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# Appendix A – Parallel programming and simulation dictionary

Due to the high computational cost involved in the optimization and simulation process, this methodology makes use of parallel programming to distribute these computationally expensive tasks. We use OpenMPI (the open software message-passing interface available on *<www.open-mpi.org>* distribute the tasks related to optimization and simulation to the processors that have been assigned to handle such tasks.

Our software architecture divides the workload into five kinds of tasks:

- Master
- *OptManager* (Optimizer Manager)
- *OptWorker* (Optimizer Worker)
- *SimManager* (Simulator Manager)
- *SimWorker* (Simulator Worker)

The manager tasks act as job schedulers that interface with the worker tasks to keep them busy but not overloaded, and also manage the packaging and unpackaging of sets of jobs that need action. In addition, we maintain a dictionary of all simulation results that serves two purposed: firstly, in the event of hardware or software failure it facilitates rapid restarts because it eliminates the need to resimulate previously obtained results, and secondly, it speeds optimization by avoiding repeated simulations.

The *Master* process packages the first optimization job and sends it to the *OptManager*, who, in turn sends it to one of the *OptWorkers*. As an *OptWorker* requires simulations to be performed for evaluation of the objective function, these are packaged into simulation jobs that are sent to the *SimManager*. The *SimManager* unpacks each simulation job into its individual simulation tasks that are queued and send to *SimWorker* as they become available. Once an *OptWorker* completes its task, the resulting forecast measurements are clustered and each cluster is used to spawn a new optimization job that is sent to the *OptManager* to begin the next cycle of optimization. This is illustrated at the Figure A.1. Once all

time steps be completed optimized *OptManager* send this message to *Master* and it finish all process.



Figure A.1: Proposed MPI architecture

During the optimization process, the same alternative of valve settings can be proposed at different periods over the optimization horizon. Thus, in order to eliminate the need for repeated simulations during the optimization, we created a dictionary of simulation results, saving valves settings, reservoir model ID, their respective forecast measurements, and the forecast NPV. Thus, before a simulation job is send to a *SimWorker*, the *SimManager* checks if the simulation has already been performed sometime in the past; if yes, then resimulation is avoided and the associated simulation results are sent from the dictionary. This serves two purposes: it avoids spending time with repeated simulations, and it allows rapid restarts in the event of hardware or software failure.

# Appendix B – UNISIM model for Eclipse simulator

The simulation model provided with UNISIM-I has been built for CMG's IMEX simulator (CMG, 2015). To be able to include a more sophisticated handling of the completions equipment and to be able to use our existing workflows, it was necessary to construct an equivalent model in Eclipse. Inevitably there are some differences between the handling of various properties in the two simulators, particularly:

- Relative permeability curves / saturation functions: By default IMEX applies an analytical smoothing at the end points, whereas Eclipse uses only the data provided. The handling of the input of irreducible oil also differs between the two simulators;
- Well indexes: Eclipse uses the Peaceman model (*Peaceman*, 1983) for calculating well indexes (alternatively the well index may be calculated externally), this option is available in IMEX but has not been used by the UNISIM-I model where instead the areal average formulation is used with a fixed value for the IMEX parameter 'geofac';
- PVT properties: The PVT model used in the IMEX model of UNISIM is largely equivalent to that obtained from the PVCO keyword in Eclipse, i.e. undersaturated oil is assumed to have a constant compressibility and viscosibility. However, the model for the viscosity of undersaturated oil is slightly different between Eclipse and IMEX (exponential versus linear).

To confirm that the Eclipse model was in agreement with the IMEX model, a scenario was tested in which the wells NA2 and NA3D were set as water injectors, attempting to inject at the maximum permissible rate and wells NA1A and RJS19 were set as producers with a simulation time frame of 10 years. Converting some of the wells to injectors helps to better test the simulator model and prevents us from rapidly reaching the pressure control. Note that this case is not intended to demonstrate an optimized production strategy.

The Eclipse and IMEX models were in agreement as to the initial total volumes of oil, water, and gas in place. The bottom-hole pressures, production and injection rates were also reasonably similar as can be observed in Figure B.1, the mnemonics WOPR, WWPR, WWIR and BHP correspond respectively to the oil and water production rates, the water injections rates and the bottom-hole pressures. The small discrepancies are likely due to the difference in definition of well-index and pressure-dependence of viscosity.



