V
Efficient Restricted Non-Elementary Route Pricing

The scope of this chapter is to present an efficient method which prices a special kind of non-elementary routes, called the ng-routes. Such non-elementary routes are built with respect to what we call the ng-sets. These ng-sets are defined for each customer and they act like a memory of each customer. For instance, an elementary route must remember every traversed customer in order to prohibit any repetition. The ng-routes have a weaker memory. Each time it arrives at a new customer, the ng-route can only remember any other traversed customer if it belongs to the memory of the current customer, otherwise the customer is forgotten. For this reason, the larger is the ng-sets, the best are the routes obtained.

The interest on the ng-routes arose from the need of obtaining better bounds than the existing relaxations available in the literature, but without dealing with the high costs incurred by pricing elementary routes.

The proposed algorithm starts by combining Righini and Salani's [72] Decremental State Space Relaxation (DSSR) technique with completion bounds. The DSSR approach allows controlling the number of labels in the dynamic programming matrix by relaxing the state-space defined by the ng-sets associated with each customer, leading to a global computation time much smaller than solving once for the original ng-sets space. The completion bounds are used to estimate a lower bound on the reduced cost of the best route a label can lead to. These bounds may prevent the algorithm to unnecessarily generate routes with non-negative reduced costs.

The algorithm is then adapted to both CARP and GVRP. We report experiments showing that the algorithm is capable of pricing ng-routes with ng-set sizes up to sixty-four for hard instances of CARP, GVRP and CVRP, a number about three times greater than what has been published so far. As mentioned earlier, the CVRP instances are solved considering them as GVRP instances. The results of the column generation algorithm also provide a clear idea of the improvements in terms of lower bounds when the ng-set

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size parameter increases, as well as the time required for computing it. In addition, several new best lower bounds are found for the GVRP, specially for large instances. Moreover, impressive lower bounds for open CARP and CVRP instances are obtained, keeping in mind that they were computed using only column generation and some classical cuts.

V.1 Column Generation

Column generation is a technique commonly used to solve a linear program with a huge number of variables (columns). The algorithm starts with a small subset of columns, which gives rise to a small linear program (LP). This LP, generally called the restricted master, can be easily solved. Next, the algorithm tries to price a new column with a reduced cost suitable to improve the current solution. This iterative process continues until no improving column exists.

As previously mentioned, the column generation presented in this chapter uses the set partitioning formulations described in Section III.1. As a minimization problem, each time one wants to find a new column to improve the current solution, this column must have necessarily a negative reduced cost. This is what the pricing subproblem must search for: columns representing routes with negative reduced costs regarding the current solution of the restricted master problem.

Given the dual variables γ and β_i, associated with the constraints (III.2) and (III.3), we can define the reduced cost of a route r as follows.

$$\bar{c}_r = c_r - \gamma - \sum_{i \in \mathcal{C}} a_r^c \beta_i. \tag{V.1}$$

Therefore, the value of the reduced cost for a route is its actual cost minus the dual variable γ and the dual variable β for each customer it visits. This equation by itself is not very interesting for the pricing algorithm, since one needs to know the reduced cost of each edge. With this in mind, this equation must be rewritten as a function of the edges. There are some different ways to handle this issue and our choice is described below.

In the case of the CARP, where each customer is an edge, the dual variable γ is associated with the constraint (III.27) and the dual variables β_e are associated with the constraints (III.28). The reduced cost of an edge e is as follows.

$$\bar{c}_e = \begin{cases} c_e - \beta_e & \text{if } e \notin \delta(\{0\}) \\ c_e - \left(\beta_e + \frac{\gamma}{2}\right) & \text{if } e \in \delta(\{0\}) \end{cases}. \tag{V.2}$$

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Moreover, if each customer is a vertex (CVRP) or a cluster (GVRP), the dual variables γ and β_i are associated to constraints (III.52) and (III.53), respectively. In both cases, we can define the reduced cost of an edge $e = (i, j)$ as follows.

$$\bar{c}_e = \begin{cases} c_e - \left(\frac{\beta_i+\beta_j}{2}\right) & \text{if } e \notin \delta(\{0\}) \\ c_e - \left(\frac{\beta_i+\gamma}{2}\right) & \text{if } e \in \delta(\{0\}) \end{cases} . \quad \text{(V.3)}$$

Now that the reduced cost is defined for a route and for an edge, some of the existing pricing relaxations will be presented.

(a) The q-Route Pricing Relaxation

Since an optimal solution of a routing problem does not include non-elementary routes, we would want to price, ideally, only elementary routes. This would correspond to solving the Elementary Shortest Path Problem with Resource Constraints (ESPPRC) as a pricing subproblem, which is known to be strongly $\mathcal{NP}$-hard [24]. However, if we choose not to deal with this level of complexity and relax the routes, allowing them to be non-elementary, the problem now corresponds to the Shortest Path Problem with Resource Constraints (SPPRC), which is weakly $\mathcal{NP}$-hard, and there are pseudo-polynomial algorithms available in the literature, as described in the seminal work of Christofides et al. [19].

We can solve this pricing relaxation using a forward dynamic programming algorithm, which is defined as follows. Let $T(d, i)$ be the minimum reduced cost of a path that starts at the depot vertex, visits a set of customers and ends at customer i with cumulative demand of d. We create a $(Q+1) \times |\mathcal{C}|$ dynamic programming matrix, where $|\mathcal{C}|$ is the number of customers and Q the total capacity of each vehicle. At first, this matrix is entirely filled with $+\infty$. Next, the recurrence shown below is used to solve the non-elementary pricing.

$$\begin{cases} T(d_i, i) = \bar{c}_{0i} \\ T(d, i) = \min_{j \in \mathcal{C}} \{T(d - d_i, j) + \bar{c}_{ji}\} \end{cases} . \quad \text{(V.4)}$$

Notice that what we call $\bar{c}_{ji}$ is the reduced cost between a pair of customers. Hence, depending on the problem tackled, it can be a single edge (CVRP and GVRP) or a sequence of edges (CARP). Furthermore, it is easy to verify that the complexity of this algorithm is $\mathcal{O}(|\mathcal{C}|^2 Q)$.

We just presented what we are calling an unrestricted non-elementary route pricing, since this algorithm does not try to forbid any kind of cycle of customers which could possibly appear in its solution. In the work of

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Christofides et al. [19], where this algorithm is presented, a label-setting extension is also described, which forbids non-elementary routes with 2-cycles ($i - j - i$). Although it is a simple extension, which does not change the complexity of this algorithm, it can improve the lower bounds significantly. Furthermore, it is natural to infer that the greater is the length of the cycle prohibited by the algorithm, the better is the improvement on the lower bound. With this in mind, the work of Irnich and Villeneuve [45] presents the general case for this algorithm, where given an input parameter s, it returns only non-elementary routes without s-cycles. As expected, the complexity of this algorithm is factorial and, for this reason, it can only be used for small values of s, as shown by Fukasawa et al. [31] for the CVRP.

(b) The ng-Route Pricing Relaxation

Recently, the ng-route relaxation was introduced in the work of Baldacci et al. [5] for the CVRP and the CVRP with Time Windows (CVRPTW), and it was later extended to the GVRP by Bartolini et al. [7], where it was used to solve transformed CARP instances. This new relaxation aims at having a better compromise between efficiently pricing non-elementary routes and obtaining good lower bounds.

For each customer $i \in \mathcal{C}$, let $N_i \subseteq \mathcal{C}$ be a subset of customers which have a relationship with i. A possible representation for this relationship can be a neighborhood relationship, i.e., N_i contains the nearest customers of i, including i. These are the ng-sets and they work like a memory for each customer. For instance, when a route is being built, by the time it arrives at a customer i, it only remembers visiting a customer j before if j belongs to the ng-set of i, N_i. Thus, if the route visits a given customer i at some point, this customer can only be visited again – forming a cycle – if another customer which "forgets" customer i is visited before. At this point, we can conclude that the size of the ng-sets is an important factor on the quality of solutions, because the larger the ng-sets, the larger will be the smallest cycles which can appear in a route. The size of each set N_i is limited by $\Delta(N_i)$, which is a parameter defined *a priori*. Obviously, this size also changes the pricing complexity, as we will show further.

Let $P = (0, \ldots, i_k, \ldots, i_p)$ be a path starting at the depot, visiting customer i_k and ending at customer i_p, and $\mathcal{C}(P)$ be the list of customers visited by path P. We can define a function $\Pi(P)$ of prohibited extensions (the "memory") of path P as follows.

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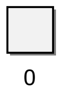
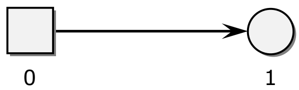
$$\Pi(P) = \left\{ i_k \in \mathcal{C}(P) : i_k \in \bigcap_{s=k+1}^{p} N_{i_s}, k = 1, \ldots, p-1 \right\} \cup \{i_p\}. \tag{V.5}$$

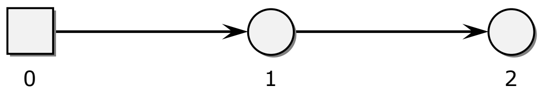
We present two examples in Figures V.1 and V.2 to illustrate the update procedure of function Π. For both examples there are three customers and the depot. The path being built starts at the depot, goes through customers 1, 2 and 3 and then tries to go back to customer 1. In the first example, shown in Figure V.1, the ng-sets for each customer are $N_1 = \{1, 2\}$, $N_2 = \{2, 1\}$ and $N_3 = \{3, 1\}$. Notice that after the extension made in Figure V.1(d), customer 1 is still present in the "memory" of the path. Therefore the extension of Figure V.1(e) is not allowed.



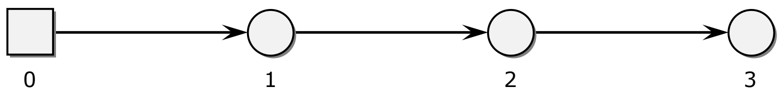
V.1(a): $\Pi_0 = \{\}$



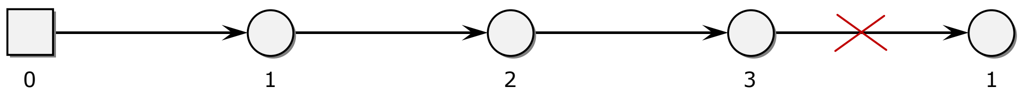
V.1(b): $\Pi_1 = \Pi_0 \cap N_1 \cup \{1\}, \Pi_1 = \{1\}$



V.1(c): $\Pi_2 = \Pi_1 \cap N_2 \cup \{2\}, \Pi_2 = \{1, 2\}$



V.1(d): $\Pi_3 = \Pi_2 \cap N_3 \cup \{3\}, \Pi_3 = \{1, 3\}$



V.1(e): Extension not allowed, $1 \in \Pi_3$

Figure V.1: Example of ng-route for $N_1 = \{1, 2\}$, $N_2 = \{2, 1\}$ and $N_3 = \{3, 1\}$.

On the other hand, in the second example, shown in Figure V.2, the ng-sets for each customer are now $N_1 = \{1, 2\}$, $N_2 = \{2, 1\}$ and $N_3 = \{3, 2\}$. The change on N_3 is enough for the path to "forget" customer 1 after the extension made in Figure V.2(d). Hence, in this case, the extension to customer 1 is now allowed, as shown in Figure V.2(e).

Given $d(P) = \sum_{i \in \mathcal{C}(P)} d_i$ as the total demand serviced by path P and $\bar{c}(P)$ as the total reduced cost of path P regarding equations (V.2) and (V.3), let $\mathcal{L}(P) = (i_p, d(P), \Pi(P), \bar{c}(P))$ be a label associated with a path P, which

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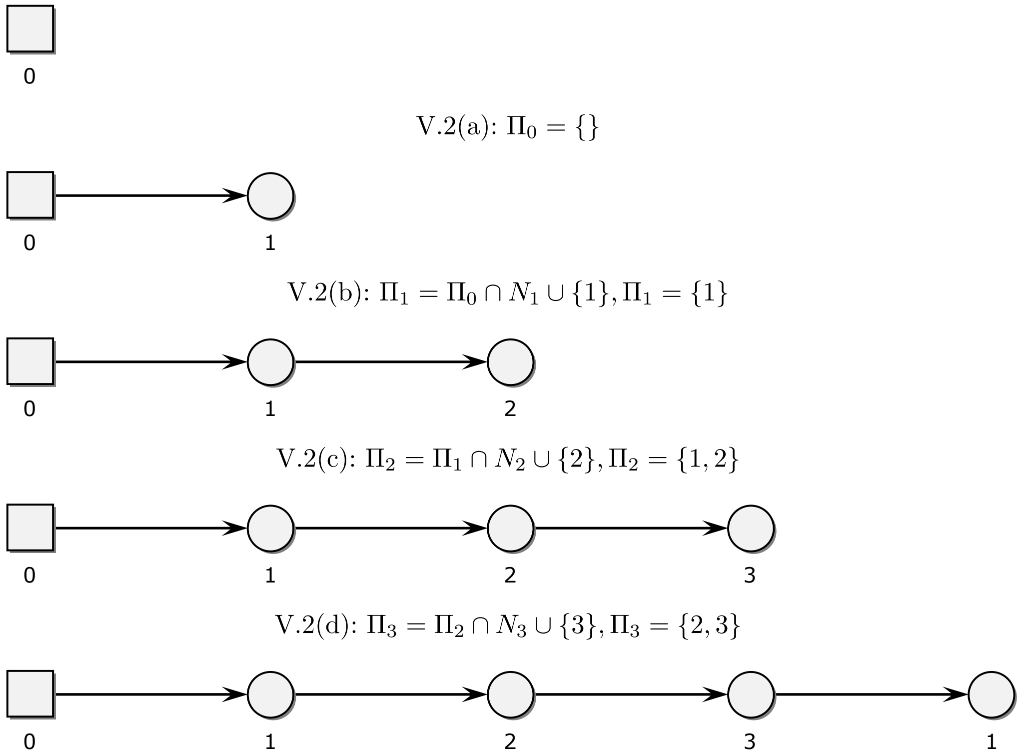

V.2(a): $\Pi_0 = \{\}$

V.2(b): $\Pi_1 = \Pi_0 \cap N_1 \cup \{1\}, \Pi_1 = \{1\}$

V.2(c): $\Pi_2 = \Pi_1 \cap N_2 \cup \{2\}, \Pi_2 = \{1, 2\}$

V.2(d): $\Pi_3 = \Pi_2 \cap N_3 \cup \{3\}, \Pi_3 = \{2, 3\}$

V.2(e): Extension allowed, $1 \notin \Pi_3$

Figure V.2: Example of ng-route for $N_1 = \{1, 2\}$, $N_2 = \{2, 1\}$ and $N_3 = \{3, 2\}$.

