I
Introduction

Among the combinatorial optimization problems studied in the literature over the last decades, Vehicle Routing Problems (VRPs) play an important role for several reasons. Since the seminal work of Dantzig and Ramser in 1959 [21], where the authors introduced a problem of oil distribution from supply stations, VRPs have become widely investigated. This interest arises from the existence of many practical applications and from the difficulty of solving them.

A large number of problems can be modeled as a vehicle routing problem, specially problems in transportation, distribution and logistics. It is remarked in Toth and Vigo [75] that a high percentage of the value added to commercial goods reflects distribution costs, therefore the use of computational methods for planning and operation is justified, as these often result in significant savings in the overall costs.

VRPs consist in finding a set of routes for a fleet of vehicles. These vehicles are located at one or more special places, called the depots. Spread over a network, there is a set of customers, each one requiring exactly one vehicle to service its demands. Notice that customers' demands may be the collection of goods, the delivery of goods or in some cases they may even be both. Typically, the solution of this problem imposes the routes to begin and end in the same depot, while meeting all customers' demands and minimizing the overall transportation cost. Besides, some operational constraints may be imposed. For instance, there may be a limitation on the capacity of each vehicle or periods of the day on which each customer can be visited.

It is important to notice that part of the difficulty of solving any vehicle routing problem comes from the fact that it generalizes the Traveling Salesman Problem (TSP), which is one of the most famous combinatorial problems and it is known to be a strongly $\mathcal{NP}$-hard problem [32]. Furthermore, depending on the operational constraints imposed, other hard problems may come into play, and the VRP will only admit solutions in the intersection of the feasible solution space of those problems. For instance, if the fleet of vehicles contains more than one vehicle, the operation of selecting which vehicle will service which customer can be considered as solving a Bin Packing Problem, which is

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also known to be an $\mathcal{NP}$-hard problem [32].

We can cite some important vehicle routing problems which have been extensively studied in the literature recently. The Capacitated Vehicle Routing Problem (CVRP) is the most well-known. In this problem, the customers are located at the vertices of the network and there is a special vertex representing the single depot. The fleet of vehicles is limited, homogeneous and the vehicles have a maximum capacity, which cannot be exceeded. The Capacitated Vehicle Routing Problem with Time Windows (CVRPTW) is a variation of the CVRP and defines time windows for each customer to be serviced. Another variation of the CVRP is the Multi Depot Capacitated Vehicle Routing Problem (MDCVRP), where more than one depot is considered. Furthermore, another vehicle routing problem, which is not a variation of the CVRP, is the Capacitated Arc Routing Problem (CARP). The main difference from the CVRP is that the customers' demands are spread along a subset of the edges of the network. These edges are called the required edges and each one must be serviced by a single vehicle. Finally, we can cite the Generalized Vehicle Routing Problem (GVRP), which is a generalization of the CVRP and the TSP. In this problem, the customers are sets of vertices, called clusters, and each one has to be serviced by a single vehicle by visiting just one vertex of the cluster.

During the last years, various types of computational approaches have been used with the intent of solving such problems. Among others, exact approaches based on solving mixed-integer linear programming formulations is one of the most successful nowadays. The best results are obtained by solving the problems using a formulation with an exponential number of columns (variables). In order to deal with this large number of columns, a technique called column generation is used. This technique was introduced in 1958 by Ford and Fulkerson [30] and it was later extended to mixed-integer programming by Gilmore and Gomory in 1961 [35] to solve the Cutting Stock Problem. It starts with a small number of columns and, at each iteration, it selects (prices) some columns to be inserted in the formulation which improve the solution. This operation is done by solving an auxiliary subproblem, called the pricing subproblem, which should be an easy problem to solve. Usually it is not true for VRPs, as the pricing subproblem can also be an $\mathcal{NP}$-hard problem. The immediate drawback of this approach is that it allows for solving only the linear relaxation of the mixed-integer formulation, i.e., all variables are considered as continuous variables.

Analyzing the structure of each problem, some valid inequalities can be devised and inserted in the formulation in order to improve the solution of

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the linear relaxation given by the column generation algorithm. The search for such inequalities is called cut separation and the algorithm which solves a formulation with cut separation is known as a cutting plane algorithm. Notice that this approach need not necessarily be used within a column generation.

Although there are some cuts which enforce the integrality of the continuous variables, they are not widely used within column generations yet. For this purpose, there is a well-known technique, called Branch-and-Bound, which enumerates all possible integer solutions through a search tree, discarding the branches with solutions greater than a known upper bound. This technique, when used together with column generation and cut separation is called *Branch-Cut-and-Price* (BCP).

The main objective of this thesis is to devise new exact algorithms for some VRPs. The focus is primarily on the CARP, however we also devise some algorithms for the GVRP, which allow us to solve other VRPs. In this work, we solve CVRP instances as GVRP instances.

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I.1 Contributions

The main contributions of this work are described as follows.

- Proposes a new exact cut separation for the CARP, which outperforms the existing separation, as shown in the experimental results.
- Develops a dual ascent heuristic for the CARP, which improves the exact cut separation, being able to be applied on large scale instances.
- Describes an efficient implementation for the state-of-the-art pricing algorithm for both CARP and GVRP, which is used with some cut separations and is able to improve several lower bounds for the GVRP, also obtaining some optimal solutions.
- Assembles all the algorithms presented in a new Branch-Cut-and-Price algorithm for the CARP, which is able to obtain new lower bounds and optimal solutions for some competitive instances.

The research presented in this thesis has also generated some published works, as the column generation with elementary and non-elementary routes for the CARP [58], the Branch-Cut-and-Price with non-elementary pricing for the CARP [59] and the dual ascent with exact cut separation for the CARP [60]. Furthermore, there is a working paper on the efficient implementation for the state-of-the-art pricing algorithm for both CARP and GVRP which is currently in its concluding stage.

I.2 Thesis Outline

This work is organized as follows.

- Chapter II describes the problems discussed in this work, as well as their literature reviews.
- Chapter III shows some of the known formulations for each problem.
- Chapter IV introduces an exact cut separation and a dual ascent heuristic. The dual ascent heuristic is used for the CARP as hot start to a cutting plane algorithm. This cutting plane algorithm uses an existing exact cut separation and the presented new one.
- Chapter V presents a column generation algorithm which uses an efficient restricted non-elementary route pricing.
- Chapter VI presents a branch-cut-and-price for the CARP which put together all the algorithms described in the preceding chapters.
- Chapter VII contains the conclusions of the work.

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