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Ali Sharafi Rad

Economic Appraisal of Flexibility in Brazilian Flex-Fuel Vehicles: A Fuzzy Real Options Approach

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DISSERTAÇÃO DE MESTRADO

Thesis presented to the Programa de Pós-Graduação em Engenharia de Produção of the Departamento de Engenharia Industrial, PUC-Rio as partial fulfillment of the requirements for the degree of Master em Engenharia de Produção.

Advisor: Prof. Carlos Patricio Samanez

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Ali Sharafi Rad, born November 03, 1974 in Tehran, Iran, graduated in *Industrial Engineering* from IUST (Iran University of Science and Technology) in 2000. After college, he began working in a factory as a production engineer for 2 years. Then he has been working more than 10 years in project planning and control in Oil and Gas industries. He received PMP (Project Management Professional) certificate since 2011. He has excellent knowledge of Project Management body of Knowledge (PMBOK) Standard.

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Abstract

Sharafi Rad, Ali; Samanez, Carlos Patricio (Advisor). **Economic Appraisal of Flexibility in Brazilian Flex-Fuel Vehicles: A Fuzzy Real Options Approach.** Rio de Janeiro, 2015. 167p. MSc. Dissertation – Departamento de Engenharia Industrial, Pontifícia Universidade Católica do Rio de Janeiro.

Brazil was drawing a lot of world attention at the time due to the production of flex-fuel vehicles introduced by its own automobile industries since 2003. Today the fleet of flexible-fuel vehicles in Brazil are the largest in the world with 23 million vehicles (ANFAVEA, 2015), following the United States with 11 million units (EIA, 2015), Canada with one million units (TPSGC , 2015), and Europe, led by Sweden with 229,400 units (BAFF, 2015). There are over 80 flex car and truck models available in Brazilian manufactured market by 14 major automakers. Brazilian flexible-fuel vehicles are optimized to run on any mix of gasoline and up to 100% hydrous ethanol. The flexibility in the choice of fuel gives a comparative advantage to this vehicle type. Different to other works done in this subject, this study seeks to value this advantage for five Brazilian geographic regions: Northern, Northeastern, Central-Western, Southeastern and Southern, determining the value of this flexibility for each region. For this purpose, fuel prices were considered as stochastic variables following a mean reversion stochastic process. Monte Carlo simulation has been utilized to predict the prices, and real options theory and a fuzzy algorithm applying triangular fuzzy Numbers (TFN) to value embedded flexibility. Since TFN offer good performance and computational efficiency and provide a more realistic modeling of the problem, they were used to model the fuel consumption rates and the distance traveled by vehicles. The results indicate that the option of flexibility adds more significant value to the owners being in Southeastern, Central-Western, and Southern regions.

Keywords

Real options; Fuzzy sets; Flex-fuel vehicle; Stochastic process; Mont Carlo simulation.

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Resumo

Sharafi Rad, Ali; Samanez, Carlos Patricio (Orientador). **Avaliação Econômica da Flexibilidade do Veículo Flex Brasileiro: Uma Abordagem através de "Fuzzy Real Options".** Rio de Janeiro, 2015. 167p. Dissertação de Mestrado – Departamento de Engenharia Industrial, Pontifícia Universidade Católica do Rio de Janeiro.

O Brasil está atraindo muita atenção do mundo devido à produção de veículos flex introduzidos pela sua indústria automobilística desde 2003. Hoje a frota de veículos flex no Brasil é a maior do mundo, com 23 milhões de unidades (ANFAVEA , 2015), seguida dos Estados Unidos com 11 milhões (EIA, 2015), Canadá com um milhão (TPSGC, 2015), e a Europa, liderada pela Suécia, com 229.400 unidades (BAFF, 2015). Há mais de 80 modelos flexíveis de carros e caminhões disponíveis no mercado brasileiro, fabricados por 14 grandes montadoras. Os veículos flex no Brasil são otimizados para funcionar com qualquer mistura de gasolina e até 100% de etanol hidratado. A flexibilidade na escolha do combustível gera uma vantagem comparativa para este tipo de veículo. Diferentemente a outros trabalhos que já trataram do assunto, este estudo busca valorar esta vantagem para as cinco regiões geográficas brasileiras: Norte, Nordeste, Centro-Oeste, Sudeste e Sul, determinando o valor desta flexibilidade para cada região. Para esse propósito, os preços dos combustíveis foram considerados como variáveis estocásticas seguindo um Movimento de Reversão à Média Aritmético. A simulação de Monte Carlo foi utilizada para prever os preços, e a Teoria de Opções Reais e um algoritmo Fuzzy, que aplica números fuzzy triangulares (NFT), foi usado para valorar a flexibilidade embutida. Desde que os NFT oferecem um bom desempenho e eficiência computacional e propiciam uma modelagem mais realista do problema, foram utilizados para modelar as taxas de consumo de combustível e a quilometragem percorrida pelos veículos estudados. Os resultados indicam que a opção de flexibilidade adiciona valor para os proprietários, sendo mais significativa nas regiões Sudeste, Centro-Oeste e Sul.

Palavras Chaves

Opções reais; Número fuzzy; Veículo flex; Processos estocásticos; Simulação de Monte Carlo.

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List of Acronyms

AAM	Average Annual Mileage
CBD	Central Business District
DCF	Discounted Cash Flow
FCF	Fuzzy Cash Flow
FCR	Fuel Consumption Rate
FFCR	Fuzzy Fuel Consumption Rate
FFV	Flex-Fuel Vehicles
FPV	Fuzzy Present Value
FSOV	Fuzzy Switch Option Value
FST	Fuzzy Sets Theory
FVMT	Fuzzy Vehicle Mileage Traveled
GBM	Geometric Brownian Motion
HFROV	Hybrid Fuzzy Real Options Valuation
MFC	Monthly Fuel Consumption
MRP	Mean Reversion Process
NPV	Net Present Value
PV	Present Value
RO	Real Options
ROA	Real Option Analysis
ROT	Real Options Theory
ROV	Real Options Valuation
TC	Traffic Congestion
TCI	Traffic Congestion Impact
TFN	Triangular Fuzzy Numbers
TIR	Traffic Index Rate
USS	Urban Spatial Structure
VMT	Vehicle Mileage Traveled

NE	Northeastern
N	Northern
CW	Central-Western
SE	Southeastern
S	Southern

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1. Introduction

Brazil first began using ethanol in vehicles as early as the 1920s, and the trend gained urgency during the oil shock of the 1970s. However, sugarcane ethanol's popularity really took off in 2003 with the introduction of flex-fuel vehicles (FFV) that run on either gasoline blends or pure ethanol. FFV gives Brazilian consumers a choice of fuel at the pump when they fuel their cars, and most are choosing sugarcane ethanol due to its price and environmental benefits. Brazilian flex cars can run by up to 100% ethanol or gasoline or by any proportion of both fuel types. More than 88.2% of sold new light cars today in Brazil are flex-fuel due to consumer demand (ANFAVEA, 2015), and these vehicles now reached more than 23 million cars in Brazil. There are over 80 flex-car and truck models available in Brazilian manufacturing market by 14 major automakers.

The flexibility of fuel choice provided by this car is an advantageous for its owner, as it happens to choose the cheapest fuel from each supplier. This option of choice (flexibility) can be understood as a project under uncertain environment while prices are free in an uncertain market. Real options theory (ROT) is a proper technique to assess this option with an exchange input (fuel switching option).

Most investment decisions share three important characteristics (Dias, 2014; Dixit and Pindyck, 1994): irreversibility, uncertainty and timing. First, the investment partially or completely is irreversible. In other words, the initial cost of investment is sunk partially; you cannot recover it all. In buying a car, if there is repentance it cannot be recovered quite the money invested, if the property is sold, its market value will be below the purchase value. The best you can do is to assess the probabilities of the alternative outcomes that can mean greater or smaller profit (or loss) for your investment. Second, there is an uncertainty about the future behavior of asset prices (fuel price) which is the main source of risk. Third, there is leeway about timing. There is flexibility to choose the fuel which has a better cost-benefit relation each time the car needs to be filled up, in this context each time the vehicle traveled 1,500 km in a month or in a range of [1167, 1833] considering vagueness of parameters value as fuzzy set.

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According to Dias (2014), real options valuation explicitly models the uncertainty through the theory of stochastic processes and considers the response flexibility (the real options that the decision maker holds) in every possible future scenario. The three main stochastic processes in finance applications are Geometric Brownian Motion (GBM), Mean Reversion Process (MRP) that is applied in this study and Poisson process generally associated with the GBM.

Despite the discrepancies among the definitions of uncertainty in the literature, it has agreed that uncertainty could be classified into Objective and Subjective as two major categories based on their causes where the stochastic process is a tool to model the objective uncertainty and fuzzy sets theory a tool to model subjective uncertainty.

The impact of “Fuzziness” is unavoidable due to the subjective assessment made by investors or inherent vagueness of some parameters. Judgment of this events or parameters may be significantly different based on their fuzzy or imprecise. Fuzzy sets theory enables us to describe and eliminate the “fuzziness” of theses subjective assessments. The traditional financial models and even real option valuation model does not take into consideration that investors face fuzzy (vague or imprecise) factors in financial analysis. The fuzzy sets theory proposed by Zadeh (1965) can be a useful tool for modeling this kind of imprecise problem. In the real world, there are times when input parameters and data cannot always be determined in the precise sense. Therefore, the fuzzy set theory naturally provides an appropriate tool in modeling the imprecision.

The main idea behind the concept of this work has been drawn from Samanez et al. (2014) which developed a Real option valuation to model the Brazilian flex-fuel vehicles flexibility value geographically. This thesis offers a method to add more realistic valuation considering the vague values as a fuzzy number for parameters of vehicle mileage travelling and fuel consumption rate where their values in real world mainly varies in a range as a vague value rather than a fixed or precise value.

This context carried out a study to value the flexibility that a flex vehicle gives to its owner comparing to the only gas-powered car. The value of this flexibility will be measured using mean reversion process to model the price. The vagueness (sometimes called imprecision, non-preciseness, ambiguity) of some parameters which is often ignored will be modelled by a fuzzy methodology and

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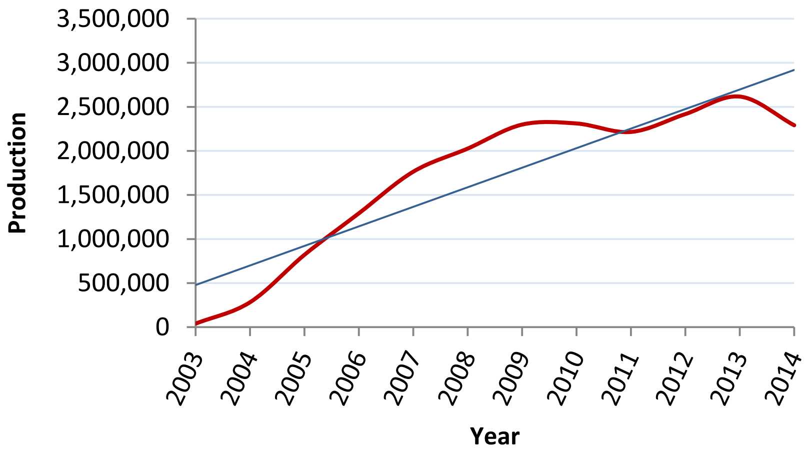
finally a fuzzy real options valuation (FROV) method will be utilized to estimate the flexibility value. The predicted price and consequently the option value will be obtained through Monte Carlo simulation applying for each geographic region of Brazil: Northeastern (NE), Northern (N), Central-Western (CO), Southeastern (SE) and Southern (S), and each sort of fuel of ethanol and gasoline due to the their variation characteristic in price. Two most popular models of flex-fuel vehicles in Brazil, VW GOL and Fiat UNO, which is supposed to have a useful lifetime of five years, is considered to evaluate and analyze the option value.

1.1. Motivation

Developing flex-fuel vehicle has launched in Brazil since 2003. Now the fleet FFV is more than 23 million and production rate in 2014 was 88.2% total automobile and light vehicles in Brazil (ANFAVEA, 2015). Brazil is the second Ethanol producer country in the world with more than 6,000 billion of gallons per year (RFA, 2014) and the first FFV producer in the world which its production rate has been increasing by the time.

Figure 1- Brazil flex-fuel vehicle production 2003-2014



Source: ANFAVEA (2015)

The idea of flex-fuel is to provide ability in using the fuel ethanol, gasoline or any mixing ratio between two fuels in the same tank for vehicles. This ability gives the owner flexibility to choose the fuel at each refueling stop. Along with uncertainty in the future fuel prices in the market, it provides the owner an option

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to choose the fuel regarding the price. It adds value to him since he always has an option to use cheaper fuel.

This thesis applied two powerful methods, real options valuation and fuzzy sets theory to analyze and evaluate this flexibility under uncertainty environment and examined a hybrid fuzzy real option method. In this regard, it attempts to answer these questions:

- How can the fuzzy numbers be applied to perform a real options valuation of FFV flexibility?
- In order to create a reliable valuation for a fuel switching option in FFV by a fuzzy sets theory, which parameters and factors should be discussed and estimated?
- What is the financial benefit of a flex-fuel car compared to a gasoline-powered car only for the owner through a hybrid fuzzy real option valuation?
- How much is the financial impact of urban spatial structure on this benefit regionally?
- How much is the financial impact of traffic congestion on this valuation in each geographical region?
- Regarding the owner location, who receives more benefit regionally?

1.2. Objectives

The purpose of this dissertation aims to develop a hybrid model of real options valuation (ROV) and fuzzy sets theory (FST) to analyze the value of Brazilian flex-fuel vehicles flexibility. This study will also construct a new approach to measure the financial impact of urban spatial structure (USS) and traffic congestion (TC) on FFV flexibility value for different geographical regions of Brazil.

Due to being vagueness in parameters influenced by USS and TC and insufficiency of traditional probability model and even real option valuation to face with this vagueness, this study proposed fuzzy sets theory to integrate imprecision and vagueness into a model as a mathematical framework under incomplete or vague environment. Two kinds of uncertainty will be faced in evaluation of FFV flexibility value. The first uncertainty is due to the fluctuation of the financial market from time to time and the second one is in due to their

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imprecise nature in reality or when the future state of a variable might not be known completely due to the lack of information. Monte Carlo simulation using mean reversion stochastic process is applied to model the first uncertainty type and fuzzy sets theory to model the second type of uncertainty in this study.

Following the whole model that is a hybrid fuzzy real options valuation (HFROV) will obtain the value of FFV flexibility for the owners in each region considering USS and TC financial impact on and, finally, the results provide us to examine fuzzy real options versus regular real options analysis by comparing their values.

1.3. Dissertation and literature review

The economic appraisal of flexibility in Brazilian FFV by a HFROV is this study's objective. Real options valuation and fuzzy sets theory as two appropriate metrics, which have been applied to model the uncertainty in many industries and stochastic processes analysis, are major tools over this study to deal with measuring the options value embedded in FFV, whereas, the focus is on fuzzy sets theory and examine employing the fuzzy approach on Real options analysis. This integrating is the objective of this study to generate a more realistic model of valuation while assessing the FFV flexibility option value using ROV has been developed before by other researcher.

Regarding the flex-fuel vehicle flexibility option valuation (regular real option valuation) using stochastic process, so far few works have performed where most of them have been achieved in Brazil due to the fact that Brazil is the largest manufacture of flex-fuel vehicles in the world. In the literature can also find many works that suggest applying the mean reversion process for estimating the price of non-financial commodities.

Kulatilaka (1993) suggested the applying of MRP in modeling the price of residual oil and natural gas that feed biofuel boilers due to the high degree of substitution between these two fuel sources. He considered that the competitive forces would tend to drive the long-run relative price to equilibrium where users will be indifferent between the two fuels. He suggested a mean-reverting specification in estimating the stochastic dynamics of the relative price of oil and gas. The author concludes that greater volatility in the relative price increases the

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value of flexibility, for instance, the real option trading. However, the increased presence of this flexible technology (the biofuel boiler industries) decreases the volatility of changes in the relative prices (there is a correlation between the increase of prices) and then potentially reduces the future value of flexibility.

By Dias (1996) the idea of MRP understood with microeconomics; due to the competitive market, the price is far below the long-term average, the firms will no longer produce the commodity and the price will rise due its scarcity in the market . In the opposite side, if the price is far above average, new producers or substitute products enter the market. It will cause its price to fall and can be concluded that the number of commodity prices has a natural tendency to revert to its long-run mean, that is, the market equilibrium average in which no matter how this reversal process is slow.

Schwartz (1997) evaluates the investment projects using three different models that consider the nature of mean reversion of the spot price of non-financial commodities, oil and copper. In the first model, he used a factor, considering Ornstein-Uhlenbeck process, the second was a version of the two-factor model of Gibson and Schwartz and the third, was an extended version of Gibson and Schwartz with adding a new stochastic factor. The analysis revealed a strong mean reversion in project evaluation for the commercial commodity prices.

Bastian-Pinto et al. (2009) measured the value of the fuel switch option of a flex fuel vehicle using the real options methodology under both Geometric Brownian motion (GBM) and Mean reversion process (MRP) for fuel prices uncertainties. They concluded that regardless of both stochastic processes, the flex option adds significant value to the car owner. The same analysis was applied by Krüger and Haglund (2013) using the option valuation, GBM and MRP stochastic processes. they attempted to establish whether the results previously found by Bastian-Pinto et al. (2009) will hold even in the European car market especially in Sweden where is the largest market for flex-fuel vehicles in this area. The results confirmed the positive value of switching option regardless of the stochastic process models, but found a higher option value with GBM than with MRP. Despite the higher total fuel cost with MRP due to some differences between countries for many reasons, for instance, Brazil is self-sufficient with regard to ethanol production while Sweden has to import most of their ethanol, the exchange rate is a major risk factor for market prices.

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This evaluation in the same approach has been done later in Brazil by Camargo et al. (2010). They applied Geometric Brownian motion stochastic process to value the option of a flex-fuel vehicle considering for two car models, Volkswagen GOL City and Fiat Palio with a lifetime of 5 to 10 years. They concluded that generated savings for owners of both vehicles is between 10% and 20% compared to its purchase price, for a life cycle of 5 to 10 years, respectively.

Due to the difference in prices for each region over a big country as Brazil, Samanez et al. (2014) introduced a geographical evaluation of economic advantages of flex-fuel car using real options valuation by simulating prices following MRP for five geographic Brazilian regions: Northern, Northeastern, Central-Western, Southeastern and Southern regions. They concluded that the option to choose the less expensive fuel between ethanol and gasoline added significant value to the flex automobile owner in all considered regions where the most benefited region by the flexibility option was the Southeastern region.

Daily real market conditions are very often uncertain and vague in several ways. When there is a lack of information or no voluntary financial data out the company or in the market, a system might not be known completely. Zadeh (1965) suggested a mathematical outline named fuzzy set theory that overcomes these inadequacies. In situation that data cannot be explained exactly as a value with specific boundary, they can be modeled simply by the fuzzy set theory.

The fuzzy approach to real option valuation was first studied by Carlsson and Fullér (2003), They combined a real option to defer project with fuzzy number. The valuation has been performed trough the Black–Scholes option pricing formula in which the present value of expected cash flow and expected cost was defined as a fuzzy number in the trapezoidal possibility distribution. They determined the optimal exercise time with the help of the possibilistic mean value and variance of fuzzy numbers. This fuzzy real option valuation has been applied in Nordic Telekom Inc. working in telecommunications industry. It was shown applying a combined fuzzy number and real option valuation is quite practical and useful. It concluded that without applying a hybrid valuation it would not be possible to formulate the genuine uncertainty and imprecision encountering in the valuation. The imprecision, which is due to judging or when the estimating future cash flows, is not stochastic in nature. They believed the

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proposed model which incorporates subjective judgments and statistical uncertainties give investors a better tools to make a decision.

Following, Wu (2004) proposed an application of fuzzy sets theory to the Black–Scholes formula and considered a fuzzy interest rate, fuzzy volatility and fuzzy stock price in a European financial option and put–call parity relationship. It provides the financial analyst to pick a European option price with an acceptable belief degree.

In relation to the decision making, Lee et al. (2005) adopted the fuzzy decision theory and Bayes' rule as a base for measuring fuzziness in the practice of option analysis. They employed 'Fuzzy Decision Space' consisting of four dimensions; fuzzy state; fuzzy sample information, fuzzy action and evaluation function to describe the decision of investors to derive a fuzzy Black-Scholes option pricing model under fuzzy environment. They concluded that without consideration of the fuzziness, the over-estimation exists in the value of risk interest rate, the expected value of variation stock price, and in the value of the call price of the in the money and at the money, and under-estimation in the value of the call price of out of the money.

Making a R&D portfolio decision is difficult, because the long lead times of R&D and the market and technology dynamics lead to unavailable or unreliable collected data for portfolio management. Wang and Hwang (2007) developed a fuzzy R&D portfolio selection model to hedge against the R&D uncertainty. Since traditional project valuation methods often underestimated the risky project, a fuzzy compound-options model was used to evaluate the value of each R&D project. The R&D portfolio selection problem was formulated as a fuzzy zero-one integer-programming model that could handle both uncertain and flexible parameters to determine the optimal project portfolio.

A Fuzzy Black-Scholes option pricing formula was applied by Thiagarajah et al. (2007) to addresses quadratic adaptive fuzzy numbers for the characteristics such as volatility parameter, interest rate and stock price to model the imprecision in project. They applied a nonlinear fuzzy numbers to model the parameter of uncertainty in the option-pricing model. Tolga et al. (2010) also applied a fuzzy Black-Sholes method in a R&D project selection based on analytic hierarchy process (AHP) and a fuzzy judgment matrix. A fuzzy ROV integrated fuzzy AHP was applied to compare the R&D projects by considering the experts' opinions.

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They also estimated the possibility value as fuzzy weights for fuzzy numbers comparison and criteria evaluation. As the experts have a significant impact on decision-making in uncertain R&D projects, they concluded that the fuzzy ROV integrated fuzzy AHP provides a proper tool to consider the expert opinions and can assist decision makers in analyzing the tradeoffs between project investments and overall portfolio value.

Collan et al. (2009) introduced "fuzzy pay-off method" as a new method in real option valuation using fuzzy number based on Datar-Mathews Method (DMM). This method uses the fuzzy sets to formulate inaccuracy that exists in human decision-making and the fuzzy algebra to handle imprecise elements in decision-making process. The main idea was according to a novel approach to real option valuation by Datar et al. (2007). In DMM model, the real option value is calculated from a pay-off distribution, derived from a probability distribution of the net present value (NPV) for a project that is generated with a (Monte Carlo) simulation. The DMM utilizes cash-flow scenarios as an input to a Monte Carlo simulation to create a pay-off distribution for the investment. In this paper, the theatrical mathematics of fuzzy real option valuation has been presented by two different fuzzy number types: trapezoidal and triangular distribution. The weighted average of positive outcomes of pay-off distribution is applied to estimate the real option valuation as fuzzy mean of the possibility distribution in which the values below zero counted as zero.

They concluded that fuzzy sets provides the advantage of a possibility to have the size of distribution decrease with a lesser degree of uncertainty. They also concluded that this new method of fuzzy mean value calculation brings simpler generic and a modular real option valuation tools.

Jing-yi et al. (2009) applied a Fuzzy Black-Sholes valuation for an investment decision analysis in the feasibility research of an agricultural industrialization processing-project.

According to the binomial lattice approach to model a real option, Wang et al. (2010) proposed a new real options analysis approach by combing binomial lattice- based model with fuzzy random variable, as named fuzzy random real options analysis (FR-ROA). In the same approach, Wang and Lee (2010) developed a fuzzy real option valuation applying fuzzy binomial lattice and concluded that (1) The FROV increases with increasing expected cash flow

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estimates; (2) FROV increases with increasing fuzzy volatility; (3) The longer the time to maturity, the greater will be the FROV; (4) An increase in risk-free rate of returns will increase the FROV; (5) The FROV will decrease if value is lost during the postponement of the investment, which can be countered by either creating business barriers for competitors or by better managing key resources.

Tolga et al. (2010) examined the valuation of a call center with two models of Fuzzy Black-Scholes and Fuzzy trinomial lattice and the results were compared. A sensitivity analysis for dividend yield and risk free interest rate also was given. Finally, it was concluded that there is not a meaningful difference between two models; however, the fuzzy trinomial lattice method offers more flexibility.

Zmeskal (2010) addressed a generalized hybrid fuzzy–stochastic binomial American real option model. The Input data, up index, down index, growth rate, initial underlying asset price, exercise price and risk-free rate were considered in a form of fuzzy numbers and result, possibility-expected option value is also determined vaguely as a fuzzy set.

A new fuzzy mean value was proposed by Ho and Liao (2011). In this paper, a fuzzy binomial approach to evaluate a project embedded with real option was developed and furthermore a new method to compute the mean value of a project's NPV characterized with right-skewed possibilistic distribution was introduced. According to the right-skewed possibilistic distribution the flexibilities retain the upside potential of profit but limit the downside risk of loss. They applied a triangular fuzzy number for the jumping up and down factors of the underlying asset’s value.

Young-Chan Lee and Seung-Seok Lee (2011) applied a more realistic setting through a fuzzy Black-Scholes option valuation by numerical value through an example of RFID (Radio Frequency Identification) investment project assuming the current value of expected cash flow and investment costs using trapezoid fuzzy number.

Hassanzadeh et al. (2012) advanced a model to address a R&D project portfolio selection problem, where there is inaccuracy in the input data, and where the fuzzy pay-off method is used as a measure of project potential/value. You et al. (2012) also applied a real option theory with Fuzzy payoff method for evaluating ERP system investment. The method introduced an active management

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in dealing with uncertainties in order to minimize the risk of failure and maximize the benefits of an ERP system to the enterprise.

Credibility theory is a construct that created to supplement the measurement of uncertainty and Collan et al. (2012) incorporated a credibility measure into the real option valuation construct of the fuzzy pay-off method. This approach is interesting as credibility theory and specifically the credibilistic expected value in which applied to problems from different areas as: portfolio optimization, facility location problem in B2C e-commerce, transportation problems, and in logistics network design.

1.4. Work Structure

This study is divided into five chapters;

Chapter 1 is containing an introduction where the problem is presented, the goals and examined the literature on the work subject.

Chapter 2 is a survey of ethanol market, with a brief history of ethanol production and flex fuel vehicle life history and launching out in the Brazilian market. It presents the ethanol and gasoline pricing structure, the competition between these fuels up to 2014, and the environmental benefits of ethanol compared to gasoline consumption.

Chapter 3 Presents theoretical basis of understanding the dissertation. In this chapter, the real options theory and fuzzy sets theory are introduced; and the outline of mean reversion process stochastic process and Monte Carlo simulation is presented.

Chapter 4 is developed to implement the theory in application including assumptions and limitation. The flex vehicle flexibility option value is evaluated by employing a practical view of fuzzy real option valuation using Monte Carlo simulation.

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Chapter 5 presents the conclusions and future prospects of the dissertation topics.

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2. Brazilian Flex-Fuel Vehicles Market

Many countries have been investing billions of dollars on developing alternative energy, in which Brazil is a unique example of successfully developing ethanol industry and possessing enormous consumers' support for this industry. Apart from its natural advantages in producing sugarcane-based ethanol, the introduction and commercialization of flex-fuel vehicles since March 2003 also played an essential role and changed the Brazil fuels market by granting consumers the capacity of making choice between ethanol and gasoline, and eventually exerted influence over the market situation of ethanol.

2.1. Ethanol as a Fuel

Ethanol, which is known as alcohol, is a flammable and colorless liquid. The molecular formula is C2H5OH (HE, 2013). It can be also found in alcoholic beverages, and is used widely as a popular alternative fuel in Brazil. Ethanol has a positive energy balance – that is, the energy content of ethanol is greater than the fossil energy used to produce it – and this balance is constantly improving with new technologies (USDOE, 2010).

Table 1 - Energy densities of fuels

Fuel Type	MJ/L	MJ/KG	Octane Number
Ethanol	24	30	108.6
E85 (85% ethanol, 15% gasoline)	25.65	33.1	100-105
Gasohol (90% gasoline, 10% ethanol)	33.18	43.54	93/94
Regular gasoline	34.2	46.4	91
Diesel	37.3	48	25

Data source: USDOE ()

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As presented in Table 1, the energy content of ethanol is 23.5 MJ/L, which is lower than gasoline's 33.18, regular gasoline's 34.8MJ/L, and diesel's 38.6MJ/L. In other words, a driver needs more ethanol in volume to reach the same destination than other fossil fuels. As an alternative to fossil fuels, ethanol is considered safer, cleaner, more sustainable and inexpensive.

Traditionally, ethanol is derived from feedstock containing natural sugar or starch that can be easily converted to sugar. There are many plants (as feedstock) that could be used to produce ethanol over the world. The United States uses corn as the major feedstock, due to its high content of starch. Europe grows sugar beet for ethanol production because of its high content of sugar. Brazil relies on sugarcane, and to be more specific, the sucrose (sugar) to derive fuel ethanol. Most of the industrial processing of sugarcane in Brazil is done through a very integrated production chain, allowing sugar production, industrial ethanol processing, and electricity generation from byproducts (Goldemberg et al., 2008).

There are two basic types of ethanol: hydrous (hydrated) and anhydrous. The former one contains 95% ethanol and 5% water, and can be used directly as fuel ethanol (E100). The latter one is often blended with gasoline, for instance, gasoline (E25) sold in Brazil contains 25% of anhydrous ethanol and 75% of pure gasoline. Both types of ethanol are available in most Brazil's gas stations.

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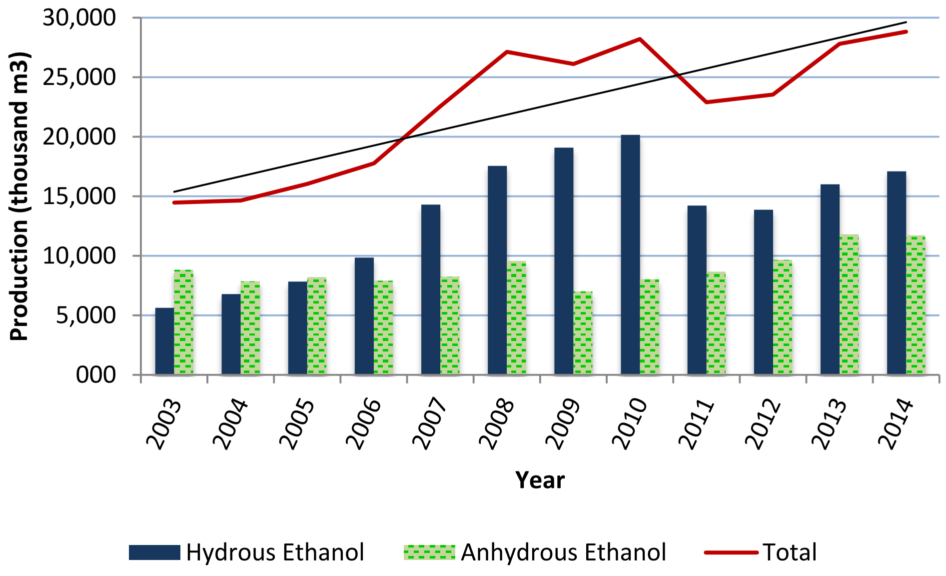
2.2. Ethanol production and sales in Brazil

Brazil with harvest production is the world's largest producer of sugarcane and the largest exporter of sugar (FAO, 2013). Brazil's ethanol production was 28.8 million m^3 in 2014 (ANO, 2015). Sugarcane serves as the exclusive source of feedstock for bioethanol production in Brazil and more than 50% of Brazil's sugarcane production converted into fuel for automobile use (Schmitz et al., 2003). Brazil has a Federal District and 26 states, which are divided into five regions (Northern, Northeastern, Central-Western, Southeastern and Southern). Sugarcane is cultivates in most Brazilian states, but the Southeast and Northeast regions contribute to more than 85% of national production (Crago et al., 2010). Brazil Anhydrous and Hydrated Ethanol production series for years 2003-2014 can be observed in Figure 2.

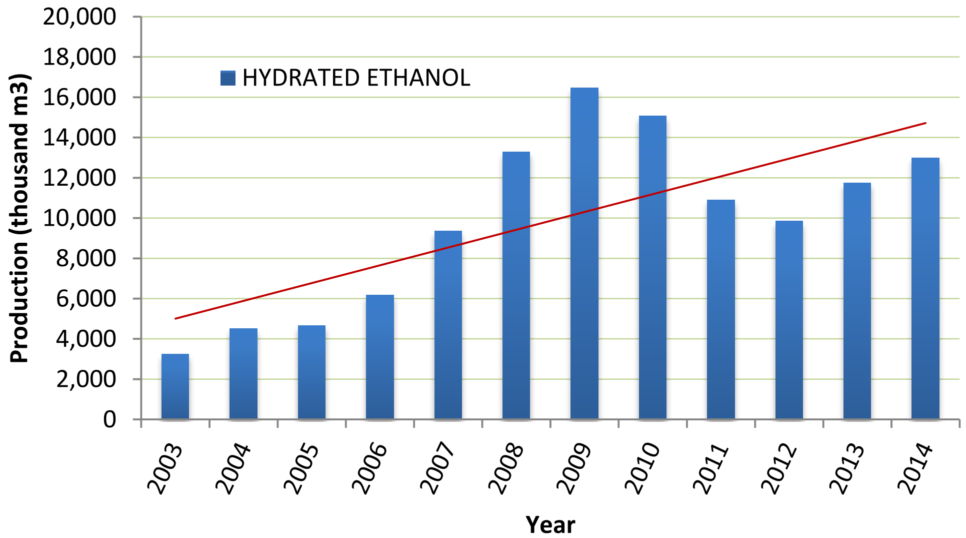
Figure 2 - Anhydrous and Hydrous ethanol production in Brazil



Data source: ANP (2015)

In 2009, the sales of fuel ethanol (Hydrated ethanol) for direct use, reached 16.5 million m^3 (ANO, 2015). As shown in Figure 3, in December 2009, the sales of hydrated ethanol in Brazil reached a record-high of 16.5 million m^3, however, the sales dropped dramatically to 9.6 million m^3 in December 2012, which might be explained by the drop in international oil price.

