3
Mathematical Model

This section describes the proposed model to solve the petroleum supply planning problem. The model is based on a fixed charge network flow structure over a discretized time representation [Wol98], where time intervals of equal duration are considered and activities allocated to a given interval must be capable of being performed within it.

The nomenclature used in our model is as follows:

Indices

c : crude oil category

cl : class of tanker

$cp(u)$: campaigns of the crude distillation unit u

p : is used interchangeably to refer to crude oil or production site

r : refinery

u : crude distillation unit

t : time period

z : terminal

Sets

C : set of crude oil categories

CAT_p : set of category for petroleum p

CDU_r : set of CDU in refinery r

CL : set of classes of tankers

CL_p : set of classes of tankers that can offload production site p

CL_z : set of classes of tankers that can operate in terminal z

CP_u : set of campaigns of CDU u

P : set of production sites or crude oils

R : set of refineries

R_z: set of refineries connected to terminal z

T : set of time periods

$TPLAN_1$, $TPLAN_2$: set of time periods to consider the strategic planning direction for the first and second months, respectively

Z : set of terminals

Z_r : set of terminals connected to refinery r

Parameters

$CAMP_{c,cp(u)}$ $[1000\ m^3/day]$: Consumption rate of category c in campaign $cp(u)$ at CDU u.

$CAPL_{c,r}$, $CAPH_{c,r}$ $[1000\ m^3]$: Lower and higher ideal storage levels of petroleum category c at refinery r. Inventory below or above this ideal range is penalized.

$CAPT_{cl}$ $[1000\ m^3]$: Average transportation capacity of class of tanker cl.

CF_{cl} $[1000\ USD]$: Freight cost of an additional tanker cl.

$CP^1_{p,r}$, $CP^2_{p,r}$ $[1000\ USD/1000\ m^3]$: Penalty for deviation from the strategic planning of crude oil p in refinery r, for the first and second months, respectively.

$CRH_{c,r}$, $CRLL_{c,r}$, $CRI_{c,r}$ $[1000\ USD/1000\ m^3]$: Penalty for having stock of petroleum category c at refinery r over interval t, high, low and infeasible, respectively.

CT_{cl} $[1000\ USD/day]$: Transportation cost of class of tanker cl per period t.

FU_{cl} : Fraction of the total number of tankers in class cl available to be used in a given time.

MSP_p $[1000\ m^3]$: Maximum storage capacity at production site p.

MSZ_z $[1000\ m^3]$: Maximum storage capacity at terminal z.

$MSZR_{z,r}$ $[1000\ m^3]$: Maximum storage capacity at terminal z alloted to refinery r.

NB_z : Number of berths in terminal z.

NT_{cl} : Number of tankers in class cl.

$P_{p,t}$ $[1000\ m^3/day]$: Production of crude oil p over interval t.

$PLAN^1_{p,r}$, $PLAN^2_{p,r}$ $[1000\ m^3]$: Amount of crude oil p planned for refinery r for the first and second months, respectively.

$VT_{i,j}[day]$: Voyage time between points i and j.

Binary variables

$bp_{p,cl,z,t}$: 1 if crude oil from production site p is sent to terminal z by class of tanker cl over interval t; 0 otherwise.

$bz_{p,cl,z,z_1,t}$: 1 if crude oil from production site p is transshipped from terminal z to terminal z_1 by class of tanker cl over interval t; 0 otherwise.

Continuous variables

df_{cl} : Number of tankers in class cl having to be freighted during the study horizon.

$dplan^1_{p,r}$, $dplan^2_{p,r}$ [1000 m^3] : Deviation from strategic planning for crude oil p in refinery r, for the first and second months, respectively.

$vbz_{p,cl,z,z_1,t}$ [1000 m^3] : Amount of crude oil from production site p transshipped from terminal z to terminal z_1 by tanker cl over interval t.

$vpz_{p,c,z,r,t}$ [1000 m^3] : Amount of crude oil p of petroleum category c that arrives at terminal z to supply refinery r over interval t.

$sp_{p,t}$ [1000 m^3] : Amount of crude oil stored in production site p over interval t.

