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Petroleum Supply Planning:  
Models, Reformulations and  
Algorithms

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**Roger Rocha**

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Reformulations and Algorithms**

**Tese de Doutorado**

Thesis presented to the Postgraduate Program in Informatics of the  
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requirements for the degree of Doutor em Informática

Advisor: Prof. Marcus Vinicius Soledade Poggi de Aragão

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To my wife Renata,  
my father Jarbas and my mother Helena

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## Resumo

Rocha, Roger; Poggi de Aragão, Marcus V.S.. **Planejamento do Suprimento de Petróleo: Modelos, Reformulações e Algoritmos**. Rio de Janeiro, 2010. 124p. Tese de Doutorado — Departamento de Informática, Pontifícia Universidade Católica do Rio de Janeiro.

A atividade de Planejamento de Suprimento de Petróleo é um elo importante para a integração da Cadeia de Suprimento de Petróleo na PETROBRAS, uma vez que é responsável por refinar as informações do planejamento estratégico a ser implementado no nível operacional. Nesta tese, esse problema é definido e explicado em detalhes e um modelo de programação Inteira Mista é proposto para resolvê-lo. Embora os resolvidores de problemas de programação Inteira Mista tenham evoluído de forma surpreendente na última década, para esta aplicação em particular, com o modelo inicial proposto não é possível obter soluções com qualidade satisfatória em tempo computacional aceitável. Desta forma, a linha de pesquisa desta tese consistiu em investigar, em detalhe, a estrutura deste problema a fim de encontrar reformulações mais adequadas e novos algoritmos para a solução deste problema. O foco principal desta tese é resolver de forma eficiente o problema de planejamento de suprimento de petróleo na PETROBRAS, no entanto, como subprodutos desse esforço são propostos um novo algoritmo de decomposição e reformulações que podem ser aplicados a uma ampla gama de problemas. No que diz respeito à realização do objetivo principal, todos os casos testados foram resolvidos de maneira eficiente através dos desenvolvimentos propostos. O novo algoritmo de decomposição se mostrou o método mais adequado para resolver as instâncias com mais de duas classes de navios operando em cada plataforma. Já para os casos com uma ou duas classes de navios, a formulação denominada *Hull Relaxation*, que tem como base uma estrutura definida neste trabalho como *Cascading Knapsack Inequalities*, é a melhor alternativa de solução. Tendo em vista estas alternativas de soluções, é implementado um algoritmo geral que automaticamente escolhe a melhor opção de solução, em função da estrutura do problema. Para a situação onde o número de classes de navios operando nas diversas plataformas varia entre um e quatro, pode-se usar mais de uma abordagem em paralelo e considerar como solução a alternativa mais rápida ou com melhor resultado. Este modelo está sendo testado na PETROBRAS e tem-se mostrado uma ferramenta eficaz para a integração de sua cadeia de suprimentos de petróleo, bem como permitindo a análise de cenários para a obtenção de soluções alternativas até então não exploradas.

## **Palavras-chave**

Planejamento do Suprimento de Petróleo; Programação Inteira;  
Algoritmo de Decomposição; Programação Disjuntiva; Reformulações.



## Abstract

Rocha, Roger; Poggi de Aragão, Marcus V.S.. **Petroleum Supply Planning: Models, Reformulations and Algorithms**. Rio de Janeiro, 2010. 124p. DSc Thesis — Departamento de Informática, Pontifícia Universidade Católica do Rio de Janeiro.

The Petroleum Supply Planning activity is an important link for the integration of the Petroleum Supply Chain at PETROBRAS as it is responsible for refining the strategic supply planning information to be used at the operational level. In this thesis we set the ground for understanding this important problem and we propose a mathematical model to solve it. Although the solvers in the last decade have evolved enormously, for this particular application we cannot get solutions with satisfactory quality in reasonable computational time with only the initial proposed model. This directed the line of research of this thesis into investigating, in detail, the structure of this problem in order to find more suitable reformulations and algorithms to tackle it. Our primary goal is to solve efficiently the petroleum supply planning problem at PETROBRAS. Nevertheless as a by-product of this endeavor, we propose a novel decomposition algorithm and reformulations based on a cascading knapsack structures that turn out to be applicable in a wide range of problems. Concerning the achievement of the main objective, we obtain good results for all instances we tested. We show that the novel decomposition algorithm is the most fitted method to solve the petroleum supply planning problem if we consider more than two tankers to offload each platform. In the case of one or two tankers to offload each platform, the hull relaxation formulation based on the cascading knapsack structure introduced after an inventory reformulation at platforms is the best option if one is to solve this problem. For the real application, these solution alternatives allow to implement a general algorithm that automatically switches to the best solution option depending on the structure of the problem. For the mixed situation, i.e., number of tanker varying from one to four, one can use more than one approaches in parallel and take the fastest or the best result obtained. This model is being tested at PETROBRAS and is showing to be an effective tool to help integrate its petroleum supply chain as well as to do what-if analysis to look for alternative solutions never thought before.

## Keywords

Petroleum Supply Planning; Integer Programming; Decomposition Algorithm; Disjunctive Programming; Reformulations.

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