Figure 3 - Hydrated ethanol sales in Brazil



Data source: ANP (2015)

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2.3. Flexible-Fuel Vehicles

Brazilian consumers had to make a choice between conventional gasoline vehicle and ethanol vehicles, until the introduction and commercialization of flexible-fuel vehicles since 2003. A flexible-fuel vehicle (flex-fuel vehicle) is designed to run on more than one fuel. The most common commercially available flex-fuel vehicle (FFV) worldwide is the ethanol flex-fuel vehicle that can run on either ethanol or gasoline or any blend of them.

Brazil has been drawing a lot of world attention at the time due to the production of flex-fuel vehicles introduced by its own automobile industries since 2003. Today the fleet of flexible-fuel vehicles in Brazil are the largest in the world with 23 million vehicles (ANFAVEA, 2015), following the United States with 11 million units (EIA, 2015), Canada with one million units (TPSGC, 2015), and Europe, led by Sweden with 229,400 units (BAFF, 2015). There are over 80 model flex-cars and trucks available in Brazilian manufactured market by 14 major automakers.

Flexible-fuel technology started being developed by Brazilian engineers near the end of 1990s. The Brazilian flex-fuel vehicles are built with an ethanol-ready engine and one fuel tank for both fuels, and they are available in a wide range of models such as sedans, pickups, and minivans. Brazilian flex-fuel vehicles are capable of running on sole hydrated ethanol (E100), or just on a blend of gasoline with 20 to 25% anhydrous ethanol (the mandatory blend since 1993), or on any arbitrary combination of both.

Table 2 - Automobile production by fuel type in Brazil, 2003-2014

Year	Gasoline	Ethanol	Flex fuel	Diesel
2003	1,561,285	34,919	49,264	182,323
2004	1,682,167	51,012	332,507	251,541
2005	1,333,221	29,402	880,941	286,685
2006	977,134	356	1,392,055	242,784
2007	767,368	-	1,936,931	275,864
2008	633,966	-	2,243,648	338,767
2009	385,756	-	2,541,153	256,573
2010	660,182	-	2,627,111	359,255
2011	467,345	-	2,550,782	427,094
2012	398,317	-	2,732,060	301,872
2013	38,716	-	2,950,611	400,677
2014	249,198	-	2,637,824	285,728

Data source: ANFAVEA (2015)

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Flex-fuel vehicles were officially introduced to Brazil in March 2003. Although in the year of 2003, only roughly 50,000 flex-fuel light vehicles were manufactured nation-wide, which only accounted for 2.9% of the total light vehicles manufactured; in the year of 2014, 2,637,824 flex-fuel light vehicles were manufactured which accounted for 88.2% of the total light vehicles produced (ANFAVEA, 2015), as presented in Table 2. As Figure 4 indicates, the flex-fuel vehicle is the only type of vehicle that experienced a huge growth since 2003. From 2003 to 2014 they represented 67.5% of Brazil's registered light motor vehicle fleet and 32% of the total fleet of light vehicles from 1975 to 2014 (ANFAVEA, 2015).

Flex-fuel vehicles have been proven successful in Brazil in the last decade. Before 2003, Brazilian drivers had to choose between traditional gasoline and ethanol vehicles, which were powered solely on one fuel. Consequently, they were limited to the fluctuations of fuel price. The introduction and commercialization of flex-fuel vehicles offer consumers options depending on the market prices and fuel efficiency. Due to the difference in fuel efficiency, hydrated ethanol (E100) price must remain lower to gasoline prices in order to be competitive. Technical estimations show that only when ethanol price is equal or lower than 70% of the gasoline price, it can be considered a better economic choice (HE, 2013).

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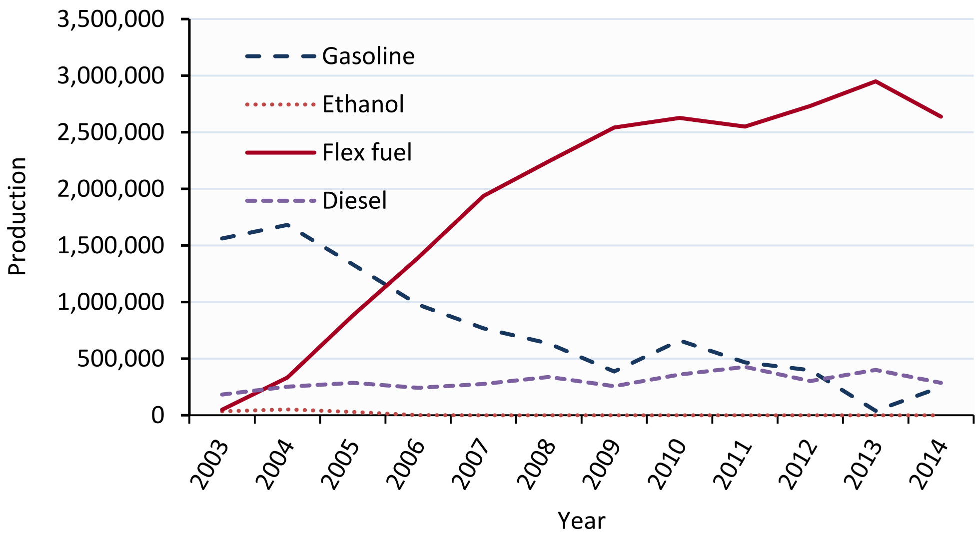
Figure 4 - Automobile production by fuel type in Brazil, 2003-2014



Data source: ANFAVEA (2015)

2.4. History of Fuel Ethanol in Brazil

Brazil has a tradition of producing and consuming fuel ethanol. The first use of sugarcane ethanol as a fuel in Brazil goes back to the late twenties and early 1930s. Due to the lack of foreign oil, the mandatory blend became as high as 50% in 1943. After the end of the war, cheap oil caused gasoline to succeed in the market.

As a result of the first oil crisis, the average price of oil increased from $2.91 per barrel in September 1973 to $12.45 per barrel in March 1975. Meanwhile the international sugar price was low. The National Alcohol Program - Pró-Álcool-('Programa Nacional do Álcool'), launched in 1975, aimed to develop its own ethanol industry and to compete with fossil fuels. Government helped to construct distilling firms close to existing sugarcane factories, enabling managers to switch between sugar and ethanol as market price fluctuated. An initial target to blend anhydrous ethanol to gasoline was made up to 22.4% (by volume). In the beginning of the Program, ethanol production costs were close to $100 per barrel, falling to $50 per barrel 10 years later due to economies of scale and technological progress.

During the second oil crisis, the average price of oil increased from $15.85 per barrel in April 1979 to $39.50 per barrel over the next 12 months. Government negotiated with car manufacturers to develop 100% alcohol fuel for vehicles. A variety of motivation were applied to attract agricultural producers, distilling firms, car manufacturers, distributors, and others to adjust their operations and help to meet the expected demand increase. Anhydrous and hydrated alcohol production levels increased from 500 million liters per year in the late 1970s to 15 billion liters per year in 1987. The alcohol fueled only cars were about 92% new car sales between 1983 and 1988.

Oil prices declined in the mid-1980s fell to below $10 per barrel in 1986; Government decreased the subsidies and then decreased the production. Economic priority shifted to manage inflation, the currency was overvalued, which caused the ethanol’s competitiveness damaged. Brazil started importing ethanol due to the spread of alcohol fueled cars and insufficient domestic production. Hence, ethanol production increasing stopped in 1986. The crisis in 1989 reduced the

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share of ethanol-fueled cars to 1.02%. International sugar prices were quite high in the 1990s, while oil price was low.

Oil price rose above $30 in 2003, reached $60 on August 2005 and peaked at $147.30 on July 2008 and world financial crisis occurred in 2008. Flexible-fuel vehicles were introduced to Brazil in March 2003, on which the government had taxed a lower rate than regular cars. Within 18 months from March 2003, flex-fuel vehicles reached rate of 15.2% of new car sales. In 2014, flex-fuel cars are already responsible for almost 88.2% of new light car sales in the Brazilian market.

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3. Theoretical Reference

3.1. Introduction

This chapter focuses on the concept of methods employed over the study. It also demonstrates a contrast between traditional valuation methods and the new generation of strategic decision analytics, namely real options analysis (with Monte Carlo simulation, stochastic forecasting process) combining with fuzzy sets theory. It briefly illustrates the advantages of using each method.

It should be noted that the analytic methods described here never completely replace the traditional approaches. Further, the new analytics complement upon the traditional approaches—as Real options analysis, simulation, and fuzzy real option are a development of a traditional model. Traditional approaches are not incorrect; they are simply incomplete when modeled under actual business conditions under uncertainty and risk. The most uncertainty models in finance could be observed through the classifications introduced by Dias (2014) as follows:

- Economic or market uncertainty

  These uncertainty types are externality uncertainties to the project. They are correlated to the general movements of the economy (markets) which continually receive new information and make fluctuate in prices of commodities and macroeconomic variables. This uncertainty mainly is modeled with stochastic processes.

- Technical or individual uncertainty

  These are specific uncertainties and endogenous (interior) to the project. They are not correlated to the general movements of the economy and it is usually necessary to focus on the information to change it. The technical uncertainty encourages investment in the profit function learning processes like a R&D project.

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- Strategic uncertainty

  They are uncertainties regarding the behavior or preferences or values of other agents that interact in an economic environment. They are endogenous and modeled with Bayesian Games Theory.

3.2. Real Options Theory

Options can be classified into two main categories, financial and real, based on whether the underlying asset is a financial or real asset. Financial assets are primarily stocks and bonds that are traded in financial markets. The options for most of these assets are listed on exchanges such as the Chicago Board Options Exchange and the American Stock Exchange. Real assets may include real estate, projects, and intellectual property, most of which are not usually traded. A real option is a right — not an obligation — to take an action on an underlying nonfinancial, real asset. The action may involve, for example, abandoning, expanding, or contracting a project or even deferring the decision until a later time. The real options can be either American, which can be exercised on or before a predetermined expiration date, or European, which can be exercised on a fixed date only. They share the same characteristics as the financial options and, therefore, the same terminology is used. Table 3 provides a comparison of financial and real options.

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Table 3 - Analogy between financial options and real options

FINANCIAL OPTION	REAL OPTION
Option exercise price	Project investment cost
Underlying asset price	Project value
Stock return	Project return
Stock price volatility	Project value volatility
Stock dividend flow	Project net cash flow
Time to expiration of the option	Time to expiration of the investment opportunity
Risk-free interest rate	Risk-free interest rate

Source: A.C. Pacheco and M.B.R.Vellasco (2009); Dias (2014)

3.2.1. Finance Option

A financial option is a right — not an obligation — to take an action (buy or sell) on an underlying financial asset (e.g., a stock) at a predetermined cost on or before a predetermined date (KODUKULA and PAPUDESU, 2006). Options are traded both on exchanges and in the over-the-counter market. There are two types of options Call and Put. A call option gives the holder the right to buy the underlying asset by a certain date for a certain price. A put option gives the holder the right to sell the underlying asset by a certain date for a certain price. The price in the contract is known as the exercise price or strike price; the date in the contract is known as the expiration date or maturity. American options can be exercised at any time up to the expiration date. European options can be exercised only on the expiration date itself. Most of the options that are traded on exchanges are American (Hull, 2012). The widely used key options terms could be summarized as:

- The call option value (C) at expiration is the maximum of two values: (1) zero and (2) the difference between the underlying asset value (S) at the time when the asset is bought at maturity and the exercise price (X) at maturity:
$$C = Max[0, S - X]$$
- The put option value (P) at expiration is the maximum of two values: (1) zero and (2) the difference between the exercise price (X) at maturity and the underlying asset value (S) when the asset is sold at maturity:
$$P = Max[0, X - S]$$
- A call option is in the money if $S – X > 0$, at the money if $S – X = 0$, and out of the money if $S – X < 0$.
- A put option is in the money if $S – X < 0$, at the money if $S – X = 0$, and out of the money if $S – X > 0$.

Figures 5 and 6 are called payoff diagrams and show the cash payoff of a call and put option, respectively, at expiration. With a call option, if the underlying asset value is less than the strike price at the time of option expiration, the option is considered to be “out of the money” that means it will not be

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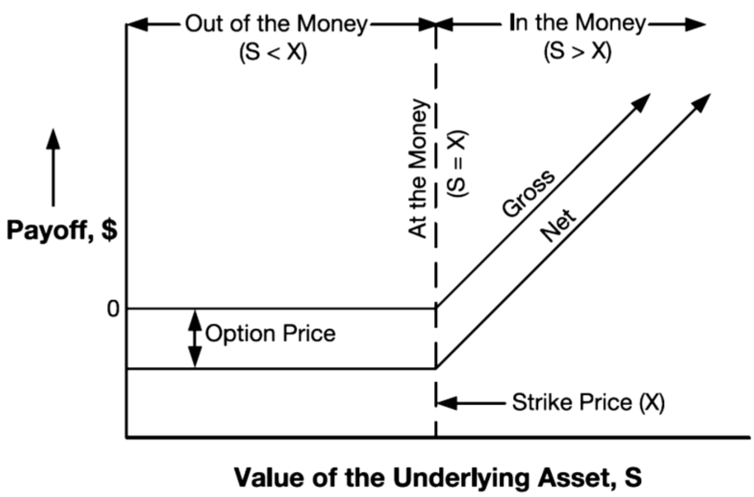
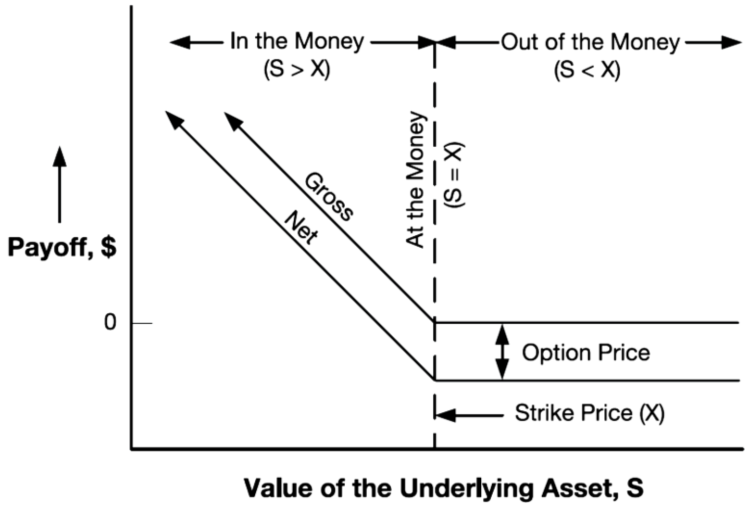
exercised. Thus, the net payoff in this case is negative. If the asset value exceeds the strike price, the option is “in the money” and, it will be exercised and the gross payoff will be positive. When the asset value is exactly equal to the strike price, the option is considered to be “at the money.”

Figure 5 - Payoff diagram for a Call option



Source: Kodukula and Papudesu (2006)

Figure 6 - Payoff diagram for a Put option



Source: Kodukula and Papudesu (2006)

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3.2.2. Real Options

Options can be classified into two broad categories, financial and real, based on whether the underlying asset is a financial or real asset. Real assets may include real estate, projects, and intellectual property, most of which are not usually traded. A real option (RO) is a right — not an obligation — to take an action on an underlying nonfinancial or real asset. The action may involve, for example, abandoning, expanding, or contracting a project or even deferring the decision until a later time. Real options analysis, as simply defined, is the application of financial options, decision sciences, corporate finance, and statistics to evaluating real or physical assets as opposed to financial assets.

According to Dias (2014), Myers (1977) was the first person who used the term of "Real Options" referring to the growth opportunities available to firms by performing a comparison of financial options. The ideas were developed in the seminal studies of Black-Scholes (1973) and Merton (1973). The first mathematical model of RO presented in the late 70s with Tourinho (1979), but the first RO textbook published in the 1990s by Dixit and Pindyck (1994) focusing on RO in continuous time using differential equations based upon an economist view approach. The following was published by Trigeorgis (1996) which focused on both approaches in discrete and continuous time based on financial theory. Since 2014, more than 50 books of the real options (textbooks and articles of collection of books) have been published Dias (2014). In Brazil, the first real options books were published by Minardi (2004) and Brasil et al. (2007). In the finance literature, the books of valuation and corporate finance usually have one or more chapters related to the Real Options valuation.

Industry analysts, experts, and academics all agree that real options provide significant insights to project evaluation that traditional types of analysis such as discounted cash flow approach cannot provide. Sometimes the simple task of thinking and framing the problem within a real options context is highly valuable. The real options can be either American, which can be exercised on or before a predetermined expiration date, or European, which can be exercised on a fixed date only. They share the same characteristics as the financial options and, therefore, the same terminology is used. Figure 7 shows the main different levers

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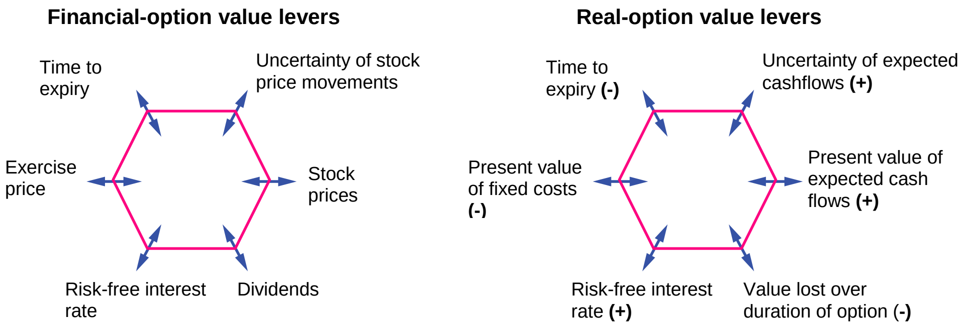
existing in financial and real options, and the increasing and decreasing impact of these levers on the real option value.

Figure 7 - Six levers of financial and real option levers



Source: Leslie and Michaels (1997)

3.2.3. Why is Real Options Analysis valuable?

Real options are important in strategic and financial analysis because traditional valuation tools such as Discounted Cash Flow (DCF) approach or Net Present Value (NPV) ignore the value of flexibility in financial evaluation. For instance, traditional valuation creates a static picture of existing investments and opportunities when exists a corporation as a set of businesses and each with an NPV. An important point is that the traditional approaches as discounted cash flow assumes a single decision pathway with fixed outcomes, and all decisions are made in the beginning without the ability to change and develop over the time.

The real options analysis (ROA) considers multiple decision pathways as a consequence of high uncertainty combining with manager’s flexibility in choosing the optimal strategies or options when new information becomes available. That is, management has flexibility to make strategy corrections at halfway point when there is uncertainty involved in the future. As information becomes available and uncertainty becomes resolved, management can choose the best strategies to implement. Traditional discounted cash flow assumes a single static decision, while real options assume a multidimensional dynamic series of decisions.

However, every real option valuation starts with the underlying asset value which is the expected payoff calculated by DCF method where the risk premium has been added to the discount rate to account the uncertainty. ROA takes the

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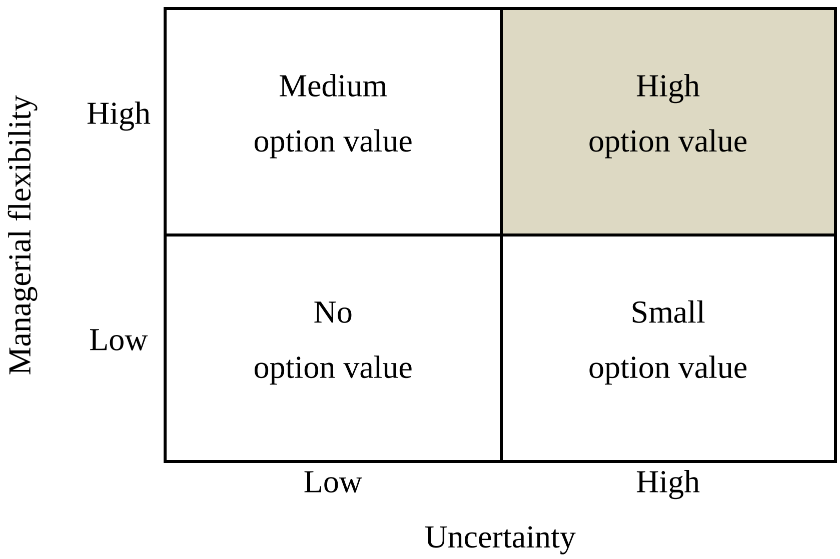
DCF to the next level and makes it a more sophisticated tool. Actually, it supplements and integrates the traditional tools into a more sophisticated valuation technique.

ROA is most valuable when there is high uncertainty with the underlying asset value and management has significant flexibility to change the course of the project in a favorable situation. When there is little uncertainty and not much room for managerial flexibility, the real options approach offers little value Figure 8.

Figure 8 - When real options provide more value



Source: Kodukula and Papudesu (2006)

ROA does not provide much value in investment decisions on projects with very high NPV, because the projects are already attractive for investment and the additional value that may be provided would not change the decision (Kodukula and Papudesu, 2006). As illustrated in Figure 9, real options offer the greatest value on projects with an NPV close to zero (either positive or negative) and high uncertainty.

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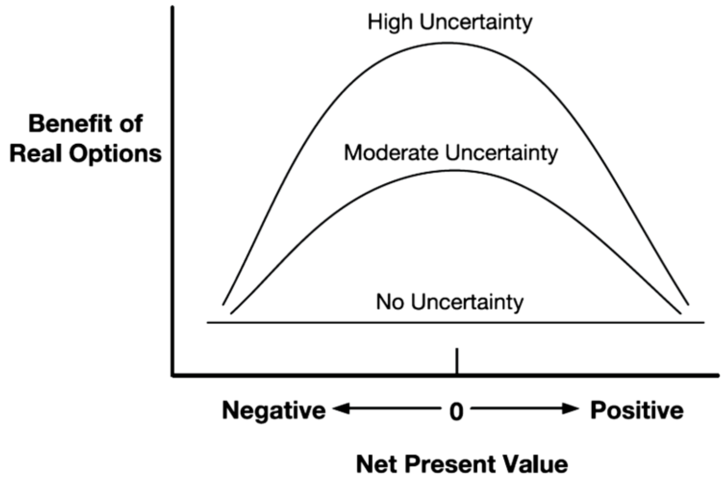
Figure 9 - Benefits of real options analysis relative to net present value



Source: Kodukula and Papudesu (2006)

3.2.4. Types of Real Options

In real situation, the real options are mainly categorized depending on the scenario which decision makers deal with (Dias, 2014; Hull, 2012; Juan Guillermo Lazo Lazo 2004). This classifying type could be indicated as:

- **Option to defer**
  One of the most important options open to a manager is the option to defer a project. It is the option of waiting for the best time to start a project and can be valued as an American call option.
- **Option to expand**
  This is the option to make further investments and increase the output if conditions are favorable.
- **Option to contract**
  This is the option to shrink the scale of a project and can be valued as an American put option.
- **Option to abandon**
  This is an option to sell or close down a project. It is an American put option on the project’s value. The strike price of the option is the liquidation (or resale) value of the project.

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- **Option to switch**

  A switching option refers to the flexibility in a project to switch from one mode of operation or asset to another. Switching options can be divided as follows (Dias, 2014):

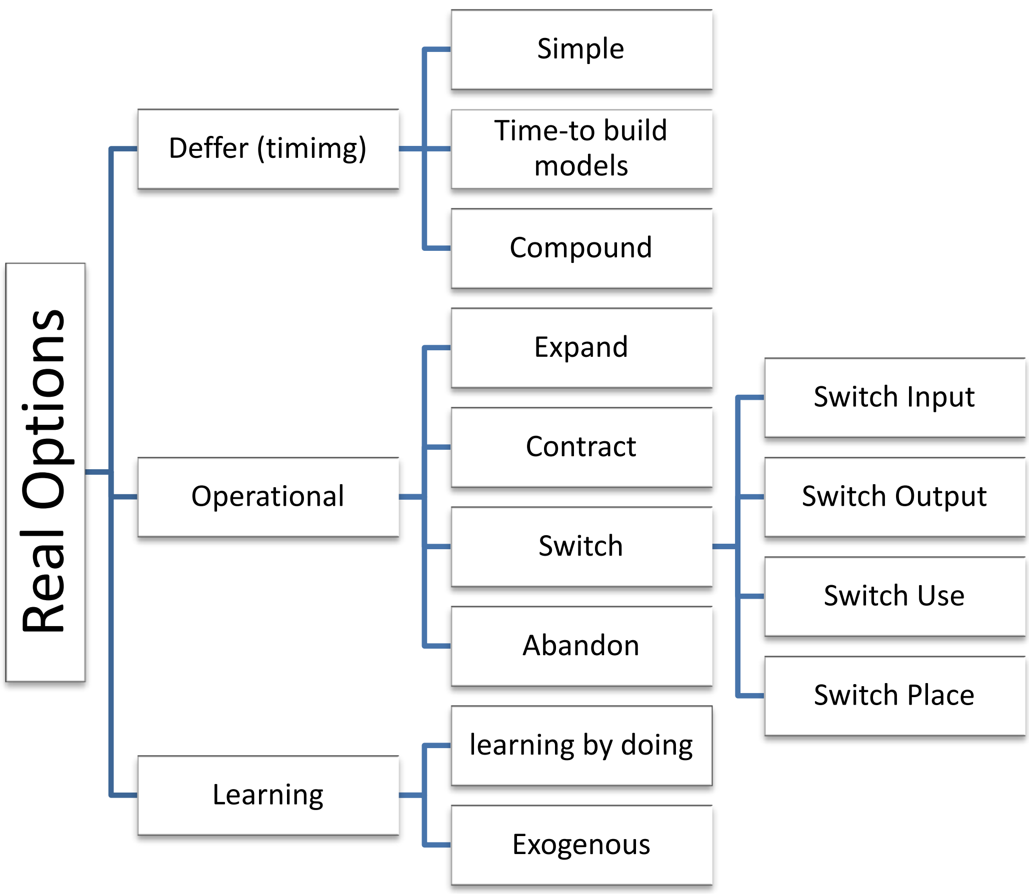
  - switch-input
  - switch-use
  - switch-output
  - switch-place

- **Options to extend**

  This option refers to possibility of extending the life of an asset by paying a fixed amount. This is a European call option on the asset's future value.

In addition to the various option types mentioned above, Dias (2014) proposed an expanded and more comprehensive version of option classification which can be observed in Figure 10:

Figure 10 - Real options classification



Source: Dias (2014)

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In this study, switch input option as switch fuel option was applied to option valuation. For this type of option, the ability to switch between using different inputs to produce the same output, that mainly are built up due to the stochastic nature of input price, is known as an input mix option or process flexibility. It gives less cost and more profit for the producer or consumer of a product or owner of asset.

For the option associated with the flex car, inputs are two types of fuels, hydrated ethanol and gasoline that are able to generate energy to move in the same vehicle (output). This option is also valuable in enabling to switch mix product according to market behavior, and still, there is no charge for the exchange of inputs, which has considerable benefits for consumers.

The calculation of the value of this option depends on the definition of the stochastic process able to depict the uncertainty of future asset price and choose the most appropriate method of modeling the distribution of occurrence probabilities of those prices. According to Bastian-Pinto et al. (2009), proper modeling of the behavior of a stochastic variable in this case is essentially, using the real options evaluation. Neglecting this consideration can cause generate misleading results, either over or under the magnitude value of the real option.

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3.2.5. Option Valuation

Financial experts and managers every day are involving in making project investment decisions. The decision may be whether to invest in a new project now or wait a while, or it may be whether to contract, expand, or abandon an ongoing project. The decision makers often are looking for tools that can help them make the right decisions. The most fundamental information needed to make such decisions is related to the value of the project in financial terms. Net present value (NPV) based on discounted cash flow (DCF) analysis is the most commonly used tool today in project valuation. You will invest in a project if the NPV of the project is positive.

These traditional methods are standard tools used by analysts and other professionals in project valuation, and they serve the purpose very well in many applications. However, these tools have certain limitations. For instance, if there is large uncertainty related to the project cash flows and the contingent decisions are involved, where the mangers have flexibility within the project, some of the values are not accounted for.

Real options analysis (ROA) is more complex comparing with traditional tools and requires a higher degree of mathematical understanding. The theoretical framework of real options solutions is complex, whereas the calculations involved are rather simple. Several methods are available to calculate option values and within each method, there are many alternative techniques to deal with the mathematic calculations. The choice depends on project complexity and available input data. The main methods include complex mathematics, which may be difficult to explain to senior management, while other methods are more known and can be illustrated easily. Some famous solution methods are as:

- Black-Scholes Equation

The famous Black-Scholes equation is:

$$C = N(d_1)S_0 - N(d_2)Xe^{-(rT)} \quad (1)$$

Where

$C = value\ of\ the\ call\ option$;

$S_0 = current\ value\ of\ the\ underlying\ asset$;

$X = cost\ of\ investment\ or\ strike\ price$

$r = risk - free\ of\ return$

$T = time\ to\ expiration$

$d_1 = [\ln(S_0/X) + (r + 0.5\sigma^2)T]/\sigma\sqrt{T}$

$d_2 = d_1 - \sigma\sqrt{T}$

$\sigma = annual\ volatility\ of\ future\ cash\ flow$
$of\ the\ underlying\ asset$

$and\ N(d_1) and\ N(d_2) are\ values\ of\ the\ standard$
$normal\ distribution\ at\ d_1\ and\ d_2$

- Simulations

This method involves simulation of thousands of paths of underlying asset value during the option life as defined by the volatility of the asset value. In the simulation method applied in this study, the option life is divided into a selected

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number of time steps, and thousands of simulations are made to identify the asset value (fuel price) at each step of each simulation.

- Binomial Trees

The binomial option pricing model which invented by COX et al. (1979), is an useful and very popular technique for pricing an option which involves constructing a binomial tree. Binomial trees model is a diagram representing different possible paths that might be followed by the value of asset over the life of an option. The underlying assumption is that the value of asset follows a random walk considering a risk-neutral preference and no-arbitrage arguments. In each time step, it has a certain probability of moving up by a certain percentage amount and a certain probability of moving down by a certain percentage amount. As the time step becomes smaller, this model is the same as the Black–Scholes model (Hull, 2012).

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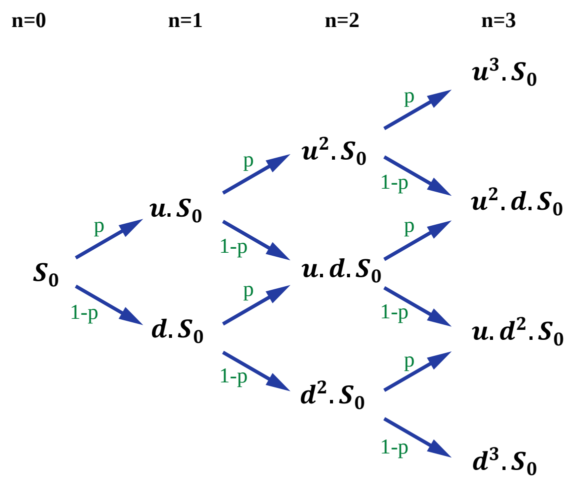
Figure 11 - Binomial tree of three steps with associated asset prices



Source: Author

At each stage, the asset price moves up by factor *u* or down by factor *d* where *u* is >1 and *d* is <1 and we assume u = 1/d. The size of these factors depends on the volatility of the underlying asset.

Option prices at every point are calculated by an equation that varies with the type of option. For instance, for European call options the equations below are applied:

$$V_N = \max(S_N - X, 0) \tag{2}$$

$$V_n = e^{-rt}(pV_u + (1-p)V_d) \tag{3}$$

Where

$$p = \frac{e^{-rt} - d}{u - d}$$

$$u = e^{\sigma\sqrt{t}}$$

$$d = \frac{1}{u}$$

$$V_u = Option\ up$$

$$V_d = Option\ down$$

And V_N is the option price at the expiry node N, X is the strike or exercise price, S_N is the stock price at the expiry node N, and n is any node before expiry node.

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3.3. Stochastic processes

In this study, the stochastic process is used for establishing simulation structures, and obtaining an evolution of pricing structure model. A stochastic process is nothing but a mathematically defined equation that can create a series of outcomes over time, outcomes that are not deterministic in nature (MUN, 2002). That is, an equation or process that does not follow any simple distinct rule such as price will increase X percent every year or revenues will increase by this factor of X plus Y percent.

3.3.1. Mean Reversion Process

Mean reversion process is the assumption that a stock's price will tend to move to the average price over the time. Mean reversion process (MRP) is widely seen in finance and used in real option models. MRP is naturally attractive to model commodity prices since they represent the economic argument that when prices are ‘too high’, demand will reduce and supply will increase. When prices are ‘too low’ the opposite will occur, again pushing prices back towards some kind of long term mean.

If a stochastic process has a long-run parameter such as a long-run production cost, then a mean reversion process is more likely suitable. The model recognizes that over time the variance tends to be pulled back to a long-run mean level of $\bar{X}$ and fluctuates around this equilibrium. The process “X” is modeled by the following stochastic equation:

$$dX = \eta(\bar{X} - X)dt + \sigma dz \qquad (4)$$

Where:
X = stochastic variable;
η = speed of reversion;
$\bar{X}$ = stochastic variable long-run mean;
σ = stochastic variable volatility;
dz = increment or Wiener differential.