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ends at customer i_p, with total demand $d(P)$, prohibited extensions $\Pi(P)$, and total reduced cost $\bar{c}(P)$. We say that a label $\mathcal{L}(P)$ can be extended to a customer i_{p+1} if $i_{p+1} \notin \Pi(P)$ and $d(P) + d_{i_{p+1}} \leq Q$. After the extension, the customer i_{p+1} becomes the last customer of a new ng-path $P' = (0, \ldots, i_p, i_{p+1})$ and a new label $\mathcal{L}(P')$ can be obtained from the label $\mathcal{L}(P)$ by the following operations:

$$\mathcal{L}(P') = (i_{p+1}, d(P) + d_{i_{p+1}}, \Pi(P) \cap N_{i_{p+1}} \cup \{i_{p+1}\}, \bar{c}(P) + \bar{c}_{i_p i_{p+1}}). \qquad \text{(V.6)}$$

These labels are computed using a forward dynamic programming algorithm which, in contrast to the q-route pricing, does not result in a pseudo-polynomial complexity. This pricing is exponential on the size of the subsets $\Delta(N_i)$ and its efficiency depends on the use of some techniques to speed up its execution.

In order to reduce the number of possible paths, a dominance rule is incorporated to the algorithm. Given the labels of two paths $\mathcal{L}(P_1) = (i_p, d(P_1), \Pi(P_1), \bar{c}(P_1))$ and $\mathcal{L}(P_2) = (i_p, d(P_2), \Pi(P_2), \bar{c}(P_2))$, path P_1 dominates path P_2 if and only if any possible extension from P_2 can be done from P_1 with a lower total reduced cost. For this to be true, the following three conditions must hold:

(i) $d(P_1) \leq d(P_2)$,

(ii) $\bar{c}(P_1) \leq \bar{c}(P_2)$ and

(iii) $\Pi(P_1) \subseteq \Pi(P_2)$.

In addition, Baldacci et al. [5] described another way to improve this dominance rule. When paths with a given capacity d are being computed, the best reduced costs for every $d' < d$ are stored in a dominance list, which is faster to check than to iterate through the dynamic programming matrix for each $d' < d$. We do not use this technique because it is not scalable, since the size of this dominance list is exponential in the value of $\Delta(N_i)$, reaching its size limit when $\Delta(N_i) \approx 13$.

(c) An Efficient Exact ng-Route Pricing Implementation

As discussed earlier, the basic ng-route pricing implementation does not allow the use of large ng-sets, which weakens the quality of the lower bounds found. To address this issue, we provide an efficient pricing implementation, adapting the Decremental State Space Relaxation (DSSR) for the ng-route pricing. This technique was introduced by Righini and Salani [72] to solve the ESPPRC. The original version of the algorithm helps reducing the number of labels to be managed during the dynamic programming algorithm which prices elementary routes. Firstly, it relaxes the elementarity of the routes and, at each iteration, identifies which customers are being repeated on the best routes found and then prohibits the repetition of these customers in subsequent iterations.

The main difference of our pricing is that instead of relaxing the elementarity of the routes, the new pricing relaxes the ng-set of each customer, therefore relaxing the ng-route restrictions. The algorithm is outlined as follows.

Basic Implementation

We start by creating a $(Q+1) \times |\mathcal{C}|$ dynamic programming matrix $\mathcal{M}$, where each entry $\mathcal{M}(d,i)$ is a bucket containing labels representing ng-paths which start at the depot and end at customer i with total demand exactly d. At first, we set $\mathcal{M}(d_i,i)$ with a single label $\mathcal{L}_i = (i, d_i, \{i\}, \bar{c}_{0i})$, $\forall i \in \mathcal{C}$, and all other entries with no label. Next, a forward dynamic programming is used to fill the matrix $\mathcal{M}$, running from $d = 1$ up to $d = Q$.

When processing the bucket $\mathcal{M}(d,i)$, the algorithm passes through all labels $\mathcal{L}(P')$ belonging to $\mathcal{M}(d - d_i, j)$, for all customers $j \in \mathcal{C}$, such that

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$d - d_i \geq 0$. As the basic ng-route pricing algorithm, the extension from $\mathcal{L}(P')$ to i can only be done if $i \notin \Pi(P')$. If this condition holds, a new label $\mathcal{L}(P)$ is then created and it must be stored in the bucket $\mathcal{M}(d, i)$. Therefore, this is the right time to check for the dominance rule, which can be verified for all the labels in $\mathcal{M}(d', i), \forall d' \leq d$. This is the place where the dominance list mentioned earlier is used in the work of Baldacci et al. [5]. Surprisingly, for large values of $\Delta(N_i)$, we have found that the algorithm runs faster if the dominance rule is tested only for labels of the same bucket, i.e., for the labels from inside $\mathcal{M}(d, i)$.

This whole procedure is shown in Algorithm V.1, where the existence of a procedure called *buildRoutes* is taken into account. This procedure iterates over each bucket of the matrix $\mathcal{M}$, checking if the best path of the bucket results in a route with negative reduced cost when it goes back to the depot.

Algorithm V.1 The Basic Dynamic Programming Algorithm

```
procedure NG-ROUTEPRICING($\mathcal{M}$, $N$)
    input: matrix $\mathcal{M}$ and ng-sets $N_i \subseteq \mathcal{C}$, $\forall i \in \mathcal{C}$
    output: the best ng-routes with respect to ng-sets $N_i$

    $\mathcal{M}(d,i) \leftarrow \emptyset, \forall i \in \mathcal{C}, d \in \{0, \dots, Q\}$
    $\mathcal{M}(d_i,i) \leftarrow \{(i, d_i, \{i\}, \bar{c}_{0i})\}, \forall i \in \mathcal{C}$
    for $d := 1, \dots, Q$ do
        for all $i \in \mathcal{C}$ do
            if $d - d_i > 0$ then
                for all $j \in \mathcal{C}$ do
                    for all $\mathcal{L}(P') \in \mathcal{M}(d - d_i, j)$ do
                        if $i \notin \Pi(P')$ then
                            $\mathcal{L}(P) \leftarrow (i, d, \Pi(P') \cap N_i \cup \{i\}, \bar{c}(P') + \bar{c}_{ji})$
                            $insertLabel \leftarrow$ true
                            for all $\mathcal{L}(P'') \in \mathcal{M}(d,i)$ do
                                if $\mathcal{L}(P)$ dominates $\mathcal{L}(P'')$ then
                                    delete $\mathcal{L}(P'')$
                                else if $\mathcal{L}(P'')$ dominates $\mathcal{L}(P)$ then
                                    $insertLabel \leftarrow$ false
                                    break
                            if $insertLabel$ then
                                $\mathcal{M}(d,i) \leftarrow \mathcal{M}(d,i) \cup \mathcal{L}(P)$
    return buildRoutes($\mathcal{M}$)
```

Decremental State-Space Relaxation

The adapted DSSR is an iterative algorithm and it works by relaxing the state space of the original subsets N_i. At each iteration k, the algorithm uses the subsets $\Gamma_i^k \subseteq N_i$ as a replacement for the N_i subsets. These subsets Γ_i^k take the role of N_i in the definition of the function Π, described in (V.5), and in

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the creation of new labels, shown in (V.6). Initially, the algorithm sets $\Gamma_i^0 = \emptyset$, $\forall i \in \mathcal{C}$, and executes the complete dynamic programming. As the best routes found by this dynamic programming are not necessarily ng-routes, they cannot be considered as the result of the pricing without verifying their feasibility. This test is performed during the update of the Γ_i^k subsets, as described as follows.

Let a cycle of customers be defined as a sub-path $H = (i, \ldots, j)$, where $i = j$, and let $\mathscr{H}(P)$ be the set of all cycles of customers in the path P. In order to evaluate if the best route R_k^* is an ng-route, the algorithm must check if there is no cycle of customers $H \in \mathscr{H}(R_k^*)$ which would not be allowed to be created if the original subsets N_i were being used. This happens only when the customer i (which creates the cycle H) is in all N_l, $\forall l \in \mathcal{C}(H)$, i.e., the customer i is in the "memory" of every other customer of the cycle. If any such cycle H is found, the subsets Γ_l^{k+1}, which are initialized as $\Gamma_l^{k+1} = \Gamma_l^k$, are then updated as $\Gamma_l^{k+1} = \Gamma_l^{k+1} \cup \{i\}$, $\forall l \in \mathcal{C}(H)$. If all cycles in the set $\mathscr{H}(R_k^*)$ respect the subsets N_i, no update is done and the subsets Γ_i^{k+1} and Γ_i^k are the same for all $i \in \mathcal{C}$. In this case, we can conclude that R_k^* is an ng-route and the pricing stops. On the other hand, if any cycle $H \in \mathscr{H}(R_k^*)$ does not respect the subsets N_i, there is at least one Γ_i^{k+1} larger than Γ_i^k and, for this reason, a new iteration of the algorithm is started.

Algorithm V.2 shows the DSSR procedure. In each iteration, it calls the basic ng-route pricing algorithm *ng-RoutePricing*, shown in Algorithm V.1, and also considers the existence of three other procedures, *selectBestRoute*, which returns the route with lowest reduced cost, *isNGRoute*, which tests if the given route is indeed an ng-route with respect to the original ng-sets and *updateNGSets*, which update the sets Γ in order to force the best route to be an ng-route.

Algorithm V.2 The DSSR Algorithm

procedure DSSRPRICING($\mathcal{M}$, N)
    input: matrix $\mathcal{M}$ and ng-sets $N_i \subseteq \mathcal{C}$, $\forall i \in \mathcal{C}$
    output: the best ng-routes with respect to ng-sets N_i

    $\Gamma_i \leftarrow \emptyset, \forall i \in \mathcal{C}$, $ng \leftarrow$ **false**, $k \leftarrow 0$
    while not ng **do**
        $\mathcal{R} \leftarrow$ ng-RoutePricing($\mathcal{M}$, Γ)
        $R_k^* \leftarrow$ selectBestRoute($\mathcal{R}$)
        if isNGRoute(R_k^*, N) **then**
            $ng \leftarrow$ **true**
        else
            updateNGSets(R_k^*, N, Γ)
        $k \leftarrow k + 1$
    return R_k^*

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It is noteworthy to mention that if the best route found is indeed an ng-route, the procedure can stop and return only this route. This is how Algorithm V.2 is implemented. However, it can also return this route together with any other route with negative reduced cost and certified as being an ng-route. Furthermore, if the best route is not an ng-route, but there exists at least one route with negative reduced cost which is an ng-route, the algorithm can stop and return these ng-routes found. In this case, we consider it as being a heuristic run of the algorithm, not an exact one. It is not a problem for the column generation, except in cases where the reduced cost of the best route is necessary for further calculations.

Completion Bounds

In order to further speed up the DSSR algorithm, at some iteration k the completion bounds are calculated for each customer i with every capacity d. As mentioned before, the completion bounds are used to estimate a lower bound on the value of a route during its creation, thus discarding any route which would not lead to a negative reduced cost. Given $T_k^*(d,i)$, the best path which starts at customer i and ends at the depot with total capacity exactly d, the completion bounds $\widehat{T}(d,i)$ are calculated as shown in (V.7) and represent the best path which starts at customer i and ends at the depot with total capacity less than or equal d.

$$\widehat{T}(d,i) = \min_{d' \leq d} \{T_k^*(d',i)\}\,. \tag{V.7}$$

It is important to observe that if the corresponding problem is represented by means of an undirected graph, $T_k^*(d,i)$ can be obtained directly from the dynamic programming matrix. On the other hand, if the problem is represented using a directed graph, in order to obtain these values, the direction of the edges has to be reversed and the last iteration of the DSSR algorithm has to be executed again. This occurs because when a route is traversed in the opposite direction on an asymmetric graph, it does not generate the same cost.

