VII
Conclusions

This thesis dealt with exact algorithms for some VRPs. The focus of the research was the CARP, but some algorithms were also extended for the GVRP, which allowed them to be tested for another problem, the CVRP. A brief review of these problems was done at the beginning of the work and then some known formulations were presented for both the CARP and the GVRP.

A first contribution of our work was related to the exact separation of the capacity cuts for the CARP. A new exact separation based on a MIP formulation was presented. The computational tests showed that this separation can perform better in practice than the previous one. Furthermore, this approach was improved by the use of a dual ascent heuristic, which can generate a large set of cuts as a hot-start for the cutting plane. Lower bounds for large scale instances were presented for the first time. Although it has not improved the bounds of any of the classical instances for the CARP, this approach can be used to assist more complex algorithms due to its low running times, as done in the proposed column generation and BCP.

As another contribution we suggested an efficient implementation for the state-of-the-art pricing algorithm, which prices a kind of restricted non-elementary route, called the ng-route. It was done using a heuristic pricing and two techniques to improve the exact pricing. The heuristic pricing is a very simple algorithm, which can run as efficiently as a pricing of non-elementary routes without any restriction. The first technique used during the exact pricing was the DSSR, which relaxes the restrictions imposed by the ng-routes rules and incrementally rebuild these rules, until a feasible solution is found, reducing the overall computational time spent. Along with this technique, completion bounds were used. They act as an estimate for the lower bound on the reduced cost a path can have. It helps the algorithm discarding any path which would lead to routes which would not be in the solution. The results confirmed the efficiency of the proposed algorithms, as it was possible to run the pricing for ng-sets with sizes never used before. Furthermore, some new best lower bounds were obtained and new optimality certificates for GVRP instances were obtained.

Finally, all the proposed algorithms were put together in a branch-cut-and-price algorithm for the CARP. The help of some improvements such as strong branching and reduced cost fixing allowed the algorithm to obtain new lower bounds for open instances, as well as prove new optimal solutions. Even with ng-sets of small sizes and a small time limit, very good results could be obtained.

VII.1 Future Work and Extensions

This thesis provides a wide range of future works. Some of them are listed below:

- The proposed pricing implementation can be tested with other families of cuts as well as with an extended formulation with capacity-indexed variables as the one presented in Pessoa et al. [69];
- A complete branch-cut-and-price for the GVRP can be created in order to try to improve the results on both GVRP and CVRP.