$strn_{c,r,t}$, $strh_{c,r,t}$, $strl_{c,r,t}$, $stri_{c,r,t}$ [1000 m^3] : Inventory of petroleum category c at refinery r over interval t, in the normal, high, low, and infeasible levels, respectively.

$stz_{c,z,t}$ [1000 m^3] : Amount of petroleum category c stored at terminal z over interval t.

$vzr_{c,z,r,t}$ [1000 m^3] : Amount of petroleum category c pumped from terminal z to refinery r over interval t.

Constraints:

Production Sites

Inventory balance at the production sites for each time period t is given by,

$$sp_{p,t} - sp_{p,t-1} - P_{p,t-1} + \sum_{z} \sum_{cl \in CL_p \wedge CL_z} CAPT_{cl} \cdot bp_{p,cl,z,t-1} = 0 \qquad (3.1)$$

$$\forall p \in P,\ t \in T$$

The inventory at each production site p and time period t must be less than or equal to its maximum storage capacity,

$$sp_{p,t} \leq MSP_p \qquad \forall p \in P,\ t \in T \tag{3.2}$$

At most one tanker should leave a given production site p at each time period t,

$$\sum_{z} \sum_{cl \in CL_p \wedge CL_z} bp_{p,cl,z,t} \leq 1 \qquad \forall p \in P, t \in T \tag{3.3}$$

At most NB_z tankers should arrive at a terminal z at each time period t,

$$\sum_{p} \sum_{cl \in CL_p \wedge CL_z} bp_{p,cl,z,t} \leq NB_z \qquad \forall z \in Z, t \in T \tag{3.4}$$

We are implicitly assuming in this inequality that the operation time at terminals is equal to one day.

Terminals

Regarding the discharging operations in each terminal, crude oil can be shipped to a terminal from a production site or from another terminal. Additionally, the volume in a tanker going to a terminal can be fully or partially unloaded in this terminal.

$$\begin{aligned} &\sum_{r \in R_z} \sum_{c \in CAT_p} vpz_{p,c,z,r,t} - \sum_{\substack{cl \in CL_p \wedge CL_z | \\ t \geq VT_{p,z}}} CAPT_{cl} \cdot bp_{p,cl,z,t-TV_{p,z}} \\ &- \sum_{cl \in CL_{z_1} \wedge CL_z} \sum_{\substack{z_1 \in Z | \\ t \geq VT_{z_1,z}}} vbz_{p,cl,z_1,z,t-TV_{z_1,z}} \\ &+ \sum_{cl \in CL_z \wedge CL_{z_1}} \sum_{z_1 \in Z} vbz_{p,cl,z,z_1,t} = 0 \\ &\qquad \forall p \in P \mid t \geq VT_{p,z},\ z \in Z,\ t \in T \end{aligned} \tag{3.5}$$

Notice that these constraints implicitly define the refineries to be supplied.

Crude oil can only be shipped from a terminal z to another terminal z_1 if it was first shipped from a production site to the terminal z,

$$\sum_{cl \in CL_z \wedge CL_{z_1}} bz_{p,cl,z,z_1,t} - bp_{p,cl,z,t-VT_{p,z}} \leq 0 \qquad (3.6)$$

$$\forall p \in P \mid t \geq VT_{p,z},\; z \in Z, r \in R_z, t \in T$$

The volume shipped between terminals is bounded by the capacity of the tanker used,

$$vbz_{p,cl,z,z_1,t} - CAPT_{cl} \cdot bz_{p,cl,z,z_1,t} \leq 0 \qquad (3.7)$$

$$\forall p \in P,\; cl \in CL,\; z, z_1 \in Z,\; t \in T$$

Inventory balance at the terminals for each time period t is given by,

$$stz_{c,z,r,t} - stz_{c,z,r,t-1} + vzr_{c,z,r,t-1} - \sum_{p|c \in CAT_p} vpz_{p,c,z,r,t-1} = 0 \qquad (3.8)$$

$$\forall c \in C,\; z \in Z,\; r \in R_z,\; t \in T$$

Observe that equation (3.8) is for each refinery r and petroleum category c. Thus for the storage capacity constraints, we need additional inequalities to account for the storage limit by refinery and for the total storage capacity in each terminal. These inequalities are written as follows,

$$\sum_c stz_{c,z,r,t} \leq MSZR_{z,r} \qquad \forall z \in Z,\; r \in R_z,\; t \in T \qquad (3.9)$$

and,

$$\sum_r \sum_{r \in R_z} stz_{c,z,r,t} \leq MSZ_z \qquad \forall z \in Z,\; t \in T \qquad (3.10)$$