Considering a problem represented by an undirected graph, the procedure which builds the completion bounds is shown in Algorithm V.3. Notice that this procedure can be called anywhere after calling the *ng-RoutePricing* procedure on line 6 of algorithm V.2.

After calculating the completion bounds, at a subsequent iteration $k' > k$ of the DSSR, they can be used to avoid the extension of a given label $\mathcal{L}(P) = (i_p, d(P), \Pi(P), \bar{c}(P))$ to a customer i_{p+1} if $\bar{c}(P)+\bar{c}_{i_p i_{p+1}}+\widehat{T}(Q-d(P), i_{p+1}) \geq 0$. This equation calculates a lower bound on the value of the reduced cost of

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Algorithm V.3 The Completion Bounds Generation

procedure GENERATECOMPLETIONBOUNDS($\mathcal{M}$)
    input: matrix $\mathcal{M}$
    output: completion bounds $\widehat{T}$

    $\widehat{T}(d,i) \leftarrow \infty, \forall i \in \mathcal{C}, d \in \{0,\dots,Q\}$
    $\widehat{T}(0,0) \leftarrow 0$
    for all $i \in \mathcal{C}$ **do**
        for $d := 1,\dots,Q$ **do**
            $\widehat{T}(d,i) \leftarrow \min(\mathcal{M}(d,i))$
            if $\widehat{T}(d-1,i) < \widehat{T}(d,i)$ **then**
                $\widehat{T}(d,i) \leftarrow \widehat{T}(d-1,i)$

any route the label can generate. Obviously, if this value is greater or equal than zero, the label cannot generate any route with a negative reduced cost, therefore it can be discarded. This test can be simply included in Algorithm V.1 together with the one in line 11.

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Heuristic Pricing

Even with the improvements described in the last section, the exact ng-route pricing still takes a long time to be executed. In the view of this, a simple but effective heuristic is developed in order to quickly price a large initial set of routes with negative reduced cost. It was inspired on the heuristic pricing done for the elementary route pricing suggested by [67]. The purpose of this heuristic is to reduce the number of calls to the exact ng-route pricing. Therefore, the heuristic ng-route pricing is used as a hot-start for the exact ng-route pricing.

The heuristic closely resembles the q-route pricing without eliminating any cycle. The main difference between the pricing algorithms is that when extending one path, the heuristic ng-route pricing respects the subsets N_i. Its data structure is also an $(Q+1)\times|\mathcal{C}|$ matrix and each entry consists of just one label. For each customer and each capacity, this label is chosen as the best one with respect to the reduced costs. Also, as the subsets N_i must be respected, each label of the dynamic programming matrix must contain the Π sets for each customer and capacity.

Notice that unlike the exact algorithm, the heuristic algorithm uses neither the dominance rules nor the speed up techniques (DSSR and completion bounds). It is straightforward to verify that the resulting complexity of this algorithm is $\mathcal{O}(n^2Q)$.

V.2 Experimental Results

For the computational experiments, all algorithms were implemented using the same configuration as described in Chapter IV. The tests compare the bounds and performance of the pricing algorithms with a variety of different configurations. The column generation starts by calling the heuristic pricing at each iteration. The heuristic pricing returns the best 20 routes with negative reduced costs. If the heuristic pricing is no longer capable of finding routes with negative reduced cost, the column generation algorithm calls the exact pricing. If the later succeeds in obtaining at least one route with negative reduced cost, the column generation procedure restarts by calling the heuristic pricing. Otherwise, the column generation stops and the current value is returned as a lower bound. The exact pricing uses the DSSR approach and needs to calculate the completion bounds at some iteration in order to speed up the DSSR. Given $\bar{c}(R_k^*)$, the reduced cost of the best route at the kth iteration of the DSSR, the exact pricing calculates completion bounds when $\bar{c}(R_k^*) > \bar{c}(R_0^*)/3$.

The results for the column generation algorithm are shown in Tables V.1-V.4. At each table, columns `Ins`, `LB` and `UB` show the name, the lower bound and the upper bound of each instance. Following these columns, the results for different values of $\Delta(N_i)$ are shown. For each X, where $\Delta(N_i) = X$, `NG=X` consists of four columns, `Value`, `Routes`, `Cuts` and `Time`, which show the value found, the number of routes, the number of cuts and the total time of each instance.

For the CARP, the algorithms were applied to the same instance datasets described in Chapter IV, except for the *egl-large* dataset. The instances of this dataset are prohibitively large for column generation. Furthermore, as a large set of cuts is known for each instance due to what was done in the previous chapter, we decided to include all the cuts found by the dual ascent heuristic and the exact separations to the master problem prior to the execution of the column generation. During the column generation, before the exact pricing is called, the exact separation of the odd-degree cutset inequalities is called and, if any violated cut is found, this cut is inserted in the master problem and the column generation restarts. The lower and upper bounds shown in Table V.1 were taken from the works of Martinelli et al. [59], Bode and Irnich [16] and Bartolini et al. [7].

For the GVRP, we applied our algorithms to the instance datasets recently generated by Bektaş et al. [8]. These instance datasets are derived from the CVRP instance datasets `A`, `B`, `P` and `M`. The transformation is performed using a method similar to that of Fischetti et al. [29], which transforms TSP

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instances into GTSP instances. The number of clusters is $t = \lceil n/\theta \rceil$, where θ is a parameter defined *a priori*. For each original CVRP instance dataset, two new instance datasets are created, using $\theta = 2$ and $\theta = 3$, resulting in 158 GVRP instances. All lower and upper bounds shown in Tables V.2 and V.3 were taken from the work of Bektaş et al. [8]. Analogously to the CARP, during the column generation, before the exact pricing is called, the capacity cuts are separated using the package CVRPSEP [56] and, if any violated cut is found, this cut is inserted in the master problem and the column generation restarts.

For the CVRP, we used the classical instance datasets `A`, `B`, `E`, `P` and `M`, available at `www.branchandcut.org` [71]. All the lower and upper bounds shown in Table V.4 were taken from the work of Baldacci et al. [4], except the upper bounds for instances `M-n200-k16` and `M-n265-k25`, which were found by running the hybrid algorithm proposed by Subramanian et al. [74].

Notice that the lower bounds obtained with `NG=32` and `NG=64` are identical for almost all instances and, in fact, these values are very close to those obtained by a column generation with an elementary route pricing algorithm, such as the results presented in [68] for the CVRP, which were found by running the algorithm developed by Pecin [67]. This is an empirical evidence that the routes found by the pricing with large ng-sets (greater than 32) are almost elementary when the average size is up to 12 or 13 customers. Moreover, the runtime is typically much smaller than that required if the elementarity constraint is imposed on the routes.

Our algorithm was not capable of solving some instances for large ng-sets within the time limit of three hours. This is probably due to the large average size of the routes that are part of an optimal solution, causing the number of labels to be treated by the dynamic programming algorithm to be prohibitive. In the work of Pecin [67], the results of elementary route pricing for classical CVRP instances with up to 101 vertices are shown, but none for larger instances. In contrast, our ng-route pricing algorithm allowed for calculating excellent lower bounds for instances greater than 100 vertices, such as `M-n151-k12`, `M-n200-k16`, and `M-n200-k17`, in which the best known lower bounds were found using complicated state-of-the-art column and cut generation algorithms.

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Table V.1: Column Generation Results for the CARP

Ins	LB	UB	NG=8				NG=16				NG=32				NG=64			
			Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time
kshs1	14661	14661	14661.0	116	5	0.140	14661.0	92	5	0.094	14661.0	92	5	0.093	14661.0	92	5	0.094
kshs2	9863	9863	9863.0	124	8	0.125	9863.0	88	8	0.093	9863.0	88	8	0.078	9863.0	88	8	0.094
kshs3	9320	9320	9320.0	167	4	0.125	9320.0	93	4	0.109	9320.0	93	4	0.094	9320.0	93	4	0.110
kshs4	11498	11498	11498.0	134	5	0.156	11498.0	99	5	0.140	11498.0	99	5	0.125	11498.0	99	5	0.156
kshs5	10957	10957	10957.0	110	4	0.140	10957.0	112	4	0.141	10957.0	112	4	0.141	10957.0	112	4	0.219
kshs6	10197	10197	10197.0	234	4	0.172	10197.0	123	4	0.110	10197.0	123	4	0.110	10197.0	123	4	0.140
gdb1	316	316	316.0	259	12	0.015	316.0	235	12	0.016	316.0	201	12	0.016	316.0	201	12	0.016
gdb2	339	339	339.0	260	8	0.031	339.0	303	8	0.031	339.0	290	8	0.031	339.0	290	8	0.031
gdb3	275	275	275.0	217	5	0.016	275.0	200	5	0.016	275.0	191	5	0.016	275.0	191	5	0.016
gdb4	287	287	287.0	126	6	0.000	287.0	86	6	0.000	287.0	86	6	0.015	287.0	86	6	0.015
gdb5	377	377	377.0	228	8	0.031	377.0	200	8	0.016	377.0	197	8	0.031	377.0	197	8	0.016
gdb6	298	298	298.0	158	5	0.016	298.0	160	4	0.015	298.0	143	4	0.016	298.0	143	4	0.016
gdb7	325	325	325.0	219	10	0.016	325.0	147	10	0.016	325.0	192	10	0.016	325.0	192	10	0.016
gdb8	348	348	346.2	623	20	0.344	346.2	518	20	0.297	346.2	498	20	0.360	346.2	470	20	0.360
gdb9	303	303	303.0	628	18	0.438	303.0	609	17	0.422	303.0	649	17	0.453	303.0	468	17	0.375
gdb10	275	275	275.0	245	12	0.031	275.0	228	12	0.031	275.0	237	13	0.031	275.0	237	13	0.047
gdb11	395	395	395.0	1105	13	0.953	395.0	925	13	0.781	395.0	1105	13	0.969	395.0	645	13	0.703
gdb12	458	458	452.7	242	11	0.062	454.0	221	11	0.063	454.0	195	11	0.078	454.0	195	11	0.093
gdb13	536	536	536.0	412	5	0.157	536.0	400	5	0.156	536.0	291	5	0.125	536.0	291	5	0.141
gdb14	100	100	100.0	151	1	0.031	100.0	165	1	0.016	100.0	135	1	0.016	100.0	135	1	0.031
gdb15	58	58	58.0	112	1	0.063	58.0	130	1	0.031	58.0	93	1	0.031	58.0	93	1	0.046
gdb16	127	127	127.0	378	6	0.078	127.0	322	6	0.078	127.0	228	7	0.063	127.0	228	7	0.078
gdb17	91	91	91.0	134	7	0.046	91.0	134	7	0.047	91.0	74	7	0.031	91.0	74	7	0.047
gdb18	164	164	164.0	565	1	0.235	164.0	685	1	0.296	164.0	385	1	0.156	164.0	365	1	0.188
gdb19	55	55	55.0	55	4	0.016	55.0	60	4	0.000	55.0	60	4	0.015	55.0	60	4	0.016
gdb20	121	121	121.0	265	9	0.062	121.0	205	10	0.062	121.0	187	10	0.047	121.0	187	10	0.063

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Table V.1: *Continued from previous page*