In general, the commodities prices are assumed to have a log-normal distribution (Samanez et al., 2014). That is, if X = ln(x), then, x = e^X, and always keeps the commodity price positive even the X value be negative. There is no sense, for example, in having a commodity price series with negative values.

According to Dixit and Pindyck (1994) for an X_t stochastic variable that follows a MRP, the mean and the variance are given by:

$$E[X_t] = \bar{X} + (X_0 - \bar{X})e^{-\eta t} \qquad (5)$$

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$$Var[X_t] = (1 - e^{-2\eta t})\sigma^2/2\eta \qquad (6)$$

3.3.2. Discretization of MRP

To realize the simulation of the stochastic process and the consequent future price estimation, it is necessary to have a discrete equation; in other words, X_t as a function of X_{t-1}. The discrete equation of the MRP real simulation for a time interval (Δt) is given, according to Dias (2008) and Samanez et al. (2014) as:

$$X_t = X_{t-1}e^{-\eta\Delta t} + \bar{X}(1 - e^{-\eta\Delta t}) + \sigma\sqrt{\frac{1 - e^{-2\eta\Delta t}}{2\eta}}N(0,1) \qquad (7)$$

According to Schwartz (1997), for a stochastic variable that follows MRP of $dX = \eta(\bar{X} - X)dt + \sigma dz$ type to be adjusted for a risk-neutral environment, a normalized risk premium π/η should be subtracted from the long- run mean, according to the following equation

$$dX = \eta\left[\left(\bar{X} - \frac{\pi}{\eta}\right) - X\right]dt + \sigma dz\ .$$

By Dias (2008) and Samanez et al. (2014), following equation presents this adaptation for an MRP risk-neutral simulation:

$$X_t = X_{t-1}e^{-\eta\Delta t} + \left[\bar{X} - \frac{\pi}{\eta}\right](1 - e^{-\eta\Delta t}) + \sigma\sqrt{\frac{1 - e^{-2\eta\Delta t}}{2\eta}}N(0,1) \qquad (8)$$

3.3.3. Estimation of MRP-Parameters

For MRP to become discrete it is necessary for the parameters estimation of stochastic Equation (4), which is, the volatility, the long-run mean and the speed of reversion and to obtain these parameters a linear regression with historical price data must be run.

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As demonstrated by Bastian-Pinto et al. (2009), by MRP mean equation (Equation 5), prices which following MRP can be described as following equation:

$$X_t = \bar{X} + (X_{t-1} - \bar{X})e^{-\eta\Delta t}$$

or (9)

$$X_t = \bar{X}(1 - e^{-\eta\Delta t}) + X_{t-1}e^{-\eta\Delta t}$$

Subtracting X_{t-1} from both sides of Equation 9 and considering the regression error:

$$X_t - X_{t-1} = \bar{X}(1 - e^{-\eta\Delta t}) + X_{t-1}(e^{-\eta\Delta t} - 1) + \varepsilon_t \;\; where \sim N(0, \sigma_\varepsilon^2) \quad (10)$$

Considering that $X_t = \ln(X_t)$ and $\bar{X} = \ln(\bar{X})$, Equation 10 can be rewritten in the following way:

$$\ln(X_t) - \ln(X_{t-1}) = \ln(\bar{X})\,(1 - e^{-\eta\Delta t}) + \ln(X_{t-1})\,(e^{-\eta\Delta t} - 1) + \varepsilon_t \quad (11)$$

Finally, the linear regression equation is given by:

$$\ln(X_t) - \ln(X_{t-1}) = a + (b - 1)\ln(X_{t-1}) + \varepsilon_t \quad (12)$$

Through Equation 12, running a regression for the logarithm of prices, a and b coefficients, can be estimated. By comparison, Equations 11 and 12, it is possible to obtain the speed of reversion and the long-run mean of MRP:

$$b - 1 = e^{-\eta\Delta t} - 1 \qquad \Rightarrow \qquad \eta = -\ln(b)\,/\Delta t \quad (13)$$

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$$a = \ln(\bar{X})\,(1 - e^{-\eta\Delta t}) \qquad \Rightarrow \qquad \bar{X} = \exp[-\frac{a}{(b-1)}] \tag{14}$$

The determination of volatility is obtained by equating the regression residual variance, Equation 7, to the variance of MRP, Equation 6:

$$\sigma_\varepsilon^2 = (1 - e^{-\eta\Delta t})\sigma^2/2\eta$$

Considering that

$$b^2 = e^{-\eta\Delta t} \quad \Rightarrow \quad \sigma_\varepsilon^2 = -(1 - b^2)[\frac{\sigma^2\Delta t}{2\ln(b)}]$$

Then

$$\sigma = \sigma_\varepsilon\sqrt{2\ln(b)\,/[(b^2-1)\Delta t]} \tag{15}$$

in which Δt is the time interval.

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3.4. Monte Carlo Simulation

Simulation is an analytical method that is aimed to imitate a real-life system, especially when other analyses are too mathematically complex or too difficult to reproduce. A simulation is an experiment, usually conducted on a computer, involving the use of random numbers. Monte Carlo simulation, or probability simulation, is a technique used to understand the impact of risk and uncertainty in financial, project management, cost, price and other forecasting models (RiskAmp, 2015). Monte Carlo simulation randomly generates values for uncertain variables repeatedly to simulate a real-life model.

Monte Carlo simulation was named after Monte Carlo, Monaco, where the primary attractions were casinos containing games of chance (MUN, 2002). Games of chance such as roulette wheels, dice, and slot machines exhibit random behavior. The random behavior in games of chance is similar to how Monte Carlo simulation selects variable values at random to simulate a model. When you roll a die, you know that a 1, 2, 3, 4, 5, or 6 will come up, but you do not know which for any particular trial. It is same with the variables that have a known or estimated range of values but an uncertain value for any particular time or event (e.g., interest rate, cost, revenue, price, inventory, discount rate).

In this study, Monte Carlo simulation permits to simulate the uncertainties that affect the fuel price. It is necessary to note that the establishing a method to assess the uncertainty needs to define a stochastic process that the variables follow according to their uncertainty.

Real Option simulation by Monte Carlo method can be performed by following steps Dias (2014):

1. Specifying the probability distributions of the input variables, including the distribution of correlations between different variables;
2. Generating a random sample (using a random number generator) of the input data distribution;
3. Performing mathematical operations by the sample generated in the previous step to calculate the result (output);
4. Repeating the above steps N times and generating N outputs;
5. Calculating average and other desired properties of the probability distribution of outputs.

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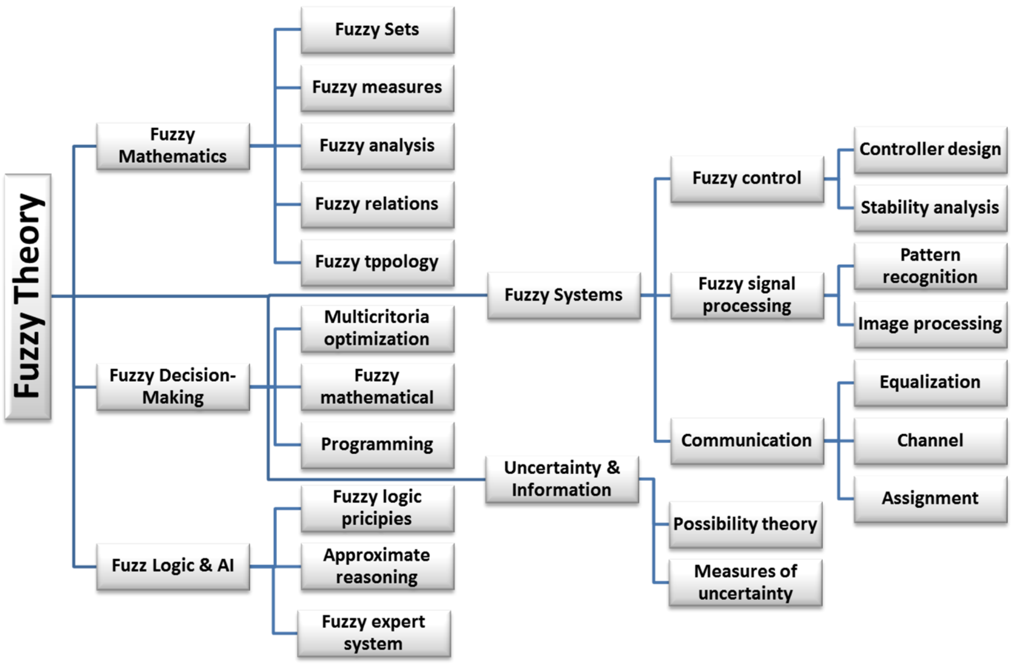
3.5. Fuzzy Theory

By using Zadeh (1965) who invented fuzzy theory, a fuzzy set is a class of objects with a continuum of grades of membership, that is, such a regular set that is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one. Accordingly, Li-Xin Wang (1997) conducted an overall classification of fuzzy theory application area using basic concept of fuzzy set or continuous membership function (Figure 12). Based on this classification there are five major areas: (i) fuzzy mathematics, where classical mathematical concepts are extended by replacing classical sets with fuzzy sets; (ii) fuzzy logic and artificial intelligence, where approximations to classical logic are introduced and expert systems are developed based on fuzzy information and approximate reasoning; (iii) fuzzy systems, which include fuzzy control and fuzzy approaches in signal processing and communications; (iv) uncertainty and information, where different kinds of uncertainties are analyzed; and (v) fuzzy decision making, which considers optimization problems with soft constraints. These five areas are not independent and there are strong interconnections among them. For example, fuzzy control applies concepts from fuzzy mathematics and fuzzy logic.

From Figure 12 can be observed that fuzzy theory is a large field and comprises a variety of sub- topics. In this text, we concentrate on fuzzy sets that could also have an interconnection with fuzzy measuring, uncertainty, and possibility. We will first study the basic concepts in fuzzy sets, then fuzzy numbers as a sub-category of fuzzy sets and then triangular distribution type of fuzzy number.

Figure 12 - Fuzzy theory classification



Source: Li-Xin Wang (1997)

Fuzzy theory is a knowledge that considered to be vague, imprecise, and with no single quantitative value, whether is due to the lack of information or parameters vague nature. Fuzzy theory states that fuzziness occurs when the boundary of a set of values are not specific. For instance, the concepts of “young”, “old”, “tall”, etc. are sort of fuzzy because the age of someone to be called

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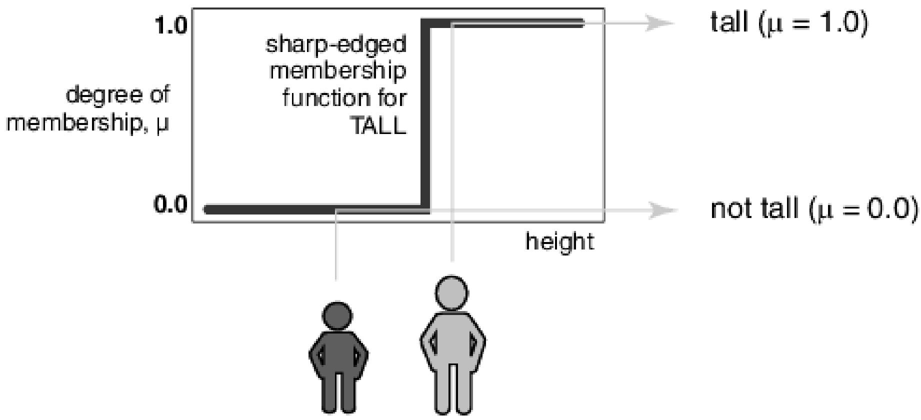
“young” or height of someone which could be mentioned as “tall” are not a fixed or crisp value. Some people, for example, consider twenty-five years old as a young age and thirty-five as not a young age, while others believe thirty-five should be considered as young age. In the concept of fuzzy theory, an age can have a membership degree of being in the “young age” set (more about the concept of fuzzy sets, membership degree, and their operations and relations will be explained in next sections).

Let us look at the mentioned example of the set of tall men. We say that people taller than or equal to 183 cm are tall. This set can be represented graphically as a classical (crisp) set as follows:

Figure 13 - Tall membership function graph in a classical set



Source: CIT (2015)

The above function describes the membership of the 'tall' set, you are either in it or you are not in it. This sharp edged membership functions work well for classical (crisp) set, but it does not work in real world well. The membership function makes no distinction between somebody who is 185 cm and someone who is 215 cm. They are both simply tall. Clearly, there is a significant difference between the two heights. The other side of this lack of distinction is the difference between a 180 and 183 cm man. This is only a difference of 3 centimeters; however, this membership function just says one is tall and the other is not.

The fuzzy set approach can provides a better representation of the tallness of a person. The below set, is defined by a continuously membership function.

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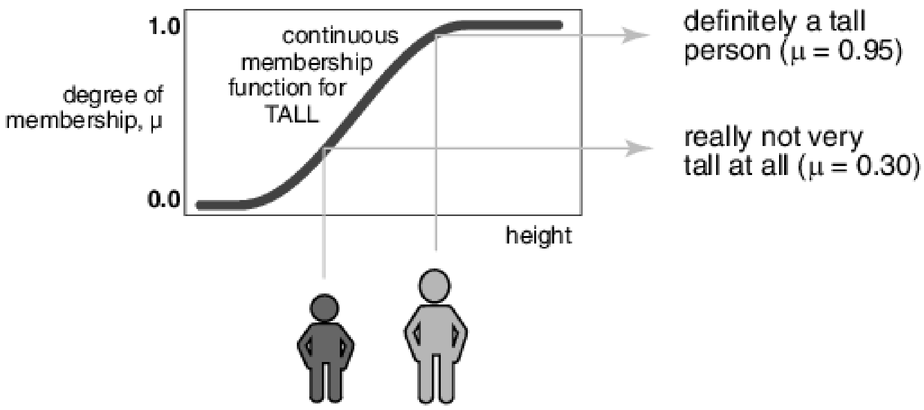
Figure 14 - Tall membership function graph in a fuzzy set



Source: CIT (2015)

The membership function defines the fuzzy set for the possible values. The vertical axis, on a scale of 0 to 1, provides the membership value of the height in the fuzzy set. Therefore, for the two people shown in the figure 14 the first person has a membership degree of 0.3 for a 155 cm man and so is not very tall. The second person has a membership degree of 0.95 for a 180 cm man and he is definitely tall. This definition of a fuzzy set is like a superset of the definition of classical sets. The grades of membership of zero and one correspond to the two possibilities of truth and false in an ordinary set.

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3.5.1. Fuzzy sets

Fuzzy sets are sets whose elements have degrees of membership contrary to classical sets that just is zero or one, yes or no, etc. Fuzzy sets were invented by Lotfi A. Zadeh in 1965 as an extension of the classical set. As following, some basic concepts and technical terms that related to fuzzy set are introduced. Many of them are extensions of the basic concepts of a classical (crisp) set, but some are unique to the fuzzy set framework.

Let X is a universal set containing all the possible elements in each particular context or application. Call a classical (crisp) set A, in the universe X which can be defined by listing all of its members or by specifying the properties that must be satisfied by the members of the set. Classical (crisp) set A can be represented as:

$$A = \{x \in X \mid x \text{ meets some conditions}\}$$

There is other method to define a set A, the membership method, which introduces a zero or one value related to the membership function (also called characteristic function) for A, denoted by $\mu_A(x)$, such that

$$\mu_A(x) = \begin{cases} 1 \; if \; x \in A \\ 0 \; if \; x \notin A \end{cases} \tag{16}$$

Membership function μ_A in crisp set maps whole members in universal set X to set {0,1};

$$\mu_A : X \rightarrow \{0,1\}$$

Definition: Fuzzy set $\tilde{A}$ in a universe of X is characterized by a membership function $\mu_{\tilde{A}}(x)$ that takes values in the interval [0, 1] (Li-Xin Wang, 1997):

$$\mu_{\tilde{A}} : X \rightarrow [0,1]$$

Then, a fuzzy set is a generalization of a classical set by allowing the membership function to take any values in the interval [0, 1] whereas the membership function of a classical set can only take two values zero and one.

The most utilized membership functions in fuzzy sets can be mentioned as:

- Triangular
- Gamma function
- Quadratic S-function
- Trapezoid
- Gaussian
- Exponential-wire function

Definition 2: A fuzzy set $\tilde{A}$ in X is represented as a set of ordered pairs of a generic element x and its membership value, that is:

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$$\tilde{A} = \{(c, \mu_A(x) \mid \text{x} \in \text{X}\} \tag{17}$$

Also, depending on whether X is continuous or discrete $\tilde{A}$ is commonly written as:

Continuous form: $\tilde{A} = \int \mu_A(x)\,/x$

The integral sign does not denote integration; it denotes the collection of all points x ∈ X with the associated membership function$\mu_A(x)$.

Discrete form: $\tilde{A} = \sum_{i=1}^{n} \mu_A(x_i)/x_i$

The summation sign does not represent arithmetic addition; it denotes the collection of all points x ∈ X with the associated membership function$\mu_A(x)$.

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3.5.2. Basic concepts

Some basic concepts and terminology associated with a fuzzy set is defined as follows. Many of them are extensions of the basic concepts of a classical (crisp) set, but some of them are unique to the fuzzy set framework.

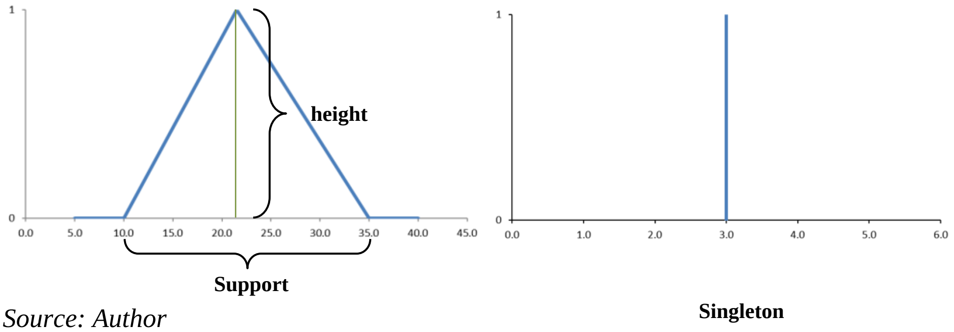
- ***The concepts of support***

The support of a fuzzy set A in the universe of X is a crisp set that contains all the elements of X that have nonzero membership values in A, that is:

$$supp(A) = \{\text{x} \in \text{X} \mid \mu_A(\text{x}) > 0\} \tag{18}$$

If the support of a fuzzy set is empty, it is called an empty fuzzy set and a fuzzy singleton if the support is only a single point in X.

Figure 15 – Fuzzy set's Support, Height and fuzzy set Singleton



Source: Author

- ***Height of a fuzzy set***

  The height of a fuzzy set is the largest membership value attained by any point.

- ***Normal fuzzy set***

  If the height of a fuzzy set equals one, it is called a normal fuzzy set.

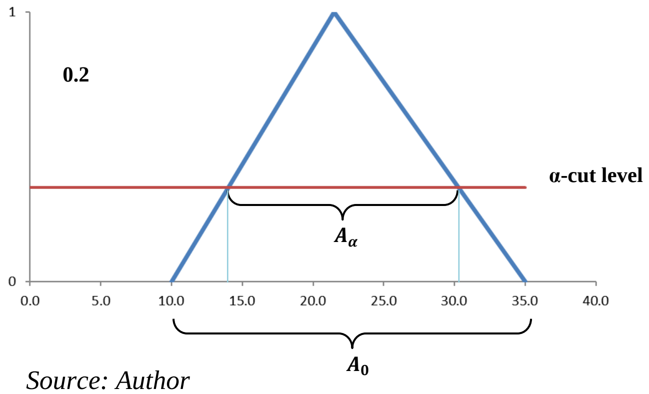
  $\exists\, x \in R, \mu_A(x) = 1$

- ***α-Cut Set***

An $\alpha - cut$ of a fuzzy set A is a crisp set of A, that contains all the elements in X which have membership values greater than or equal to α in A, that is,

$$A_\alpha = \{x \in X \,|\, \mu_A(x) \geq \alpha\} \tag{19}$$

Figure 16 - α − cut of a fuzzy set



Source: Author

For example, for α = 0.2, the $\alpha - cut$ of the fuzzy set $A_{0.2}$ (Figure 16) is a crisp set of [14, 30.5].

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- ***Convex set***

A fuzzy set A is convex if and only if its α-cut A, is a convex set for any α in the interval (0, 1] .The following lemma gives an equivalent definition of a convex fuzzy set.

Lemma[1]: A fuzzy set A in R^n is convex if and only if (Li-Xin Wang, 1997)

$$\mu_A[\lambda x_1 + (1 - \lambda)x_2] \geq \min[\mu_A(x_1), \mu_A(x_2)] \tag{20}$$

for all $x_1, x_2 \in R^n$ and all $\lambda \in [0,1]$.

3.5.3. Main Operations on Fuzzy Sets

The basic concepts introduced in previous section were concerned to only a single fuzzy set. Now let A and B be fuzzy sets defined in the universal discourse X. There are several operations on fuzzy sets and in this study according to Li-Xin Wang (1997) main operations were considered as follows:

- ***The equality of two fuzzy sets***

We say A and B are equal if and only if $\mu_A(x) = \mu_B(x)$ for all x ∈ X.

- ***Containment of two fuzzy sets***

We say B contains A, denoted by A ⊂ B, if and only if $\mu_A(x) \leq \mu_B(x)$ for all x ∈ X.

- ***Complement of two fuzzy sets***

The complement of A is a fuzzy set $\bar{A}$ in X whose membership function is defined as

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x) \tag{21}$$

[1] This definition is similar to the quasiconcave function property in economics area.

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- ***Union of two fuzzy sets***

The union of A and B is a fuzzy set in X, denoted by A $\cup B$, whose membership function is defined as

$$\mu_{A\cup B}(x) = \max[\mu_A(x), \mu_B(x)] \tag{22}$$

- ***Intersection of two fuzzy sets***

The intersection of A and B is a fuzzy set $A \cap B$ in X with membership function:

$$\mu_{A\cap B}(x) = \min[\mu_A(x), \mu_B(x)] \tag{23}$$

3.5.4. Fuzzy Numbers

A fuzzy number is:

> A fuzzy set of the *real line* with a *normal*, (fuzzy) *convex* and *continuous membership function* of a *bounded support*.

The α-cut of A which is indicated as A_α or sometime A^{α} is a closed interval for every $\alpha \in (0,1]$ and the support of A for A^{+0} is bounded.

3.5.5. Triangular Fuzzy Number

All the membership functions of input variables are selected, more or less subjectively, by the expertise based on his experience and the meaning of the function related to the real physical system. Uniform shape(s) of membership functions are usually desirable for computational efficiency, simple memory, and easy analysis. The most common choices of simple and efficient membership functions in finance area are triangular and trapezoidal distribution. As the triangular fuzzy number (TFN) presents a good performance and computational efficiency and provides a realistic model, in this text, it will be utilized as membership function for the vague parameters in analysis.

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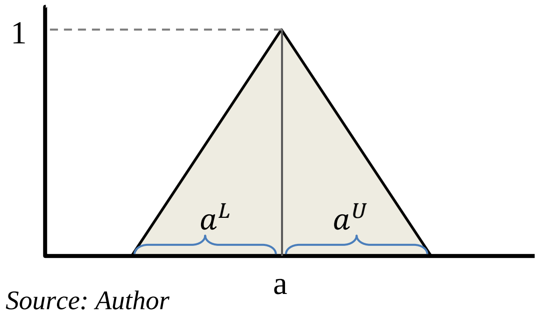
A fuzzy set A is called Triangular fuzzy number with peak (or center) a, left width $a^L > 0$ and right width $a^U > 0$ if its membership function has the following form

$$\begin{cases} 1 - \dfrac{a-x}{a^L} \ if \ a - a^L \leq x \leq a \\ 1 - \dfrac{x-a}{a^U} \ if \ a \leq x \leq a + a^U \\ 0 \qquad otherwise \end{cases} \tag{24}$$

and we use the notation $A = (a, a^L, a^U)$. In this format, the support of A is (a − a^L, a +a^L).

Figure 17 - Triangular fuzzy number in form of (a, a^L, a^U)



Source: Author

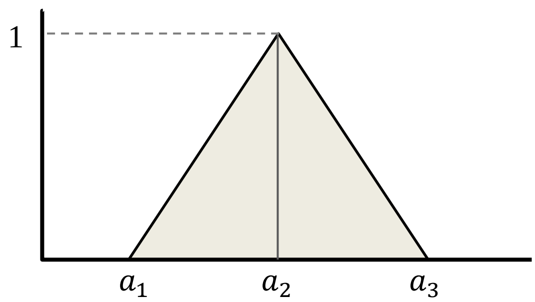
It can easily be verified that α-level intervals of A, obtain by

$$A_\alpha = [a_\alpha^L , a_\alpha^U] = [a - (1-\alpha)a^L, a + (1+\alpha)a^U], \quad \forall\alpha \in [0,1] \tag{25}$$

A triangular fuzzy number can also be represented with three points as:

$$A = (a_1, a_2, a_3)$$

Figure 18 - Triangular fuzzy number in form of (a_1, a_2, a_3)



Source: Author

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This representation is interpreted as below membership function:

$$\begin{cases} \dfrac{x-a_1}{a_2-a_1} & if\ a_1 \leq x \leq a_2 \\ \dfrac{a_3-x}{a_3-a_2} & if\ a_2 \leq x \leq a_3 \\ 0 & otherwise \end{cases} \tag{26}$$

The α-cut interval of the triangular fuzzy numbers A is defined as:

$$A_\alpha = [a_\alpha^L, a_\alpha^U] = [(a_2 - a_1)\alpha - a_1, -(a_3 - a_2)\alpha + a_3] \tag{27}$$

3.5.6. Triangular fuzzy numbers- arithmetic operations

The algorithms between any two fuzzy numbers were introduced by Zadeh (1968) and Santos (1970). Suppose triangular fuzzy numbers $\tilde{a}$ and $\tilde{b}$ are defined as,

$$\tilde{a} = [\tilde{a}_\alpha^L, \tilde{a}_\alpha^U] \ , \quad \tilde{b} = [\tilde{b}_\alpha^L, \tilde{b}_\alpha^U]$$

Then $\tilde{a} \oplus \tilde{b}$, $\tilde{a} \ominus \tilde{b}$ and $\tilde{a} \otimes \tilde{b}$ are also fuzzy numbers and their α-cut level sets are defined as:

- **Addition**

$$(\tilde{a} \oplus \tilde{b})_\alpha = \tilde{a}_\alpha \oplus \tilde{b}_\alpha = [\tilde{a}_\alpha^L + \tilde{b}_\alpha^L, \tilde{a}_\alpha^U + \tilde{b}_\alpha^U] \tag{28}$$

- **Subtraction**

$$(\tilde{a} \ominus \tilde{b})_\alpha = \tilde{a}_\alpha \ominus \tilde{b}_\alpha = [\tilde{a}_\alpha^L - \tilde{b}_\alpha^U, \tilde{a}_\alpha^U - \tilde{b}_\alpha^L] \tag{29}$$

- **Multiplication**

$$(\tilde{a} \otimes \tilde{b})_\alpha = \tilde{a}_\alpha \otimes \tilde{b}_\alpha = \\ [\min\{\tilde{a}_\alpha^L \tilde{b}_\alpha^L, \tilde{a}_\alpha^L \tilde{b}_\alpha^U, \tilde{a}_\alpha^U \tilde{b}_\alpha^L, \tilde{a}_\alpha^U \tilde{b}_\alpha^U\}, \max\{\tilde{a}_\alpha^L \tilde{b}_\alpha^L, \tilde{a}_\alpha^L \tilde{b}_\alpha^U, \tilde{a}_\alpha^U \tilde{b}_\alpha^L, \tilde{a}_\alpha^U \tilde{b}_\alpha^U\}] \tag{30}$$

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And for $m \in R$ then

$$m \otimes \tilde{a}_\alpha = [\min\{m\tilde{a}_\alpha^L, m\tilde{a}_\alpha^U\}, \max\{m\tilde{a}_\alpha^L, m\tilde{a}_\alpha^U\}] \tag{31}$$

for all α ∈ [0,1].

- **Division**

If the α-cut set $\tilde{b}_\alpha$ of $\tilde{b}$ does not contain zero for all α ∈ [0,1], then $\tilde{a} \oslash \tilde{b}$ is also a fuzzy number and its α-cut set is defined as:

$$(\tilde{a} \oslash \tilde{b})_\alpha = \tilde{a}_\alpha \oslash \tilde{b}_\alpha = [\min\{\tilde{a}_\alpha^L/\tilde{b}_\alpha^L, \tilde{a}_\alpha^L/\tilde{b}_\alpha^U, \tilde{a}_\alpha^U/\tilde{b}_\alpha^L, \tilde{a}_\alpha^U/\tilde{b}_\alpha^U\}, \max\{\tilde{a}_\alpha^L/\tilde{b}_\alpha^L, \tilde{a}_\alpha^L/\tilde{b}_\alpha^U, \tilde{a}_\alpha^U/\tilde{b}_\alpha^L, \tilde{a}_\alpha^U/\tilde{b}_\alpha^U\}] \tag{32}$$

Numerical example: Suppose triangular fuzzy numbers $\tilde{a}$ and $\tilde{b}$ are defined as:

$$\tilde{a} = (a_1, a_2, a_3) = (-3, 2, 4), \quad \tilde{b} = (b_1, b_2, b_3) = (-1, 0, 6)$$

Then, α-cut intervals for two triangular fuzzy numbers $\tilde{a}$ and $\tilde{b}$ are

$$\tilde{a} = [\tilde{a}_\alpha^L, \tilde{a}_\alpha^U] = [(a_2 - a_1)\alpha + a_1, -(a_3 - a_2)\alpha + a_3] = [5\alpha - 3, -2\alpha + 4]$$

$$\tilde{b} = [\tilde{b}_\alpha^L, \tilde{b}_\alpha^U] = [(b_2 - b_1)\alpha + b_1, -(b_3 - b_2)\alpha + b_3] = [\alpha - 1, -6\alpha + 6]$$

Then, the results of adding two α-cut interval of $\tilde{a}$ and $\tilde{b}$ can be shown as:

$$\tilde{a}_\alpha \oplus \tilde{b}_\alpha = [6\alpha - 4, -8\alpha + 10]$$

Especially for $\alpha = 0$ and $= 1$:

$$\tilde{a}_0 \oplus \tilde{b}_0 = [-4, 10]$$
$$\tilde{a}_1 \oplus \tilde{b}_1 = [2, 2]$$

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In final step by the result of $\tilde{a}_\alpha \oplus \tilde{b}_\alpha$ given in the example, three points from this procedure come together in a triangular fuzzy number as:

$$\tilde{a} \oplus \tilde{b} = (-4, 2, 10)$$

3.5.7. Fuzzy Ranking or Defuzzification

When we got the results through the operations of fuzzy set, which continue to remain the fuzzy form, we will then deffuzify or transform the fuzzy value into a crisp value (ranked value). This is normally done after all fuzzy operations are performed by reason this fact the fuzzy result must be used in the real world. For every evaluation methods, the key factor is decision making. Fuzzy numbers have to be ranked just because of this fact.

There are different methods for ranking a fuzzy set according to its case study condition. Usually by reducing the whole of any analysis to a single number, much of the information is lost, hence in this study, a method based on the possibilistic mean value or fuzzy mean value which introduced by Carlsson and Fuller (2001) is applied and attempted to minimize this loss;

Let $\mathrm{A} = (\mathrm{a}, \alpha, \beta)$ be a triangular fuzzy number with center a, left-width $\alpha > 0$ and rightwidth $\beta > 0$ then fuzzy mean value is computed by the equation:

$$E(A) = a + (\frac{\beta - \alpha}{6}) \tag{33}$$

3.6. Fuzzy Present Value Equation

The present-value method as an alternative evaluation is very popular because future expenditures or receipts are transformed into present equivalent amount. That is, all of the future cash flows associated with an alternative equation are converted into a present value (PV). Each cash inflow/outflow is discounted back to its PV, and then they are summed (Samanez, 2006):

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$$PV = \sum_{t=0}^{n} \frac{CF_t}{(1+i)^t} \tag{34}$$

Which

$t: the\ time\ of\ cash\ flow$

$i: the\ discounted\ rate$

$CF_t: the\ cash\ flow\ at\ time\ t$

$PV: the\ conventional\ present\ value$

If $\tilde{c}_t$ is fuzzy cash flow at time t and $\tilde{P}$ fuzzy present value, then we will have following equation:

$$\tilde{P} = \sum_{t=0}^{n} \frac{\tilde{c}_t}{(1+i)^t} = \tilde{c}_0 + \frac{\tilde{c}_1}{(1+i)^1} + \cdots + \frac{\tilde{c}_n}{(1+i)^n}$$

$$(\boldsymbol{P_\alpha^L}, \boldsymbol{P_\alpha^U}) = \left(c_0^{L(\alpha)}, c_0^{U(\alpha)}\right) + \frac{\left(c_1^{L(\alpha)}, c_1^{L(\alpha)}\right)}{(1+i)^1} + \cdots + \frac{\left(c_n^{L(\alpha)}, c_n^{L(\alpha)}\right)}{(1+i)^n} \tag{35}$$

which

$$\boldsymbol{P_\alpha^L} = c_0^{L(\alpha)} + \frac{c_1^{L(\alpha)}}{(1+i)^1} + \cdots + \frac{c_n^{L(\alpha)}}{(1+i)^n}$$

$$\boldsymbol{P_\alpha^U} = c_0^{U(\alpha)} + \frac{c_1^{U(\alpha)}}{(1+i)^1} + \cdots + \frac{c_n^{U(\alpha)}}{(1+i)^n}$$

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4. Methodology, application and results

The owner of a car with flex-fuel technology can choose to fill up the car with gasoline, ethanol or any mixture of both fuels in any proportion. The decision depends on the price of the trend of inputs (fuel) in order to minimize the expenses while driving by car.