Refineries

Figure 3.1 illustrates the range of each inventory variable at a given refinery as well as how the inventory balance is modeled. The middle portion of the figure represents the ideal inventory of a given category in the refinery. The upper portion corresponds to high inventory and, therefore, is penalized since it can give rise to logistical problems in the refinery. The low inventory is depicted by the lower portion and, in the same way, is penalized because the refinery may need to shut down some units. The variables $stri_{c,r,t}$ are associated with the highest penalty, and have an interesting interpretation as they indicate to the user the need to import more oil to supply this particular refinery. The equations describing the inventory balance are then as follows,

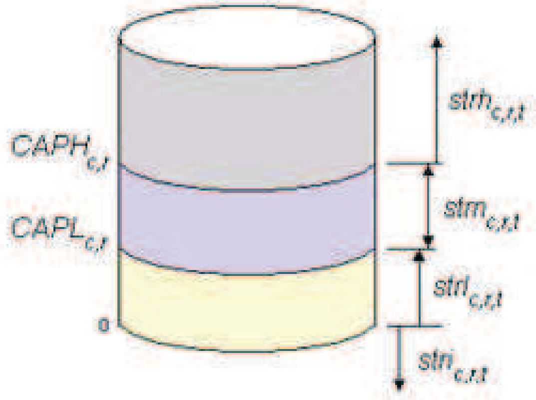

Figure 3.1: Schematic representation of the inventory in a given refinery

$$\begin{aligned} & strn_{c,r,t} + strh_{c,r,t} - strl_{c,r,t} - stri_{c,r,t} - strn_{c,r,t-1} \\ & - strh_{c,r,t-1} + strl_{c,r,t-1} + stri_{c,r,t-1} - zr_{c,z,r,t-1} \\ & + \sum_{cp \in CP_u} \sum_{u \in CDU_r} CAMP_{cp(u),c,t} = 0 \\ & \qquad \forall c \in C,\ z \in Z,\ r \in R_z,\ t \in T \end{aligned} \tag{3.11}$$

In addition, there are bounds on the inventory variables,

$$CAPL_{c,r} \leq strn_{c,r,t} \leq CAPH_{c,r} \qquad \forall c \in C,\ r \in R,\ t \in T \tag{3.12}$$

$$strl_{c,r,t} \leq CAPL_{c,r} \qquad \forall c \in C,\ r \in R,\ t \in T \tag{3.13}$$

Deviation from the strategic planning direction

Solution deviation from the strategic planning is written for each month as follows,
First month,

$$dplan1_{p,r} + \sum_{c \in CAT_p} \sum_{z|r \in R_z} \sum_{t \in TPLAN_1} pz_{p,c,z,r,t} \geq PLAN^1_{p,r} \quad (3.14)$$

$$\forall p \in P,\ r \in R$$

$$dplan1_{p,r} - \sum_{c \in CAT_p} \sum_{z|r \in R_z} \sum_{t \in TPLAN_1} pz_{p,c,z,r,t} \geq -PLAN^1_{p,r} \quad (3.15)$$

$$\forall p \in P,\ r \in R$$

Second month,

$$dplan2_{p,r} + \sum_{c \in CAT(p)} \sum_{z|r \in R(z)} \sum_{t \in TPLAN_2} pz_{p,c,z,r,t} \geq PLAN^2_{p,r} \quad (3.16)$$

$$\forall p \in P,\ r \in R$$

$$dplan2_{p,r} - \sum_{c \in CAT(p)} \sum_{z|r \in R(z)} \sum_{t \in TPLAN_2} pz_{p,c,z,r,t} \geq -PLAN^2_{p,r} \quad (3.17)$$

$$\forall p \in P,\ r \in R$$

Tanker management

The maximum number of additional tankers required during the time horizon for each class of tanker *cl* is estimated by,

