented by the quantiles of 5% and 95% of the scenarios, in blue lines and the average, in black.

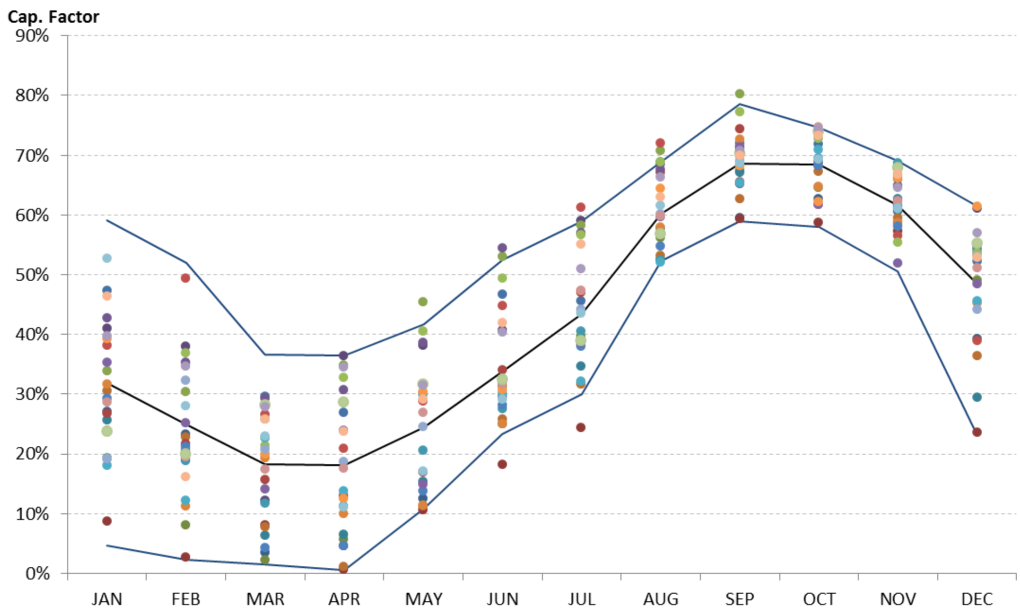

Figure A – Historical wind data (dots) contrasted with the seasonal average and confidence interval (quantiles of 5% and 95%) of simulated data (lines).

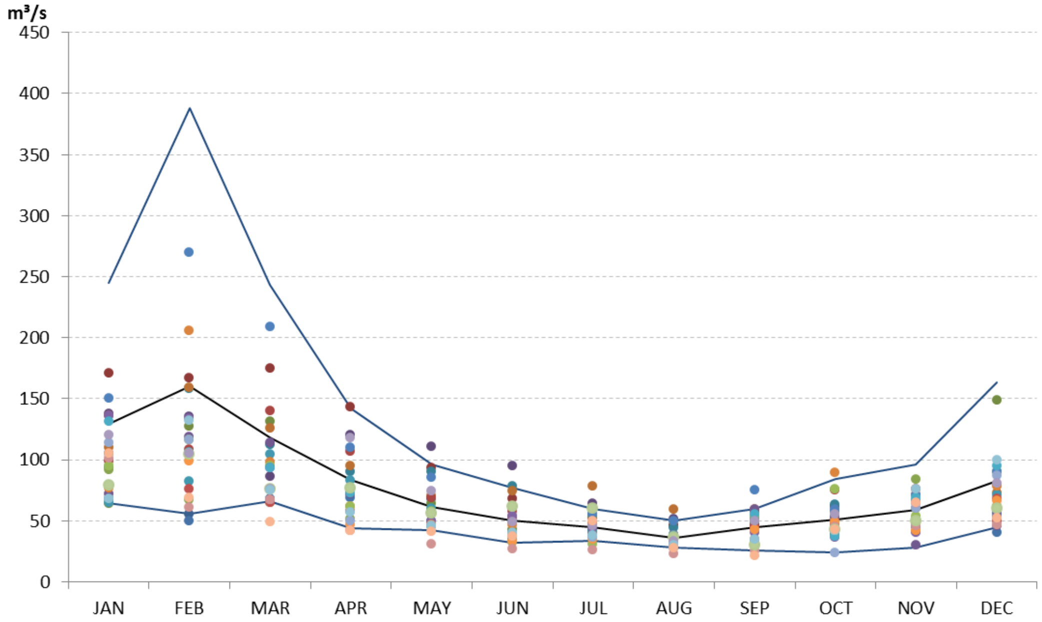

Figure B – Historical inflow data (dots) contrasted with the seasonal average and confidence interval (quantiles of 5% and 95%) of simulated data (lines).