Fuel prices are free in the market in Brazil. There are some factors which effect on the final price for the consumer, for instance, gasoline consumer price composition consists of: cost price about 32% impact, CIDE, PIS/PASEP and COFINS about 11% impact, ICMS about 28% impact, cost of Anhydrous Ethanol with about 11% impact and Distribution and Resale about 17% impact (PETROBRAS, 2015).

In general, the fuel price at the pump can be divided, basically, in four components: cost price, taxes, logistic and profit margin (Samanez et al., 2014). The first is the selling price without taxes and freight at the mill/refinery before leaving to the fuel distributors. The second are the federal and state taxes (PIS/PASEP, COFINS, ICMS and CIDE). The logistic is the freight cost from the mill/refinery to the distributors and from there to the oil station. Finally, the profit margin forms part of that of the distributor and of that of the oil station. Special attention should be given to ICMS – Tax of goods and service circulation- (*IMPOSTO SOBRE CIRCULAÇÃO DE MERCADORIAS* e *SERVIÇOS)*, which varies by state or by fuel type. The state of São Paulo, for example, taxes ethanol at 12% and gasoline at 25%, quite different from the state of Paraná where taxes for the same are 18% and 28%, respectively. In other state, for instance, the state of Pará, the tax on ethanol reaches 30%. Other factors can be pointed out as determinants of consumer fuel price differences. Among them, market size and logistic costs according to the distance between the producing state and final consumer can also be included.

As explained, some of these factors vary from one state to another and by the fuel type. Furthermore, for instance, the location of plants or refineries in any state or region influences on fuel prices considering the transportation cost. Consequently, the option value of fuel flexibility will be different. Hence,

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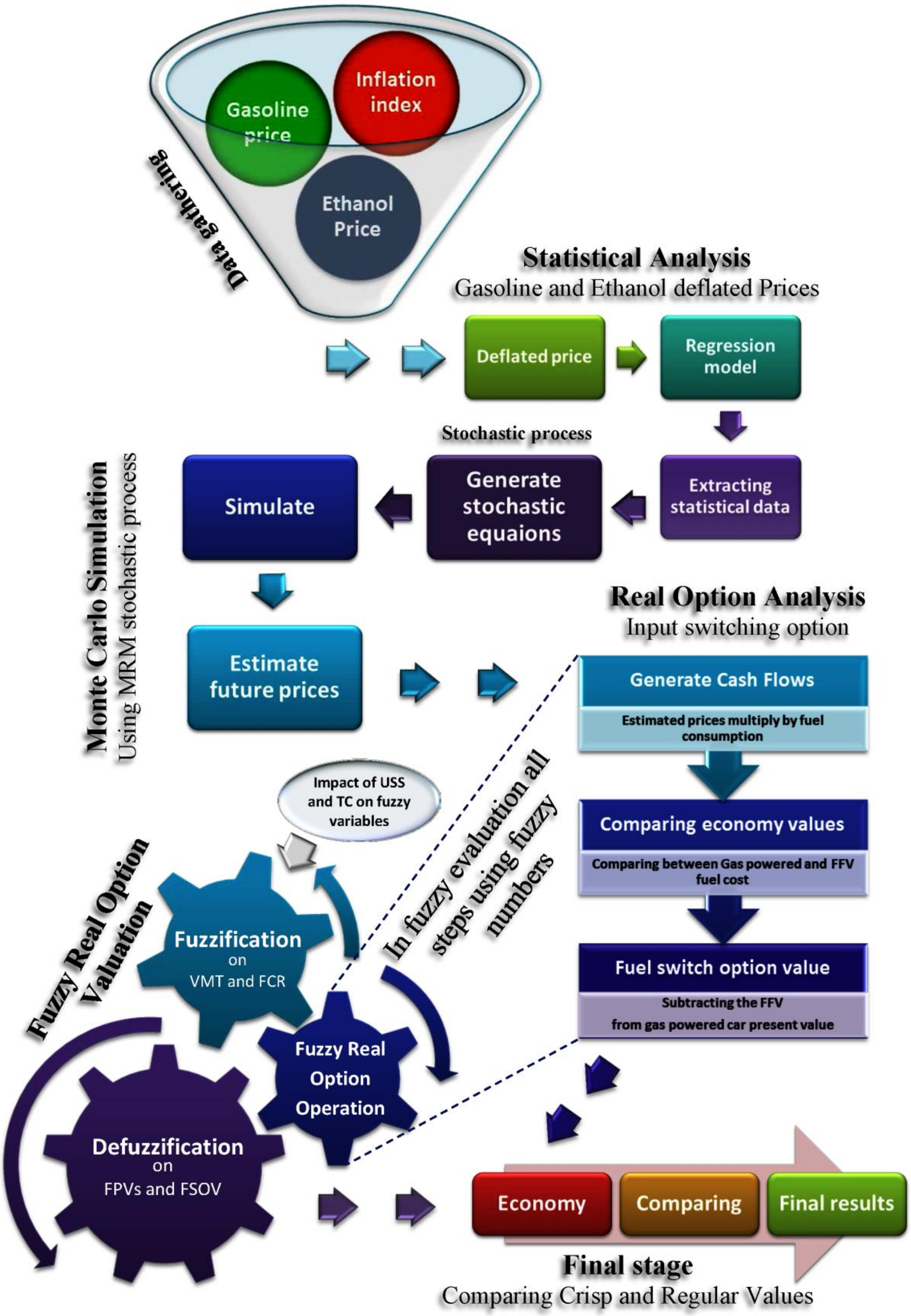
according to these differences in option values, this work will evaluate the option of the flex vehicles flexibility for each geographic region in Brazil apart:

Figure 19 – Fuzzy fuel switch option valuation flow diagram



Source: Author

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Northern (N), Northeastern (NE), Central-Western (CW), Southeastern (SE) and Southern (S) regions. In the following, both the regular real option and fuzzy real option valuation using the mean reversion process and Monte Carlo simulation will be examined. In the final step, not only the economy analysis of flex car over gas powered car option value is measured, but also the consumer preference in purchasing a vehicle in each region is specified. The main steps of fuel switch option valuation by regular and fuzzy real option approach is illustrated by Figure 19.

The following section presents the assumptions, the data surveying and limitations of methodology.

4.1. Assumptions

To evaluate the option flexibility, it is necessary to define input parameters and specifications of flex-fuel vehicles (FFV) in the model, as well as car's lifetime, the risk free rate and the risk premium, etc.

The vehicle assumed will be 1.0 cc. Due to FENABRAVE (Federação Nacional da Distribuição de Veículos Automotores) report, the two best-selling car models in domestic market in 2014 are Fiat UNO and VW GOL (Volkswagen GOL). Both are available in version 1.0 cc and flex-fuel technology. The quantity of sold car for these two models, in domestic market in 2014, is 122,269 units for Fiat UNO and 183,368 units for VW GOL. Their characteristics are shown in Table 4.

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Table 4 - Specifications of selected vehicles

Vehicle	**Condition**	**VW GOL 1.0**	**FIAT UNO**
		1.0 8V Special	**1.0 8V Vivace**
Vehicle value (R$)		31,240	29,220
Gasoline consumption rate (km/liter)	City	11.6	11.8
	Highway	13.9	15.0
Ethanol consumption rate (km/liter)	City	7.7	8.2
	Highway	9.6	10.4
Tank capacity (liters)		55	48
Monthly kilometers ran (km)		1500	1500
Maxim power-gasoline (km/h)		72	73
Maxim power-ethanol (km/h)		76	75

Source: Fiat (2015), INMETRO (2015) and Volkswagen (2015)

Other necessary data for the calculation of option value can be observed as following:

• Asset lifetime: 5 years;
• Risk-free rate (r) : 15.26%[2] p.a., monthly equivalent 1.71%[3] (National Treasury, 2015);
• Risk-premium (π) : 8.66% p.a., monthly equivalent 0.69% (Damodaran, 2015).

4.1.1. Fuel Consumption Rate

All new light vehicles sold are required to present the automobile's fuel consumption information. It includes all passenger cars as four wheel drives and light commercial vehicles up to 3.5 tones gross vehicle mass. The label indicates the vehicle's fuel consumption rate (FCR) in kilometer running per one liters (Km/L) or the fuel consumption per 100 km running and maybe its emissions of carbon dioxide (CO2) in grams per kilometer (g/km). The fuel consumption rate normally presents two or three types of information. It can be different from one country to another country. In Brazil, widely, just the fuel consumption rate in highway and city driving conditions are indicated. Manufacturers mainly do not present the combined fuel consumption rate on the labels possibly due to difficulties in estimating of combined condition.

- City:
  A "city" estimate represents urban driving, in which a vehicle is started in the morning (after being parked all night) and driven in stop-and-go traffic.
- Highway:
  A "highway" estimate represents a mixture of rural and interstate highway driving in a warmed-up vehicle, typical of longer trips in free-flowing traffic.

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[2] 15.26% p.a. is the yield of LTN (Letra do Tesouro Nacional) - National Treasury – with maturity in 01/01/2021.
[3] Because the annual risk-free rate is based on 252 days per year, the monthly risk-free rate is calculated as follows:

$$1.71 = (1.1616)^{\frac{30}{252}}$$

- Combined:

  A "combined" estimate represents a combination of city driving and highway driving.

Living inside a city does not mean that the ultimate car’s fuel consumption rate is exclusively equal to city driving type. During the month or year, the car is running outside the city for traveling, mission, or any other purpose. In many developing countries, recently, governments are expanding the highways system inside the city due to the increasing rate of car production, huge population of vehicles on roads and streets and possibly the majority due to traffic congestion problems. Running along a highway when there is no traffic inside the city, the fuel consumption rate will not be as much as the city driving type as supposed.

The vehicle’s fuel consumption will certainly varies car-by-car and even for a specific driver time by time. It could be varied based on where you drive, how you drive, the vehicle type and other parameters like temperature. Hence, it is impossible to predict fuel economy precisely for all drivers in all environments. Moreover, the following factors can decrease or increase vehicle’s FCR:

- Aggressive driving (hard acceleration and braking)
- Excessive idling, accelerating, and braking in stop-and-go traffic
- Cold weather (engines are more efficient when warmed up)
- Driving with a heavy load or with the air conditioner running
- Improperly tuned engine or underinflated tires
- Use of remote starters

USDOE (2014) publishes a fuel cost estimate report for various vehicle models each year. They proposed a combined driving model to estimate the fuel consumption rate by combination of city driving (55%) and highway driving (45%) for most of light vehicles.

According to this ratio and data from Table 4, the combined fuel consumption rate for VW GOL and Fiat UNO was estimated and presented in Table 5.

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Table 5 - Combined fuel consumption rate

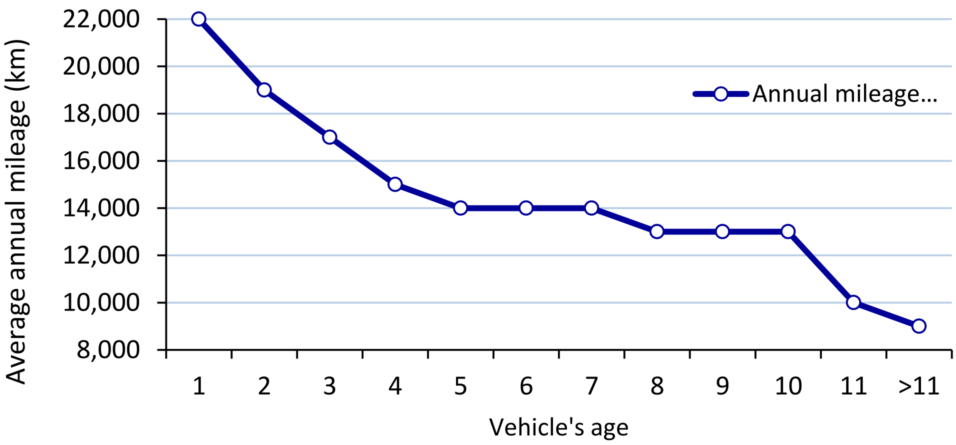
Vehicle	Condition	Ratio	VW GOL 1.0 1.0 8V Special	FIAT UNO 1.0 8V Vivace
Gasoline consumption rate (km/liter)	City	55%	11.6	11.8
	Highway	45%	13.9	15.0
	Combined	-	**12.6**	**13.2**
Ethanol consumption rate (km/liter)	City	55%	7.7	8.2
	Highway	45%	9.6	10.4
	Combined	-	**8.6**	**9.2**
Maxim relation between prices (ethanol/gasoline)[4]			0.677	0.694

Source: Author calculation based on INMETRO (2015)and USDOE (2014)

4.1.2. Vehicle Mileage Traveled

Bastos (2011) presents a research of the metropolitan region of São Paulo which estimated the average annual mileage for light vehicles and attempted to find a relationship between automobile’s age and annual mileage in this region. The distribution of this annual average mileage is presented in Figure 20.

Figure 20 - Distribution of annual mileage (in km) according to the age of the vehicle



*Source:*Bastos (2011)

[4]In Brazil, because Flex-fuel cars were modified from cars powered only by ethanol, their engines are typically more powerful when using hydrated ethanol, so the best performance is in using this fuel. However, the yield of this fuel, which measure per kilometer, is lower compared to gasoline and causes an increased expenditure of fuel for the vehicle to run the same mileage that would run by gasoline. As ethanol and gasoline have different consumption rate (fuel efficiency), the CEPEA-USP (Centro de Estudos Avançados em Economia Aplicada) recommends an economic ratio for selecting the fuel type equal to 70% for ethanol price per gasoline price.

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By Figure 20, new vehicle runs in the first year about 22,000 kilometers, and as the vehicle gets older the mileage decreases accordingly. Vehicles with more than eleven years of fabrication time, runs an average of 9,000 km.

Based on this estimation, average annual mileage for first five years of vehicle's age as assumed for the asset lifetime (vehicle life expectancy) in this study will be in a range of 22,000 – 14,000 km per year. Consequently, average of this range gives the average of vehicle mileage traveled per year and the average VMT per month dividing by 12 months.

$$\text{Average of vehicle kilometers ran per year} = \frac{14{,}000+22{,}000}{2} = 18{,}000\ km$$

$$\text{Average of vehicle kilometers ran per month} = \frac{18{,}000}{12} = 1500\ km$$

Now, based on the 1500 km ran per month and urban fuel consumption rate from previous section, we are able to calculate the monthly fuel use (liter) for each vehicle and fuel type:

$$\text{Monthly fuel use} = \frac{\text{Average kilometers ran per month } (km)}{\text{Urban fuel consumption rate } (\text{km}/L)}$$

Table 6 - Monthly fuel consumption (liter) by vehicle and fuel type

Vehicle	**VW GOL 1.0**	**FIAT UNO**
	1.0 8V Special	**1.0 8V Vivace**
Monthly gasoline use (liters)	118.8	113.3
Monthly Ethanol use (liters)	175.3	163.2

Source: Author

In this study it is supposed the vehicle owner of flex technology will always make a great choice at the time of fueling and optimize the lowest monthly cash flow through the minimizing the fuel cost during the vehicle lifetime. In this study, a European option valuation that expires every 1500 kilometers ran in a

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month is applied to estimate the FFV flexibility value. That is, consumer chooses one fuel type at every refueling time until it is covered 1,500 km in a month. The supply decision in the month *i* will be absolutely independent from the decision of other month *j*, for $\forall(i,j) \in [1,60], i \neq j$. Furthermore, there is no cost of fuel exchange between each period. The summary of whole characteristics of selected vehicles is represented by the following table:

Table 7 - Summary of selected vehicles specifications

Vehicle	Condition	VW GOL 1.0 1.0 8V Special	FIAT UNO 1.0 8V Vivace
Vehicle value (R$)		31,240	29,220
Urban gasoline consumption (km/liter)	City	11.6	11.8
	Highway	13.9	15.0
	Combined	12.6	13.9
Urban ethanol consumption (km/liter)	City	7.7	8.2
	Highway	9.6	10.4
	Combined	8.6	9.2
Monthly kilometers ran (km)		1500	1500
Monthly gasoline use (liters)		118.8	113.3
Monthly Ethanol use (liters)		175.3	163.2

Source: Author based on Fiat (2015), INMETRO (2015), and Volkswagen (2015)

4.2. Data gathering

In this work, the monthly time series of the hydrated ethanol and gasoline prices for the consumer were used for each geographic regions: Northern (N), Northeastern (NE), Central-Western (CW), Southeastern (SE) and Southern (S) from July 2001 to December 2014 obtained from the website of Agência Nacional do Petróleo ANP (2014) (see Appendix 7.1.1 and 7.1.2). It contains 162 observations for each region and fuel type. These values are given in Real (currency) per liter and include the cost price of each fuel, federal taxes PIS / PASEP, COFINS and CIDE, state ICMS, as well as logistics and margin costs.

This series was deflated by the general price index, IGP-DI, provided by Fundação Getúlio Vargas, as well as the monthly and annual references from July 2001 to December 2014. It is a national index and calculated monthly based on

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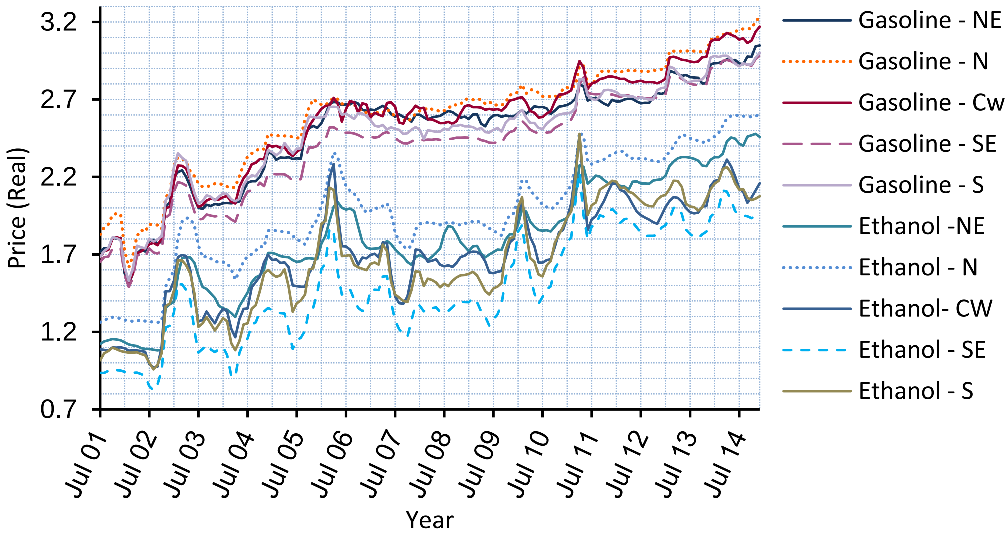
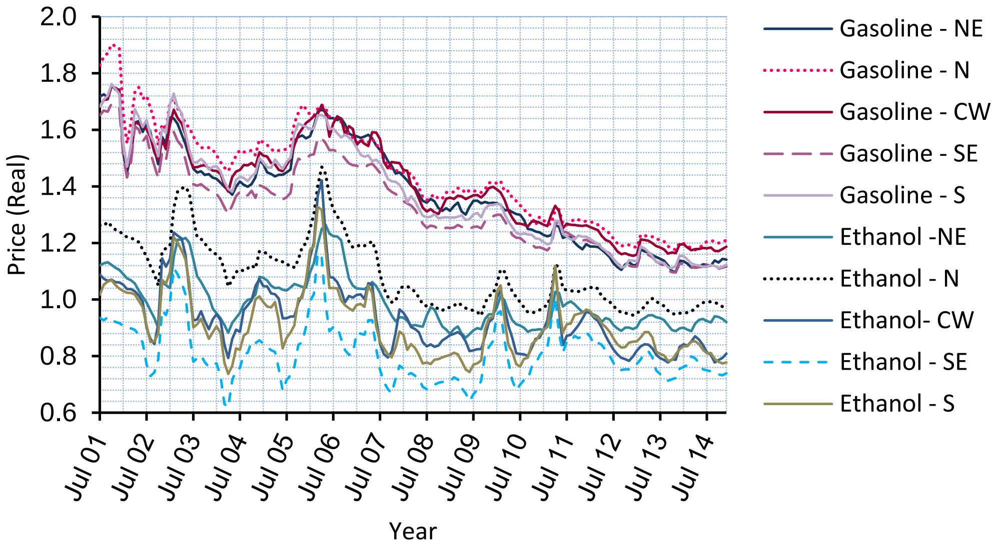
the weighted average of three other price indices: 60% of the wholesale price index (IPA), 30% of the consumer price index (CPI) and 10% of national index prices of construction cost (INCC). These series of historical fuel prices and deflated prices (from July 2001) can be seen in Figures 21 and 22 as follows:

Figure 21 – Trend of monthly average price of ethanol and gasoline, by Brazilian geographic region



Source: Author based on ANP (2014)

Figure 22 – Trend of deflated monthly average price of ethanol and gasoline, by Brazilian geographic region



Source: Author based on ANP (2014)

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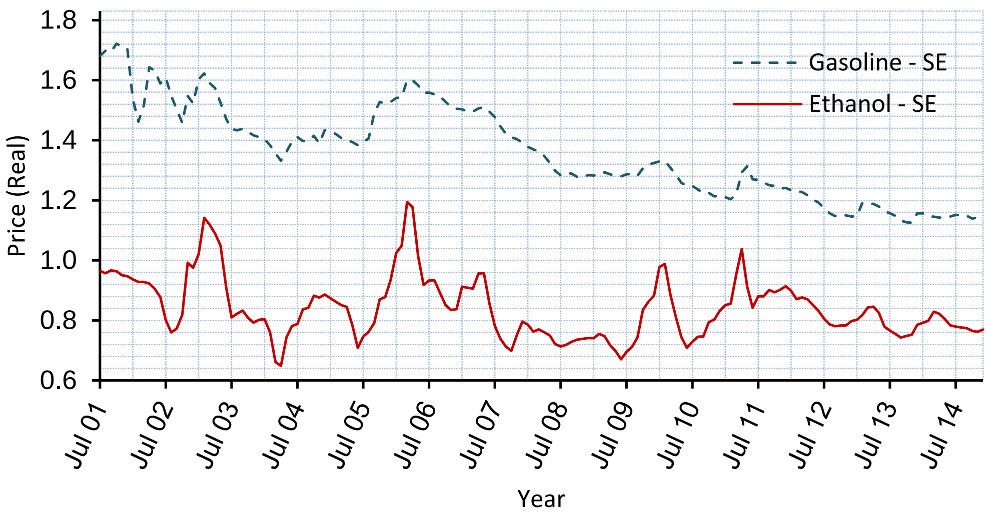
Separately, the results of the ethanol and gasoline deflated price series for the Southern region is presented in Figure 23. It can be observed in Figures 22 and 23 that the deflated prices of both fuels fluctuate considerably around their means up to 2005, when a big increase occurred for ethanol as well as for gasoline. After this year, prices moved back to levels not much below the previous years. This behavior, around the mean, suggests an MRP application in price modelling. However, not only a graphic analysis should be performed for this model determination, but also statistical tests should be conducted as well. In next session, the deflated price series will be used to determinate the stochastic process parameters and perform the fuel price simulation.

Figure 23 - Trend of deflated monthly average price of ethanol and gasoline for Southeastern region



Source: Author based on ANP (2014)

4.3. Limitations

In next sections, fuel consumption rate (FCR) and vehicle mileage traveled (VMT) factors will be transformed as fuzzy numbers. Then geographic impacts (on FCR and VMT) as urban traffic congestion and urban spatial structure for each region will be estimated. Due to the lack of data for all cities in Brazil, data modeling is developed based on the available information which is not included all cities in each region. We considered 37 cities among all regions based on the research and study’s result from Ojima (2007) and due to lack of information the

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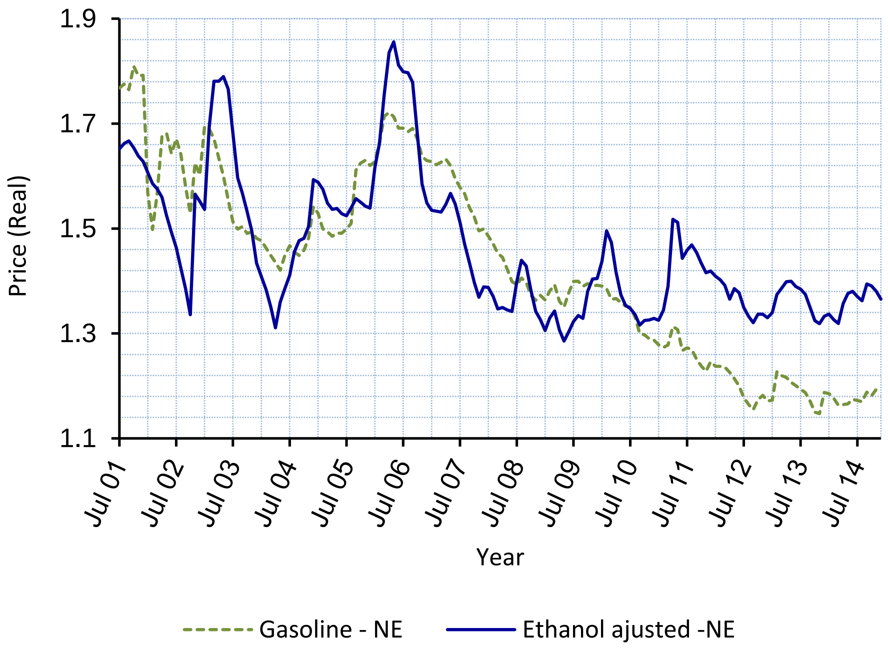
factor's impact of urban road density and the urban public bus accessibility didn't take to consideration in this analysis where in next researches depending the availability of data it can be performed.

4.4. An overall view of opportunity associated with Flex-Fuel Vehicle

If multiply the factor of 1 / 0.70 by deflated price of ethanol and compared the evolution of its price to deflated gasoline price, both based in July 2001, it can totally be observed in which periods there are advantageous of ethanol consumption in each region. This adjusted value, which is multiplied to the deflated price of ethanol series, is limitation of exercising the flexibility associated by FFV. By Figures 24-28, it can be examined roughly in which periods switching between each fuel type has more advantage for vehicle owner by geographic region over the period of analysis.

Figure 24 – Trend of adjusted ethanol and deflated gasoline prices for Northeastern region



Source: Author based on ANP (2014)

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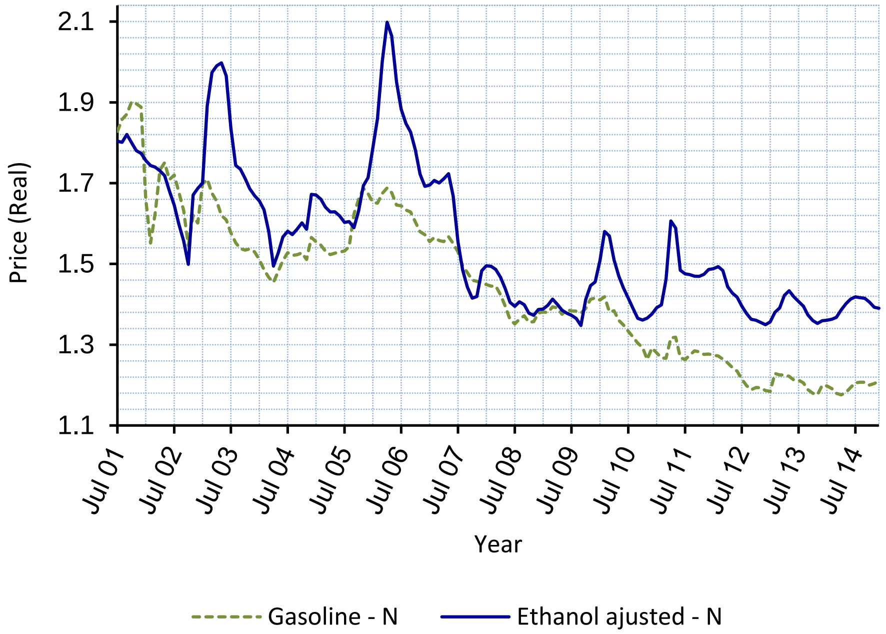
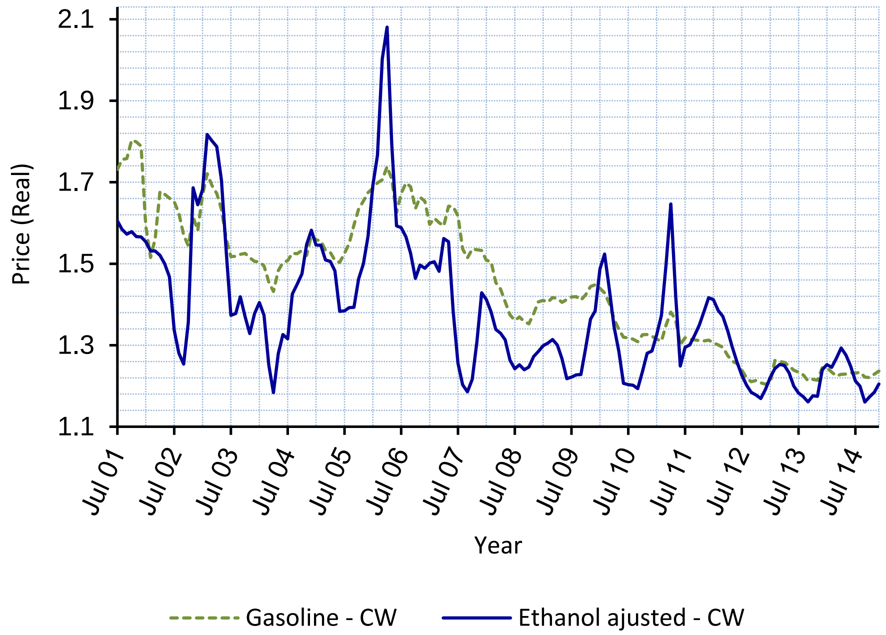
Figure 25 – Trend of adjusted ethanol and deflated gasoline prices for Northern region



Source: Author based on ANP (2014)

Figure 26 – Trend of adjusted ethanol and deflated gasoline prices for Central-Western region



Source: Author based on ANP (2014)

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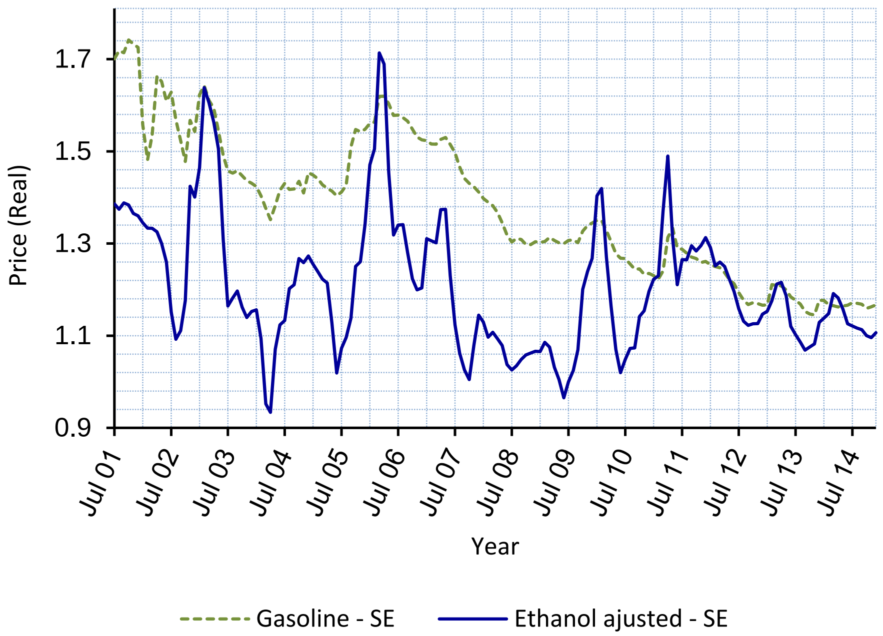
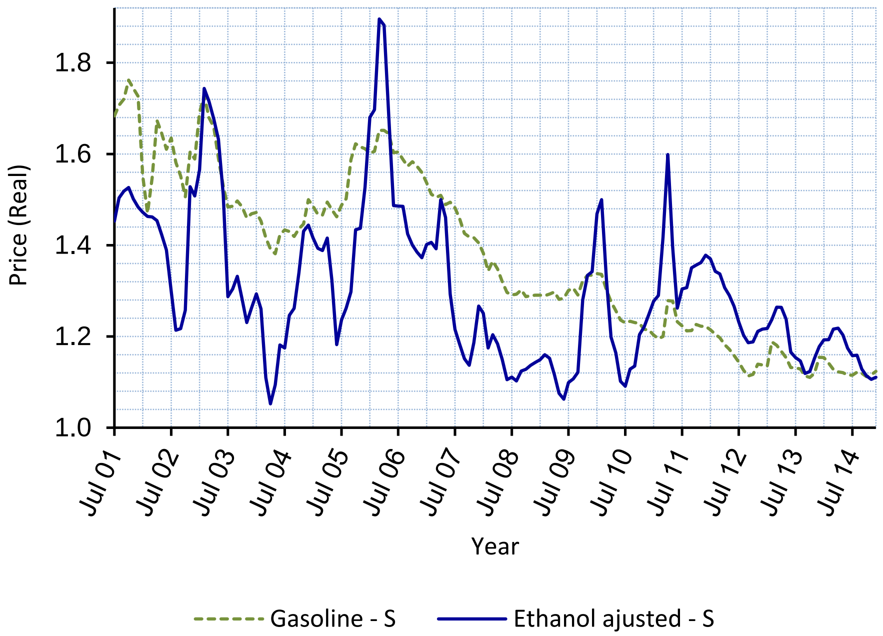
Figure 27 – Trend of adjusted ethanol and deflated gasoline prices for Southeastern region



Source: Author based on ANP (2014)

Figure 28 – Trend of adjusted ethanol and deflated gasoline prices for Southern region



Source: Author based on ANP (2014)

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4.5. Monte Carlo Simulation using MRP stochastic process

To assess the value of FFV flexibility using real option theory, it needs to predict the fuel price over the vehicle lifetime. Monte Carlo simulation as a powerful method permits to predict the prices through a specified stochastic process. Determining the appropriate stochastic process for simulating price is first step in Monte Carlo simulation. Since Bastian-Pinto et al. (2008) and Samanez et al. (2014) analyzed and approved MRP in valuation of flex-fuel vehicle flexibility option as a proper stochastic process model. In this study, this stochastic process type will be applied to model the fuel price parameters as well.