$$\begin{aligned} df_{cl} \geq & \sum_{p \in P} \sum_{z \in Z} \sum_{t - VT_{p,z} \leq t_1 \leq t} bp_{p,cl,z,t} \\ & + \sum_{p \in P} \sum_{z \in Z} \sum_{z_1 \in Z} \sum_{t - VT_{z,z_1} \leq t_1 \leq t} bz_{p,cl,z,z_1,t} - \lceil FU_{cl} \cdot NT_{cl} \rceil \quad (3.18) \\ & \forall cl \in CL,\ t \in T \end{aligned}$$

Variables df_{cl} are used in the objective function to minimize the number of extra tankers of each class *cl* needed during the study horizon. It is important to point out that the ship routing is not being considered in this study. However, we use the parameter FU_{cl} to have an estimate on the number of tankers available at each time period. In this study, we set $FU_{cl} = 0.50$, representing that we have only half of the tankers available in each class of tanker as we are considering that the other half is already busy transporting crude oil to terminals.

Objective function

The model seeks to minimize the total cost, which involves shipping costs, penalty for inventory shortage or surplus, penalty for deviation from the strategic planning, and freight cost for additional tanker,

$$\begin{aligned} Min \sum_{p} \sum_{cl \in CL_p \wedge CL_z} \sum_{z} \sum_{t} CT_{cl} \cdot VT_{p,z} \cdot bp_{p,cl,z,t} \\ + \sum_{p} \sum_{cl \in CL_z \wedge CL_{z_1}} \sum_{z} \sum_{z_1} \sum_{t} CT_{cl} \cdot VT_{p,z} \cdot bz_{p,cl,z,z_1,t} \\ + \sum_{c} \sum_{r} \sum_{t} CRH_{c,r} \cdot strh_{c,r,t} + \sum_{c} \sum_{r} \sum_{t} CRL_{c,r} \cdot strl_{c,r,t} \\ + \sum_{c} \sum_{r} \sum_{t} CRI_{c,r} \cdot stri_{c,r,t} \\ + \sum_{p} \sum_{r} CP^{1}_{p,r} dplan^{1}_{p,r} + \sum_{p} \sum_{r} CP^{2}_{p,r} dplan^{2}_{p,r} \\ + \sum_{cl} CF_{cl} \cdot df_{cl} \end{aligned} \tag{3.19}$$

3.1 Solution Example

We present part of the solution of a test instance to give some insights about the decisions that the model handles. Figure 3.2 summarizes the offload scheduling at platform $P1$. As we can see, the important questions confronted at the platform offload scheduling are twofold: avoiding that the inventory reaches its maximum capacity with subsequent platform shutdown, and preventing a tanker to be sent before having sufficient inventory to fill it up completely. Additionally, each tanker assigned to offload each platform has its terminal destination determined. The figure in the terminals is slightly complicated since for each discharge of a tanker the model has to determine how its volume will be split between possible refineries and categories. In the case shown in Figure 3.3, the terminal $T1$ is only linked to one refinery, and therefore the representation becomes easier as the model only needs to manage the classification into categories. When it comes to the refineries, the most relevant issue is to keep the inventory inside the safe region delimited by horizontal lines inside the graphs in Figure 3.4. As pointed out in the modeling subsection, every time the inventory goes beyond these limits the solution is penalized.

