

Rafael Martinelli Pinto

**Exact Algorithms for Arc and
Node Routing Problems**

TESE DE DOUTORADO

DEPARTAMENTO DE INFORMÁTICA
Programa de Pós-Graduação em
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August 2012

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Problems**

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Thesis presented to the Programa de Pós-Graduação em
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as partial fulfillment of the requirements for the degree of
Doutor.

Advisor: Prof. Marcus Vinicius Soledade Poggi de Aragão

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Rafael Martinelli Pinto

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To my mother and my son.

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Resumo

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Os problemas de roteamento estão entre os problemas combinatórios mais difíceis de encontrar limites melhores do que os existentes ou de provar novas soluções ótimas. Nesta tese, são abordados o *Capacitated Arc Routing Problem* (CARP) e o *Generalized Vehicle Routing Problem* (GVRP). Em ambos os problemas, existe um conjunto de clientes os quais estão espalhados por um grafo dado, onde cada cliente possui uma demanda que deve ser atendida por exatamente um veículo de um conjunto de veículos idênticos. Os custos de travessia e o vértice de depósito são dados. O objetivo é encontrar rotas que coletam todas as demandas com custo mínimo, sem exceder a capacidade de nenhum veículo. No CARP, os clientes são um subconjunto de arestas, chamadas de arestas *requireds*, e para o GVRP, cada cliente é um subconjunto de vértices, chamado de grupo, onde cada grupo deve ser atendido visitando-se exatamente um vértice deste grupo. Além disto, vale notar que quando cada grupo possui apenas um vértice, o problema passa a ser o *Capacitated Vehicle Routing Problem* (CVRP). Primeiramente, são investigados métodos para melhorar os limites inferiores de instâncias de grande porte. É proposta a exploração da velocidade de uma heurística *dual ascent* para gerar cortes de capacidade. Em seguida, é apresentado um algoritmo de geração de colunas com um *pricing* eficiente para um tipo especial de rota não-elementar. O *pricing* proposto combina a técnica *Decremental State-Space Relaxation* (DSSR) com limites de complemento. Estas técnicas permitem o fortalecimento da regra de dominância entre as rotas, reduzindo drasticamente o número total de rótulos utilizados pela programação dinâmica. Finalmente, um algoritmo de *branch-cut-and-price* é criado o qual usa a geração de colunas e a separação de cortes previamente apresentadas. Além disto, este *branch-cut-and-price* é implementado usando *strong branching* e fixação por custo reduzido. Ao fim de cada parte, são apresentados resultados computacionais os quais avaliam a qualidade dos algoritmos propostos, os quais obtêm novos limites inferiores para um grande número de instâncias do CARP e do GVRP.

Palavras-chave

Problemas de Roteamento; Geração de Colunas; Separação de Cortes; Branch-Cut-and-Price; Programação Inteira.

Abstract

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Routing problems stand among the hardest combinatorial problems to find high quality bounds or to prove new optimal solutions. In this thesis, we tackle the Capacitated Arc Routing Problem (CARP) and the Generalized Vehicle Routing Problem (GVRP). For both problems, there are a set of customers spread over a given graph, where each customer has a demand which must be serviced by exactly one vehicle from a set of identical vehicles. The traversal costs and a depot vertex are given. The objective is to find routes that collect all the demands, without exceeding the capacity of any vehicle, at minimum cost. For the CARP, the customers are a subset of edges, called the required edges, and for the GVRP, each customer is a subset of vertices, called clusters, where each cluster must be serviced by visiting exactly one vertex of it. Furthermore, it is noteworthy that when every cluster contains just a single vertex, the problem is the Capacitated Vehicle Routing Problem (CVRP). Firstly, we investigate methods to improve lower bounds for large scale instances. We propose to explore the speed of a new dual ascent heuristic to generate capacity cuts. The quality of the cuts found is next improved with a new exact separation which is used in the linear program resolution that follows the dual heuristic. Following, we present a column generation algorithm with an efficient pricing for a special kind of non-elementary routes. The proposed pricing algorithm combines Decremental State-Space Relaxation (DSSR) technique with completion bounds. These techniques allow the strengthening of the domination rule between routes, drastically reducing the total number of labels used during the dynamic programming. Finally, we devise a branch-cut-and-price algorithm which uses the previously presented column generation and cut separation. Moreover, this branch-cut-and-price is implemented using strong branching and reduced cost fixing. At the end of each part, we present computational experiments which evaluate the quality of the proposed algorithms and show new best lower bounds for a large number of CARP and GVRP instances.

Keywords

Routing Problems; Column Generation; Cut Separation; Branch-Cut-and-Price; Integer Programming.

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