Figure B.1: Liquid production and injection rates along with bottomhole pressures as determined by Eclipse and IMEX simulation models.

# Appendix C - Multi-segment well

There are some ways of representing the donwhole inflow control devices. In this thesis we choose to use the Eclipse simulator to perform the reservoir behaviors, because this simulator allow us to represent the smart valves by multisegmented wells. Knowing that the type of the valves interfere on the valves aperture settings, this appendix describes was divided in two parts, as follow:

- 1. Physical description and types of flow control valves;
- 2. Multi-segment wellbore model of valves.

All information in this appendix was obtained by Schlumberger employers, once this company provides smart valves to many reservoir fields.

### Physical description and types of flow control valves

Flow control valves allow the creation of a 'choke' restriction to the flow with a smaller (often much smaller) cross-sectional area than that of the tubing. Flow control valves can be installed either as annular flow control valves or inline flow control valves. The annular flow control valve controls the flow into or out of the tubing and can be used e.g. for zonal isolation. The inline flow control valve controls the flow along the tubing and can be used e.g. to control flow from multilateral wells or along horizontal wells.

A wide variety of flow control valves are available, with many common features and some differences. The flow control valves available from Schlumberger all share the following properties:

- Tubing retrievable (wireline retrievable available for gas lift applications);
- Can be used for either production or injection;
- Can be installed as either annular control valves or inline control valves;
- Sand control;
- Suitable for environments with scale deposition.

More specific features, relevant to the project, of selected valves are: *TRFC-HM AP/LP* 

- Hydraulic control with a single line;
- 11 discrete choke positions including fully open and closed.

Here the hydraulic control uses only a single line, with pressure pulses being used to cycle through the available positions. The valve cross-sectional area of the choke positions can be customized to the reservoir.

Odin

- Hydraulic control with two lines;
- 8 discrete choke positions including fully open and closed;
- Mechanical override possible.

Here the hydraulic control uses two lines; it is possible to cycle through the choke positions by applying a differential pressure between the two control lines. Where multiple flow control valves are to be installed in a single well, one of the two control lines can be shared.

### TRFC-E

- Electrical control with single electrical cable that can be shared with other control valves and monitoring equipment;
- Infinitely adjustable choke.

Electrical control allows for continuum of valve cross-sectional area, and avoids the need to cycle through choke positions.

In Brazil the Odin valve is being used as part of the intelligent completions architecture for the Lula field. The electrical control valve TRFC-E is a next-generation technology and has not yet been used in Brazil.

In addition to the flow control valve it is possible to install a sliding sleeve device. This provides an additional opportunity to mitigate against valve failure, although a slickline intervention would be required. Chemical injection lines should also be installed to help prevent/remove scale formation.

#### Multi-segment wellbore model of valves

The multi-segment wellbore model of Eclipse allows for a more detailed description of fluid flow through the wellbore. It is particularly useful for modeling

the complicated topology of multilateral wells and gives a more complete model of multi-phase flow in horizontal wells. In this project it is the built-in model of the pressure loss due to flow through flow control valves. Constructing a multi segment wellbore model requires us to generate the topology of the flow-paths, this can be done either manually or using Petrel to automatically generate this from a description of the installed completion equipment.

The flow control valve model as implemented in Eclipse (through the WSEGVALV keyword) can model the pressure loss through the valve due to both acceleration of the fluid through the constriction and any frictional loss through the valve.

A simple calculation can give us some insight into the exact relationship between the cross-sectional area of the valve and the restriction of the flow that can be expected. An extremely simple model for the (single-phase) production from a reservoir is that the flow rate is proportional to some pressure drop between the reservoir pressure and the wellbore pressure, i.e. the flow rate,  $Q = \lambda \Delta p$  for some constant  $\lambda$  that depends on the productivity of the reservoir and on the well connectivity and where  $\Delta p$  is the pressure difference driving the flow. The presence of the constriction in the valve leads to an additional drop between the pressure in the reservoir and the wellbore, and so the restricted flow rate is

$$q = \lambda \left( \Delta p - \frac{\rho v^2}{2C^2} \right),$$

where v is the flow velocity through the constriction,  $\rho$  is the fluid density, and *C* is a flow coefficient for the valve.