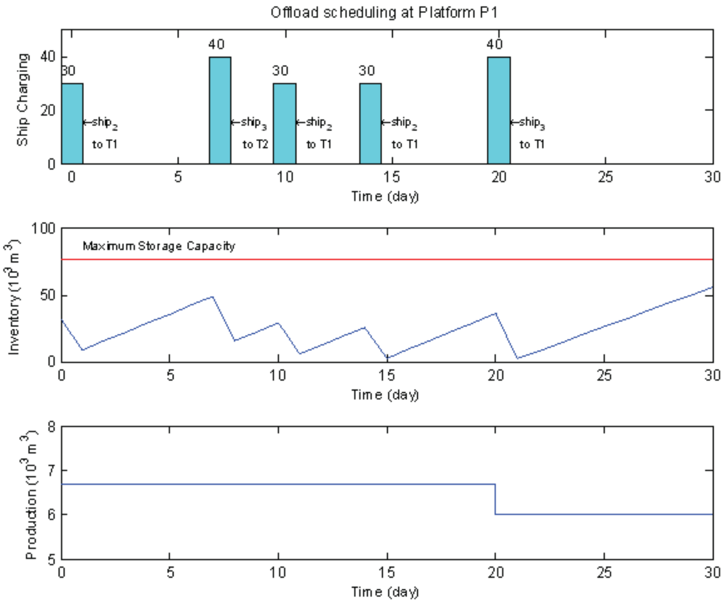

Figure 3.2: Summary of offload scheduling at Platform P1

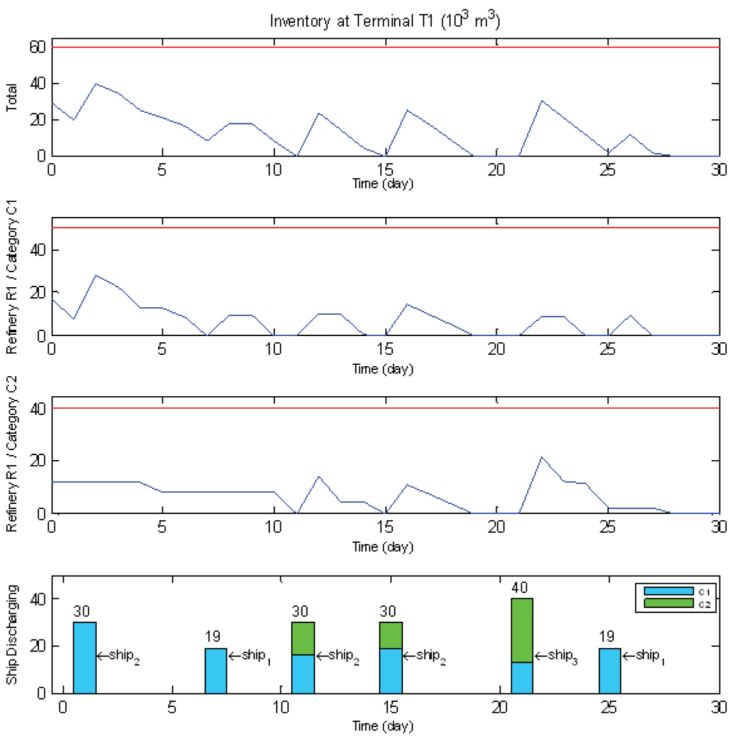

Figure 3.3: Summary of the inventory and discharging activity at Terminal T1

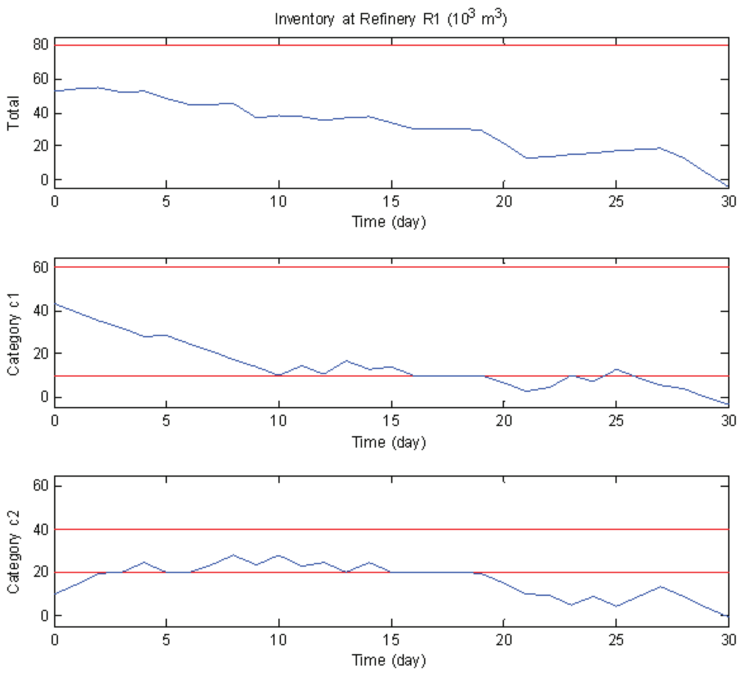

Figure 3.4: Summary of the inventory levels at Refinery R1

3.2 Flexibility of Campaigns

In this model we study the effects of considering flexible dates for the initial campaigns defined by the user. In addition to the initial campaigns given, we ask the user to inform the earliest time, the latest time, and the duration of each campaign. If it is worth to change the campaign dates, the model will perform it and can even break up the original campaign into the earliest and latest dates, on condition that the crude oil estimate profiles match better with the refineries consumption. It should be noticed that every time the model divides a campaign, a changeover cost has to be paid. Hence, this is carried out only if it is absolutely necessary. Figure 