Ins	LB	UB	NG=8				NG=16				NG=32				NG=64			
			Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time
gdb21	156	156	156.0	414	7	0.141	156.0	334	6	0.094	156.0	294	7	0.094	156.0	254	7	0.110
gdb22	200	200	200.0	352	7	0.156	200.0	332	6	0.156	200.0	292	5	0.125	200.0	252	6	0.171
gdb23	233	233	233.0	372	1	0.219	233.0	372	1	0.219	233.0	332	1	0.203	233.0	272	1	0.219
1A	173	173	173.0	2715	20	6.531	173.0	2704	20	7.610	173.0	2380	20	7.500	173.0	1396	20	5.750
1B	173	173	173.0	1762	20	2.937	173.0	1410	20	2.719	173.0	1483	20	2.906	173.0	1277	20	2.813
1C	245	245	241.3	483	18	0.406	241.3	603	18	0.547	241.3	425	18	0.531	241.3	373	18	0.656
2A	227	227	227.0	2107	12	3.906	227.0	2138	11	5.016	227.0	1250	11	6.047	227.0	1323	11	6.641
2B	259	259	257.5	2030	25	3.500	258.0	2021	25	5.063	258.0	1708	25	8.188	258.0	1559	25	35.95
2C	457	457	457.0	406	16	0.328	457.0	398	16	0.297	457.0	345	16	0.266	457.0	372	16	0.406
3A	81	81	81.0	2371	25	3.109	81.0	2555	23	4.016	81.0	2085	24	4.422	81.0	1378	29	2.891
3B	87	87	87.0	1740	23	1.562	87.0	1782	26	1.766	87.0	1378	26	1.391	87.0	1363	27	1.563
3C	138	138	136.0	462	11	0.156	136.0	473	12	0.172	136.0	447	12	0.172	136.0	495	13	0.219
4A	400	400	400.0	8603	37	86.44	400.0	9467	37	113.3	400.0	10767	37	192.8	400.0	8227	36	171.1
4B	412	412	412.0	4116	34	22.05	412.0	5396	36	37.22	412.0	6076	34	52.02	412.0	3936	34	32.20
4C	428	428	428.0	3105	33	13.33	428.0	3705	33	17.19	428.0	3065	33	14.41	428.0	3085	33	17.22
4D	530	530	524.9	2103	50	6.329	525.3	2037	51	6.297	525.4	1816	52	6.766	525.4	1813	52	15.33
5A	423	423	423.0	4147	25	24.13	423.0	5187	26	35.42	423.0	4827	26	37.08	423.0	3847	26	37.49
5B	446	446	444.2	3629	44	18.19	444.2	3848	41	22.63	444.2	4163	40	29.77	444.2	2703	40	39.89
5C	474	474	469.3	2716	42	11.34	469.5	3068	46	14.67	469.5	2805	39	15.20	469.5	2551	44	28.47
5D	577	577	572.1	1433	27	3.672	572.3	1233	19	3.735	572.8	1456	63	6.063	572.8	1450	58	11.80
6A	223	223	223.0	2603	20	7.531	223.0	2820	21	8.891	223.0	2650	21	8.813	223.0	2587	20	42.49
6B	233	233	229.0	1948	22	4.234	229.8	2168	75	6.344	229.8	2076	58	7.250	229.8	1919	65	8.938
6C	317	317	310.8	610	22	0.765	311.1	506	23	0.813	311.1	593	23	1.016	311.1	522	22	1.110
7A	279	279	279.0	4507	23	23.22	279.0	4019	21	24.31	279.0	5376	21	40.17	279.0	3619	21	34.83
7B	283	283	283.0	3616	21	14.11	283.0	3576	21	17.49	283.0	3815	21	19.36	283.0	3135	21	25.95
7C	334	334	328.5	1744	43	7.656	328.9	1655	53	6.625	328.9	1609	52	6.078	328.9	1593	60	18.05
8A	386	386	386.0	3167	23	15.57	386.0	3127	23	16.44	386.0	3567	23	21.36	386.0	2147	23	15.81

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Table V.1: *Continued from previous page*

Ins	LB	UB	NG=8				NG=16				NG=32				NG=64			
			Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time
8B	395	395	395.0	2496	24	9.610	395.0	3116	21	13.05	395.0	3216	22	14.92	395.0	1916	21	9.891
8C	521	521	517.2	1049	46	2.687	517.2	1054	46	2.688	517.2	1057	46	2.766	517.2	938	46	3.328
9A	323	323	323.0	9427	54	140.0	323.0	11226	54	213.3	323.0	10767	53	242.8	323.0	11467	54	395.7
9B	326	326	326.0	9556	49	127.5	326.0	10656	49	193.1	326.0	16016	50	638.9	326.0	18216	49	1626
9C	332	332	332.0	5325	44	43.17	332.0	6665	44	66.33	332.0	6025	45	61.63	332.0	6525	44	91.94
9D	391	391	383.9	3328	79	15.80	383.9	3318	94	20.71	383.9	3276	85	17.86	383.9	3283	112	27.11
10A	428	428	428.0	10329	44	212.9	428.0	15513	44	577.4	428.0	20027	44	1392	428.0	23827	44	3103
10B	436	436	436.0	7216	59	108.9	436.0	7696	59	140.6	436.0	8396	58	179.5	436.0	8876	58	234.4
10C	446	446	446.0	6485	52	83.19	446.0	6405	51	85.28	446.0	7465	50	115.8	446.0	7665	51	151.9
10D	526	526	524.6	2639	60	15.89	524.6	2521	50	15.30	524.6	2485	60	16.86	524.6	2395	58	18.02
C01	4105	4150	4094.5	3646	101	15.61	4100.3	3376	95	23.55	4101.3	3369	87	19.53	4101.3	2820	94	23.27
C02	3135	3135	3135.0	2081	32	3.469	3135.0	2013	32	4.094	3135.0	1853	32	3.578	3135.0	1640	32	4.281
C03	2575	2575	2548.7	2426	54	5.390	2548.7	2429	55	6.594	2548.8	1927	59	4.016	2548.8	1757	51	4.813
C04	3478	3510	3474.7	4164	57	13.78	3475.1	4149	62	17.69	3476.6	4413	72	22.53	3476.6	3224	62	47.99
C05	5365	5365	5319.9	1956	64	4.844	5323.3	1897	57	5.328	5323.4	1963	57	5.953	5323.5	1756	57	6.110
C06	2535	2535	2515.9	2720	56	5.219	2518.8	2426	63	8.016	2519.8	2514	63	11.94	2519.8	2092	56	93.89
C07	4075	4075	4022.1	1975	54	3.469	4022.5	2008	54	3.875	4022.5	1843	54	4.485	4022.5	1970	54	5.156
C08	4090	4090	4039.5	2262	91	6.313	4039.5	2198	81	6.406	4039.5	2001	74	5.938	4039.5	1636	76	7.891
C09	5233	5260	5222.5	4905	69	27.39	5224.3	3873	73	25.69	5224.8	3862	78	26.78	5224.8	3642	82	33.52
C10	4700	4700	4620.7	1410	69	3.031	4623.8	1398	65	4.016	4624.0	1216	65	3.219	4624.0	1109	65	3.422
C11	4583	4635	4573.7	5676	158	34.33	4574.2	5894	154	40.50	4575.0	5402	187	41.27	4575.0	5659	190	61.88
C12	4209	4240	4172.7	3392	72	11.59	4173.6	4067	76	15.13	4173.8	3558	71	14.49	4173.8	2773	71	14.84
C13	2955	2955	2911.0	1735	40	2.953	2911.0	1526	40	2.937	2911.0	1543	40	3.485	2911.0	1304	40	8.641
C14	4030	4030	3990.7	1799	35	3.219	3990.7	1985	35	4.188	3990.7	1778	37	4.719	3990.7	1439	35	4.922
C15	4912	4940	4890.2	6512	92	43.34	4892.3	6846	95	58.41	4892.7	7024	86	66.98	4892.7	6083	94	83.84
C16	1475	1475	1470.0	2201	25	3.594	1470.0	1559	28	4.297	1470.0	1256	25	2934	1470.0	1216	25	21.67
C17	3555	3555	3550.0	1444	44	1.922	3550.0	1245	44	1.813	3550.0	1032	44	1.672	3550.0	966	44	2.375

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Table V.1: *Continued from previous page*

Ins	LB	UB	NG=8				NG=16				NG=32				NG=64			
			Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time
C18	5577	5620	5557.9	8764	115	97.83	5559.6	9199	115	134.8	5559.6	9188	115	153.3	—	—	—	—
C19	3096	3115	3082.0	2791	80	7.937	3084.2	2568	77	16.34	3091.5	2467	103	32.00	3091.5	2144	81	223.0
C20	2120	2120	2120.0	4452	59	10.16	2120.0	3683	53	7.735	2120.0	3223	38	7.875	2120.0	2670	38	71.25
C21	3960	3970	3956.2	5333	50	18.81	3956.3	4448	50	16.92	3959.0	4693	53	23.49	3959.0	4470	54	36.86
C22	2245	2245	2245.0	2420	36	2.937	2245.0	2086	36	3.515	2245.0	1673	36	3.844	2245.0	2220	36	13.00
C23	4032	4085	**4035.9**	6725	134	50.80	**4042.8**	6287	133	225.6	**4044.4**	7200	135	4943	**4046.2**	6060	136	6000
C24	3384	3400	3379.4	4573	83	20.28	3379.4	4908	53	22.72	3381.3	5886	64	41.13	3381.2	4225	61	50.02
C25	2310	2310	2310.0	1406	28	1.313	2310.0	1584	28	1.610	2310.0	970	28	1.047	2310.0	1042	28	1.656
D01	3215	3215	3215.0	8421	54	67.35	3215.0	10840	53	118.5	3215.0	10856	61	134.9	3215.0	12990	54	316.2
D02	2520	2520	2520.0	3618	25	9.266	2520.0	4276	25	13.41	2520.0	4744	25	17.91	2520.0	2845	25	22.11
D03	2065	2065	2065.0	5906	31	19.06	2065.0	6161	30	23.08	2065.0	5749	31	26.06	2065.0	3228	31	22.94
D04	2785	2785	2785.0	7941	52	46.91	2785.0	9448	51	84.33	2785.0	10169	51	129.5	2785.0	7774	49	200.5
D05	3935	3935	3935.0	4870	49	22.80	3935.0	4790	49	23.66	3935.0	4870	49	26.53	3935.0	4170	49	27.88
D06	2125	2125	2125.0	5378	30	17.27	2125.0	5941	30	23.16	2125.0	6861	30	38.69	2125.0	3482	36	16.30
D07	3115	3115	3045.2	4114	52	25.03	3046.2	5164	58	41.95	3049.4	5437	60	396.6	3049.7	3187	51	938.8
D08	2995	3045	**3001.0**	7165	42	43.70	**3003.6**	7212	41	69.72	**3010.9**	7083	53	80.70	**3010.9**	5607	56	203.0
D09	4120	4120	4120.0	11462	50	121.0	4120.0	14270	50	267.5	4120.0	19502	48	698.3	4120.0	18342	50	931.4
D10	3340	3340	3331.6	5185	33	26.23	3331.7	4906	33	20.28	3331.8	4213	33	1111	—	—	—	—
D11	3745	3745	3745.0	12155	71	146.8	3745.0	11215	71	160.9	3745.0	12575	71	232.8	3745.0	10794	71	233.7
D12	3310	3310	3310.0	8855	49	61.59	3310.0	9775	49	86.56	3310.0	10688	48	121.1	3310.0	7015	48	70.17
D13	2535	2535	2535.0	4279	37	12.91	2535.0	4769	36	16.97	2535.0	3461	36	12.38	2535.0	2848	36	13.33
D14	3272	3280	3271.7	5179	46	28.95	3271.8	5510	50	42.08	—	—	—	—	—	—	—	—
D15	3990	3990	3990.0	18902	55	355.1	3990.0	20299	54	436.4	3990.0	24021	54	657.0	—	—	—	—
D16	1060	1060	1060.0	5622	19	20.44	1060.0	6662	16	132.7	—	—	—	—	—	—	—	—
D17	2620	2620	2620.0	2874	27	5.672	2620.0	2659	26	5.985	2620.0	1936	26	5.437	2620.0	1700	26	7.437
D18	4165	4165	4165.0	18322	67	392.2	4165.0	19485	66	530.6	4165.0	19922	67	658.8	4165.0	23942	67	1482
D19	2393	2400	2372.6	9522	45	54.30	—	—	—	—	—	—	—	—	—	—	—	—

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Table V.1: *Continued from previous page*

Ins	LB	UB	NG=8				NG=16				NG=32				NG=64			
			Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time
D20	1870	1870	1870.0	6911	33	23.36	1870.0	8744	34	58.75	1870.0	7562	34	46.17	1870.0	3979	34	1020
D21	2985	3050	2972.4	11003	60	121.4	2974.5	11880	58	124.0	2974.5	14951	38	311.2	2974.5	11555	41	287.5
D22	1865	1865	1865.0	6840	32	20.22	1865.0	9213	32	43.09	1865.0	5867	32	78.89	1865.0	3147	34	25.09
D23	3114	3130	**3116.8**	21716	97	671.3	**3116.8**	21246	99	10033	—	—	—	—	—	—	—	—
D24	2676	2710	2672.9	11907	65	105.7	2673.0	17033	89	410.8	—	—	—	—	—	—	—	—
D25	1815	1815	1815.0	3501	24	6.563	1815.0	3841	58	10.33	1815.0	3376	24	356.5	1815.0	1888	24	5.813
E01	4885	4910	4855.3	3275	73	14.19	4856.2	3788	72	19.61	4856.9	3350	74	17.50	4856.9	2886	76	38.80
E02	3990	3990	3965.4	2043	51	3.766	3965.8	1807	51	4.891	3966.0	1811	51	5.516	3966.0	1416	51	7.375
E03	2015	2015	2015.0	3253	30	6.860	2015.0	3372	30	5.563	2015.0	2839	30	6.313	2015.0	2353	32	11.77
E04	4155	4155	4132.7	4477	111	17.91	4133.6	4052	93	26.02	4133.7	4197	99	26.31	4133.7	3225	107	29.61
E05	4585	4585	4573.7	2301	78	7.000	4573.8	2308	65	5.953	4577.0	2129	65	6.531	4577.0	1785	65	6.563
E06	2055	2055	2055.0	1668	25	2.391	2055.0	1704	25	2.125	2055.0	1315	25	1.688	2055.0	1429	25	7.641
E07	4155	4155	4064.0	1146	64	2.188	4066.3	1114	66	3.000	4066.3	1031	63	4.437	4066.3	837	63	5.860
E08	4710	4710	4677.5	2219	72	4.985	4680.6	1821	73	4.328	4680.6	1681	73	4.250	4680.6	1622	73	4.891
E09	5780	5820	5773.4	5622	145	38.17	5773.9	4903	133	33.52	5774.7	5079	146	39.39	5774.7	3983	143	33.66
E10	3605	3605	3605.0	1411	58	2.094	3605.0	1486	58	3.313	3605.0	1098	58	1.969	3605.0	1294	58	2.703
E11	4637	4655	4631.1	5275	125	36.66	4632.2	4948	125	36.08	4632.6	5263	126	50.52	4632.6	5445	126	66.53
E12	4180	4180	4128.6	2576	87	6.954	4128.7	2512	87	10.00	4128.9	2761	88	8.906	4128.9	1948	86	9.453
E13	3345	3345	3312.5	1709	60	3.344	3313.0	1438	60	3.969	3313.6	1364	60	129.6	3313.6	1306	60	90.58
E14	4115	4115	4092.0	1868	61	3.579	4092.1	1755	56	3.485	4092.1	1336	55	10.88	4092.1	1365	54	12.61
E15	4189	4205	4182.0	7049	81	48.84	4184.5	7601	75	63.39	4186.6	7099	80	297.5	—	—	—	—
E16	3755	3775	**3759.8**	3205	69	6.969	**3760.1**	2619	69	7.203	**3760.1**	2810	69	7.672	**3760.1**	1808	69	16.24
E17	2740	2740	2740.0	1248	38	1.297	2740.0	1144	37	1.156	2740.0	1146	39	1.328	2740.0	890	38	1.188
E18	3825	3835	3825.0	6194	86	36.80	**3825.6**	7368	97	62.08	**3825.7**	6336	92	58.81	**3825.8**	6325	91	73.25
E19	3222	3235	3215.7	3881	109	14.03	3217.4	3735	99	14.70	3219.2	2860	98	16.08	3219.4	2606	94	81.28
E20	2802	2825	2798.9	3314	65	9.547	2798.9	2846	67	9.047	2798.9	2727	60	8.360	2798.9	2302	62	15.58
E21	3728	3730	3727.6	4189	54	12.91	3727.6	4015	54	13.95	3727.6	4019	54	16.03	3727.6	3667	57	647.8