In this regard, for gasoline and ethanol, respectively, the regression equations will comply with the following equations:

$$\ln e_t - \ln e_{t-1} = a_e + (b_e - 1)\ln e_{t-1} + \varepsilon_t \tag{36}$$

$$\ln g_t - \ln g_{t-1} = a_g + \left(b_g - 1\right)\ln g_{t-1} + \varepsilon_t \tag{37}$$

The results with the parameters of linear regression of the natural logarithm of the deflated prices using Ms. Excel for all geographic regions are shown in Table 8. The details of regression for each geographic region and each fuel type can be seen in appendix 7.2.

Table 8 - Linear regression results for ethanol and gasoline by geographic regions

	NE		**N**		**CW**		**SE**		**S**	
Parameter	**Ethanol**	**Gas**	**Ethanol**	**Gas**	**Ethanol**	**Gas**	**Ethanol**	**Gas**	**Ethanol**	**Gas**
a	-0.002	0.002	0.001	0.004	-0.008	0.004	-0.024	0.003	-0.010	0.002
b-1	-0.049	-0.016	-0.031	-0.018	-0.081	-0.018	-0.103	-0.019	-0.081	-0.013
b	0.951	0.984	0.969	0.982	0.919	0.982	0.897	0.981	0.919	0.987
σ_ε	0.027	0.021	0.026	0.019	0.048	0.021	0.053	0.019	0.049	0.021

Source: Author

From section 3.3.3, the main formulas for calculating the parameters of the MRP stochastic process are summarized in table 9 as follows:

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Table 9 - Summary of formulas of MRP parameters

Estimated parameter	**Equations**
Speed of reversion	$\eta = -\ln(b)\,/\Delta t$
Volatility	$\sigma = \sigma_{\varepsilon}\sqrt{2\ln(b)\,/[(b^2-1)\Delta t]}$
Long-run mean	$\bar{X} = \exp[-a/(b-1)]$

* a and b are results of running a linear regression for natural log of deflated prices.

Substituting the values of $a, b, and\ \sigma_{\varepsilon}$ from Table 8 and considering $\Delta t = 1$ (time series are monthly), the estimated parameters for monthly deflated prices series is obtained, as follows in Table 10.

Table 10 – Values of MRP parameters by geographic regions

Estimated parameters	**NE**		**N**		**CW**		**SE**		**S**	
	Ethanol	**Gas**	**Ethanol**	**Gas**	**Ethanol**	**Gas**	**Ethanol**	**Gas**	**Ethanol**	**Gas**
Speed of reversion	0.050	0.016	0.031	0.018	0.084	0.018	0.109	0.019	0.085	0.013
Volatility	0.028	0.021	0.027	0.020	0.050	0.021	0.055	0.020	0.051	0.021
Long-run mean	0.961	1.164	1.029	1.243	0.907	1.236	0.793	1.162	0.888	1.131
ρ_{eg}	0.442		0.433		0.510		0.476		0.505	

Source: Author

The parameters show the ethanol in all regions reverts rapidly to its long-run average than gasoline, and that the volatility of gasoline is lower than that of ethanol. Both fuels have low volatility, a range from 3% to 6% for ethanol and 2% for gasoline. This low volatility may come to reduce the value of option (Brealey et al., 2011). The value of an option increases with increasing in volatility and decreases with decreasing in volatility. The correlation, which is calculated through the results of $\ln(deflated\ price)$, has value between 40% and 50%. According to Bastian-Pinto et al. (2009), Camargo et al. (2010), Dias (2014), and Samanez et al. (2014), and among others, the high correlation between the price of assets is also a factor that decreases the value of flexibility.

With the MRP parameters value estimated and presented in Table 10, it is possible to conduct the fuel price simulation for a 60-month period (life expectancy of the flex-fuel vehicle in this study). The discrete transformation of

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Equation 8, appropriate for a risk-neutral simulation, can be expressed for ethanol and gasoline, respectively, by Equations 38 and 39.

$$E_t = E_{t-1}e^{-\eta_E \Delta t} + \left[\bar{E} - \frac{\pi}{\eta_E}\right]\left(1 - e^{-\eta_E \Delta t}\right) + \sigma_E \sqrt{\frac{1 - e^{-2\eta_E \Delta t}}{2\eta_E}} N(0,1) \quad (38)$$

$$G_t = G_{t-1}e^{-\eta_G \Delta t} + \left[\bar{G} - \frac{\pi}{\eta_G}\right]\left(1 - e^{-\eta_G \Delta t}\right) + \sigma_G \sqrt{\frac{1 - e^{-2\eta_G \Delta t}}{2\eta_G}} N(0,1) \quad (39)$$

In the simulation process, the stochastic part of the discrete transformed Equations 38 and 39 were considered as inputs, that is, how its randomly nature is introduced. However, since fuel prices are correlated, a new stochastic factor that able to introduce this correlation was incorporated in the simulation model. The known Cholesky decomposition was used (Dias, 2008; Samanez et al., 2014), where a stochastic factor of ethanol (ε_y) was associated with gasoline stochastic fact (or contemporary it is possible). This factor is given by the following equation:

$$\varepsilon_y(t) = \rho\varepsilon_G(t) + \sqrt{1 - \rho^2}\varepsilon_I(t) \quad (40)$$

where

$$\varepsilon_G(t) \sim N(0,1);\ \varepsilon_I(t) \sim N(0,1)$$

The ρ is the fuel prices series correlation coefficient, $\varepsilon_G(t)$ and $\varepsilon_I(t)$ are two uncorrelated standard normal distribution random variables.

This new stochastic factor (Equation 40) that incorporates price correlation was uniquely associated with the variable E_t, used in generation of simulated values for ethanol, remaintaining $\varepsilon_G(t) \sim N(0,1)$ in generation of simulated values for gasoline (G_t). In other words, Equation 38 is transformed as Equation 41 below:

$$E_t = E_{t-1}e^{-\eta_E \Delta t} + \left[\bar{E} - \frac{\pi}{\eta_E}\right]\left(1 - e^{-\eta_E \Delta t}\right) + \sigma_E \sqrt{\frac{1 - e^{-2\eta_E \Delta t}}{2\eta_E}} \varepsilon_y \quad (41)$$

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The Monte Carlo simulation of fuel prices will be made using the transformed discrete Equations 39 and 41 for gasoline and ethanol, respectively. For instance, adopting a monthly risk premium (π) of 0.69% and considering that $X_{t-1} = \ln(X_{t-1})$ and $\bar{X} = \ln(\bar{X})$, for the Northeastern region (NE) the discrete transformation for ethanol and gasoline prices in period *t* would be given by

$$G_t = \ln(g_{t-1})\, e^{-0.016\times 1} + \left[\ln(1.164) - \frac{0.0069}{0.016}\right] \times (1 - e^{-0.016\times 1}) + 0.021 \times \sqrt{\frac{1 - e^{-2\times 0.016\times 1}}{2\times 0.016}} \times N(0,1)$$

$$E_t = \ln(e_{t-1})\, e^{-0.050\times 1} + \left[\ln(0.961) - \frac{0.0069}{0.050}\right] \times (1 - e^{-0.050\times 1}) + 0.028 \times \sqrt{\frac{1 - e^{-2\times 0.050\times 1}}{2\times 0.050}} \times \varepsilon_y$$

For period t, to estimate ethanol (e_t) and gasoline (g_t) prices, must consider that $X_t = \exp[X_t - 0.5\, Var(X_t)]$ and then the equations can be written for each fuel type as follows:

$$e_t = \exp\{E_t - 0.5[(1 - e^{-2\eta_E \Delta t})\, \sigma_E^2 / 2\eta_E]\} \quad (42)$$

$$g_t = \exp\{G_t - 0.5[(1 - e^{-2\eta_G \Delta t})\, \sigma_G^2 / 2\eta_G]\} \quad (43)$$

Again, with the Northeastern region (NE) as an example, the ethanol (e_t) and gasoline (g_t) prices are given by:

$$e_t = \exp\{E_t - 0.5 \times [(1 - e^{-2\times 0.050\times t}) \times 0.028^2/2 \times 0.050]\}$$
$$g_t = \exp\{G_t - 0.5 \times [(1 - e^{-2\times 0.016\times t}) \times 0.021^2/2 \times 0.016]\}$$

Accordingly, a Monte Carlo simulation of fuel prices was applied using the transformed discrete Equations 38 and 41. In the computational process of price

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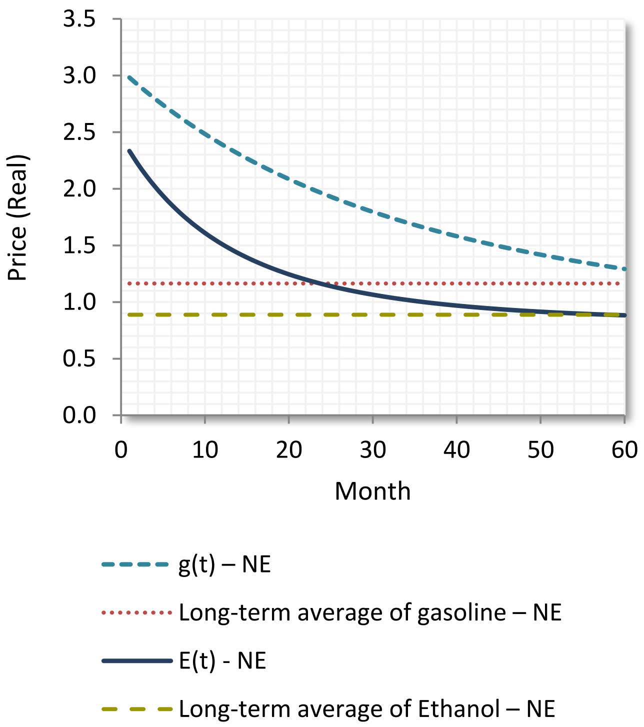
simulation, 10,000 interactions through the @RISK software were performed. As a result of the simulation, and as an illustration, it can be observed by Figure 29-33 that for all regions the long-run mean of the ethanol and gasoline simulated prices have an exponential decay along the time. This can be explained by the price reversion tendency to their long-run mean (market equilibrium level), decrease in the normalized risk premium, since it is a risk-neutral simulation.

Figure 29 - Behavior of simulated fuel prices for the Northeastern Region



Source: Author

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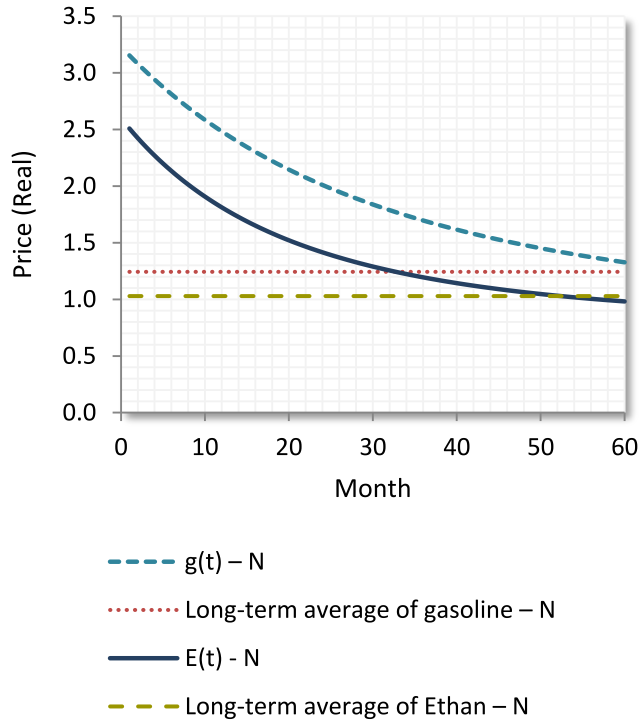
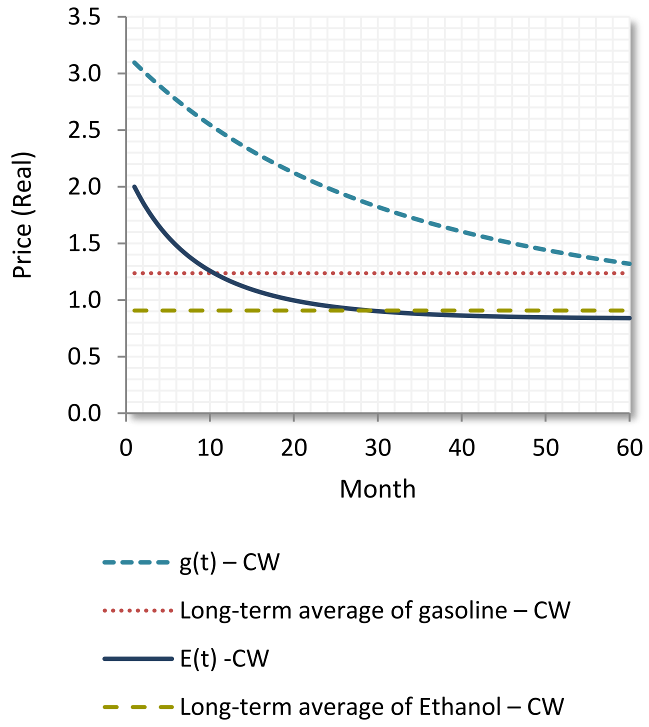
Figure 30 - Behavior of simulated fuel prices for the Northern Region



Source: Author

Figure 31 - Behavior of simulated fuel prices for the Central-Western Region



Source: Author

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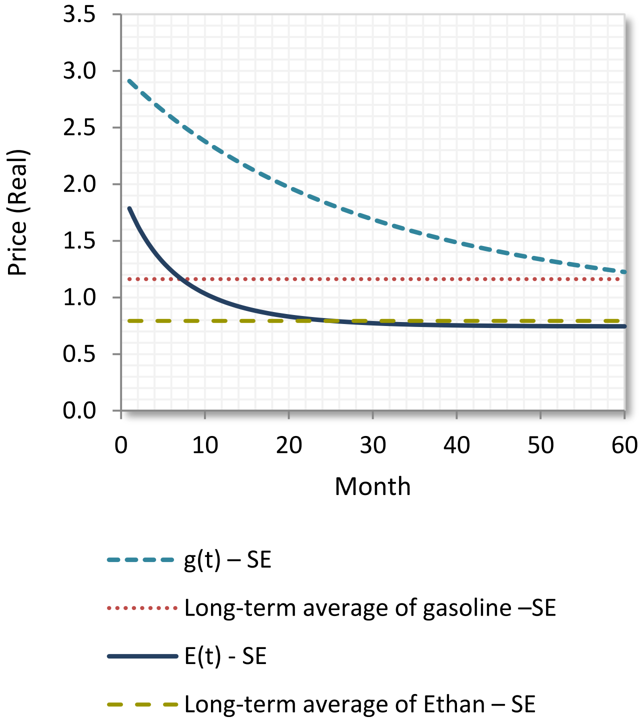
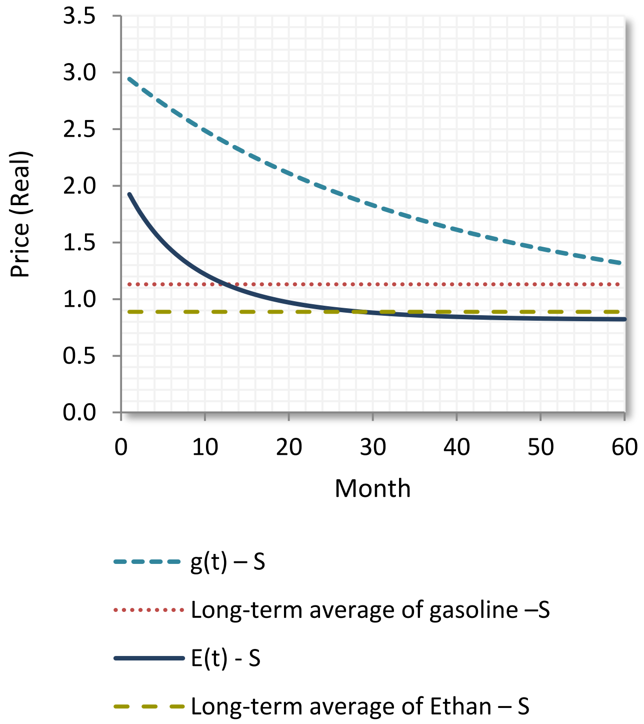
Figure 32 - Behavior of simulated fuel prices for the Southeastern Region



Source: Author

Figure 33 - Behavior of simulated fuel prices for the Southern Region



Source: Author

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4.6. Calculation of FFV Fuel Switch Option Value

By estimated future fuel prices, it is possible to obtain the cash flow of the car owners expenses traveled exclusively by gasoline, or with flex motor, for two automobile models, VW GOL and FIAT UNO, according to the assumption given in Table 7.

The cash flow generated is monthly, as well as the price series. For the generation of the initial cash flow (when t = 0), it was assumed that the consumer is indifferent about consumption fuel type, ethanol or gasoline, while the 70% price relation between fuel was kept. Considering fuel monthly expenses for both vehicle types (Table 7), the remaining cash flows (for t = 1, …, 60) are determined in the following way:

- **VW GOL**

  Cash flow for gasoline in period *t*:

$$\text{CF\_VW GOL}_{g_t} = g_t \times 118.718$$

  Cash flow for ethanol in period t:

$$\text{CF\_VW GOL}_{e_t} = e_t \times 175.336$$

- **FIAT UNO**

  Cash flow for gasoline in period t:

$$\text{CF\_FIAT UNO}_{g_t} = g_t \times 113.293$$

  Cash flow for ethanol in period t:

$$\text{CF\_FIAT UNO}_{e_t} = e_t \times 163.221$$

The economic choice of fuel option is associated with monthly expenses (cash flow) of both fuel types, where the one with the lowest cash flow as less expensive fueling alternative for the owner will be chosen along with the vehicle life expectancy each month. Hence, the final cash flow for each vehicle in period t is given by:

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- **VW GOL**

$$\text{Final CF\_VW GOL}_t = \min(CF\_VW\_GOL_{g_t}; CF\_VW\_GOL_{e_t})$$

- **FIAT UNO**

$$\text{Final } CF\_FIAT\ UNOt_t = \min\left(\text{CF\_FIAT\_UNO}_{g_t}; \text{CF\_FIAT\_UNO}_{e_t}\right)$$

Finally, total expenses of flex-fuel vehicle as a sum of present value of cash flows will be determined in the following way:

- **VW GOL**

$$PV\ of\ total\ fuel\ expenses\ of\ VW\ GOL = \sum_{t=1}^{60} \frac{\text{Final CF\_VW\_GOL}_t}{(1+r)^t}$$

- **FIAT UNO**

$$PV\ of\ total\ fuel\ expenses\ of\ FIAT\ UNO = \sum_{t=1}^{60} \frac{\text{Final } CF\_FIAT\_UNOt_t}{(1+r)^t}$$

It is assumed the simulation was risk-neutral, the risk-free rate is used (r = 1.71% per month) as discount factor.

The fuel switch option value is given by the difference between the present value (PV) of total fuel expenses of automobile traveled only by gasoline and the present value of total fuel expenses of flex-fuel car. The results by vehicle type and by geographic region, for the present value of fuel expenses, for the switch option value and for the economy of using the flex-fuel car are presented in Tables 11 and 12.

Table 11 - PV of fuel expenses, SOV and economy for VW GOL by geographic region

	VW GOL				
Values (R$)	**NE**	**N**	**CW**	**SE**	**S**
PV car moved only by gasoline	9046.65	9358.60	9248.31	8612.94	9114.94
PV flex-fuel car	8460.07	9358.60	7184.04	6122.19	6985.37
Switch option value	586.58	0.00	2064.27	2490.75	2129.57
Economy (SOV/A. price)*	**-1.88%**	**0.00%**	**-6.61%**	**-7.97%**	**-6.82%**

* *SOV: Switch Option Value; A. price: Automobile price*
Source: Author

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Table 12 - PV of fuel expenses, SOV and economy for Fiat UNO by geographic region

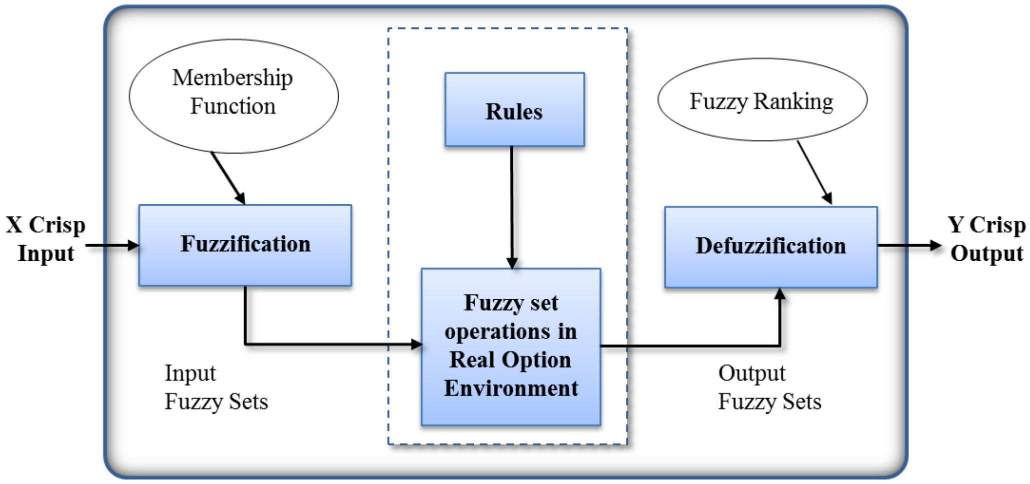
	Fiat UNO				
Values (R$)	**NE**	**N**	**CW**	**SE**	**S**
PV car moved only by gasoline	8633.26	8930.96	8825.71	8219.37	8698.43
PV flex-fuel car	7921.24	8930.96	6687.65	5699.17	6502.70
Switch option value	712.02	0.00	2138.06	2520.21	2195.73
Economy (SOV/A. price)*	**-2.44%**	**0.00%**	**-7.32%**	**-8.62%**	**-7.51%**

* *SOV: Switch Option Value; A. price: Automobile price*
Source: Author

4.7. Fuzzy Real Option Valuation

The traditional financial model does not take into consideration that investors face fuzzy (vague or imprecise) factors in financial analysis. The fuzzy set theory, which introduced by Zadeh (1965), is a useful tool for modeling this kind of imprecise condition. In the real world, there are times when input parameters and data cannot always be determined in the precise value due to their nature or lack of information. It is appropriate for these parameters to be treated as an imprecise value and evaluated through a fuzzy set that can result a proper evaluation. Fuzzy set theory naturally provides an adapted tool in modeling the imprecise values.

Figure 34 – Fuzzy real option modeling system



Source: Author based on Pacheco and Vellasco (2009)

A fuzzy System involves the non-fuzzy or crisp values as an input that is usually the results from measurements or observations. These crisp data (input) is mapped into the fuzzy sets in *fuzzification* stage. The next stage includes fuzzy

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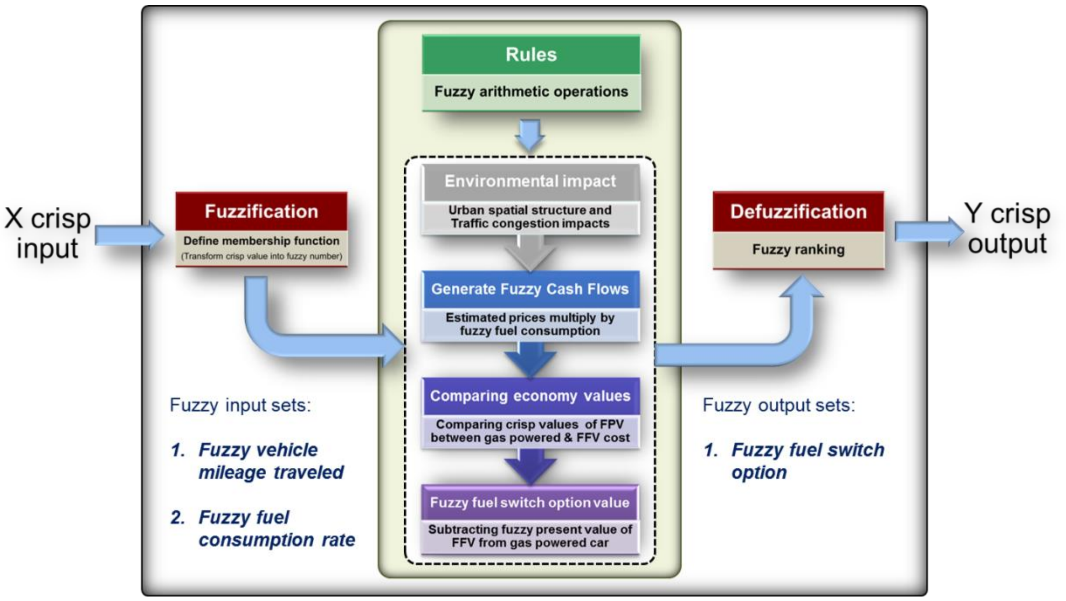
operation that is relevant to the given case. Once the output fuzzy set is obtained through the fuzzy operation process, the interpretation of that information is performed in defuzzification or fuzzy ranking (see section 3.5.7).

Fuzzy real option valuation will be applied in three conditions; 1) Through the fuzzy vehicle mileage traveled parameter (FVMT) considering the urban spatial structure (USS) impact; 2) Through the fuzzy fuel consumption rate parameter (FFCR) considering the traffic congestion (TC) impact; and 3) Through both fuzzy variables simultaneously considering the USS.

The main steps of fuel switch option valuation in a fuzzy environment has been illustrated in Figure 35.

Figure 35 - Fuzzy fuel switch option valuation system of FFV flexibility



Source: Author

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4.7.1. Fuzzy Vehicle Mileage Traveled

Bastos (2011) proposed an annual average mileage (AAM) for light vehicles in Brazil according to the relation between vehicle's age and annual mileage. According to Figure 20 the range of AAM, for 5 years of life expectancy of vehicle is from 14,000 to 22,000 km per year. It means this range for one month can be calculated from 1,167 to 1,833 km as lower and upper limits of monthly average mileage. If m ssupposed to be range of mileage traveled of a vehicle per month, then:

$$m = [1167{,}1833]$$

Considering the average between these two values, we are able to generate a triangular fuzzy mileage traveled of a vehicle:

$$\frac{1167 + 1833}{2} = 1500$$

Then

$$\tilde{m} = (m_1, m_2, m_3) = (1167, 1500, 1833)$$

Which

m_1: $minimum\ value\ of\ vehicle\ mileage\ traveled\ per\ month$ (Lower limit)
m_2: $most\ possibility\ of\ vehicle\ mileage\ traveled\ per\ month$
m_1: $maximum\ value\ of\ vehicle\ mileage\ traveled\ per\ month\ (Upper\ limit)$

Regarding the vehicle mileage travelling and the impact of urban spatial structure on travel demand, Bento et al. (2002) conducted a survey. They carried out this study to address the question that how the measures of urban form including city shape, road density, and the spatial distribution of population affect the annual miles driven. At the end, the authors found that the population centrality has a significant impact on annual household vehicle miles traveled as well as city shape, road density, and (in rail cities) the rail route miles supplied.

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- City Shape

The theory suggests that trip distances should be longer in long, narrow cities than in circular cities with radial road networks. To measure how much an urbanized area deviates from a circular city it should be surrounded each city with an ellipse equal in area to the urbanized area of the city, and measure the major and minor axis of the ellipse. The ratio of the minor to the major axis is the measure of city shape. It ranges between 0 and 1, with 1 indicating a circular city (Bento et al., 2002).

- Population Centrality

To create a measure of population centrality that is less correlated with city area, the percent of population living within x percent of the distance from the Central Business District (CBD) is plotted to edge of the urbanized area against x and compute the area between this curve and a 45- degree line representing a uniformly distributed population. Higher values of population centrality indicate that a larger fraction of the population lives near the CBD (Bento et al., 2002).

Based on the above assumptions, they measured the elasticity of the Vehicle mileage traveled with respect to each variable. The quantitative impact of these variables on annual average VMT was assessed as: a 10% increase in population centrality, through its effect on vehicle choice, reduces annual VMT by 1.5%, while a 10% increase in the index that indicates how circular a city is, and reduces annual VMT by 0.4%. In cities with a rail system, a 10% increase in rail route miles reduces annual VMTs by 0.2%. The summary of these marginal effects can be seen in Table 13 as follows:

Table 13 - Marginal effects of USS on annual VMT

Increase of …	Variable	Total Percentage Impact
10%	Population Centrality	-1.50%
10%	City Shape	-0.40%
10%	Supply of Rail Transit	-0.20%

Source: Bento et al. (2002)

The data of population centrality and city Shape properties can be obtain from Ojima (2007) for 37 cities in Brazil spreading over all states and regions. It

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will be utilized as input for adjusting the urban spatial structure effect on the VMT for each region. He developed a survey to address the Brazilian urbanization and the construction of urban dispersion indicators. He achieved the density, fragmentation, linearity (City Shape) and centrality of the social and spatial dimensions of 37 cities in Brazil. Table 14 summarized the results.

Based on Table 13 and 14 we can obtain the impact of population centrality and city shape on VMT for each city and consequently for each region. In this study, the household numbers of each city is considered as parameter to weight each city and measure its impact on the region located. Summary of this calculation is presented in Tables 15-17. It should be noted that the average annual vehicle mileage traveled has been performed over São Paulo city, so all calculation for other cities regarding the measuring and estimation has been adjusted based on São Paulo urban specifications.

To measure the impact of the supply of rail transit system on VMT, the existing of the train (subway) system, the total train system length, and the quantity of passenger per day in each city has been considered. First, the passenger ratio per population is estimated, then, this ratio multiplied by the marginal effect of supply of rail transit system for each city. In next step, by multiplying the total impact percentage (sum of the adjusted impact of population centrality, city shape, and supply of transit system) and the fuzzy vehicle mileage traveled (FVMT), the adjusted FVMT for each city is obtained. Multiplying the new FVMT by the weighted value factor (the adjusting factor for each region) gives the region share of the adjusted FVMT per each city. In final step, the final FVMT is achieved through rolling up all adjusted values for each region. All data and calculations are presented in Tables 13-18.