3.5 illustrates how the assignment of campaigns works. As we can notice, campaign 1 is initially set up to start on day 1 and finish on day 6. Nevertheless, the model could, for instance, move it to start on day 4 and finish on day 9. In the same way, campaign 2 is originally programmed to start on day 7 and finish on day 15. However, the model could split it into two campaigns, one starting on day 1 and finishing on day 3, and another taking place from day 10 to 15.

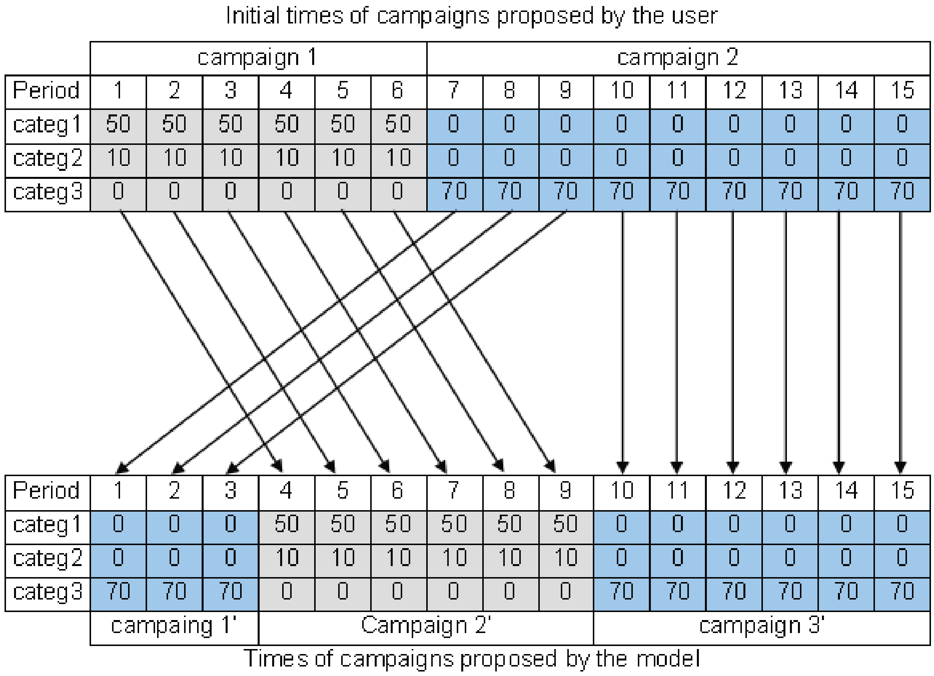

Figure 3.5: Flexibility of Campaigns at Refineries

In order to model the flexibility of campaigns, we need the following additional definitions:
Parameters

CS : Set up cost for CDU campaign changes.

DC_{cp} : Duration of campaign cp.

TR_{cp} : Release date for campaign cp.

TD_{cp} : Deadline for completing campaign cp.

Binary variables

$bcp_{u,cp,t}$: 1 if CDU u processes campaign cp over interval t; 0 otherwise.

$bsu_{u,t}$: 1 if a set-up is necessary in CDU u at time t; 0 otherwise.