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Table V.1: *Continued from previous page*

Ins	LB	UB	NG=8				NG=16				NG=32				NG=64			
			Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time
E22	2470	2470	2465.9	1880	60	6.437	2466.1	1823	56	10.06	2466.6	1636	56	43.70	2466.6	1494	56	130.1
E23	3686	3710	3684.8	5211	114	32.78	**3688.0**	5620	114	48.80	**3688.4**	5672	114	77.92	**3688.4**	4631	114	116.5
E24	4001	4020	3999.2	4386	95	20.80	**4004.8**	4566	98	24.38	**4006.4**	4789	106	30.30	**4006.4**	3731	101	25.47
E25	1615	1615	1615.0	956	20	0.578	1615.0	1155	21	1.063	1615.0	662	20	1.047	1615.0	662	20	1.875
F01	4040	4040	4040.0	7975	93	78.02	4040.0	10155	85	123.4	4040.0	10915	94	195.7	4040.0	9135	88	195.7
F02	3300	3300	3300.0	6067	49	26.00	3300.0	5381	49	24.27	3300.0	7296	49	52.39	3300.0	3718	49	20.05
F03	1665	1665	1665.0	6982	36	21.27	1665.0	7249	36	25.33	1665.0	4442	39	48.56	1665.0	2949	38	6398
F04	3476	3485	3475.8	11068	90	133.4	3476.0	9914	89	345.1	**3476.5**	10814	82	6415	—	—	—	—
F05	3605	3605	3605.0	4154	65	17.69	3605.0	5074	65	27.69	3605.0	5171	66	31.27	3605.0	3453	65	19.94
F06	1875	1875	1875.0	3149	28	6.469	1875.0	3353	28	7.860	1875.0	2812	28	7.765	1875.0	1973	28	7.906
F07	3335	3335	3335.0	2594	59	6.657	3335.0	2948	59	8.235	3335.0	2238	59	6.438	3335.0	1773	59	7.969
F08	3695	3705	3692.8	5015	69	23.53	3693.1	5542	69	31.17	3693.1	5781	69	60.55	3693.1	4492	69	68.80
F09	4730	4730	4730.0	11522	88	151.9	4730.0	15582	88	344.7	4730.0	15702	87	449.7	4730.0	15243	87	598.1
F10	2925	2925	2925.0	3345	44	8.406	2925.0	3680	44	10.70	2925.0	2588	44	8.250	2925.0	2248	44	18.61
F11	3835	3835	3835.0	12694	76	169.1	3835.0	15695	76	292.8	3835.0	20413	76	675.9	3835.0	21295	76	1484
F12	3390	3395	3386.1	7611	66	46.42	3386.1	9017	66	67.42	3386.2	7785	66	58.83	3386.3	4978	66	286.8
F13	2855	2855	2855.0	2723	48	6.922	2855.0	3004	48	8.687	2855.0	2730	48	8.125	2855.0	2106	48	8.156
F14	3330	3330	3330.0	4082	50	14.97	3330.0	5078	50	231.9	3330.0	5333	50	2018	3330.0	3024	50	885.8
F15	3560	3560	3560.0	16941	74	301.4	3560.0	23295	70	589.8	3560.0	21400	70	566.7	3560.0	19554	71	1027
F16	2725	2725	2725.0	7115	38	25.92	2725.0	7537	37	36.48	2725.0	8646	37	3381	—	—	—	—
F17	2055	2055	2055.0	2981	31	5.906	2055.0	3206	31	8.453	2055.0	1497	31	3.141	—	—	—	—
F18	3063	3075	3061.6	12888	68	136.4	3061.9	16706	65	703.6	—	—	—	—	—	—	—	—
F19	2500	2525	2490.9	8545	65	57.25	—	—	—	—	—	—	—	—	—	—	—	—
F20	2445	2445	2445.0	8308	54	46.80	2445.0	9674	54	74.11	2445.0	9913	54	92.11	2445.0	6610	54	92.63
F21	2930	2930	2930.0	9647	51	66.03	2930.0	11328	51	111.1	2930.0	11865	51	138.5	2930.0	6428	51	66.74
F22	2075	2075	2075.0	5529	50	17.830	2075.0	6951	56	84.11	2075.0	4756	49	323.6	—	—	—	—
F23	2994	3005	**2996.9**	19076	153	413.7	**2998.8**	27193	112	3012	—	—	—	—	—	—	—	—

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Table V.1: *Continued from previous page*

Ins	LB	UB	NG=8				NG=16				NG=32				NG=64			
			Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time
F24	3210	3210	3210.0	13141	88	173.9	3210.0	16598	102	960.1	3210.0	22045	88	9437	—	—	—	—
F25	1390	1390	1390.0	2023	22	2.610	1390.0	2092	22	4.031	1390.0	736	22	3.766	1390.0	736	22	7.562
e1-A	3548	3548	3548.0	3116	55	26.73	3548.0	2898	55	23.16	3548.0	2623	55	18.63	3548.0	1717	55	26.05
e1-B	4498	4498	4472.5	1877	78	9.297	4472.5	1531	78	10.88	4473.4	1555	78	8.828	4473.4	1356	78	10.10
e1-C	5595	5595	5541.8	1312	70	6.938	5544.8	1227	70	6.860	5544.8	1012	70	5.485	5544.8	894	70	9.922
e2-A	5018	5018	5006.0	4177	82	46.77	5008.3	3980	84	132.6	5008.3	3734	84	692.3	—	—	—	—
e2-B	6305	6317	6291.7	2623	81	22.53	6296.6	2773	78	32.39	6298.7	2576	78	380.5	6299.3	2508	78	96.41
e2-C	8335	8335	8270.6	1895	80	13.05	8274.4	1777	80	12.42	8274.4	1666	80	12.24	8274.4	1366	80	16.50
e3-A	5898	5898	5895.9	5959	75	103.7	5895.9	5989	75	104.1	5895.9	6794	75	179.7	5895.9	5567	75	535.5
e3-B	7711	7775	7696.9	3091	77	38.44	7701.0	3087	77	63.86	7702.8	2857	76	45.08	7704.6	2450	76	90.83
e3-C	10244	10292	10180.0	2034	84	17.86	10183.6	2183	84	20.55	10183.9	1882	84	17.86	10184.1	1708	84	20.44
e4-A	6408	6444	6389.7	6453	105	123.1	6390.7	7517	116	608.8	—	—	—	—	—	—	—	—
e4-B	8935	8961	8884.8	2714	72	51.38	8889.3	2755	71	47.66	8890.4	2576	71	52.72	8890.4	2547	71	92.75
e4-C	11493	11550	11465.3	2146	77	28.25	11467.5	2418	78	29.27	11467.5	2326	77	34.60	11467.5	1918	77	33.02
s1-A	5018	5018	5014.5	5674	143	100.3	5014.5	8792	143	188.0	5014.5	7544	143	167.2	5014.5	6216	143	413.2
s1-B	6388	6388	6377.4	3272	154	35.28	6378.0	2932	154	31.91	6378.0	3753	154	54.77	6378.0	3109	154	237.9
s1-C	8518	8518	8484.4	1821	156	17.03	8487.1	1663	153	14.34	8487.2	1903	153	17.56	8487.2	1452	153	37.09
s2-A	9825	9884	9800.4	7940	142	310.6	9801.6	7711	145	330.6	9801.6	8474	139	498.5	9801.6	7903	145	2464
s2-B	13017	13100	12965.9	5049	165	138.8	12972.5	5060	153	209.0	12976.3	4782	185	201.4	12978.6	4698	175	223.1
s2-C	16425	16425	16347.9	2979	164	70.55	16355.8	3268	155	88.17	16358.0	3622	156	93.00	16358.4	3009	156	106.8
s3-A	10145	10220	10140.7	9409	162	623.3	10144.7	9585	178	895.4	**10147.4**	9717	203	1964	—	—	—	—
s3-B	13648	13682	13618.7	5279	151	182.1	13621.9	5027	183	213.2	13623.3	5270	189	240.1	13623.5	4924	185	323.8
s3-C	17188	17188	17096.8	3615	138	103.0	17112.2	3165	146	89.74	17113.3	3199	150	90.34	17113.7	3011	145	102.6
s4-A	12143	12268	12127.4	9100	159	536.8	12133.0	8910	137	640.4	12136.6	9971	141	1639	12137.1	9402	141	4629
s4-B	16098	16283	16065.5	6326	325	299.0	16076.7	5636	384	377.3	16078.0	5283	343	376.4	16078.6	5585	358	497.1
s4-C	20430	20481	20384.8	4220	158	176.6	20394.2	4144	151	196.4	20397.2	4054	147	199.7	20397.2	3951	151	320.3

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Table V.2: Column Generation Results for the GVRP ($\theta = 2$)

Ins	LB	UB	NG=8				NG=16				NG=32				NG=64			
			Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time
A-n32-k5-C16-V2	519.0	519	516.3	248	4	0.501	516.7	190	1	0.324	516.7	190	1	0.322	516.7	190	1	0.558
A-n33-k5-C17-V3	451.0	451	451.0	315	20	0.346	451.0	216	27	0.288	451.0	202	20	0.289	451.0	202	20	0.359
A-n33-k6-C17-V3	465.0	465	465.0	299	3	0.254	465.0	208	3	0.263	465.0	164	3	0.239	465.0	164	3	0.348
A-n34-k5-C17-V3	489.0	489	489.0	252	0	0.230	489.0	172	0	0.299	489.0	143	0	0.198	489.0	143	0	0.270
A-n36-k5-C18-V2	505.0	505	500.9	437	8	0.681	504.5	300	2	0.486	504.5	223	5	0.359	504.5	223	5	0.477
A-n37-k5-C19-V3	432.0	432	432.0	490	0	0.365	432.0	472	0	0.560	432.0	256	0	0.341	432.0	256	0	0.433
A-n37-k6-C19-V3	584.0	584	584.0	379	6	0.450	584.0	313	6	0.536	584.0	225	4	0.463	584.0	225	4	0.744
A-n38-k5-C19-V3	476.0	476	476.0	386	5	0.474	476.0	322	5	0.389	476.0	219	5	0.319	476.0	219	5	0.435
A-n39-k5-C20-V3	557.0	557	543.8	559	55	0.630	547.1	443	45	0.581	547.3	368	51	0.595	547.3	368	51	0.717
A-n39-k6-C20-V3	544.0	544	544.0	423	29	0.640	544.0	355	19	0.538	544.0	290	18	0.471	544.0	290	18	0.690
A-n44-k6-C22-V3	608.0	608	608.0	517	21	0.624	608.0	365	0	0.387	608.0	321	0	0.585	608.0	321	0	1.002
A-n45-k6-C23-V4	613.0	613	608.4	633	46	0.759	608.4	635	29	0.864	608.4	402	39	0.757	608.4	402	39	1.046
A-n45-k7-C23-V4	674.0	674	662.6	523	51	0.631	663.1	547	50	0.658	663.2	350	50	0.675	663.2	350	50	0.986
A-n46-k7-C23-V4	593.0	593	590.7	634	21	0.786	591.8	541	19	0.949	591.8	412	22	1.073	591.8	412	22	1.783
A-n48-k7-C24-V4	667.0	667	655.0	609	68	1.029	655.4	591	58	1.143	655.5	410	56	0.954	655.5	410	56	1.288
A-n53-k7-C27-V4	603.0	603	603.0	894	28	1.409	603.0	937	28	1.611	603.0	757	58	1.644	603.0	715	58	2.706
A-n54-k7-C27-V4	690.0	690	689.5	1010	111	1.971	690.0	899	91	2.009	690.0	625	54	1.253	690.0	625	54	1.961
A-n55-k9-C28-V5	699.0	699	699.0	644	36	1.082	699.0	658	31	1.775	699.0	467	32	2.281	699.0	467	32	3.951
A-n60-k9-C30-V5	769.0	769	769.0	834	12	1.229	769.0	734	21	1.469	769.0	596	20	1.138	769.0	596	20	1.701
A-n61-k9-C31-V5	638.0	638	635.3	784	68	1.548	636.2	807	101	1.909	636.3	756	111	1.708	636.3	756	111	2.080
A-n62-k8-C31-V4	740.0	740	740.0	1176	201	3.190	740.0	1150	157	2.701	740.0	915	127	2.677	740.0	915	127	3.848
A-n63-k10-C32-V5	801.0	801	794.0	808	68	2.057	794.0	858	65	1.384	794.0	741	57	1.446	794.0	725	66	1.766
A-n63-k9-C32-V5	900.3	912	**906.8**	904	109	2.430	**907.0**	807	104	1.578	**907.0**	740	99	3.004	**907.0**	642	92	2.770
A-n64-k9-C32-V5	763.0	763	763.0	958	0	1.800	763.0	978	0	1.460	763.0	827	0	1.553	763.0	675	0	1.462
A-n65-k9-C33-V5	682.0	682	681.6	997	66	1.547	681.5	1040	73	2.436	681.5	989	71	1.808	681.5	783	65	1.992
A-n69-k9-C35-V5	680.0	680	671.4	1026	94	1.766	672.6	940	63	2.317	672.6	948	72	2.053	672.6	782	63	3.729