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Table 14 - Brazilian cities: Density, Fragmentation, Circularity and Centrality

				A	B
Urbans	**Region**	**Density**	**fragmentation**	**Circularity (City shape)**	**Centrality**
São Paulo	SE	0.22441	0.55188	0.40245	0.16402
Rio de Janeiro	SE	0.49560	0.53597	0.24284	0.11054
Salvador	NE	0.26499	0.68718	0.46159	0.30058
Belo Horizonte	SE	0.47156	0.65475	0.46343	0.08393
Fortaleza	NE	0.54578	0.62054	0.46166	0.17169
Brasília	CW	0.64495	0.78049	0.41886	0.08125
Curitiba	S	0.50934	0.75328	0.45449	0.07321
Recife	NE	0.38288	0.80801	0.4209	0.07078
Porto Alegre	S	0.47888	0.68555	0.40209	0.11971
Belém	N	0.31464	0.69242	0.4588	0.06871
Goiânia	CW	0.50816	0.62786	0.43896	0.05345
Campinas	SE	0.55500	0.58724	0.46638	0.2022
São Luís	NE	0.48408	0.67359	0.48566	0.12003
Maceió	NE	0.38987	0.88398	0.4831	0.18328
Natal	NE	0.33485	0.74589	0.45206	0.11277
João Pessoa	NE	0.48009	0.83982	0.45901	0.13733
São José dos Campos	SE	0.63034	0.87837	0.39492	0.29092
Ribeirão Preto	SE	0.53713	0.70592	0.39704	0.16496
Sorocaba	SE	0.57631	0.67075	0.48185	0.20669
Aracaju	NE	0.68414	0.54236	0.46218	0.06201
Londrina	S	0.55529	0.7511	0.44428	0.1302
Santos	SE	0.54050	0.57844	0.36622	0.11149
Joinville	S	0.87803	0.87785	0.4647	0.32008
São José do Rio Preto	SE	0.63034	0.73589	0.45796	0.16953
Caxias do Sul	S	0.52397	0.75485	0.39724	0.33917
Jundiaí	SE	0.56192	0.62104	0.44429	0.08992
Florianópolis	S	0.65224	0.74103	0.41454	0.07797
Maringá	S	0.01202	0.75407	0.46528	0.0631
Vitória	SE	0.59490	0.57307	0.40935	0.09403
Volta Redonda	SE	0.57188	0.73	0.44861	0.14835
Blumenau	S	0.89802	0.70888	0.37606	0.24552
Ipatinga	SE	0.58543	0.70988	0.45829	0.15514
Criciúma	S	0.68656	0.63325	0.4768	0.17274
Itajaí	S	0.64718	0.62003	0.42739	0.26382
Cabo Frio	SE	0.71780	0.59788	0.4679	0.21717
Moji Mirim	SE	0.51595	0.75271	0.47192	0.30267
Guaratinguetá	SE	0.55924	0.75355	0.49071	0.29877

Source: Ojima (2007)

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Table 15 – Region weighted value and the rail system passenger ratio

		C	K= C/(total households for each region)	D	E	F=E/D
Urbans	**Region**	**households**	**Weighted value for each region** (based on households) **%**	**Population**	**Rail system passenger per day**	**Passenger ratio per population %**
São Paulo	SE	5,000,541	41%	17,596,957	7,300,000	41.48%
Rio de Janeiro	SE	3,295,702	27%	10,870,155	1,260,000	11.59%
Salvador	NE	791,007	23%	2,959,434	50,000	1.69%
Belo Horizonte	SE	1,151,418	9%	4,210,662	234,000	5.56%
Fortaleza	NE	692,926	20%	2,821,761	18,000	0.64%
Brasília	CW	701,028	61%	2,623,303	130,000	4.96%
Curitiba	S	728,859	25%	2,502,129	0	0
Recife	NE	849,458	25%	3,238,736	244,900	7.56%
Porto Alegre	S	1,065,320	37%	3,436,431	192,000	5.59%
Belém	N	412,634	100%	1,965,794	0	0
Goiânia	CW	447,284	39%	1,560,625	0	0
Campinas	SE	610,616	5%	2,119,322	0	0
São Luís	NE	221,409	6%	945,280	0	0
Maceió	NE	220,414	6%	865,717	11,000	1.27%
Natal	NE	241,998	7%	961,638	9,300	0.97%
João Pessoa	NE	212,388	6%	828,712	10,100	1.22%
São José dos Campos	SE	319,772	3%	1,172,423	0	0
Ribeirão Preto	SE	173,083	1%	603,452	0	0
Sorocaba	SE	242,659	2%	873,329	0	0
Aracaju	NE	178,052	5%	703,983	0	0
Londrina	S	162,867	6%	564,768	0	0
Santos	SE	395,757	3%	1,350,446	0	0
Joinville	S	160,270	6%	566,106	0	0
São José do Rio Preto	SE	120,894	1%	395,379	0	0
Caxias do Sul	S	158,949	6%	518,069	0	0
Jundiaí	SE	140,029	1%	496,413	0	0
Florianópolis	S	207,661	7%	698,447	0	0
Maringá	S	116,631	4%	399,356	0	0
Vitória	SE	373,646	3%	1,327,342	0	0
Volta Redonda	SE	153,483	1%	530,317	0	0
Blumenau	S	112,126	4%	380,273	0	0
Ipatinga	SE	90,418	1%	341,608	0	0
Criciúma	S	67,556	2%	238,867	0	0
Itajaí	S	95,286	3%	326,236	0	0
Cabo Frio	SE	59,885	0%	204,939	0	0
Moji Mirim	SE	55,382	0%	196,551	0	0
Guaratinguetá	SE	58,742	0%	213,180	0	0

Source: Ojima (2007) *and each city's rail and metro system websites*

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Table 16 – Adjusted marginal effects of Rail system, City shape, and Centrality based on São Paulo city.

		G = (F/FS -1)/10%* (-0.2%)	H = (A/AS-1)/10%* (-0.4%)	I = (B/BS-1)/10%* (-1.5%)	N = G+H+I
Urbans	**Region**	**Impact of rail system %**	**Impact of City shape %**	**Impact of Centrality %**	**Total Impact on VTM %**
São Paulo	SE	0.0%	0.0%	0.0%	0.0%
Rio de Janeiro	SE	1.4%	1.6%	4.9%	7.9%
Salvador	NE	1.9%	-0.6%	-12.5%	-11.2%
Belo Horizonte	SE	1.7%	-0.6%	7.3%	8.5%
Fortaleza	NE	2.0%	-0.6%	-0.7%	0.7%
Brasília	CW	1.8%	-0.2%	7.6%	9.2%
Curitiba	S	2.0%	-0.5%	8.3%	9.8%
Recife	NE	1.6%	-0.2%	8.5%	10.0%
Porto Alegre	S	1.7%	0.0%	4.1%	5.8%
Belém	N	2.0%	-0.6%	8.7%	10.2%
Goiânia	CW	2.0%	-0.4%	10.1%	11.7%
Campinas	SE	2.0%	-0.6%	-3.5%	-2.1%
São Luís	NE	2.0%	-0.8%	4.0%	5.2%
Maceió	NE	1.9%	-0.8%	-1.8%	-0.6%
Natal	NE	2.0%	-0.5%	4.7%	6.1%
João Pessoa	NE	1.9%	-0.6%	2.4%	3.8%
São José dos Campos	SE	2.0%	0.1%	-11.6%	-9.5%
Ribeirão Preto	SE	2.0%	0.1%	-0.1%	2.0%
Sorocaba	SE	2.0%	-0.8%	-3.9%	-2.7%
Aracaju	NE	2.0%	-0.6%	9.3%	10.7%
Londrina	S	2.0%	-0.4%	3.1%	4.7%
Santos	SE	2.0%	0.4%	4.8%	7.2%
Joinville	S	2.0%	-0.6%	-14.3%	-12.9%
São José do Rio Preto	SE	2.0%	-0.6%	-0.5%	0.9%
Caxias do Sul	S	2.0%	0.1%	-16.0%	-14.0%
Jundiaí	SE	2.0%	-0.4%	6.8%	8.4%
Florianópolis	S	2.0%	-0.1%	7.9%	9.7%
Maringá	S	2.0%	-0.6%	9.2%	10.6%
Vitória	SE	2.0%	-0.1%	6.4%	8.3%
Volta Redonda	SE	2.0%	-0.5%	1.4%	3.0%
Blumenau	S	2.0%	0.3%	-7.5%	-5.2%
Ipatinga	SE	2.0%	-0.6%	0.8%	2.3%
Criciúma	S	2.0%	-0.7%	-0.8%	0.5%
Itajaí	S	2.0%	-0.2%	-9.1%	-7.4%
Cabo Frio	SE	2.0%	-0.7%	-4.9%	-3.5%
Moji Mirim	SE	2.0%	-0.7%	-12.7%	-11.4%
Guaratinguetá	SE	2.0%	-0.9%	-12.3%	-11.2%

F, A, and B are the specifications of São Paulo city.

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Table 17 - Adjusted VMT for each city and region.

		P_1= 1167*N	P_3 = 1833*N	L=P_1*K	U=P_3*K
Urbans	**Region**	**VMT Lower limit** (based on city)	**VMT Upper limit** (based on city)	**VMT weighted Lower limit** (based on region)	**VMT weighted Upper limit** (based on region)
São Paulo	SE	1167	1833	477	749
Rio de Janeiro	SE	1259	1979	339	533
Salvador	NE	1036	1629	241	378
Belo Horizonte	SE	1265	1988	119	187
Fortaleza	NE	1175	1846	239	375
Brasília	CW	1274	2001	778	1222
Curitiba	S	1281	2013	325	510
Recife	NE	1283	2016	320	503
Porto Alegre	S	1234	1939	457	719
Belém	N	1285	2020	1285	2020
Goiânia	CW	1304	2049	508	798
Campinas	SE	1142	1794	57	89
São Luís	NE	1227	1929	80	125
Maceió	NE	1159	1822	75	118
Natal	NE	1238	1946	88	138
João Pessoa	NE	1211	1903	75	119
São José dos Campos	SE	1055	1659	28	43
Ribeirão Preto	SE	1190	1869	17	26
Sorocaba	SE	1135	1784	23	35
Aracaju	NE	1292	2030	68	106
Londrina	S	1221	1919	69	109
Santos	SE	1250	1965	40	64
Joinville	S	1016	1597	57	89
São José do Rio Preto	SE	1178	1851	12	18
Caxias do Sul	S	1004	1577	55	87
Jundiaí	SE	1264	1987	14	23
Florianópolis	S	1280	2012	92	145
Maringá	S	1290	2028	52	82
Vitória	SE	1264	1986	39	61
Volta Redonda	SE	1201	1888	15	24
Blumenau	S	1106	1738	43	68
Ipatinga	SE	1193	1875	9	14
Criciúma	S	1172	1842	28	43
Itajaí	S	1081	1698	36	56
Cabo Frio	SE	1126	1769	6	9
Moji Mirim	SE	1034	1625	5	7
Guaratinguetá	SE	1036	1628	5	8

Source: Author

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In Final step, the results of urban spatial structure impact estimation on FVMT for each region are achieved by the following table:

Table 18 – Adjusted FVMT considering the USS impact

Region		**Lower limit** M_1	**Most Likely** M_2	**Upper limit** M_3
Northeastern	NE	1185	1524	1862
Northern	N	1285	1652	2020
Central-Western	CW	1285	1653	2020
Southeastern	SE	1202	1546	1890
Southern	S	1214	1561	1908

$M_1 = rolling\ up\ the\ L\ values\ (from\ table\ 17)\ for\ each\ region$
$M_3 = rolling\ up\ the\ U\ values\ (from\ table\ 17)\ for\ each\ region$
$M_2 = (M_1 + M_3)/2$

Source: Author

Based on the urban fuel consumption rate (km/liter) for each fuel type and vehicle, we can calculate the monthly fuel consumption in fuzzy number for each region. For example for Northeastern region and VW GOL vehicle, the fuzzy monthly fuel consumptions are obtained as:

VW GOL - Gasoline consumption rate:	12.6	km/liter
VW GOL - Ethanol consumption rate:	8.6	km/liter

Monthly fuel consumption for Northeastern region for Gasoline and Ethanol:

$$NE_G = (1185, 1524, 1862)/12.6 = (93.8, 120.6, 147.4)$$
$$NE_E = (1185, 1524, 1862)/8.6 = (138.5, 178.1, 217.7)$$

Following table presents the Fuzzy MFC for each region and fuel type:

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Table 19 - FMFC for VW GOL by fuel type and geographic region

Fuel	Region	VW GOL Lower limit	Most Likely	Upper limit
Gasoline	NE	93.8	120.6	147.4
	N	101.7	130.8	159.8
	CW	101.7	130.8	159.9
	SE	95.2	122.4	149.6
	S	96.1	123.6	151.0
Ethanol	NE	138.5	178.1	217.7
	N	150.2	193.1	236.1
	CW	150.2	193.2	236.1
	SE	140.6	180.7	220.9
	S	142.0	182.5	223.1

Source: Author

Table 20 - FMFC for Fiat UNO by fuel type and geographic region

Fuel	Region	Fiat UNO Lower limit	Most Likely	Upper limit
Gasoline	NE	89.5	115.1	140.6
	N	97.1	124.8	152.5
	CW	97.1	124.8	152.6
	SE	90.8	116.8	142.7
	S	91.7	117.9	144.1
Ethanol	NE	128.9	165.8	202.6
	N	139.8	179.8	219.8
	CW	139.9	179.8	219.8
	SE	130.8	168.2	205.6
	S	132.2	169.9	207.7

Source: Author

Considering monthly fuzzy fuel consumption (Table 19 and 20), the fuzzy cash flows (for t = 1, …, 60) are determined in the following way for each region. For example for NE region:

- **VW GOL – Northeastern region**

  Fuzzy cash flow for gasoline in period t:

$$\mathrm{FCF\_VW\_GOL}_{g_t} = g_t \times (93.8, 120.6, 147.4)$$

  Fuzzy cash flow for ethanol in period t:

$$\mathrm{FCF\_VW\_GOL}_{e_t} = e_t \times (138.5, 178.1, 217.7)$$

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- **FIAT UNO – Northeastern region**

  Fuzzy cash flow for gasoline in period t:

$$\text{FCF\_FIAT\_UNO}_{g_t} = g_t \times (89.5, 115.1, 140.6)$$

  Fuzzy cash flow for ethanol in period t:

$$\text{FCF\_FIAT\_UNO}_{e_t} = e_t \times (128.9, 165.8, 202{,}6)$$

The economic choice of fuel option is associated with monthly expenses (cash flow) of both fuel types, where the one with the lowest cash flow will be chosen along with the vehicle expectancy life each month. In fuzzy set system, for comparing and choosing lowest fuzzy cash flow, first we must rank the fuzzy number then compare and choose the lower value. After determining the lowest value, the fuzzy cash flow with lowest value must be considered for next operation. This ranking value is only to determine which fuzzy cash flow must be chosen to produce the final fuzzy cash flow.

To rank the fuzzy number we calculate the fuzzy mean value (possibility mean value) through the Equation 33. For instance, If consider the fuzzy cash flow of gasoline for VW GOL as (a_1, a_2, a_3) then the fuzzy mean value is calculated as:

$$\text{FCF\_VW\_GOL}_{g_t} = (a_1, a_2, a_3) \Rightarrow \text{CF\_VW\_GOL}_{g_t} = (a_2, a^L, a^U)$$

Which

$$a^L = a_2 - a_1$$

$$a^U = a_3 - a_2$$

Then fuzzy mean value is

$$E\left(\text{FCF\_VW\_GOL}_{g_t}\right) = a_2 + \left(\frac{a^L - a^U}{6}\right)$$

Numerical example of fuzzy cash flow of gasoline in Northeastern region;

For $t = 1$:

$$\text{FCF\_VW\_GOL}_{g_1} = (280.2{,}360.3, 440.3) \Rightarrow \text{FCF\_VW\_GOL}_{g_1} = (360.3, 80.1, 80.1)$$

Then, in this case:

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$$E\left(\text{FCF\_VW\_GOL}_{g_1}\right) = 360.3 + \frac{80.1 - 80.1}{6} = 360.3$$

Accordingly, the final fuzzy cash flow for each vehicle in period t is given by:

- **VW GOL**

$$\text{Final FCF\_VW\_GOL}_{\text{t}} = \textit{The Fuzz Cash Flow associated with } \min(E(F\text{CF\_VW\_GOL}_{g_t}); E(\text{FCF\_VW\_GOL}_{e_t}))$$

- **FIAT UNO**

$$\text{Final FCF\_Fiat\_UNO}_{\text{t}} = \textit{The Fuzz Cash Flow associated with } \min(E(\text{CF\_Fiat\_UNO}_{g_t}); E(\text{CF\_Fiat\_UNO}_{e_t}))$$

Total fuzzy expenses of flex fuel vehicle as a sum of the fuzzy present values (FPV) of FCF will be determined in the following way:

- **VW GOL**

$$FPV\ of\ total\ fuel\ expenses\ of\ VW\ GOL = \sum_{t=1}^{60} \frac{\text{Final FCF\_VW\_GOL}_{\text{t}}}{(1+r)^t}$$

- **FIAT UNO**

$$FPV\ of\ total\ fuel\ expenses\ of\ FIAT\ UNO = \sum_{t=1}^{60} \frac{\text{Final F}CF\_FIAT\_UNOt_{\text{t}}}{(1+r)^t}$$

The fuzzy fuel switch option value is given by the difference between the fuzzy present values (FPV) of total fuzzy fuel expenses of automobile traveled only by gasoline and flex automobile. The results by vehicle type and by geographic region, for the fuzzy present value (FPV) of fuel expenses, and for fuzzy switch option (FSO) value are presented in Tables 21 and 22.

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Table 21 - Fuzzy present value of fuel expenses, and fuzzy switch option value for VW GOL by geographic region

		VW GOL				
Values (R$)		**NE**	**N**	**CW**	**SE**	**S**
FPV car moved only by gasoline	Upper limit	11230.44	12599.98	12453.40	10849.94	11596.94
	Most Likely	9188.54	10309.08	10189.14	8877.22	9488.41
	Lower limit	7146.64	8018.17	7924.89	6904.51	7379.87
FPV flex-fuel car	Upper limit	10502.26	12599.98	9673.74	7712.28	8887.49
	Most Likely	8592.76	10309.08	7914.88	6310.05	7271.58
	Lower limit	6683.26	8018.17	6156.01	4907.81	5655.68
Fuzzy Switch option value	Upper limit	4547.18	4581.81	6297.38	5942.13	5941.26
	Most Likely	595.78	0.00	2274.27	2567.18	2216.82
	Lower limit	-3355.62	-4581.81	-1748.85	-807.77	-1507.62

Source: Author

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Table 22 - Fuzzy present value of fuel expenses, and fuzzy switch option value for Fiat UNO by geographic region

		Fiat UNO				
Values (R$)		**NE**	**N**	**CW**	**SE**	**S**
FPV car moved only by gasoline	Upper limit	10717.26	12024.23	11884.34	10354.15	11067.02
	Most Likely	8768.67	9838.01	9723.55	8471.58	9054.83
	Lower limit	6820.08	7651.78	7562.76	6589.01	7042.65
FPV flex-fuel car	Upper limit	9833.36	12024.23	9005.31	7179.38	8273.39
	Most Likely	8045.48	9838.01	7367.98	5874.04	6769.14
	Lower limit	6257.59	7651.78	5730.65	4568.70	5264.89
Fuzzy Switch option value	Upper limit	4459.67	4372.45	6153.69	5785.45	5802.13
	Most Likely	723.19	0.00	2355.57	2597.54	2285.69
	Lower limit	-3013.29	-4372.45	-1442.55	-590.38	-1230.75

Source: Author

Finally, to make decision, comparing values, and estimate the economy ratio, the fuzzy numbers must be defuzzify to a crisp value. This defuzzification as described in section 3.5.7 is performed by ranking the fuzzy number into real number. Hence, the economy percentage will be obtained by dividing the crisp fuzzy switch option value on the automobile price.

$$Economy\ = -\frac{E(Fuzzy\ switch\ option\ value)}{Automibile\ price}$$

Tables 23 and 24 show the results by vehicle type and geographic region, for the crisp fuzzy present value of fuel expenses, the crisp fuzzy switch option value and the economy considering the urban spatial structure impact on the vehicle mileage traveled.

Table 23 - Crisp value of FPV, FSOV, and economy for VW GOL by geographic region considering the USS impact on VMT

	VW GOL				
Values (R$)	**NE**	**N**	**CW**	**SE**	**S**
PV car moved only by gasoline	9188.54	10309.08	10189.14	8877.22	9488.41
PV flex-fuel car	8592.76	10309.08	7914.88	6310.05	7271.58
Switch option value	595.78	0.00	2274.27	2567.18	2216.82
Economy (SOV/A. price)*	**-1.91%**	**0.00%**	**-7.28%**	**-8.22%**	**-7.10%**

* *SOV: Switch Option Value; A. price: Automobile price*
Source: Author

Table 24 - Crisp value of FPV, FSOV, and economy for Fiat UNO by geographic region considering the USS impact on VMT

	Fiat UNO				
Values (R$)	**NE**	**N**	**CW**	**SE**	**S**
PV car moved only by gasoline	8768.67	9838.01	9723.55	8471.58	9054.83
PV flex-fuel car	8045.48	9838.01	7367.98	5874.04	6769.14
Switch option value	723.19	0.00	2355.57	2597.54	2285.69
Economy (SOV/A. price)*	**-2.47%**	**0.00%**	**-8.06%**	**-8.89%**	**-7.82%**

* *SOV: Switch Option Value; A. price: Automobile price*
Source: Author

4.7.2. Fuzzy Fuel Consumption Rate

In section 4.1.1, the fuel consumption rate (FCR) value was achieved through consideration of 55% city and 45% highway and presented in Table 5. This average 12.6 km/L is only a value deepens to many factors like traffic

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congestion, deriving method, driver personality, vehicle type, and also seasons. Maybe for this reason, many automobile manufacturers just prefer to put the FCR adjusted to the city and highway rather than a fix or constant estimated value of city and highway combination. Due to the changing in fuel consumption rate value in real environment, it could be considered as an interval between the city and highway-driving mode not just a crisp value. For instance, for VW GOL vehicle and for gasoline can be between 11.6 and 13.9 based on data from Table 5.

Fuzzy sets theory which introduced by Zadeh (1965) was specifically designed to enable us to consider the variable or parameters nature as they are and as a range or interval instead of an exact number in proper time.

Based on the value presented in Table 5 we are able to define fuzzy fuel consumption rate for each car and its fuel type. If consider $\widetilde{VG}, \widetilde{VE}, \widetilde{FG}, and\ \widetilde{FE}$ as fuzzy variable for VW GOL gasoline, VW GOL ethanol, Fiat UNO gasoline, and Fiat UNO ethanol fuel type respectively, the fuzzy number can be generated as follows. For instance:

$$\widetilde{VG} = (vg_1, vg_2, vg_3) = (11.6, 12.6, 13.9)$$

Where

$$vg_1 = \ Gasoline\ city\ consumption\ rate\ km/L$$

$$vg_3 = \ Gasoline\ chighway\ onsumption\ rate\ km/L$$

$$vg_2 = \ 55\%\ vg_1 + 45\% vg_3$$

Accordingly, for other variables we have:

$$\widetilde{VE} = (ve_1, ve_2, ve_3) = (7.7, 8.6, 9.6)$$

$$\widetilde{FG} = (fg_1, fg_2, fg_3) = (11.8, 13.2, 15.0)$$

$$\widetilde{FE} = (fe_1, fe_2, fe_3) = (8.2, 9.2, 10.4)$$

Traffic congestion is recognized throughout the world as a growing problem. It also has been steadily increasing in Brazil. In São Paulo, September 2012, the traffic tailbacks in and out of the city extend for a total of 180km (112 miles) on average (BBC, 2012) or Recife, Rio de Janeiro and Salvador are among

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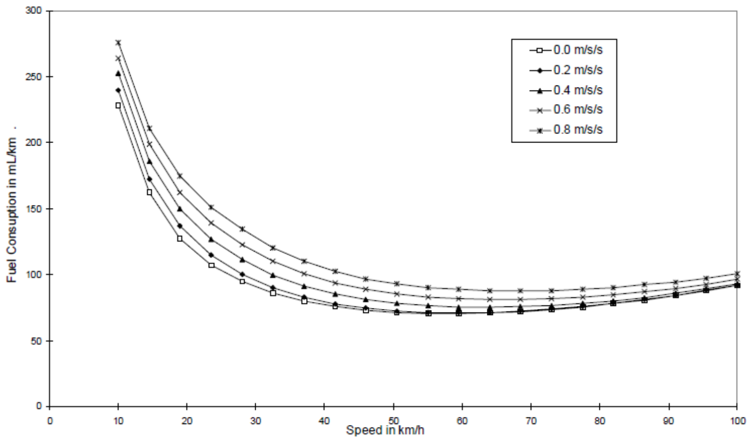
the 10 cities having the worst traffic congestion in the world (TomTom, 2015). Traffic congestion has a considerable influence o vehicle consumption rate or fuel efficiency which can be expressed as fuel economy (FE), in traveled distance per unit volume of fuel (liter). Traffic congestion affects vehicle fuel economy through the lower average travel speed and increased vehicle speed variability (accelerations and decelerations) (Bigazzi et al., 2012).

Figure 36 - Effect of acceleration noise on passenger car fuel consumption (mL/km)



Source: Greenwood and Bennett (1996)

Greenwood and Bennett (1996) outlined a developed methodology to predict the additional fuel consumption due to the traffic congestion. They found that driver behavior could be represented through the standard deviation of acceleration, that is, the acceleration noise. These experiments were conducted to measure acceleration noise with a number of different vehicles and drivers under varying levels of congestion. A Monte-Carlo simulation model was developed. It uses the acceleration noise in conjunction with a mechanistic fuel model to predict the fuel consumption of vehicles at different levels of congestion. The model was found to give consistent and reasonable results when applied to different classes of roads. The results are show that there is 50 percent increase in the passenger car fuel consumption under heavily congested conditions.

Accordingly, the data of traffic index rate (TIR) (in percentage) for the cities was gathered from TomTom (2015). For the cities were not included in the traffic index list, it is assumed 15% , 13% , 10% traffic index rate for cities having more

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than 1 million, between 200,000-1,000,000 and less than 200,000 population, respectively. This data can be observed in Table 25.

By multiplying the traffic index rate of each city by traffic congestion effect on FCR (50 percent), the traffic increase weighted for each city is estimated. It is supposed that the traffic congestion will just influence on the city consumption rate. Hence, we multiply this weighted value as an increase rate by the city driving mode. For calculating the region adjusted fuel consumption rate, weighted value of each city based on its own household's population is considered. The result for VW GOL has been reported on Table 26. By rolling up the estimated values for each region, the lower limit value of fuzzy FCR for each region is obtained.

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Table 25 - Urban traffic index rate

		A	B = A * 50%
Urbans	**Region**	**Traffic Index rate**	**Traffic increase weight**
São Paulo	SE	33%	16.5%
Rio de Janeiro	SE	51%	25.5%
Salvador	NE	46%	23.0%
Belo Horizonte	SE	27%	13.5%
Fortaleza	NE	35%	17.5%
Brasília	CW	21%	10.5%
Curitiba	S	19%	9.5%
Recife	NE	45%	22.5%
Porto Alegre	S	25%	12.5%
Belém	N	15%	7.5%
Goiânia	CW	15%	7.5%
Campinas	SE	15%	7.5%
São Luís	NE	13%	6.5%
Maceió	NE	13%	6.5%
Natal	NE	13%	6.5%
João Pessoa	NE	13%	6.5%
São José dos Campos	SE	15%	7.5%
Ribeirão Preto	SE	10%	5.0%
Sorocaba	SE	13%	6.5%
Aracaju	NE	13%	6.5%
Londrina	S	10%	5.0%
Santos	SE	15%	7.5%
Joinville	S	10%	5.0%
São José do Rio Preto	SE	10%	5.0%
Caxias do Sul	S	10%	5.0%
Jundiaí	SE	10%	5.0%
Florianópolis	S	10%	5.0%
Maringá	S	10%	5.0%
Vitória	SE	15%	7.5%
Volta Redonda	SE	10%	5.0%
Blumenau	S	10%	5.0%
Ipatinga	SE	10%	5.0%
Criciúma	S	10%	5.0%
Itajaí	S	10%	5.0%
Cabo Frio	SE	10%	5.0%
Moji Mirim	SE	10%	5.0%
Guaratinguetá	SE	10%	5.0%

Source: TomTom (2015)

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Table 26 - City fuel consumption rate of VW GOL considering TC impact

		W	$C_G = 11.6 * (1 - B)$	$C_E = 7.7 * (1 - B)$	$L_G = C_G * W$	$L_E = C_E * W$
Urbans	**Region**	**Weighted value for each region** (based on households) **%**	**Gasoline** (City FCR) **Adjusted By TIR**	**Ethanol** (City FCR) **Adjusted By TIR**	**Gasoline** (City FCR) **Adjusted by region WV**	**Ethanol** (City FCR) **Adjusted by region WV**
São Paulo	SE	41%	9.7	6.4	4.0	2.6
Rio de Janeiro	SE	27%	8.6	5.7	2.3	1.5
Salvador	NE	23%	8.9	5.9	2.1	1.4
Belo Horizonte	SE	9%	10.0	6.7	0.9	0.6
Fortaleza	NE	20%	9.6	6.4	1.9	1.3
Brasília	CW	61%	10.4	6.9	6.3	4.2
Curitiba	S	25%	10.5	7.0	2.7	1.8
Recife	NE	25%	9.0	6.0	2.2	1.5
Porto Alegre	S	37%	10.2	6.7	3.8	2.5
Belém	N	100%	10.7	7.1	10.7	7.1
Goiânia	CW	39%	10.7	7.1	4.2	2.8
Campinas	SE	5%	10.7	7.1	0.5	0.4
São Luís	NE	6%	10.8	7.2	0.7	0.5
Maceió	NE	6%	10.8	7.2	0.7	0.5
Natal	NE	7%	10.8	7.2	0.8	0.5
João Pessoa	NE	6%	10.8	7.2	0.7	0.4
São José dos Campos	SE	3%	10.7	7.1	0.3	0.2
Ribeirão Preto	SE	1%	11.0	7.3	0.2	0.1
Sorocaba	SE	2%	10.8	7.2	0.2	0.1
Aracaju	NE	5%	10.8	7.2	0.6	0.4
Londrina	S	6%	11.0	7.3	0.6	0.4
Santos	SE	3%	10.7	7.1	0.3	0.2
Joinville	S	6%	11.0	7.3	0.6	0.4
São José do Rio Preto	SE	1%	11.0	7.3	0.1	0.1
Caxias do Sul	S	6%	11.0	7.3	0.6	0.4
Jundiaí	SE	1%	11.0	7.3	0.1	0.1
Florianópolis	S	7%	11.0	7.3	0.8	0.5
Maringá	S	4%	11.0	7.3	0.4	0.3
Vitória	SE	3%	10.7	7.1	0.3	0.2
Volta Redonda	SE	1%	11.0	7.3	0.1	0.1
Blumenau	S	4%	11.0	7.3	0.4	0.3
Ipatinga	SE	1%	11.0	7.3	0.1	0.1
Criciúma	S	2%	11.0	7.3	0.3	0.2
Itajaí	S	3%	11.0	7.3	0.4	0.2
Cabo Frio	SE	0%	11.0	7.3	0.1	0.0
Moji Mirim	SE	0%	11.0	7.3	0.0	0.0
Guaratinguetá	SE	0%	11.0	7.3	0.1	0.0

Source: Author

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Finally, considering the 55% city and 45% high condition mode for calculating combined condition, while the highway fuel consumption rate is unchanged, the adjusted fuzzy fuel consumption rate for each fuel type and region is achieved as shown in Table 27.

Table 27 - Adjusted fuzzy FCR for VW GOL by fuel type and geographic region considering the TC impact

		VW GOL		
		M_1	M_2	M_3
Fuel	**Region**	**Lower limit**	**Most Likely**	**Upper limit**
Gasoline	NE	9.7	11.6	13.9
	N	10.7	12.2	13.9
	CW	10.5	12.0	13.9
	SE	9.7	11.6	13.9
	S	10.6	12.1	13.9
Ethanol	NE	6.4	7.9	9.6
	N	7.1	8.2	9.6
	CW	7.0	8.2	9.6
	SE	6.4	7.9	9.6
	S	7.0	8.2	9.6

$M_1 = rolling\ up\ the\ L\ values\ (from\ table\ 26) for\ each\ fuel\ type\ and\ region$
$M_3 = Highway\ cunsumption\ rate\ for\ each\ fuel\ type$
$M_2 = 55\%\ M_1 + 45\%\ M_3$

Source: Author

By applying the same operation for Fiat UNO vehicle, the adjusted fuzzy fuel consumption rate can be estimated. The results are shown in Table 28.

Table 28 - Adjusted fuzzy FCR for Fiat UNO by fuel type and geographic region considering the TC impact

		Fiat UNO		
Fuel	**Region**	**Lower limit**	**Most Likely**	**Upper limit**
Gasoline	NE	9.8	12.2	15.0
	N	10.9	12.8	15.0
	CW	10.7	12.6	15.0
	SE	9.9	12.2	15.0
	S	10.7	12.7	15.0
Ethanol	NE	6.8	8.4	10.4
	N	7.6	8.9	10.4
	CW	7.4	8.8	10.4
	SE	6.9	8.5	10.4
	S	7.5	8.8	10.4

Source: Author

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Based on the vehicle mileage traveled from table 7, the fuzzy monthly fuel consumption for each region and fuel type can be estimated. For example, for Northeastern region and VW GOL vehicle, the values are obtained by:

Vehicle mileage traveled: 1,500 km/ month

Monthly fuel consumption (MFC) for Northeastern region: ME_G and ME_E

$$ME_G = 1500/(9.7, 11.6, 13.9) = (107.9, 129.5, 155.0)$$
$$ME_E = 1500/(6.4, 7.9, 9.6) = (156.3, 191.0, 233.5)$$

The following tables present the fuzzy MFC for each region and fuel type after considering the traffic congestion impact:

Table 29 - Fuzzy MFC for VW GOL by fuel type, and region considering TC impact

		VW GOL		
Fuel	**Region**	**Lower limit**	**Most Likely**	**Upper limit**
Gasoline	NE	107.9	129.5	155.0
	N	107.9	123.4	139.8
	CW	107.9	124.6	142.6
	SE	107.9	129.4	154.7
	S	107.9	124.3	142.0
Ethanol	NE	156.3	191.0	233.5
	N	156.3	182.1	210.6
	CW	156.3	183.8	214.9
	SE	156.3	190.8	233.0
	S	156.3	183.4	213.9

Source: Author

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Table 30 - Fuzzy MFC for Fiat UNO by fuel type, and region considering TC impact

Fuel	Region	Fiat UNO Lower limit	Most Likely	Upper limit
Gasoline	NE	100.0	123.3	152.3
	N	100.0	117.6	137.4
	CW	100.0	118.7	140.2
	SE	100.0	123.2	152.0
	S	100.0	118.5	139.6
Ethanol	NE	144.2	177.7	219.2
	N	144.2	169.5	197.8
	CW	144.2	171.1	201.8
	SE	144.2	177.5	218.8
	S	144.2	170.7	200.8

Source: Author

Considering monthly fuzzy fuel consumption (Table 29 and 30), the fuzzy cash flows (for t = 1, …, 60) are determined, in the following way for each region. How illustration, for Northeastern region, the calculus is:

- **VW GOL – Northeastern region**

  Fuzzy cash flow for gasoline in period t:

$$\text{FCF\_VW\_GOL}_{g_t} = g_t \times (107.9, 129.5, 155.0)$$

  Fuzzy cash flow for ethanol in period t:

$$\text{FCF\_VW\_GOL}_{e_t} = e_t \times (156.3, 191.0, 233.5)$$

- **FIAT UNO – Northeastern region**

  Fuzzy cash flow for gasoline in period t:

$$\text{FCF\_FIAT\_UNO}_{g_t} = g_t \times (100.0, 123.3, 152.3)$$

  Fuzzy cash flow for ethanol in period t:

$$\text{FCF\_FIAT\_UNO}_{e_t} = e_t \times (144.2, 177.7, 219.2)$$

The economic choice of fuel option is associated with monthly expenses (cash flow) of both fuel types, where the one with the lowest cash flow will be chosen along with the vehicle life expectancy each month. As described in section 3.5.7, for comparing and choosing lowest fuzzy cash flow, first the ranked fuzzy number must be estimated, then compare the results and choose the lower

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value. Accordingly, the final fuzzy cash flow for each vehicle in period t is given by:

- **VW GOL**

$$\text{Final FCF\_VW\_GOL}_{\mathrm{t}} = \mathit{The\ Fuzz\ Cash\ Flow\ associated\ with}\ \min(E(F\text{CF\_VW\_GOL}_{g_t}); E(\text{FCF\_VW\_GOL}_{e_t}))$$

- **FIAT UNO**

$$\text{Final FCF\_Fiat\_UNO}_{\mathrm{t}} = \mathit{The\ Fuzz\ Cash\ Flow\ associated\ with}\ \min(E(\text{CF\_Fiat\_UNO}_{g_t}); E(\text{CF\_Fiat\_UNO}_{e_t}))$$

Total fuzzy expenses of flex-fuel vehicle as a sum of fuzzy present value (FPV) of FCF, will be generated in following equations:

- **VW GOL**

$$FPV\ of\ total\ fuel\ expenses\ of\ VW\ GOL = \sum_{t=1}^{60} \frac{\text{Final FCF\_VW\_GOL}_{\mathrm{t}}}{(1+r)^t}$$

- **FIAT UNO**

$$FPV\ of\ total\ fuel\ expenses\ of\ FIAT\ UNO = \sum_{t=1}^{60} \frac{\text{Final F}CF\_FIAT\_UNOt_{\mathrm{t}}}{(1+r)^t}$$

The fuzzy fuel switch option value is given by the difference between the fuzzy present values (FPV) of total fuzzy fuel expenses of automobile traveled only by gasoline and flex automobile. Tables 31 and 32 present the results according to the vehicle type and geographic region, for the values of fuzzy present value (FPV) of fuel expenses, and the fuzzy switch option value.