The inequalities describing the flexibility of campaigns are as follows:
Assignment of production campaign to time slots within valid time windows,

$$\sum_{\substack{cp \in CP(u)| \\ TR_{cp} \leq t \leq TD_{cp}}} bcp_{u,cp,t} = 1 \qquad \forall u,\ t \in T \tag{3.20}$$

The duration of each campaign must hold,

$$\sum_{t=TR_{cp}}^{TD_{cp}} bcp_{u,cp,t} = DC_{cp} \qquad \forall u \in CDU, cp \in CP(u) \qquad (3.21)$$

If a change of campaign takes place from time period t to $t+1$, then a crude distillation set-up is necessary,

$$bsu_{u,t} + bcp_{u,cp,t+1} - bcp_{u,cp,t} \geq 0 \qquad \forall u, t, cp \in CP(u) \qquad (3.22)$$

Moreover, we incorporate the changeover costs, $\sum_u \sum_t CS \cdot bsu_{u,t}$, to the objective function given in (3.19).

Figures 3.6-3.7 show the advantage of flexible campaigns. We observe that when the flexibility of campaigns is considered we avoid in some time periods stockout of some petroleum categories in the refineries.

This concept of flexible campaigns is not considered yet in practice in the Petroleum Supply Planning activity. However, we introduce it in this model to draw the attention to the importance of considering this aspect of the problem since, as verified by Figures 3.6-3.7, it allows us to manage our resources in a more economical way.

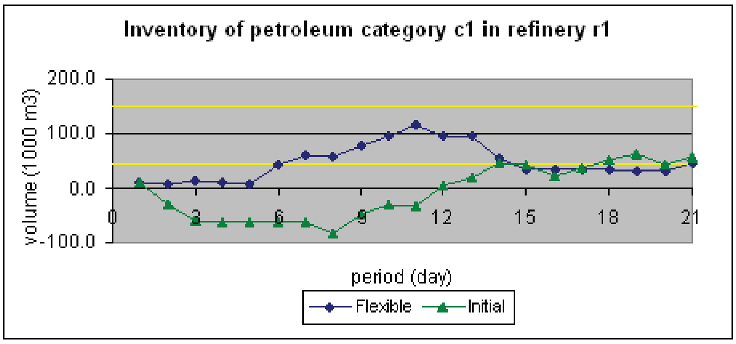

Figure 3.6: Comparison between the inventory evolution of category c1 at refinery r1 using initial and flexible campaigns

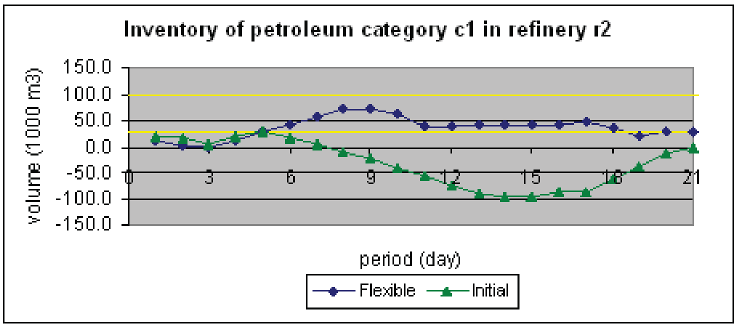

Figure 3.7: Comparison between the inventory evolution of category c1 at refinery r2 using initial and flexible campaigns

3.3 Changeover Cut

As shown by Yee and Shah [Yee98], the presence of changeovers in an MILP scheduling model may lead to a large relaxation gap. To overcome this difficulty, usually some cut constraints are added to enforce that a minimum number of changeover tasks must be performed. It is easy to show that scheduling problems with changeover costs present a large integrality gap that increases the computation burden in Branch-and-Bound algorithms. The example in Figure 3.8 motivates this discussion and sheds some light on the possible outcome when this aspect is disregarded.