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Table V.2: *Continued from previous page*

Ins	LB	UB	NG=8				NG=16				NG=32				NG=64			
			Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time
A-n80-k10-C40-V5	957.4	997	**982.4**	1595	71	4.184	**982.7**	1507	65	4.768	**983.0**	1619	56	6.152	**982.8**	1485	90	7.868
B-n31-k5-C16-V3	441.0	441	441.0	310	8	0.269	441.0	165	8	0.213	441.0	165	8	0.214	441.0	165	8	0.271
B-n34-k5-C17-V3	472.0	472	472.0	390	5	0.416	472.0	247	5	0.390	472.0	232	5	0.311	472.0	232	5	0.423
B-n35-k5-C18-V3	626.0	626	626.0	436	18	0.474	626.0	290	4	0.328	626.0	267	5	0.423	626.0	267	5	0.605
B-n38-k6-C19-V3	451.0	451	451.0	384	4	0.515	451.0	336	7	0.462	451.0	230	5	0.345	451.0	230	5	0.431
B-n39-k5-C20-V3	357.0	357	357.0	523	5	0.807	357.0	421	4	0.860	357.0	348	7	0.752	357.0	348	7	1.119
B-n41-k6-C21-V3	481.0	481	481.0	490	31	0.802	481.0	429	6	0.756	481.0	274	11	0.719	481.0	274	11	1.100
B-n43-k6-C22-V3	483.0	483	481.8	792	54	1.090	482.0	642	42	0.843	482.0	470	46	1.086	482.0	470	46	1.533
B-n44-k7-C22-V4	540.0	540	540.0	637	35	0.786	540.0	591	31	1.239	540.0	501	40	0.741	540.0	501	40	0.890
B-n45-k5-C23-V3	497.0	497	497.0	815	16	1.308	497.0	492	0	0.807	497.0	466	0	0.963	497.0	451	0	1.057
B-n45-k6-C23-V4	478.0	478	474.2	567	25	0.713	474.5	577	22	0.884	474.5	462	15	0.749	474.5	475	23	0.790
B-n50-k7-C25-V4	449.0	449	449.0	740	0	1.654	449.0	602	0	0.911	449.0	454	0	0.576	449.0	442	0	2.372
B-n50-k8-C25-V5	916.0	916	912.6	537	58	0.996	913.2	487	38	0.748	913.2	386	25	0.838	913.2	386	25	1.098
B-n51-k7-C26-V4	651.0	651	651.0	654	4	0.925	651.0	472	4	0.971	651.0	416	4	0.782	651.0	416	4	1.158
B-n52-k7-C26-V4	450.0	450	450.0	838	11	1.567	450.0	812	11	1.294	450.0	571	13	1.000	450.0	582	11	1.460
B-n56-k7-C28-V4	486.0	486	484.7	1022	29	2.461	486.0	1032	20	1.904	486.0	742	16	1.788	486.0	742	16	2.451
B-n57-k7-C29-V4	751.0	751	751.0	940	40	1.456	751.0	915	35	1.857	751.0	692	25	1.488	751.0	692	25	2.127
B-n57-k9-C29-V5	942.0	942	942.0	606	22	0.772	942.0	695	24	0.924	942.0	666	34	1.358	942.0	666	34	1.790
B-n63-k10-C32-V5	816.0	816	809.0	953	45	1.726	809.0	1084	58	2.609	809.0	863	53	2.430	809.0	862	52	1.887
B-n64-k9-C32-V5	509.0	509	509.0	952	5	1.560	509.0	900	1	2.240	509.0	770	1	2.045	509.0	740	1	2.310
B-n66-k9-C33-V5	808.0	808	808.0	1037	53	1.992	808.0	1055	50	2.065	808.0	978	78	2.317	808.0	922	55	2.017
B-n67-k10-C34-V5	673.0	673	667.7	1087	77	2.046	667.8	1192	67	2.136	667.7	1079	65	2.243	667.7	943	70	2.237
B-n68-k9-C34-V5	704.0	704	704.0	1020	18	2.747	704.0	998	3	2.370	704.0	920	0	7.059	704.0	1063	44	6.034
B-n78-k10-C39-V5	803.0	803	803.0	1420	57	3.339	803.0	1365	48	3.335	803.0	1246	36	3.237	803.0	1216	48	5.456
P-n16-k8-C8-V5	239.0	239	239.0	15	0	0.007	239.0	15	0	0.007	239.0	15	0	0.012	239.0	15	0	0.007
P-n19-k2-C10-V2	147.0	147	147.0	73	0	0.105	147.0	42	0	0.106	147.0	42	0	0.158	147.0	42	0	0.145

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Table V.2: *Continued from previous page*

Ins	LB	UB	NG=8				NG=16				NG=32				NG=64			
			Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time
P-n20-k2-C10-V2	154.0	154	154.0	82	0	0.168	154.0	38	0	0.126	154.0	38	0	0.133	154.0	38	0	0.164
P-n21-k2-C11-V2	160.0	160	160.0	130	0	0.321	160.0	42	0	0.253	160.0	42	0	0.251	160.0	42	0	0.415
P-n22-k2-C11-V2	162.0	162	162.0	120	0	0.216	162.0	65	0	0.229	162.0	65	0	0.230	162.0	65	0	0.351
P-n22-k8-C11-V5	314.0	314	314.0	48	0	1.240	314.0	47	0	1.241	314.0	47	0	1.230	314.0	47	0	1.245
P-n23-k8-C12-V5	312.0	312	312.0	54	10	0.026	312.0	57	8	0.028	312.0	57	8	0.028	312.0	57	8	0.030
P-n40-k5-C20-V3	294.0	294	294.0	506	12	1.000	294.0	434	21	0.912	294.0	225	27	0.741	294.0	225	27	1.233
P-n45-k5-C23-V3	337.0	337	337.0	576	19	1.003	337.0	597	9	1.344	337.0	369	13	1.240	337.0	369	13	2.064
P-n50-k7-C25-V4	353.0	353	350.5	572	42	1.244	350.6	560	35	1.520	350.6	404	31	1.369	350.6	404	31	2.036
P-n50-k8-C25-V4	378.4	392	**385.7**	372	18	0.913	**385.8**	391	17	0.749	**385.8**	304	11	1.052	**385.8**	304	11	1.457
P-n50-k10-C25-V5	410.0	410	407.9	392	56	0.974	409.0	412	44	1.241	409.0	341	27	0.820	409.0	341	27	1.252
P-n51-k10-C26-V6	427.0	427	427.0	406	0	0.424	427.0	391	3	0.482	427.0	325	0	0.368	427.0	325	0	0.528
P-n55-k7-C28-V4	361.0	361	355.7	749	30	1.771	355.7	712	27	1.963	355.7	494	28	1.380	355.7	494	28	1.605
P-n55-k8-C28-V4	361.0	361	359.6	658	60	1.644	359.7	717	37	1.937	359.6	467	39	1.637	359.6	467	39	2.494
P-n55-k10-C28-V5	415.0	415	411.9	522	51	1.035	412.1	515	45	1.037	412.2	374	47	1.155	412.2	374	47	1.609
P-n55-k15-C28-V8	545.3	555	**555.0**	219	4	0.399	**555.0**	212	4	0.385	**555.0**	216	3	0.350	**555.0**	216	3	0.538
P-n60-k10-C30-V5	433.0	443	**435.4**	519	39	1.760	**435.5**	491	43	2.339	**435.3**	426	39	2.540	**435.4**	426	39	3.621
P-n60-k15-C30-V8	553.9	565	**564.7**	374	32	0.607	**564.8**	336	21	0.614	**564.8**	294	26	0.591	**564.8**	294	26	0.674
P-n65-k10-C33-V5	487.0	487	484.8	750	99	2.051	485.4	669	87	2.042	485.4	688	78	1.922	485.4	579	69	3.345
P-n70-k10-C35-V5	485.0	485	485.0	765	87	2.366	485.0	695	60	2.889	485.0	741	63	2.033	485.0	595	61	17.53
P-n76-k4-C38-V2	383.0	383	379.4	2027	26	23.54	380.7	1835	11	24.45	380.7	1641	10	110.8	380.7	1383	10	2185
P-n76-k5-C38-V3	405.0	405	400.1	1596	60	14.63	401.4	1753	81	17.46	401.6	1588	65	24.58	401.6	1253	59	40.28
P-n101-k4-C51-V2	455.0	455	452.4	4739	51	146.0	452.8	5071	26	342.7	—	—	—	—	—	—	—	—
M-n101-k10-C51-V5	542.0	542	540.4	2077	22	19.85	540.4	2182	17	19.08	540.4	1953	17	19.57	540.5	2153	15	41.26
M-n121-k7-C61-V4	707.7	719	**710.3**	4773	156	112.9	**710.7**	6086	170	131.0	**710.8**	6850	176	1656	**710.9**	4887	177	7919
M-n151-k12-C76-V6	629.9	659	**649.5**	3272	296	52.39	**650.0**	3554	275	62.55	**649.9**	3356	208	52.22	**650.2**	2971	244	86.25
M-n200-k16-C100-V8	744.9	791	**777.8**	4180	382	88.92	**778.4**	4366	284	109.9	**778.8**	4313	231	128.7	**778.8**	4241	217	192.9

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Table V.3: Column Generation Results for the GVRP ($\theta = 3$)

Ins	LB	UB	NG=8				NG=16				NG=32				NG=64			
			Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time
A-n32-k5-C11-V2	386.0	386	382.3	112	5	0.303	382.3	84	7	0.269	382.3	84	7	0.262	382.3	84	7	0.361
A-n33-k5-C11-V2	315.0	315	315.0	109	3	0.203	315.0	85	3	0.180	315.0	85	3	0.180	315.0	85	3	0.238
A-n33-k6-C11-V2	370.0	370	370.0	92	0	0.180	370.0	58	0	0.177	370.0	58	0	0.177	370.0	58	0	0.258
A-n34-k5-C12-V2	419.0	419	418.3	137	19	0.392	418.3	107	16	0.475	418.3	107	16	0.471	418.3	107	16	0.785
A-n36-k5-C12-V2	396.0	396	385.9	192	14	0.458	385.9	104	8	0.323	385.9	104	8	0.323	385.9	104	8	0.440
A-n37-k5-C13-V2	347.0	347	347.0	187	0	0.395	347.0	106	0	0.700	347.0	106	0	0.712	347.0	106	0	1.317
A-n37-k6-C13-V2	431.0	431	431.0	138	4	0.244	431.0	84	0	0.209	431.0	84	0	0.198	431.0	84	0	0.295
A-n38-k5-C13-V2	367.0	367	367.0	158	0	0.287	367.0	110	0	0.401	367.0	110	0	0.400	367.0	110	0	0.679
A-n39-k5-C13-V2	364.0	364	358.0	222	18	0.523	359.2	162	12	0.706	359.2	162	12	0.705	359.2	162	12	1.148
A-n39-k6-C13-V2	403.0	403	403.0	172	2	0.375	403.0	113	2	0.294	403.0	113	2	0.293	403.0	113	2	0.395
A-n44-k6-C15-V2	503.0	503	503.0	206	0	0.450	503.0	152	0	0.737	503.0	152	0	0.735	503.0	152	0	1.244
A-n45-k6-C15-V3	474.0	474	474.0	250	5	0.510	474.0	155	5	0.558	474.0	155	5	0.555	474.0	155	5	0.847
A-n45-k7-C15-V3	475.0	475	475.0	208	0	0.480	475.0	137	0	0.304	475.0	137	0	0.302	475.0	137	0	0.398
A-n46-k7-C16-V3	462.0	462	462.0	303	18	0.545	462.0	206	14	0.650	462.0	206	14	0.651	462.0	206	14	1.041
A-n48-k7-C16-V3	451.0	451	449.4	262	7	0.535	451.0	138	5	4.554	451.0	138	5	4.558	451.0	138	5	6.942
A-n53-k7-C18-V3	440.0	440	440.0	525	0	1.148	440.0	246	0	1.073	440.0	214	3	1.042	440.0	214	3	1.800
A-n54-k7-C18-V3	482.0	482	482.0	355	8	0.870	482.0	285	8	0.979	482.0	256	4	1.051	482.0	256	4	1.831
A-n55-k9-C19-V3	473.0	473	473.0	346	20	0.787	473.0	361	19	1.159	473.0	202	21	1.355	473.0	202	21	2.572
A-n60-k9-C20-V3	595.0	595	592.8	480	23	1.086	593.5	451	28	1.604	593.5	305	29	1.566	593.5	305	29	2.665
A-n61-k9-C21-V4	473.0	473	473.0	354	7	0.787	473.0	368	7	0.914	473.0	280	7	1.086	473.0	280	7	1.859
A-n62-k8-C21-V3	596.0	596	593.0	486	74	1.376	594.0	447	67	1.499	594.8	347	54	1.306	594.8	347	54	1.868
A-n63-k10-C21-V4	593.0	593	591.7	432	8	0.952	592.3	361	12	0.841	592.3	305	9	0.805	592.3	305	9	1.101
A-n63-k9-C21-V3	625.6	642	**629.7**	462	19	1.317	**636.3**	384	15	1.417	**636.3**	300	7	1.007	**636.3**	300	7	1.605
A-n64-k9-C22-V3	536.0	536	536.0	649	28	1.496	536.0	653	29	1.387	536.0	523	63	2.148	536.0	523	63	3.037
A-n65-k9-C22-V3	500.0	500	500.0	414	0	0.875	500.0	388	0	0.882	500.0	303	0	0.749	500.0	303	0	0.987
A-n69-k9-C23-V3	520.0	520	515.3	640	39	1.916	519.8	432	23	2.049	520.0	313	16	1.577	520.0	313	16	2.347