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Table 31 - FPV of fuel Expenses and FSOV for VW GOL by region considering the traffic congestion impact

		VW GOL				
Values (R$)		**NE**	**N**	**CW**	**SE**	**S**
FPV car moved only by gasoline	Upper limit	11808.82	11020.12	11110.23	11220.80	10900.44
	Most Likely	9871.90	9726.97	9705.63	9390.20	9544.81
	Lower limit	8223.34	8506.90	8406.65	7829.10	8285.41
FPV flex-fuel car	Upper limit	11204.36	11020.12	8803.22	8135.64	8521.03
	Most Likely	9219.78	9726.97	7531.97	6662.87	7308.04
	Lower limit	7580.08	8506.90	6402.03	5455.76	6224.98
Fuzzy Switch option value	Upper limit	4228.74	2513.22	4708.20	5765.04	4675.46
	Most Likely	652.12	0.00	2173.66	2727.33	2236.77
	Lower limit	-2981.03	-2513.22	-396.57	-306.53	-235.62

Source: Author

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Table 32 - FPV of fuel Expenses and FSOV for Fiat UNO by region considering the traffic congestion impact

		Fiat UNO				
Values (R$)		**NE**	**N**	**CW**	**SE**	**S**
FPV car moved only by gasoline	Upper limit	11608.67	10833.34	10921.92	11030.62	10715.69
	Most Likely	9395.74	9271.82	9248.76	8937.54	9096.11
	Lower limit	7620.29	7883.06	7790.16	7254.97	7677.81
FPV flex-fuel car	Upper limit	10641.29	10833.34	8266.44	7639.56	8001.45
	Most Likely	8621.54	9271.82	7008.60	6197.76	6800.36
	Lower limit	6998.11	7883.06	5909.56	5036.09	5746.14
Fuzzy Switch option value	Upper limit	4610.56	2950.28	5012.35	5994.53	4969.55
	Most Likely	774.20	0.00	2240.16	2739.78	2295.75
	Lower limit	-3021.00	-2950.28	-476.28	-384.59	-323.64

Source: Author

Finally, to make decision, comparing values, and estimate the economy (flex/ Gas) vale the fuzzy numbers must be defuzzify to a crisp value. The economy percentage will be obtained by dividing the crisp fuzzy switch option value on the automobile price.

$$Economy = -\frac{E(Fuzzy\ switch\ option\ value)}{Automibile\ price}$$

Tables 33 and 34 present the results by vehicle type and geographic region, for the crisp fuzzy present value of fuel expenses, the crisp fuzzy switch option value and the economy considering the traffic congestion impact on the fuel consumption rate.

Table 33 - Crisp value of FPV of fuel Expenses, FSOV and economy for VW GOL by geographic region considering the TC impact

	VW GOL				
Values (R$)	**NE**	**N**	**CW**	**SE**	**S**
PV car moved only by gasoline	9929.38	9739.15	9729.18	9434.48	9566.46
PV flex-fuel car	9210.35	9726.97	7526.02	6663.52	7302.43
Switch option value	652.12	0.00	2173.66	2727.33	2236.77
Economy (SOV/A. price)*	**-2.09%**	**0.00%**	**-6.96%**	**-8.73%**	**-7.16%**

* *SOV: Switch Option Value; A. price: Automobile price*
Source: Author

Table 34 - Crisp value of FPV of fuel Expenses, FSOV and economy for Fiat UNO by geographic region considering the TC impact

	Fiat UNO				
Values (R$)	**NE**	**N**	**CW**	**SE**	**S**
PV car moved only by gasoline	9461.79	9300.61	9275.22	8984.22	9120.59
PV flex-fuel car	8628.40	9271.82	7017.89	6219.49	6809.43
Switch option value	774.20	0.00	2240.16	2739.78	2295.75
Economy (SOV/A. price)*	**-2.65%**	**0.00%**	**-7.67%**	**-9.38%**	**-7.86%**

* *SOV: Switch Option Value; A. price: Automobile price*
Source: Author

4.7.3. Fuzzy Vehicle Mileage Traveled and Fuzzy Fuel Consumption Rate

In two previous sections, establishing a fuzzy number for two parameters of vehicle mileage traveled and fuel consumption rate was proposed. Also, measuring the environmental impacts on these fuzzy parameters was achieved. Finally, a fuzzy number approach was applied to estimate fuel switch option flexibility value embedded in flex-fuel vehicles.

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Following, the assessing the impact of combination of two fuzzy parameters on FFV option value is demonstrated. It is assumed that fuzzy numbers were generated and the impact of environmental condition is included (Tables 18, 27, and 28). Now the monthly fuzzy fuel consumption must be calculated. Since, for each region there are two fuzzy numbers, monthly fuzzy fuel consumption will be obtained through the following way:

If consider $\tilde{m}$ as fuzzy vehicle mileage traveled and $\tilde{f}$ as fuzzy fuel consumption rate, then $\tilde{c}$ the monthly fuzzy fuel consumption is calculated by Equation 44 as:

$$\tilde{c} = (c_1, c_2, c_3) \quad or \quad \tilde{c}_\alpha = [c_\alpha^L, c_\alpha^U]$$
$$\tilde{m} = (m_1, m_2, m_3) \quad or \quad \tilde{m}_\alpha = [m_\alpha^L, m_\alpha^U]$$
$$\tilde{f} = (f_1, f_2, f_3) \quad or \quad \tilde{f}_\alpha = [f_\alpha^L, f_\alpha^U]$$

$$\begin{aligned}\tilde{c}_\alpha = \tilde{m}_\alpha \oslash \tilde{f}_\alpha = [&\min\{m_\alpha^L/f_\alpha^L, m_\alpha^L/f_\alpha^U, m_\alpha^U/f_\alpha^L, m_\alpha^U/f_\alpha^U\},\\ &\max\{m_\alpha^L/f_\alpha^L, m_\alpha^L/f_\alpha^U, m_\alpha^U/f_\alpha^L, m_\alpha^U/f_\alpha^U\}]\end{aligned} \tag{44}$$

Numerical example for VW GOL, Northeastern region, and gasoline, the FVMT ($\tilde{m}$) and FFCR ($\tilde{f}$):

$$\tilde{m} = (1185, 1524, 1862) \Rightarrow \tilde{m}_\alpha = [1185 + 339\alpha, 1862 - 339\alpha]$$

$$\tilde{f} = (9.7, 11.6, 13.9) \Rightarrow \tilde{f}_\alpha = [9.7 + 1.9\alpha, 13.9 - 2.3\alpha]$$

Then through the Equation 44, the results are achieved as:

Table 35 – α – cut levels of $\tilde{m}_\alpha$, $\tilde{f}_\alpha$ and $\tilde{c}_\alpha$ for $\alpha \in [0,1]$.

	$\tilde{m}_\alpha$		$\tilde{f}_\alpha$		$\tilde{c}_\alpha$	
α	m_α^L	m_α^U	f_α^L	f_α^U	c_α^L	c_α^U
0	1185.0	1862.1	9.7	13.9	**85.2**	**192.4**
0.1	1218.8	1828.2	9.9	13.7	89.2	185.2
0.2	1252.7	1794.4	10.1	13.4	93.2	178.4
0.3	1286.5	1760.5	10.2	13.2	97.4	171.8
0.4	1320.4	1726.7	10.4	13.0	101.8	165.4
0.5	1354.2	1692.8	10.6	12.7	106.3	159.3
0.6	1388.1	1659.0	10.8	12.5	111.0	153.3
0.7	1422.0	1625.1	11.0	12.3	115.8	147.6
0.8	1455.8	1591.2	11.2	12.0	120.9	142.1
0.9	1489.7	1557.4	11.4	11.8	126.1	136.7
1	1523.5	1523.5	11.6	11.6	**131.6**	**131.6**

Source: Author

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From $\alpha - cut$ level calculations in Table 35, the fuzzy value of $\tilde{c}$ for VW GOL, Northeastern region, and gasoline fuel type will be obtained according to the value of $\alpha = 0 \; and \; \alpha = 1$ as:

$$\tilde{c} = (85.2, 131.6, 192.4)$$

Following tables present the fuzzy MFC for each region and fuel type after performing fuzzy operation for all monthly fuzzy fuel consumption variables:

Table 36 – Fuzzy MFC for VW GOL by fuel type and region considering USSI and TCI

		VW GOL		
Fuel	**Region**	**Lower limit**	**Most Likely**	**Upper limit**
Gasoline	NE	85.2	131.6	192.4
	N	92.5	135.9	188.2
	CW	92.5	137.3	192.0
	SE	86.5	133.4	194.8
	S	87.4	129.4	180.6
Ethanol	NE	123.4	194.0	289.8
	N	133.9	200.6	283.5
	CW	133.9	202.5	289.3
	SE	125.3	196.7	293.5
	S	126.5	191.0	272.1

Source: Author

Table 37 - Fuzzy MFC for Fiat UNO by fuel type and region considering USSI and TCI

		Fiat UNO		
Fuel	**Region**	**Lower limit**	**Most Likely**	**Upper limit**
Gasoline	NE	79.0	125.2	189.1
	N	85.7	129.6	185.0
	CW	85.7	130.8	188.8
	SE	80.2	127.0	191.5
	S	81.0	123.3	177.6
Ethanol	NE	113.9	180.4	272.1
	N	123.6	186.7	266.3
	CW	123.6	188.5	271.7
	SE	115.6	182.9	275.6
	S	116.8	177.7	255.5

Source: Author

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Considering monthly fuzzy fuel consumption (Table 36 and 37), the fuzzy cash flows (for t = 1, …, 60) are calculated in the following way for each region; for instance, Northeastern region:

- **VW GOL – Northeastern region**

  Fuzzy cash flow for gasoline in period t:

$$\text{FCF\_VW\_GOL}_{g_t} = g_t \times (85.2, 131.6, 192.4)$$

  Fuzzy cash flow for ethanol in period t:

$$\text{FCF\_VW\_GOL}_{e_t} = e_t \times (123.4, 194.0, 289.8)$$

- **FIAT UNO – Northeastern region**

  Fuzzy cash flow for gasoline in period t:

$$\text{FCF\_FIAT\_UNO}_{g_t} = g_t \times (79.0, 125.2, 189.1)$$

  Fuzzy cash flow for ethanol in period t:

$$\text{FCF\_FIAT\_UNO}_{e_t} = e_t \times (113.9, 180.4, 272.1)$$

The economic choice of fuel option is associated with monthly expenses (cash flow) of both fuel types, where the one with the lowest cash flow will be chosen along with the vehicle life expectancy each month. For comparing and choosing lowest fuzzy cash flow, first we must rank the fuzzy number then compare and choose the lower value. Accordingly, the final fuzzy cash flow for each vehicle in period t is given by:

- **VW GOL**

$$\text{Final FCF\_VW\_GOL}_{\text{t}}$$
$$= \textit{The Fuzz Cash Flow associated with } \min(E(F\text{CF\_VW\_GOL}_{g_t}); E(\text{FCF\_VW\_GOL}_{e_t}))$$

- **FIAT UNO**

$$\text{Final FCF\_Fiat\_UNO}_{\text{t}}$$
$$= \textit{The Fuzz Cash Flow associated with } \min(E(\text{CF\_Fiat\_UNO}_{g_t}); E(\text{CF\_Fiat\_UNO}_{e_t}))$$

Total fuzzy expenses of flex-fuel vehicle as a sum of fuzzy present value (FPV) of FCF, will be generated in following equations:

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- **VW GOL**

$$FPV\ of\ total\ fuel\ expenses\ of\ VW\ GOL = \sum_{t=1}^{60} \frac{\text{Final FCF\_VW\_GOL}_{\text{t}}}{(1+r)^t}$$

- **FIAT UNO**

$$FPV\ of\ total\ fuel\ expenses\ of\ FIAT\ UNO = \sum_{t=1}^{60} \frac{\text{Final F}CF\_FIAT\_UNOt_{\text{t}}}{(1+r)^t}$$

The fuzzy fuel switch option value is given by the difference between the fuzzy present values (FPV) of total fuzzy fuel expenses of automobile traveled only by gasoline and flex automobile. The results by vehicle type and by geographic region, for the fuzzy present value (FPV) of fuel expenses, and for the fuzzy switch option value are presented in Tables 38 and 39.

Table 38 - FPV of fuel expenses and FSOV for VW GOL by geographic region considering the USS and TC impact

		VW GOL				
Values (R$)		**NE**	**N**	**CW**	**SE**	**S**
FPV car moved only by gasoline	Upper limit	14659.37	14836.98	14960.58	14135.13	13868.64
	Most Likely	10026.73	10714.86	10692.98	9678.34	9935.89
	Lower limit	6496.25	7288.46	7203.67	6276.15	6708.25
FPV flex-fuel car	Upper limit	13909.01	14836.98	11854.05	10248.67	10841.31
	Most Likely	9364.38	10714.86	8298.20	6867.32	7607.48
	Lower limit	5988.09	7288.46	5485.91	4373.58	5040.03
Fuzzy Switch option value	Upper limit	8671.29	7548.52	9474.67	9761.55	8828.61
	Most Likely	662.35	0.00	2394.78	2811.02	2328.41
	Lower limit	-7412.76	-7548.52	-4650.39	-3972.52	-4133.06

Source: Author

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Table 39 - FPV of fuel expenses and FSOV for Fiat UNO by geographic region considering the USS and TC impact

		Fiat UNO				
Values (R$)		**NE**	**N**	**CW**	**SE**	**S**
FPV car moved only by gasoline	Upper limit	14410.91	14585.51	14707.01	13895.55	13633.58
	Most Likely	9543.11	10213.49	10189.63	9211.78	9468.81
	Lower limit	6019.85	6753.97	6675.40	5815.90	6216.31
FPV flex-fuel car	Upper limit	13210.02	14585.51	11131.25	9623.75	10180.25
	Most Likely	8756.77	10213.49	7721.59	6387.93	7079.00
	Lower limit	5528.35	6753.97	5063.91	4037.15	4652.34
Fuzzy Switch option value	Upper limit	8882.56	7831.53	9643.10	9858.41	8981.24
	Most Likely	786.34	0.00	2468.05	2823.85	2389.82
	Lower limit	-7190.16	-7831.53	-4455.85	-3807.85	-3963.94

Source: Author

When all fuzzy calculations are done, to make decision, comparing values, and estimate the economy value, the fuzzy numbers must be defuzzified to a crisp value. Accordingly, the economy percentage will be obtained by dividing the crisp fuzzy switch option value on the automobile price.

$$Economy = -\frac{E(Fuzzy\ switch\ option\ value)}{Automibile\ price}$$

Tables 40 and 41 show the results by vehicle type and geographic region, for the crisp present value of fuel expenses, the crisp switch option value and the economy considering the urban spatial structure and traffic congestion impacts.

Table 40 – Crisp value of FPV of fuel expenses, FSOV and economy for VW GOL by geographic region considering USSI and TCI

	VW GOL				
Values (R$)	**NE**	**N**	**CW**	**SE**	**S**
PV car moved only by gasoline	10221.45	10830.81	10816.91	9826.27	10046.95
PV flex-fuel car	9353.35	10714.86	8303.99	6895.15	7613.93
Switch option value	662.35	0.00	2394.78	2811.02	2328.41
Economy (SOV/A. price)*	**-2.12%**	**0.00%**	**-7.67%**	**-9.00%**	**-7.45%**

* *SOV: Switch Option Value; A. price: Automobile price*
Source: Author

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Table 41 - Crisp value of FPV of fuel expenses, FSOV and economy for Fiat UNO by geographic region considering USSI and TCI

	Fiat UNO				
Values (R$)	**NE**	**N**	**CW**	**SE**	**S**
PV car moved only by gasoline	9747.24	10365.57	10314.97	9359.28	9581.25
PV flex-fuel car	8776.72	10213.49	7763.44	6455.07	7118.61
Switch option value	786.34	0.00	2468.05	2823.85	2389.82
Economy (SOV/A. price)*	**-2.69%**	**0.00%**	**-8.45%**	**-9.66%**	**-8.18%**

* *SOV: Switch Option Value; A. price: Automobile price*
Source: Author

4.8. Results Aanalysis

4.8.1. General

The results showed that when the automobile price is considered, the option value represents the economy between 0% and 10% of the vehicle price. The economy for VW GOL was between 0% and 9.0% and 0.3% to 9.7% for Fiat UNO. The Northern Region was the less favorable among all the five regions. The flexibility option evaluation presented no economy in this region. Possibly, the low economy in the Northern region is due to the high value of Long-run mean of ethanol, R$ 1.029 per liter, very close to the value of gasoline, R$ 1,243 per liter. In addition, this region contains the lowest ethanol’s speed of reversion 0.031 among other regions and very close to value of gasoline’s speed of reversion, 0.018. The lowest volatility among all regions belongs to the Northern region with 0.027. The Southeastern region with most economy, has the lowest long-run mean, the highest speed of reversion and volatility of ethanol among all regions with 0.793, 0.109 and 0.055 respectively

Samanez et al. (2014) in the same case examined the influence of the long-run mean, speed of reversion, and volatility upon the switch option value. The results demonstrate if the gasoline’s long-run mean keeps constant and ethanol’s long-run mean varies, option value falling with increasing the ethanol’s long-run mean. They also illustrated that by analyzing the sensitivity of the option value with respect to the speed of reversion of ethanol, it is concluded that the faster

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price of reverting to the long-run mean for ethanol, results the higher the option value. By Bastian-Pinto et al. (2008), Camargo et al. (2010), and Samanez et al. (2014) in the similar case, they showed, as expected in real option theory, the increase in volatility increases the option value in all cases analyzed.

Comparing the results in Tables 40 and 41, it can be seen the vehicle Fiat UNO has a little more option value than VW GOL. This difference is due to a little more fuel efficiency percentage of the Fiat UNO engine for ethanol comparing with gasoline. This fuel efficiency can be obtained by dividing the ethanol consumptions rate over the gasoline consumption rate for each vehicle, which in this study was represented as 67.7% and 69.4% for VW GOL and Fiat UNO respectively in Table 5.

By the results, either for the VW GOL or for Fiat UNO, the Southeastern region has more economy and following Central-Western and Southern regions. It shows that vehicle type does not have any effect on the region's economy ranking. For Northern region, there is no economy and for Northeastern region is 2.12% for VW GOl and 2.69% for Fiat UNO.

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4.8.2. Comparing Fuzzy Results with the Regular Option Values

- **Fuzzy Vehicle Mileage Traveled (FVMT)**

In section 4.7.1 due to the inherent imprecise of VMT a fuzzy number as FVMT, through a monthly traveling average range, was defined. Considering the urban spatial structure impact for each city and consequently each region, the fuzzy number was differentiated along all regions. This difference caused a new option value for each region. Tables 42 and 43 present the impact of the urban spatial structure effect on evaluation along all regions by measuring the differences between regular and fuzzy evaluation approach.

Table 42 - Differences between the fuzzy and regular option valuations for VW GOL based on VMT considering the USS impact

		VW GOL				
Values (R$)	**	**NE**	**N**	**CW**	**SE**	**S**
Car moved only by gasoline	$CFPV_{(G)}$	9188.54	10309.08	10189.14	8877.22	9488.41
	$PV_{(G)}$	9046.65	9358.60	9248.31	8612.94	9114.94
	Dif. $_{(G)}$	**141.89**	**950.48**	**940.83**	**264.28**	**373.47**
Flex-fuel car	$CFPV_{(F)}$	8592.76	10309.08	7914.88	6310.05	7271.58
	$PV_{(F)}$	8460.07	9358.60	7184.04	6122.19	6985.37
	Dif. $_{(F)}$	**132.69**	**950.48**	**730.83**	**187.86**	**286.21**
Switch option value	CFV	595.78	0.00	2274.27	2567.18	2216.82
	$PV_{(S)}$	586.58	0.00	2064.27	2490.75	2129.57
	Dif. $_{(S)}$	**9.20**	**0.00**	**210.00**	**76.43**	**87.26**
Economy (SOV/A. price)*	EF	-1.91%	0.00%	-7.28%	-8.22%	-7.10%
	ER	-1.88%	0.00%	-6.61%	-7.97%	-6.82%
	Dif. $_{(E)}$	**-0.03%**	**0.00%**	**-0.67%**	**-0.24%**	**-0.28%**

** SOV: Switch Option Value; A. price: Automobile price*

*** CFPV: Crisp Fuzzy Present Value; PV: Present Value; CFV: Crisp Fuzzy Value; Dif.: Difference; EF: Economy by fuzzy approach; ER: Economy by regular analysis ;(G): Gas powered automobile; (F): Flex-fuel vehicle; (S) Switch Option Value; (E):Economy.*

Source: Author

For both vehicle types as can be seen in Table 42 and 43, there is significant increasing value among the regions that represents the importance of using the fuzzy evaluation approach and considering the impact of USS in evaluation of FFV option flexibility. The most increase in present values was related to the Northern region and in the following the Central-Western, Southern, Southeastern, Northeastern regions whereas regarding the switch option value most increasing was occurred in Central-Western region and no value for the Northern region. Increasing in option value rose due to the increasing in the fuel consumption. It shows the sensitivity of option value to the total fuel consumption in this study. The economy has increased at the same rate of switch option values. In regular option evaluation, Southern region was the second and Central-Western region was the third region having the most economy value among all regions. However, considering the USS impact through a fuzzy evaluation approach changed the region economy ranking. According to the new approach, Central-

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Western region is in the second and Southern region is in the third position. For rest of regions, the economy ranking is the same as regular option evaluation.

The following table presents the same data for Fiat UNO vehicle:

Table 43 - Differences between the fuzzy and regular option valuations for Fiat UNO based on VMT considering the USS impact

		Fiat UNO				
Values (R$)	**	**NE**	**N**	**CW**	**SE**	**S**
Car moved only by gasoline	$CFPV_{(G)}$	8768.67	9838.01	9723.55	8471.58	9054.83
	$PV_{(G)}$	8633.26	8930.96	8825.71	8219.37	8698.43
	Dif. $_{(G)}$	**135.41**	**907.05**	**897.84**	**252.21**	**356.40**
Flex-fuel car	$CFPV_{(F)}$	8045.48	9838.01	7367.98	5874.04	6769.14
	$PV_{(F)}$	7921.24	8930.96	6687.65	5699.17	6502.70
	Dif. $_{(F)}$	**124.24**	**907.05**	**680.34**	**174.88**	**266.44**
Switch option value	CFV	723.19	0.00	2355.57	2597.54	2285.69
	$PV_{(S)}$	712.02	0.00	2138.06	2520.21	2195.73
	Dif. $_{(S)}$	**11.17**	**0.00**	**217.51**	**77.33**	**89.97**
Economy (SOV/A. price)*	EF	-2.47%	0.00%	-8.06%	-8.89%	-7.82%
	ER	-2.44%	0.00%	-7.32%	-8.62%	-7.51%
	Dif. $_{(E)}$	**-0.04%**	**0.00%**	**-0.74%**	**-0.26%**	**-0.31%**

* *SOV: Switch Option Value; A. price: Automobile price*

*** CFPV: Crisp Fuzzy Present Value; PV: Present Value; CFV: Crisp Fuzzy Value; Dif.: Difference; EF: Economy by fuzzy approach; ER: Economy by regular analysis ;(G): Gas powered automobile; (F): Flex-fuel vehicle;(S) Switch Option Value; (E):Economy.*

Source: Author

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- **Fuzzy Fuel Consumption Rate (FFCR)**

Fuzzy sets theory enables us to represent the inherent uncertainty and vagueness of variables in a mathematical model. Due to the vagueness in the fuel consumption rate value, as described in section 4.7.2, a fuzzy model considering the city, combined and highway-driving conditions as lower limit, most likely, and upper limit values was introduced. Two fuzzy numbers as gasoline and ethanol consumption rate for each vehicle generated. Considering the impact of traffic congestion (TC) and then applying this impact on FCR for each region and vehicle, the fuzzy real option value was concluded. Following tables present the difference between fuzzy option value considering the traffic congestion impact and regular option value.

Table 44 - Differences between the fuzzy and regular option valuations for VW GOL based on FCR considering the TC impact

		VW GOL				
Values (R$)	**	**NE**	**N**	**CW**	**SE**	**S**
Car moved only by gasoline	$CFPV_{(G)}$	9929.38	9739.15	9729.18	9434.48	9566.46
	$PV_{(G)}$	9046.65	9358.60	9248.31	8612.94	9114.94
	Dif. $_{(G)}$	**882.73**	**380.55**	**480.87**	**821.54**	**451.53**
Flex-fuel car	$CFPV_{(F)}$	9210.35	9726.97	7526.02	6663.52	7302.43
	$PV_{(F)}$	8460.07	9358.60	7184.04	6122.19	6985.37
	Dif. $_{(F)}$	**750.29**	**368.37**	**341.98**	**541.32**	**317.06**
Switch option value	CFV	652.12	0.00	2173.66	2727.33	2236.77
	$PV_{(S)}$	586.58	0.00	2064.27	2490.75	2129.57
	Dif. $_{(S)}$	**65.54**	**0.00**	**109.39**	**236.58**	**107.20**
Economy (SOV/A. price)*	EF	-2.09%	0.00%	-6.96%	-8.73%	-7.16%
	ER	-1.88%	0.00%	-6.61%	-7.97%	-6.82%
	Dif. $_{(E)}$	**-0.21%**	**0.00%**	**-0.35%**	**-0.76%**	**-0.34%**

* *SOV: Switch Option Value; A. price: Automobile price*

*** CFPV: Crisp Fuzzy Present Value; PV: Present Value; CFV: Crisp Fuzzy Value; Dif.: Difference; EF: Economy by fuzzy approach; ER: Economy by regular analysis ;(G): Gas powered automobile; (F): Flex-fuel vehicle;(S) Switch Option Value; (E):Economy.*

Source: Author

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For both VW GOL and Fiat UNO vehicles, the impact of traffic congestion on final values was positive, despite the fact that the increasing value was different for each of them. For the PV of gas-powered car VW GOL, the most increasing value was related to Northeastern region and in following Southeastern, Central-Western, Southern, and Northern regions whereas the most increasing value in switch option value was quite different. Southeastern region is the region with most increased switch option value while Northern region has no increasing.

For FCR variable considering the TC impact, the increasing percentage rate for the economy ratio is same as switch option value's increasing rate. However, the difference in economy values did not change the region's economy ranking.

The Southeastern reign still is the region with most option value among other regions comparing with regular option value. Southern, Central-Western, Northeastern, and Northern regions are the next respectively.

Table 45 - Differences between the fuzzy and regular option valuations for Fiat UNO based on FCR considering the TC impact

		Fiat UNO				
Values (R$)	**	**NE**	**N**	**CW**	**SE**	**S**
Car moved only by gasoline	$CFPV_{(G)}$	9461.79	9300.61	9275.22	8984.22	9120.59
	$PV_{(G)}$	8633.26	8930.96	8825.71	8219.37	8698.43
	Dif. $_{(G)}$	**828.53**	**369.66**	**449.51**	**764.85**	**422.16**
Flex-fuel car	$CFPV_{(F)}$	8628.40	9271.82	7017.89	6219.49	6809.43
	$PV_{(F)}$	7921.24	8930.96	6687.65	5699.17	6502.70
	Dif. $_{(F)}$	**707.16**	**340.87**	**330.25**	**520.32**	**306.73**
Switch option value	CFV	774.20	0.00	2240.16	2739.78	2295.75
	$PV_{(S)}$	712.02	0.00	2138.06	2520.21	2195.73
	Dif. $_{(S)}$	**62.17**	**0.00**	**102.09**	**219.57**	**100.03**
Economy (SOV/A. price)*	EF	-2.65%	0.00%	-7.67%	-9.38%	-7.86%
	ER	-2.44%	0.00%	-7.32%	-8.62%	-7.51%
	Dif. $_{(E)}$	**-0.21%**	**0.00%**	**-0.35%**	**-0.75%**	**-0.34%**

* *SOV: Switch Option Value; A. price: Automobile price*

** *CFPV: Crisp Fuzzy Present Value; PV: Present Value; CFV: Crisp Fuzzy Value; Dif.: Difference; EF: Economy by fuzzy approach; ER: Economy by regular analysis ;(G): Gas powered automobile; (F): Flex-fuel vehicle;(S) Switch Option Value; (E):Economy.*

Source: Author

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- **Combination of FVMT and FFCR**

In two previous parts, an analogy between fuzzy switch option value and regular option value considering the influences of USSI and TC impact was performed. In section 4.7.3, it was explained how combination of these two factor can be applied as a fuzzy set number. After applying fuzzy arithmetic operations, ultimately the final fuzzy switch option value was obtained. As described, to compare and make decision by fuzzy set numbers it must be ranked through a proper deffuzication formula to a crisp value. Tables 46 and 47 present the difference between fuzzy and regular option value for VW GOL and Fiat UNO vehicles. Tables 48 and 49 show the increasing percentage rate between two methods as well. It shows how much applying a fuzzy set theory considering the USS and TC affects the regular real option valuation.

Table 46 - Differences between the fuzzy and regular option valuations for VW GOL based on VMT and FCR considering USS and TC impacts

Values (R$)	**	VW GOL NE	N	CW	SE	S
Car moved only by gasoline	$CFPV_{(G)}$	10221.45	10830.81	10816.91	9826.27	10046.95
	$PV_{(G)}$	9046.65	9358.60	9248.31	8612.94	9114.94
	Dif. (G)	**1174.80**	**1472.22**	**1568.60**	**1213.33**	**932.02**
Flex-fuel car	$CFPV_{(F)}$	9353.35	10714.86	8303.99	6895.15	7613.93
	$PV_{(F)}$	8460.07	9358.60	7184.04	6122.19	6985.37
	Dif. (F)	**893.29**	**1356.26**	**1119.95**	**772.96**	**628.56**
Switch option value	CFV	662.35	0.00	2394.78	2811.02	2328.41
	$PV_{(S)}$	586.58	0.00	2064.27	2490.75	2129.57
	Dif. (S)	**75.77**	**0.00**	**330.52**	**320.27**	**198.85**
Economy (SOV/A. price)*	EF	-2.12%	0.00%	-7.67%	-9.00%	-7.45%
	ER	-1.88%	0.00%	-6.61%	-7.97%	-6.82%
	Dif. (E)	**-0.24%**	**0.00%**	**-1.06%**	**-1.03%**	**-0.64%**

* *SOV: Switch Option Value; A. price: Automobile price*

*** CFPV: Crisp Fuzzy Present Value; PV: Present Value; CFV: Crisp Fuzzy Value; Dif.: Difference; EF: Economy by fuzzy approach; ER: Economy by regular analysis ;(G): Gas powered automobile; (F): Flex-fuel vehicle;(S) Switch Option Value; (E):Economy.*

Source: Author

By comparing the results between two methods, we have about 10.2% to 17% increasing value in the expense of car moved only by gasoline and 9.0% to 15.6% increasing on flex-fuel car present value for VW GOL vehicle among all regions where this range regarding the switch option value is about 0% to 16% (see Table 48). The most increasing value is related to the Central-Western region with 16 percent increasing in all parts. Northern region has about 15.7% increasing in the car moved only by gasoline and 14.5% for the flex-fuel car present value whereas this percentage was zero for switch option value. The crisp economy values regarding the magnitude order is almost same as regular option evaluation except Southern and Central-Western regions in which their economy ranking order is substituted.