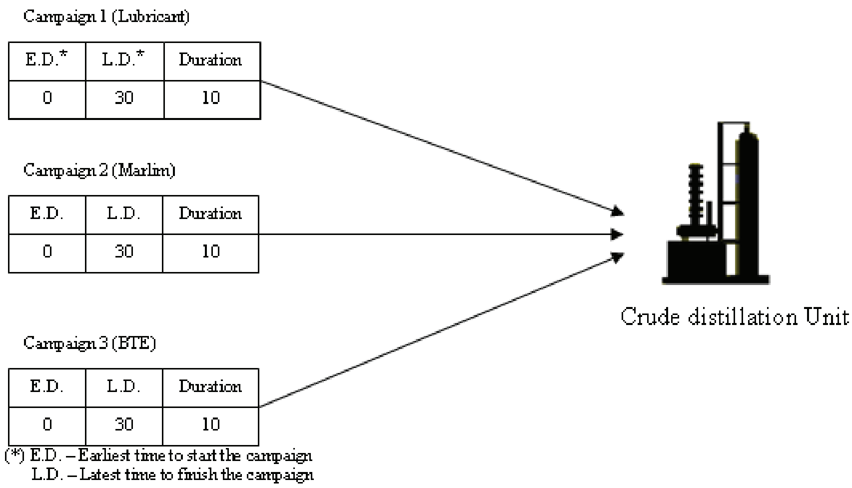

Figure 3.8: Motivated example to justify the changeover cuts

It is likely that the LP relaxation solution to this example happens to be,

$$bcp_{u,cp,t} = \frac{1}{3} \qquad \forall cp \in CP_u, t \tag{3.23}$$

Thus, from (3.22),

$$bsu_{u,t} = 0 \qquad \forall t$$

Therefore, the changeover costs will be zero in the objective function. However, we can verify by inspection that at least two changeovers will be necessary, as three campaigns were initially assigned to this crude distillation unit. We can generalize this idea and write the following cuts for each crude distillation unit,

$$\sum_t bsu_{u,t} \geq |NCP_u| - 1 \qquad \forall u \tag{3.24}$$

where, $|NCP_u|$, is the number of campaigns in $CDU\ u$.

To demonstrate the importance of adding changeover cuts to our formulation, in the sequel we present some computational results on three instances whose sizes are detailed in Table 3.1. All the results were obtained by setting the integrality tolerance gap to 10%.

Table 3.1: Dimension of test instances

	Instances		
Elements	#1	#2	#3
Production sites	6	6	11
Terminals	4	4	5
Refineries	5	5	6
CDUs	6	6	7
Crude oil categories	3	3	3
Ship classes	3	3	6
Horizon (days)	10	10	60

Table 3.2: Computational results for solving instance 1 using changeover cuts

Instance 1	**Without changeover cuts**	**With changeover cuts**
no. of constraints	1550	1553
no. of variables	2740	2740
no. of binary variables	940	940
no. of visited nodes	>> 1000000	202
CPU(s)	>> 100000	3
LP relaxation solution (Z_{lp})	321.40	464.73
Best solution (Z_o)	-	605.50
Initial integrality gap*	47 %	23 %

* $\left(\frac{Z_* - Z_{lp}}{Z_*}\right) \times 100\%, Z_* = \min\{Z_0\}$

Table 3.3: Computational results for solving instance 2 using changeover cuts

Instance 2	**Without changeover cuts**	**With changeover cuts**
no. of constraints	1550	1553
no. of variables	2740	2740
no. of binary variables	940	940
no. of visited nodes	551	313
CPU(s)	4	3
LP relaxation solution (Z_{lp})	539.00	832.00
Best solution (Z_o)	1111.00	1104.50
Initial integrality gap	51 %	25 %

Table 3.4: Computational results for solving instance 3 using changeover cuts

Instance 3	**Without changeover cuts**	**With changeover cuts**
no. of constraints	13847	13855
no. of variables	46992	46992
no. of binary variables	22123	22123
no. of visited nodes	>>1000000	631243
CPU(s)	>>100000	32760
LP relaxation solution (Z_{lp})	16484.03	17471.83
Best solution (Z_o)	-	25743.72
Initial integrality gap	35 %	32 %

Although these test instances do not represent the complexity of the real problem, the results in Tables 3.2-3.4 make clear the strength of the cuts in (3.24). After adding the changeover cuts, the integrality gap was less than half the initial value of the original formulation for instance tests 1 and 2. For instance 3, although the decrease in the integrality gap was smaller, the changeover cuts made it possible to solve this very large problem. Moreover, the number of changeover cuts introduced is negligible compared to the size of the original formulation, and thus the time to solve the Linear Relaxation at each node of the Branch-and-Bound is not affected.