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Table V.3: *Continued from previous page*

Ins	LB	UB	NG=8				NG=16				NG=32				NG=64			
			Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time
A-n80-k10-C27-V4	679.4	710	**706.5**	779	17	2.132	**708.8**	797	32	2.326	**708.8**	636	23	2.394	**708.8**	636	23	3.278
B-n31-k5-C11-V2	356.0	356	354.5	133	4	0.187	354.5	66	2	0.147	354.5	66	2	0.147	354.5	66	2	0.180
B-n34-k5-C12-V2	369.0	369	369.0	154	8	0.305	369.0	94	4	0.334	369.0	94	4	0.334	369.0	94	4	0.546
B-n35-k5-C12-V2	501.0	501	501.0	171	4	0.434	501.0	108	7	0.389	501.0	108	7	0.389	501.0	108	7	0.618
B-n38-k6-C13-V2	370.0	370	370.0	144	0	0.277	370.0	93	0	0.255	370.0	93	0	0.262	370.0	93	0	0.356
B-n39-k5-C13-V2	280.0	280	280.0	193	0	0.402	280.0	149	0	0.481	280.0	149	0	0.471	280.0	149	0	0.799
B-n41-k6-C14-V2	407.0	407	407.0	156	4	0.316	407.0	146	4	0.547	407.0	146	4	0.559	407.0	146	4	0.907
B-n45-k5-C15-V2	410.0	410	410.0	297	0	0.860	410.0	153	0	0.419	410.0	153	0	0.422	410.0	153	0	0.606
B-n43-k6-C15-V2	343.0	343	343.0	347	21	1.017	343.0	165	14	1.044	343.0	165	14	1.041	343.0	165	14	1.956
B-n45-k6-C15-V2	336.0	336	336.0	227	0	0.388	336.0	150	0	0.373	336.0	150	0	0.374	336.0	150	0	0.547
B-n44-k7-C15-V3	395.0	395	394.3	239	4	0.615	394.3	156	4	0.516	394.3	156	4	0.515	394.3	156	4	0.790
B-n50-k8-C17-V3	598.0	598	598.0	310	27	0.530	598.0	240	21	0.457	598.0	216	20	0.447	598.0	216	20	0.532
B-n51-k7-C17-V3	511.0	511	511.0	285	1	0.560	511.0	204	1	0.574	511.0	200	1	0.525	511.0	200	1	0.789
B-n50-k7-C17-V3	393.0	393	393.0	312	1	0.778	393.0	244	3	0.831	393.0	178	1	1.195	393.0	178	1	1.970
B-n52-k7-C18-V3	359.0	359	359.0	469	2	1.026	359.0	370	5	1.157	359.0	309	7	1.022	359.0	309	7	1.569
B-n57-k9-C19-V3	681.0	681	674.0	527	51	1.667	674.0	419	30	1.265	674.0	372	35	1.185	674.0	372	35	1.495
B-n56-k7-C19-V3	356.0	356	355.0	544	6	1.375	355.5	411	6	1.310	355.5	238	6	0.874	355.5	238	6	1.326
B-n57-k7-C19-V3	558.0	558	558.0	460	11	1.423	558.0	410	6	1.464	558.0	316	10	1.560	558.0	316	10	2.717
B-n63-k10-C21-V3	599.0	599	599.0	425	0	1.519	599.0	340	0	1.565	599.0	257	0	1.974	599.0	257	0	3.888
B-n66-k9-C22-V3	609.0	609	595.8	611	44	1.531	596.2	637	22	1.601	596.5	451	26	2.232	596.5	451	26	3.226
B-n64-k9-C22-V4	452.0	452	452.0	424	0	1.333	452.0	363	0	1.013	452.0	268	0	0.964	452.0	268	0	1.492
B-n67-k10-C23-V4	558.0	558	551.0	475	8	1.387	551.0	423	7	1.137	551.0	338	8	1.190	551.0	338	8	1.921
B-n68-k9-C23-V3	523.0	523	523.0	667	35	1.621	523.0	495	12	1.760	523.0	326	7	2.202	523.0	326	7	4.188
B-n78-k10-C26-V4	606.0	606	606.0	756	35	1.724	606.0	721	15	3.002	606.0	568	10	1.954	606.0	568	10	3.034
P-n16-k8-C6-V4	170.0	170	170.0	17	0	0.013	170.0	17	0	0.008	170.0	17	0	0.007	170.0	17	0	0.014
P-n19-k2-C7-V1	111.0	111	111.0	26	0	0.137	111.0	26	0	0.078	111.0	26	0	0.077	111.0	26	0	0.160

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Table V.3: *Continued from previous page*

Ins	LB	UB	NG=8				NG=16				NG=32				NG=64			
			Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time
P-n20-k2-C7-V1	117.0	117	117.0	21	0	0.128	117.0	21	0	0.091	117.0	21	0	0.088	117.0	21	0	0.115
P-n21-k2-C7-V1	117.0	117	117.0	22	0	0.115	117.0	22	0	0.108	117.0	22	0	0.108	117.0	22	0	0.222
P-n22-k2-C8-V1	111.0	111	111.0	25	0	0.103	111.0	25	0	0.102	111.0	25	0	0.102	111.0	25	0	0.141
P-n22-k8-C8-V4	249.0	249	249.0	31	0	1.170	249.0	31	0	1.177	249.0	31	0	1.177	249.0	31	0	1.221
P-n23-k8-C8-V3	174.0	174	174.0	34	0	0.022	174.0	34	0	0.023	174.0	34	0	0.022	174.0	34	0	0.025
P-n40-k5-C14-V2	213.0	213	212.2	250	12	0.944	212.2	132	11	0.969	212.2	132	11	0.973	212.2	132	11	1.741
P-n45-k5-C15-V2	238.0	238	237.7	264	9	1.209	237.7	137	8	0.794	237.7	137	8	0.791	237.7	137	8	1.289
P-n50-k7-C17-V3	261.0	261	259.0	286	16	1.059	259.0	209	4	0.809	259.0	177	11	0.962	259.0	177	11	1.510
P-n50-k8-C17-V3	262.0	262	262.0	206	0	0.447	262.0	170	0	0.505	262.0	190	0	0.558	262.0	190	0	0.850
P-n50-k10-C17-V4	292.0	292	292.0	204	3	0.434	292.0	164	3	0.478	292.0	154	3	0.422	292.0	154	3	0.529
P-n51-k10-C17-V4	309.0	309	305.5	209	6	0.375	305.5	187	6	0.335	305.5	167	5	0.316	305.5	167	5	0.413
P-n55-k7-C19-V3	271.0	271	267.8	385	7	1.704	267.9	327	5	1.230	267.9	264	5	1.169	267.9	264	5	1.492
P-n55-k8-C19-V3	274.0	274	271.6	356	14	1.453	271.6	295	16	1.439	271.6	223	10	1.091	271.6	223	10	1.485
P-n55-k10-C19-V4	301.0	301	300.8	274	13	0.717	301.0	266	16	0.663	301.0	185	11	0.584	301.0	185	11	0.718
P-n55-k15-C19-V6	378.0	378	378.0	141	0	0.250	378.0	161	0	0.289	378.0	130	0	0.267	378.0	130	0	0.374
P-n60-k10-C20-V4	325.0	325	320.7	312	20	1.112	320.7	267	25	1.214	320.7	234	22	1.157	320.7	234	22	1.709
P-n60-k15-C20-V5	380.0	382	**380.3**	172	9	0.562	**381.5**	172	10	0.675	**381.5**	156	9	0.693	**381.5**	156	9	1.053
P-n65-k10-C22-V4	372.0	372	369.1	383	11	1.326	369.4	383	18	1.689	369.4	280	16	1.131	369.4	280	16	1.579
P-n70-k10-C24-V4	385.0	385	382.0	514	34	2.401	382.6	429	33	1.721	382.6	343	28	1.597	382.6	343	28	2.423
P-n76-k4-C26-V2	309.0	309	303.4	1114	28	13.39	303.5	1039	16	20.35	303.5	716	16	29.99	303.5	716	16	48.04
P-n76-k5-C26-V2	309.0	309	309.0	856	0	6.453	309.0	821	0	13.61	309.0	497	0	7.282	309.0	497	0	11.65
P-n101-k4-C34-V2	370.0	370	367.3	3014	43	85.14	367.8	3122	54	304.1	—	—	—	—	—	—	—	—
M-n101-k10-C34-V4	458.0	458	456.3	1181	50	11.09	456.9	1183	61	13.44	456.9	943	58	17.63	456.9	964	51	45.84
M-n121-k7-C41-V3	527.0	527	526.9	2883	42	69.99	527.0	2629	23	147.7	527.0	3287	34	10001	527.0	2241	32	11210
M-n151-k12-C51-V4	465.6	483	**482.3**	2229	107	37.62	**483.0**	2142	126	38.88	**483.0**	2095	105	56.54	**483.0**	1972	155	103.1
M-n200-k16-C67-V6	563.1	605	**592.4**	2504	161	78.75	**593.6**	2400	119	58.65	**594.0**	2445	147	68.53	**594.2**	2606	119	152.5

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Table V.4: Column Generation Results for the CVRP

Ins	LB	UB	NG=8				NG=16				NG=32				NG=64			
			Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time
A-n32-k5	784	784	784.0	1017	61	0.948	784.0	1009	38	0.860	784.0	779	55	0.610	784.0	779	55	0.793
A-n33-k5	661	661	661.0	818	66	0.569	661.0	748	43	0.484	661.0	649	44	0.489	661.0	674	45	1.793
A-n33-k6	742	742	740.3	825	49	0.578	740.3	821	68	0.776	740.3	752	57	0.552	740.3	667	59	0.784
A-n34-k5	778	778	775.5	926	69	0.981	776.0	958	95	1.134	776.2	826	98	0.906	776.0	764	59	0.891
A-n36-k5	799	799	798.4	1221	77	1.130	798.7	1352	60	1.218	798.7	1221	61	1.135	798.7	981	53	1.298
A-n37-k5	669	669	667.3	1530	89	1.554	667.3	1560	76	1.703	667.2	1330	79	1.354	667.4	1178	69	1.901
A-n37-k6	949	949	937.0	915	63	1.181	940.4	997	44	0.893	940.1	985	55	3.378	940.7	883	83	3.055
A-n38-k5	730	730	723.4	1204	124	1.232	723.4	1290	83	1.367	723.4	1220	77	1.287	723.4	1279	85	1.893
A-n39-k5	822	822	818.0	1545	182	2.164	818.1	1529	271	2.804	819.2	1424	194	2.104	819.0	1261	199	1.804
A-n39-k6	831	831	824.5	1264	113	1.310	825.2	1225	84	1.262	825.2	1198	88	1.418	825.2	1024	91	1.737
A-n44-k6	937	937	934.9	1120	174	1.321	936.8	1311	142	1.586	936.8	1254	163	1.765	936.8	1219	183	1.903
A-n45-k6	944	944	941.2	1340	107	2.015	941.8	1384	170	2.218	942.5	1276	86	1.844	942.5	1181	134	3.841
A-n45-k7	1146	1146	1140.5	1329	203	1.970	1141.5	1224	236	2.036	1142.3	1323	167	2.016	1142.4	1143	192	2.171
A-n46-k7	914	914	914.0	1582	188	2.019	914.0	1294	103	1.873	914.0	1401	203	2.739	914.0	1353	203	5.508
A-n48-k7	1073	1073	1071.6	1460	157	2.100	1071.8	1482	173	2.365	1072.0	1626	170	3.144	1072.0	1381	157	5.438
A-n53-k7	1010	1010	1003.7	2233	223	4.908	1004.2	2159	134	3.824	1004.5	2389	111	4.539	1004.5	2110	122	8.414
A-n54-k7	1167	1167	1155.6	1852	268	4.217	1156.8	1999	275	5.150	1157.1	2219	245	4.688	1156.6	1860	279	5.446
A-n55-k9	1073	1073	1068.5	1384	158	2.551	1069.4	1332	204	2.974	1069.4	1461	200	3.928	1069.3	1387	179	5.175
A-n60-k9	1354	1354	1345.0	1891	186	3.847	1345.6	1817	173	3.703	1345.7	1925	198	4.327	1345.7	1947	171	6.334
A-n61-k9	1034	1034	1023.7	1577	255	3.501	1024.6	1478	288	4.066	1024.6	1534	312	5.264	1024.6	1770	295	8.005
A-n62-k8	1288	1288	1280.9	2567	288	8.051	1282.1	2690	204	9.503	1282.0	2489	212	9.422	1282.1	2577	207	19.43
A-n63-k9	1616	1616	1608.6	1888	259	4.420	1609.1	2051	232	5.168	1610.3	2199	206	5.681	1610.8	2153	224	6.641
A-n63-k10	1314	1314	1302.4	1715	183	3.868	1303.9	1794	149	4.163	1304.0	1817	132	4.730	1304.0	2046	147	7.308

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Table V.4: *Continued from previous page*

Ins	LB	UB	NG=8				NG=16				NG=32				NG=64			
			Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time
A-n64-k9	1401	1401	1387.8	2069	177	5.159	1389.6	2046	158	5.269	1390.0	2208	152	6.275	1390.1	2268	161	10.42
A-n65-k9	1174	1174	1168.5	2074	269	4.643	1169.0	1905	223	4.844	1169.0	1892	190	4.423	1169.0	2098	222	5.948
A-n69-k9	1159	1159	1144.2	2338	228	4.688	1145.2	2344	231	5.015	1145.4	2156	221	5.417	1145.4	2367	214	8.407
A-n80-k10	1763	1763	1756.5	3192	217	11.76	1757.0	3184	200	12.03	1756.7	3238	174	11.50	1756.8	3070	191	15.00
B-n31-k5	672	672	672.0	931	14	0.474	672.0	940	11	0.595	672.0	746	10	0.741	672.0	746	10	0.980
B-n34-k5	788	788	784.9	1435	76	1.259	785.0	1335	80	1.179	785.2	1202	95	1.298	785.2	1351	96	9.624
B-n35-k5	955	955	955.0	1347	22	1.030	955.0	1396	29	0.948	955.0	1164	26	0.897	955.0	1014	31	1.246
B-n38-k6	805	805	804.1	1187	51	0.907	804.1	1185	44	1.075	804.1	1146	43	1.340	804.1	1048	37	1.657
B-n39-k5	549	549	549.0	2003	46	3.173	549.0	1382	37	18.01	549.0	1408	46	5.338	549.0	1294	42	306.7
B-n41-k6	829	829	828.5	1243	86	1.384	828.6	1236	80	1.494	828.8	1316	70	1.467	828.8	1023	83	1.567
B-n43-k6	742	742	737.3	1414	109	1.821	737.6	1383	136	2.016	737.7	1637	97	2.083	737.6	1336	129	2.120
B-n44-k7	909	909	909.0	1588	132	1.861	909.0	1269	37	1.482	909.0	1484	49	1.997	909.0	1352	88	2.171
B-n45-k5	751	751	750.6	2301	81	3.802	751.0	2323	90	7.787	751.0	2276	98	31.35	751.0	1963	65	12.70
B-n45-k6	678	678	678.0	1612	98	3.213	678.0	1540	82	4.320	678.0	1466	95	5.385	678.0	1526	88	3.676
B-n50-k7	741	741	741.0	2156	58	3.114	741.0	2034	64	2.483	741.0	2265	57	3.885	741.0	1976	63	4.278
B-n50-k8	1312	1312	1303.4	1597	181	2.381	1303.6	1630	138	2.853	1303.7	1799	136	3.019	1303.7	1541	137	3.882
B-n51-k7	1032	1032	1026.8	1994	135	3.833	1027.2	1965	121	3.348	1027.3	1737	110	3.233	1027.2	1833	98	3.922
B-n52-k7	747	747	746.3	2304	109	4.313	746.5	2105	104	3.694	746.5	1968	79	3.027	746.5	1996	83	8.596
B-n56-k7	707	707	705.0	2562	65	3.994	705.0	2480	51	4.173	705.0	1928	24	3.901	705.0	2599	60	13.82
B-n57-k7	1153	1153	1153.0	2409	168	7.198	1153.0	2030	85	6.627	1153.0	2170	40	7.708	1153.0	1934	51	7.625
B-n57-k9	1598	1598	1596.0	2022	171	3.563	1596.2	1805	189	3.969	1596.2	2049	131	4.047	1596.2	1799	133	5.268
B-n63-k10	1496	1496	1487.2	1840	172	4.447	1487.4	1847	167	4.690	1487.4	1937	152	4.843	1487.4	2202	165	6.355
B-n64-k9	861	861	860.5	2645	199	5.895	860.5	2639	214	6.630	860.6	2369	137	5.117	860.6	2738	237	9.453

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Table V.4: *Continued from previous page*

Ins	LB	UB	NG=8				NG=16				NG=32				NG=64			
			Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time
B-n66-k9	1316	1316	1308.9	2552	236	7.893	1308.9	2525	247	7.698	1309.2	2342	218	8.401	1309.4	2667	193	20.40
B-n67-k10	1032	1032	1027.6	2369	128	6.346	1028.1	2283	147	5.750	1028.1	2511	131	6.055	1028.1	2499	136	9.254
B-n68-k9	1272	1272	1263.8	2854	246	9.085	1263.9	2698	206	10.15	1263.9	2546	256	8.934	1263.9	2979	273	18.90
B-n78-k10	1221	1221	1217.0	3134	257	12.22	1217.4	3133	187	11.37	1217.5	3377	182	13.95	1217.6	3478	187	15.54
E-n13-k4	247	247	247.0	61	0	0.020	247.0	78	0	0.025	247.0	78	0	0.025	247.0	78	0	0.027
E-n22-k4	375	375	375.0	403	35	0.150	375.0	368	35	0.148	375.0	310	35	0.158	375.0	310	35	0.220
E-n23-k3	569	569	569.0	889	2	19.73	569.0	544	13	18.14	569.0	328	13	14.71	569.0	328	13	15.70
E-n30-k3	534	534	518.5	1643	61	2.391	523.4	1535	58	2.882	523.4	910	46	2.619	523.4	910	46	3.692
E-n31-k7	379	379	378.3	918	13	0.572	378.5	996	14	0.648	378.5	560	14	0.409	378.5	560	14	0.491
E-n33-k4	835	835	835.0	1330	72	6.385	835.0	1154	66	5.374	835.0	1010	45	6.749	835.0	1117	48	14.79
E-n51-k5	521	521	519.3	2031	153	4.953	519.4	2004	116	4.358	519.4	1700	107	4.946	519.5	1725	146	6.393
E-n76-k7	682	682	671.1	3082	144	13.26	671.3	3177	112	15.86	671.9	2922	93	17.95	671.9	2888	121	30.19
E-n76-k8	735	735	726.8	2836	303	14.46	727.0	2908	261	15.07	727.3	2812	224	12.40	727.3	2770	199	18.58
E-n76-k10	830	830	817.1	1932	143	5.277	817.4	1921	190	6.704	818.0	1860	174	5.941	818.0	2317	161	11.12
E-n76-k14	1021	1021	1006.9	1514	47	3.578	1007.5	1353	70	3.145	1007.5	1381	72	3.145	1007.5	1321	74	4.510
E-n101-k8	815	815	804.4	5224	392	48.57	804.9	5180	451	58.38	805.1	5328	339	69.69	805.1	5213	415	171.8
E-n101-k14	1067	1067	1053.7	2898	151	10.31	1054.1	3031	154	11.86	1055.0	2835	168	11.02	1055.0	2878	168	14.09
P-n16-k8	450	450	448.0	61	7	0.017	448.0	60	7	0.016	448.0	60	7	0.021	448.0	60	7	0.017
P-n19-k2	212	212	212.0	436	1	0.278	212.0	289	1	0.398	212.0	207	2	0.242	212.0	207	2	0.273
P-n20-k2	216	216	213.0	559	3	0.398	215.5	383	5	0.365	215.5	288	4	0.325	215.5	288	4	0.414
P-n21-k2	211	211	211.0	606	6	0.511	211.0	409	0	0.447	211.0	298	0	0.440	211.0	298	0	0.672
P-n22-k2	216	216	216.0	581	14	0.522	216.0	475	5	0.672	216.0	361	5	0.565	216.0	361	5	0.851
P-n22-k8	603	603	603.0	198	10	0.033	603.0	150	0	0.028	603.0	165	0	0.032	603.0	165	0	0.038

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Table V.4: *Continued from previous page*

Ins	LB	UB	NG=8				NG=16				NG=32				NG=64			
			Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time	Value	Routes	Cuts	Time
P-n23-k8	529	529	529.0	193	0	0.040	529.0	162	0	0.033	529.0	129	0	0.029	529.0	129	0	0.032
P-n40-k5	458	458	457.9	1201	161	1.508	458.0	1310	143	1.658	458.0	1185	146	1.787	458.0	1156	140	1.879
P-n45-k5	510	510	507.0	1377	181	2.623	507.3	1353	150	2.676	507.3	1428	181	2.816	507.3	1308	189	3.847
P-n50-k7	554	554	551.5	1200	145	2.691	552.0	1273	148	2.978	551.7	1245	134	2.616	551.7	1240	137	3.689
P-n50-k8	631	631	617.2	877	73	1.475	618.0	895	47	1.561	618.0	854	52	1.683	618.0	886	55	2.449
P-n50-k10	696	696	689.4	779	39	1.009	689.6	783	40	1.085	689.6	759	47	1.082	689.6	797	43	2.897
P-n51-k10	741	741	736.4	850	92	1.169	736.3	951	120	1.080	736.3	1076	119	1.472	736.1	996	102	1.868
P-n55-k7	568	568	559.2	1476	55	3.391	559.8	1330	85	2.965	559.8	1384	97	4.326	559.8	1326	59	8.831
P-n55-k10	694	694	681.8	947	73	1.514	681.9	943	102	1.609	681.9	960	89	1.595	681.9	1038	100	2.415
P-n55-k15	989	989	972.5	587	103	0.937	972.8	584	67	0.844	972.8	560	70	0.911	972.8	591	81	1.009
P-n55-k8	588	588	572.4	1427	201	3.294	573.0	1423	96	4.594	573.1	1352	90	3.294	573.1	1441	90	4.394
P-n60-k10	744	744	738.9	1086	59	1.827	739.7	1038	65	1.977	739.7	1111	47	2.151	739.7	1037	73	4.801
P-n60-k15	968	968	963.5	830	77	1.164	963.6	773	42	1.246	963.6	704	44	1.002	963.6	808	41	1.258
P-n65-k10	792	792	787.1	1338	86	3.130	788.3	1277	101	2.855	788.3	1291	101	2.994	788.3	1378	95	4.135
P-n70-k10	827	827	814.3	1508	164	4.686	815.3	1503	138	5.588	815.3	1617	124	4.969	815.3	1704	144	6.800
P-n76-k4	593	593	589.2	5308	232	63.95	589.9	5480	183	85.47	590.1	5193	154	309.8	590.0	5340	179	6333
P-n76-k5	627	627	617.8	4091	297	32.15	617.9	4013	172	27.99	618.6	4017	199	40.30	618.6	3857	138	67.71
P-n101-k4	681	681	678.6	12741	555	613.5	678.7	12797	445	1097	—	—	—	—	—	—	—	—
M-n101-k10	820	820	820.0	4024	204	10.90	820.0	4948	48	16.04	820.0	4728	59	68.90	820.0	3718	4	333.3
M-n121-k7	1034	1034	1032.5	8244	213	147.5	1032.6	12719	193	6271	—	—	—	—	—	—	—	—
M-n151-k12	1003	1015	1000.4	6974	234	114.2	1001.4	6963	207	118.2	1002.2	7351	68	167.7	1002.7	7020	203	827.4
M-n200-k16	1256.4	1286	1252.5	8610	161	213.9	1252.8	8619	113	247.0	1253.3	8620	116	313.2	1254.1	8963	129	1879
M-n200-k17	1256.8	1275	1255.0	8288	323	232.6	1255.3	8669	319	227.0	1255.6	8832	316	318.5	1256.2	8740	304	10054