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It can observed that applying the fuzzy set method and considering the environmental impacts, besides giving an ability to treat with variables as their vagueness or imprecise inherent, it presents more precise value regarding the present values and also the switch option values and economy ratio. The new approach gives the owner a more precise value to make decision and investment.

The same data and evaluation result for Fiat UNO was presented in tables 47 and 49 as follows.

Table 47 - Differences between the fuzzy and regular option valuations for Fiat UNO based on VMT and FCR considering USS and TC impacts

		Fiat UNO				
Values (R$)	**	**NE**	**N**	**CW**	**SE**	**S**
Car moved only by gasoline	$CFPV_{(G)}$	9747.24	10365.57	10314.97	9359.28	9581.25
	$PV_{(G)}$	8633.26	8930.96	8825.71	8219.37	8698.43
	Dif. $_{(G)}$	**1113.98**	**1434.62**	**1489.26**	**1139.91**	**882.82**
Flex-fuel car	$CFPV_{(F)}$	8776.72	10213.49	7763.44	6455.07	7118.61
	$PV_{(F)}$	7921.24	8930.96	6687.65	5699.17	6502.70
	Dif. $_{(F)}$	**855.48**	**1282.53**	**1075.80**	**755.91**	**615.90**
Switch option value	CFV	786.34	0.00	2468.05	2823.85	2389.82
	$PV_{(S)}$	712.02	0.00	2138.06	2520.21	2195.73
	Dif. $_{(S)}$	**74.32**	**0.00**	**329.99**	**303.64**	**194.09**
Economy (SOV/A. price)*	EF	-2.69%	0.00%	-8.45%	-9.66%	-8.18%
	ER	-2.44%	0.00%	-7.32%	-8.62%	-7.51%
	Dif. $_{(E)}$	**-0.25%**	**0.00%**	**-1.13%**	**-1.04%**	**-0.66%**

* *SOV: Switch Option Value; A. price: Automobile price*

*** CFPV: Crisp Fuzzy Present Value; PV: Present Value; CFV: Crisp Fuzzy Value; Dif.: Difference; EF: Economy by fuzzy approach; ER: Economy by regular analysis ;(G): Gas powered automobile; (F): Flex-fuel vehicle;(S) Switch Option Value; (E):Economy.*

Source: Author

Table 48 - Increasing percentage rate of the fuzzy sets vs. regular real options analysis considering USS and TC impact for VW GOL

	VW GOL				
Values in percentage	**NE**	**N**	**CW**	**SE**	**S**
Car moved only by gasoline	13.0%	15.7%	17.0%	14.1%	10.2%
Flex-fuel car	10.6%	14.5%	15.6%	12.6%	9.0%
Switch option value	12.9%	0.0%	16.0%	12.9%	9.3%
Economy	12.9%	0.0%	16.0%	12.9%	9.3%

Source: Author

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Table 49 - Increasing percentage rate of the fuzzy sets vs. regular real options analysis considering USS and TC impact for Fiat UNO

	Fiat UNO				
Values in percentage	**NE**	**N**	**CW**	**SE**	**S**
Car moved only by gasoline	12.9%	16.1%	16.9%	13.9%	10.1%
Flex-fuel car	10.8%	14.4%	16.1%	13.3%	9.5%
Switch option value	10.4%	0.0%	15.4%	12.0%	8.8%
Economy	10.4%	0.0%	15.4%	12.0%	8.8%

Source: Author

5. Conclusion and Research Perspective

5.1. Conclusion

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Many countries have been investing billions of dollars on developing alternative energy, in which Brazil is a unique example of successfully developing ethanol industry and being a pioneer in using flex fuel vehicles in the world. Introduction and commercialization of flex-fuel vehicles since March 2003 played an essential role and changed the Brazil fuels market by granting consumers the capacity of making choice between ethanol and gasoline. Vehicles with this technology are able to run with gasoline, hydrated ethanol or any mix portion of both fuels. In 2014, these cars represented more than 88.2% of the marketed new light vehicles in Brazil. This technology under an uncertain and risk market makes an investment problem on minimizing the cost and maximizing the profit in fuel switching option flexibility. Real options valuation approach is an adequate tool to assess this investment problem. Based on this approach as an input switch option, the flexibility value of an automobile with a flex-fuel technology comparing to a vehicle moved only with gasoline has been analyzed. To quantify the cost of fueling, two major parameters, fuel consumption rate and monthly car’s mileage plus fuel prices per each month h been considered.

Due to the variation in fuel consumption rate and car’s mileage, which usually in real world varies in a range (interval), this study proposed the fuzzy sets theory to deal with this imprecise value. A hybrid fuzzy real option value has been applied to estimate the fuel switch option value embedded in Brazilian flex vehicles. This thesis was aimed to investigate whether fuzzy numbers can apply to real option valuation of flex fuel vehicle's flexibility and how applicable this method is comparing with the regular real option valuation. For this investigation, the mean reversion stochastic process was used to model the market uncertainty in gasoline and ethanol prices. Monte Carlo simulation was applied to predict the future fuel prices. To estimate the value of embedded flexibility in flex cars the fuzzy and regular real option valuation both analysis have been conducted in this study. Figure 19 maps the evaluation process by a schematic diagram of fuzzy and regular real option valuation. It shows the interconnection between these two methods and other option assessing stages.

By the range of annual mileage of the car's life expectancy introduced by Bastos (2011), fuzzy vehicle mileage traveled was introduced to the model. The city, highway and the 55% and 45% of their combination respectively was considered to fuzzify the fuel consumption rate as a triangular fuzzy number for each vehicle type as well. After adjusting the environmental effects (the urban spatial structure and traffic congestion), which affect the option value, on fuzzy parameters the fuzzy switch option value was estimated.

Results indicated that the option to choose the less expensive fuel between ethanol and gasoline added significant value to the flex automobile owner in Southeastern, Central-Western, Southern, Northeastern regions and has no economy value in Northern region. The most benefited region by the option flexibility was the Southeastern region that presented an economy between 9.0% and 9.66% for VW GOL and Fiat UNO respectively if compared to the automobile's price. The obtained results showed that the use of flex-fuel technology gives not only environmental advantage but also economic benefits to its owner in most regions.

Comparing between the fuzzy approach considering the environmental impacts and regular real option values indicated that there is significant difference between values, for instance, for the geographic reigns switch option values the differences are 0% to 16% for VW GOL and 0% to 15.4% for Fiat UNO vehicle. Appling this new approach to economic appraisal of FFV flexibility option value discovered a substitution in the region economy raking. It indicated that Central-Western and Southern regions have the second and third economy value among all regions whereas with accord to the regular option valuation Central-Western region was third and Southern region was second.

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The fuzzy sets approach used in this study showed that applying this hybrid method makes more precise and accurate value for the fuel switch option and other results. Regardless of ranking the geographic regions by economy factor ratio, it gives the car’s owner more reliable value to make decision by considering the vehicle mileage and fuel consumption as an interval value (fuzzy number) as they are in the real world.

5.2. Future research

The developed model in this study adds new valuation tool to the flex fuel vehicle’s flexibility evaluation that can serve as a base for other purposes. For advanced users of fuzzy valuations it could be interesting to consider the application of fuzzy numbers on other elements of the valuation. Applying fuzzy numbers to other input variables in a cash flow forecast could be valuable. Hence, the entire valuation could be fuzzified to represent a pure possibility approach and possibly an improved result. The possible mathematical ranking model for substituting fuzzy mean value formula could be interesting to investigate. Also, involving fuzzy logic with fuzzy numbers, in which is not utilized much in finance area, is a sector that could improve the fuzzy-finance evaluations.

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Appendix A - Applied data

Gasoline price and deflation calculation by geographic region from July 2001 to December 2014

Month	IGP-DI (%)	Deflator	Price at the time					deflated price base on July 2001				
			NE	N	CO	SE	S	NE	N	CO	SE	S
Jul 01	1.62	1.0000	1.718	1.828	1.681	1.651	1.683	1.718	1.828	1.681	1.651	1.683
Aug 01	0.90	1.0090	1.741	1.875	1.721	1.683	1.722	1.725	1.858	1.705	1.668	1.707
Sep 01	0.38	1.0128	1.737	1.895	1.730	1.685	1.742	1.715	1.871	1.708	1.664	1.720
Oct 01	1.45	1.0275	1.808	1.954	1.802	1.738	1.811	1.760	1.902	1.754	1.692	1.762
Nov 01	0.76	1.0353	1.803	1.964	1.811	1.743	1.806	1.742	1.897	1.749	1.683	1.744
Dec 01	0.18	1.0372	1.807	1.959	1.803	1.738	1.791	1.742	1.888	1.739	1.676	1.727
Jan 02	0.19	1.0392	1.578	1.726	1.604	1.567	1.613	1.519	1.661	1.543	1.508	1.552
Feb 02	0.18	1.0410	1.508	1.616	1.525	1.490	1.531	1.448	1.552	1.465	1.432	1.470
Mar 02	0.11	1.0422	1.581	1.694	1.575	1.548	1.613	1.517	1.625	1.512	1.486	1.548
Apr 02	0.70	1.0495	1.706	1.819	1.707	1.693	1.756	1.625	1.733	1.627	1.613	1.674
May 02	1.11	1.0611	1.731	1.858	1.720	1.699	1.747	1.631	1.751	1.621	1.602	1.646
Jun 02	1.74	1.0796	1.719	1.845	1.739	1.683	1.738	1.593	1.709	1.611	1.559	1.610
Jul 02	2.05	1.1017	1.786	1.895	1.765	1.739	1.801	1.621	1.720	1.602	1.578	1.635
Aug 02	2.36	1.1277	1.795	1.892	1.775	1.713	1.783	1.592	1.678	1.574	1.519	1.581
Sep 02	2.64	1.1575	1.770	1.889	1.760	1.705	1.796	1.529	1.632	1.520	1.473	1.552
Oct 02	4.21	1.2062	1.784	1.864	1.802	1.722	1.817	1.479	1.545	1.494	1.428	1.506
Nov 02	5.84	1.2767	2.011	2.069	1.998	1.937	2.048	1.575	1.620	1.565	1.517	1.604
Dec 02	2.70	1.3111	2.035	2.099	2.008	1.957	2.083	1.552	1.601	1.532	1.493	1.589
Jan 03	2.17	1.3396	2.202	2.275	2.168	2.107	2.265	1.644	1.698	1.619	1.573	1.691
Feb 03	1.59	1.3609	2.232	2.327	2.274	2.167	2.352	1.640	1.710	1.671	1.592	1.729
Mar 03	1.66	1.3835	2.243	2.317	2.271	2.159	2.324	1.621	1.675	1.642	1.561	1.680
Apr 03	0.41	1.3891	2.201	2.299	2.254	2.144	2.305	1.584	1.655	1.622	1.544	1.659
May 03	-0.67	1.3798	2.139	2.235	2.185	2.062	2.195	1.550	1.620	1.584	1.494	1.590
Jun 03	-0.70	1.3702	2.061	2.205	2.070	1.975	2.096	1.504	1.609	1.511	1.441	1.530
Jul 03	-0.20	1.3674	2.001	2.157	2.007	1.925	2.029	1.463	1.577	1.467	1.408	1.484
Aug 03	0.62	1.3759	1.994	2.136	2.020	1.930	2.044	1.449	1.553	1.468	1.403	1.486
Sep 03	1.05	1.3904	2.021	2.138	2.048	1.958	2.081	1.454	1.538	1.473	1.408	1.497
Oct 03	0.44	1.3965	2.012	2.143	2.060	1.951	2.073	1.441	1.534	1.475	1.397	1.484
Nov 03	0.48	1.4032	2.026	2.157	2.055	1.946	2.049	1.444	1.537	1.465	1.386	1.460
Dec 03	0.60	1.4116	2.020	2.159	2.055	1.949	2.074	1.431	1.530	1.456	1.381	1.469
Jan 04	0.80	1.4229	2.031	2.149	2.070	1.955	2.094	1.427	1.510	1.454	1.374	1.472
Feb 04	1.08	1.4383	2.032	2.137	2.078	1.947	2.090	1.413	1.486	1.445	1.354	1.453
Mar 04	0.93	1.4516	2.031	2.129	2.040	1.925	2.056	1.399	1.466	1.405	1.326	1.416
Apr 04	1.15	1.4683	2.034	2.135	2.029	1.912	2.043	1.386	1.454	1.382	1.302	1.392
May 04	1.46	1.4898	2.042	2.208	2.135	1.985	2.058	1.371	1.482	1.433	1.332	1.381
Jun 04	1.29	1.5090	2.111	2.276	2.192	2.060	2.147	1.399	1.508	1.453	1.365	1.423

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Jul 04	1.14	1.5262	2.163	2.332	2.225	2.107	2.188	1.417	1.528	1.458	1.381	1.434
Aug 04	1.31	1.5462	2.173	2.352	2.282	2.114	2.211	1.405	1.521	1.476	1.367	1.430
Sep 04	0.48	1.5536	2.173	2.366	2.291	2.126	2.205	1.399	1.523	1.475	1.368	1.419
Oct 04	0.53	1.5618	2.200	2.386	2.316	2.163	2.243	1.409	1.528	1.483	1.385	1.436
Nov 04	0.82	1.5747	2.255	2.379	2.315	2.141	2.278	1.432	1.511	1.470	1.360	1.447
Dec 04	0.52	1.5828	2.361	2.478	2.407	2.222	2.374	1.492	1.566	1.521	1.404	1.500
Jan 05	0.33	1.5881	2.349	2.469	2.396	2.221	2.359	1.479	1.555	1.509	1.399	1.485
Feb 05	0.40	1.5944	2.311	2.466	2.401	2.217	2.340	1.449	1.547	1.506	1.390	1.468
Mar 05	0.99	1.6102	2.325	2.466	2.388	2.218	2.361	1.444	1.531	1.483	1.377	1.466
Apr 05	0.51	1.6184	2.323	2.465	2.392	2.217	2.419	1.435	1.523	1.478	1.370	1.495
May 05	-0.25	1.6144	2.327	2.465	2.354	2.202	2.386	1.441	1.527	1.458	1.364	1.478
Jun 05	-0.45	1.6071	2.316	2.458	2.336	2.175	2.350	1.441	1.529	1.454	1.353	1.462
Jul 05	-0.40	1.6007	2.320	2.452	2.362	2.180	2.381	1.449	1.532	1.476	1.362	1.487
Aug 05	-0.79	1.5880	2.318	2.450	2.381	2.186	2.385	1.460	1.543	1.499	1.377	1.502
Sep 05	-0.13	1.5860	2.476	2.575	2.451	2.315	2.518	1.561	1.624	1.545	1.460	1.588
Oct 05	0.63	1.5960	2.513	2.649	2.528	2.390	2.589	1.575	1.660	1.584	1.498	1.622
Nov 05	0.33	1.6012	2.530	2.701	2.568	2.390	2.587	1.580	1.687	1.604	1.493	1.616
Dec 05	0.07	1.6023	2.516	2.683	2.603	2.400	2.582	1.570	1.674	1.624	1.498	1.611
Jan 06	0.72	1.6139	2.546	2.667	2.639	2.438	2.585	1.578	1.653	1.635	1.511	1.602
Feb 06	-0.06	1.6129	2.603	2.662	2.659	2.441	2.590	1.614	1.650	1.649	1.513	1.606
Mar 06	-0.45	1.6057	2.672	2.689	2.659	2.520	2.652	1.664	1.675	1.656	1.569	1.652
Apr 06	0.02	1.6060	2.684	2.711	2.711	2.521	2.653	1.671	1.688	1.688	1.570	1.652
May 06	0.38	1.6121	2.681	2.702	2.673	2.504	2.651	1.663	1.676	1.658	1.553	1.644
Jun 06	0.67	1.6229	2.664	2.671	2.557	2.480	2.601	1.642	1.646	1.576	1.528	1.603
Jul 06	0.17	1.6256	2.668	2.673	2.638	2.485	2.608	1.641	1.644	1.623	1.529	1.604
Aug 06	0.41	1.6323	2.668	2.666	2.690	2.486	2.592	1.635	1.633	1.648	1.523	1.588
Sep 06	0.24	1.6362	2.685	2.665	2.682	2.480	2.574	1.641	1.629	1.639	1.516	1.573
Oct 06	0.81	1.6495	2.673	2.645	2.615	2.471	2.611	1.621	1.604	1.585	1.498	1.583
Nov 06	0.57	1.6589	2.633	2.621	2.676	2.457	2.609	1.587	1.580	1.613	1.481	1.573
Dec 06	0.26	1.6632	2.627	2.615	2.667	2.453	2.594	1.579	1.572	1.604	1.475	1.560
Jan 07	0.43	1.6703	2.635	2.598	2.583	2.461	2.567	1.578	1.555	1.546	1.473	1.537
Feb 07	0.23	1.6742	2.631	2.621	2.615	2.454	2.531	1.572	1.566	1.562	1.466	1.512
Mar 07	0.22	1.6779	2.645	2.614	2.602	2.459	2.525	1.576	1.558	1.551	1.466	1.505
Apr 07	0.14	1.6802	2.659	2.613	2.592	2.480	2.536	1.583	1.555	1.543	1.476	1.509
May 07	0.16	1.6829	2.642	2.638	2.678	2.491	2.506	1.570	1.568	1.591	1.480	1.489
Jun 07	0.26	1.6873	2.607	2.616	2.683	2.472	2.521	1.545	1.550	1.590	1.465	1.494
Jul 07	0.37	1.6935	2.589	2.591	2.654	2.451	2.510	1.529	1.530	1.567	1.447	1.482
Aug 07	1.39	1.7171	2.604	2.566	2.552	2.432	2.504	1.517	1.494	1.486	1.416	1.458
Sep 07	1.17	1.7372	2.588	2.568	2.544	2.417	2.477	1.490	1.478	1.464	1.391	1.426
Oct 07	0.75	1.7502	2.577	2.555	2.601	2.416	2.482	1.472	1.460	1.486	1.380	1.418
Nov 07	1.05	1.7686	2.557	2.577	2.625	2.429	2.505	1.446	1.457	1.484	1.373	1.416
Dec 07	1.47	1.7946	2.601	2.608	2.660	2.444	2.522	1.449	1.453	1.482	1.362	1.405
Jan 08	0.99	1.8123	2.602	2.626	2.643	2.443	2.506	1.436	1.449	1.458	1.348	1.383
Feb 08	0.38	1.8192	2.585	2.629	2.647	2.437	2.444	1.421	1.445	1.455	1.340	1.343
Mar 08	0.70	1.8319	2.570	2.648	2.572	2.441	2.501	1.403	1.445	1.404	1.332	1.365
Apr 08	1.12	1.8525	2.584	2.637	2.571	2.440	2.497	1.395	1.424	1.388	1.317	1.348

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May 08	1.88	1.8873	2.590	2.634	2.560	2.444	2.492	1.372	1.396	1.356	1.295	1.320
Jun 08	1.89	1.9230	2.594	2.620	2.547	2.439	2.493	1.349	1.362	1.325	1.268	1.296
Jul 08	1.12	1.9445	2.611	2.628	2.549	2.437	2.511	1.343	1.352	1.311	1.253	1.291
Aug 08	-0.38	1.9371	2.626	2.641	2.556	2.443	2.504	1.356	1.363	1.319	1.261	1.293
Sep 08	0.36	1.9441	2.619	2.667	2.543	2.447	2.531	1.347	1.372	1.308	1.259	1.302
Oct 08	1.09	1.9653	2.600	2.664	2.560	2.453	2.530	1.323	1.356	1.303	1.248	1.287
Nov 08	0.07	1.9666	2.583	2.669	2.611	2.454	2.536	1.313	1.357	1.328	1.248	1.290
Dec 08	-0.44	1.9580	2.590	2.699	2.656	2.455	2.527	1.323	1.378	1.356	1.254	1.291
Jan 09	0.01	1.9582	2.573	2.705	2.664	2.453	2.527	1.314	1.381	1.360	1.253	1.290
Feb 09	-0.13	1.9556	2.603	2.698	2.651	2.453	2.522	1.331	1.380	1.356	1.254	1.290
Mar 09	-0.84	1.9392	2.604	2.703	2.651	2.450	2.507	1.343	1.394	1.367	1.263	1.293
Apr 09	0.04	1.9400	2.544	2.699	2.648	2.438	2.517	1.311	1.391	1.365	1.257	1.297
May 09	0.18	1.9435	2.527	2.673	2.634	2.432	2.490	1.300	1.375	1.355	1.251	1.281
Jun 09	-0.32	1.9373	2.573	2.687	2.639	2.421	2.487	1.328	1.387	1.362	1.250	1.284
Jul 09	-0.64	1.9249	2.597	2.663	2.634	2.419	2.504	1.349	1.383	1.368	1.257	1.301
Aug 09	0.09	1.9266	2.600	2.665	2.638	2.424	2.516	1.350	1.383	1.369	1.258	1.306
Sep 09	0.25	1.9314	2.588	2.662	2.628	2.418	2.493	1.340	1.378	1.361	1.252	1.291
Oct 09	-0.04	1.9306	2.597	2.683	2.653	2.465	2.552	1.345	1.390	1.374	1.277	1.322
Nov 09	0.07	1.9320	2.590	2.729	2.693	2.490	2.577	1.341	1.413	1.394	1.289	1.334
Dec 09	-0.11	1.9299	2.590	2.732	2.699	2.498	2.575	1.342	1.416	1.399	1.294	1.334
Jan 10	1.01	1.9494	2.613	2.750	2.709	2.534	2.608	1.340	1.411	1.390	1.300	1.338
Feb 10	1.09	1.9706	2.631	2.796	2.716	2.562	2.633	1.335	1.419	1.378	1.300	1.336
Mar 10	0.63	1.9830	2.608	2.739	2.681	2.535	2.583	1.315	1.381	1.352	1.278	1.303
Apr 10	0.72	1.9973	2.630	2.764	2.626	2.505	2.549	1.317	1.384	1.315	1.254	1.276
May 10	1.57	2.0287	2.656	2.759	2.615	2.492	2.549	1.309	1.360	1.289	1.228	1.256
Jun 10	0.34	2.0356	2.651	2.745	2.584	2.480	2.516	1.302	1.349	1.269	1.218	1.236
Jul 10	0.22	2.0400	2.652	2.720	2.587	2.484	2.509	1.300	1.333	1.268	1.218	1.230
Aug 10	1.10	2.0625	2.643	2.720	2.609	2.487	2.544	1.281	1.319	1.265	1.206	1.233
Sep 10	1.10	2.0852	2.606	2.719	2.624	2.490	2.566	1.250	1.304	1.258	1.194	1.231
Oct 10	1.03	2.1066	2.628	2.725	2.688	2.517	2.588	1.247	1.294	1.276	1.195	1.229
Nov 10	1.58	2.1399	2.654	2.704	2.731	2.532	2.602	1.240	1.264	1.276	1.183	1.216
Dec 10	0.38	2.1481	2.658	2.775	2.733	2.546	2.608	1.237	1.292	1.272	1.185	1.214
Jan 11	0.98	2.1691	2.665	2.777	2.744	2.560	2.610	1.229	1.280	1.265	1.180	1.203
Feb 11	0.96	2.1899	2.679	2.775	2.759	2.570	2.617	1.223	1.267	1.260	1.174	1.195
Mar 11	0.61	2.2033	2.706	2.791	2.863	2.623	2.644	1.228	1.267	1.299	1.190	1.200
Apr 11	0.50	2.2143	2.797	2.916	2.948	2.797	2.831	1.263	1.317	1.331	1.263	1.279
May 11	0.01	2.2145	2.785	2.920	2.906	2.842	2.829	1.258	1.319	1.312	1.283	1.277
Jun 11	-0.13	2.2116	2.693	2.805	2.768	2.743	2.725	1.218	1.268	1.252	1.240	1.232
Jul 11	-0.05	2.2105	2.703	2.792	2.804	2.736	2.704	1.223	1.263	1.268	1.238	1.223
Aug 11	0.61	2.2240	2.712	2.834	2.812	2.733	2.696	1.219	1.274	1.264	1.229	1.212
Sep 11	0.75	2.2407	2.691	2.879	2.831	2.735	2.718	1.201	1.285	1.263	1.221	1.213
Oct 11	0.40	2.2497	2.673	2.885	2.838	2.739	2.759	1.188	1.282	1.262	1.218	1.226
Nov 11	0.43	2.2593	2.661	2.883	2.848	2.731	2.762	1.178	1.276	1.261	1.209	1.222
Dec 11	-0.16	2.2557	2.698	2.880	2.848	2.732	2.757	1.196	1.277	1.263	1.211	1.222
Jan 12	0.30	2.2625	2.687	2.883	2.842	2.724	2.748	1.188	1.274	1.256	1.204	1.215
Feb 12	0.07	2.2641	2.688	2.880	2.832	2.717	2.728	1.187	1.272	1.251	1.200	1.205

Mar 12	0.56	2.2768	2.700	2.878	2.834	2.726	2.726	1.186	1.264	1.245	1.197	1.197
Apr 12	1.02	2.3000	2.705	2.887	2.817	2.729	2.719	1.176	1.255	1.225	1.187	1.182
May 12	0.91	2.3209	2.702	2.886	2.810	2.720	2.720	1.164	1.243	1.211	1.172	1.172
Jun 12	0.69	2.3369	2.687	2.885	2.816	2.717	2.707	1.150	1.235	1.205	1.163	1.158
Jul 12	1.52	2.3724	2.676	2.883	2.821	2.713	2.714	1.128	1.215	1.189	1.144	1.144
Aug 12	1.29	2.4030	2.679	2.881	2.812	2.709	2.706	1.115	1.199	1.170	1.127	1.126
Sep 12	0.88	2.4242	2.678	2.881	2.813	2.709	2.699	1.105	1.188	1.160	1.117	1.113
Oct 12	-0.31	2.4167	2.716	2.886	2.812	2.711	2.699	1.124	1.194	1.164	1.122	1.117
Nov 12	0.25	2.4227	2.743	2.891	2.805	2.713	2.760	1.132	1.193	1.158	1.120	1.139
Dec 12	0.66	2.4387	2.735	2.894	2.816	2.722	2.774	1.121	1.187	1.155	1.116	1.137
Jan 13	0.31	2.4463	2.746	2.898	2.834	2.732	2.777	1.123	1.185	1.158	1.117	1.135
Feb 13	0.20	2.4512	2.885	3.012	2.972	2.846	2.912	1.177	1.229	1.212	1.161	1.188
Mar 13	0.31	2.4588	2.876	3.012	2.976	2.849	2.903	1.170	1.225	1.210	1.159	1.181
Apr 13	-0.06	2.4573	2.866	3.011	2.968	2.846	2.869	1.166	1.225	1.208	1.158	1.168
May 13	0.32	2.4651	2.852	3.012	2.956	2.834	2.843	1.157	1.222	1.199	1.150	1.153
Jun 13	0.76	2.4839	2.858	3.012	2.953	2.817	2.812	1.151	1.213	1.189	1.134	1.132
Jul 13	0.14	2.4874	2.845	3.015	2.945	2.803	2.814	1.144	1.212	1.184	1.127	1.131
Aug 13	0.46	2.4988	2.842	3.012	2.941	2.795	2.821	1.137	1.205	1.177	1.119	1.129
Sep 13	1.36	2.5328	2.840	3.010	2.945	2.793	2.821	1.121	1.188	1.163	1.103	1.114
Oct 13	0.63	2.5487	2.804	3.007	2.973	2.796	2.830	1.100	1.180	1.166	1.097	1.110
Nov 13	0.28	2.5559	2.805	3.006	2.975	2.799	2.861	1.097	1.176	1.164	1.095	1.119
Dec 13	0.69	2.5735	2.928	3.087	3.074	2.900	2.971	1.138	1.200	1.194	1.127	1.154
Jan 14	0.40	2.5838	2.934	3.095	3.088	2.911	2.981	1.136	1.198	1.195	1.127	1.154
Feb 14	0.85	2.6058	2.936	3.105	3.085	2.911	2.974	1.127	1.192	1.184	1.117	1.141
Mar 14	1.48	2.6443	2.944	3.120	3.106	2.949	2.982	1.113	1.180	1.175	1.115	1.128
Apr 14	0.45	2.6562	2.960	3.123	3.130	2.955	2.982	1.114	1.176	1.178	1.112	1.123
May 14	-0.45	2.6443	2.951	3.125	3.118	2.947	2.964	1.116	1.182	1.179	1.114	1.121
Jun 14	-0.63	2.6276	2.955	3.138	3.107	2.933	2.931	1.125	1.194	1.182	1.116	1.115
Jul 14	-0.55	2.6132	2.933	3.150	3.087	2.930	2.914	1.122	1.205	1.181	1.121	1.115
Aug 14	0.06	2.6147	2.928	3.156	3.096	2.929	2.933	1.120	1.207	1.184	1.120	1.122
Sep 14	0.02	2.6153	2.977	3.157	3.065	2.925	2.927	1.138	1.207	1.172	1.118	1.119
Oct 14	0.59	2.6307	2.975	3.157	3.080	2.918	2.925	1.131	1.200	1.171	1.109	1.112
Nov 14	1.14	2.6607	3.043	3.203	3.137	2.961	2.970	1.144	1.204	1.179	1.113	1.116
Dec 14	0.38	2.6708	3.048	3.231	3.170	2.983	3.001	1.141	1.210	1.187	1.117	1.124

Source: ANP (2014)

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Ethanol price and deflation calculation by geographic region from July 2001 to December 2014

Month	IGP-DI (%)	Deflator	Price at the time					deflated price base on July 2001				
			NE	N	CO	SE	S	NE	N	CO	SE	S
Jul 01	1.62	1.0000	1.718	1.828	1.681	1.651	1.683	1.718	1.828	1.681	1.651	1.683
Aug 01	0.90	1.0090	1.741	1.875	1.721	1.683	1.722	1.725	1.858	1.705	1.668	1.707
Sep 01	0.38	1.0128	1.737	1.895	1.730	1.685	1.742	1.715	1.871	1.708	1.664	1.720
Oct 01	1.45	1.0275	1.808	1.954	1.802	1.738	1.811	1.760	1.902	1.754	1.692	1.762
Nov 01	0.76	1.0353	1.803	1.964	1.811	1.743	1.806	1.742	1.897	1.749	1.683	1.744
Dec 01	0.18	1.0372	1.807	1.959	1.803	1.738	1.791	1.742	1.888	1.739	1.676	1.727
Jan 02	0.19	1.0392	1.578	1.726	1.604	1.567	1.613	1.519	1.661	1.543	1.508	1.552
Feb 02	0.18	1.0410	1.508	1.616	1.525	1.490	1.531	1.448	1.552	1.465	1.432	1.470
Mar 02	0.11	1.0422	1.581	1.694	1.575	1.548	1.613	1.517	1.625	1.512	1.486	1.548
Apr 02	0.70	1.0495	1.706	1.819	1.707	1.693	1.756	1.625	1.733	1.627	1.613	1.674
May 02	1.11	1.0611	1.731	1.858	1.720	1.699	1.747	1.631	1.751	1.621	1.602	1.646
Jun 02	1.74	1.0796	1.719	1.845	1.739	1.683	1.738	1.593	1.709	1.611	1.559	1.610
Jul 02	2.05	1.1017	1.786	1.895	1.765	1.739	1.801	1.621	1.720	1.602	1.578	1.635
Aug 02	2.36	1.1277	1.795	1.892	1.775	1.713	1.783	1.592	1.678	1.574	1.519	1.581
Sep 02	2.64	1.1575	1.770	1.889	1.760	1.705	1.796	1.529	1.632	1.520	1.473	1.552
Oct 02	4.21	1.2062	1.784	1.864	1.802	1.722	1.817	1.479	1.545	1.494	1.428	1.506
Nov 02	5.84	1.2767	2.011	2.069	1.998	1.937	2.048	1.575	1.620	1.565	1.517	1.604
Dec 02	2.70	1.3111	2.035	2.099	2.008	1.957	2.083	1.552	1.601	1.532	1.493	1.589
Jan 03	2.17	1.3396	2.202	2.275	2.168	2.107	2.265	1.644	1.698	1.619	1.573	1.691
Feb 03	1.59	1.3609	2.232	2.327	2.274	2.167	2.352	1.640	1.710	1.671	1.592	1.729
Mar 03	1.66	1.3835	2.243	2.317	2.271	2.159	2.324	1.621	1.675	1.642	1.561	1.680
Apr 03	0.41	1.3891	2.201	2.299	2.254	2.144	2.305	1.584	1.655	1.622	1.544	1.659
May 03	-0.67	1.3798	2.139	2.235	2.185	2.062	2.195	1.550	1.620	1.584	1.494	1.590
Jun 03	-0.70	1.3702	2.061	2.205	2.070	1.975	2.096	1.504	1.609	1.511	1.441	1.530
Jul 03	-0.20	1.3674	2.001	2.157	2.007	1.925	2.029	1.463	1.577	1.467	1.408	1.484
Aug 03	0.62	1.3759	1.994	2.136	2.020	1.930	2.044	1.449	1.553	1.468	1.403	1.486
Sep 03	1.05	1.3904	2.021	2.138	2.048	1.958	2.081	1.454	1.538	1.473	1.408	1.497
Oct 03	0.44	1.3965	2.012	2.143	2.060	1.951	2.073	1.441	1.534	1.475	1.397	1.484
Nov 03	0.48	1.4032	2.026	2.157	2.055	1.946	2.049	1.444	1.537	1.465	1.386	1.460
Dec 03	0.60	1.4116	2.020	2.159	2.055	1.949	2.074	1.431	1.530	1.456	1.381	1.469
Jan 04	0.80	1.4229	2.031	2.149	2.070	1.955	2.094	1.427	1.510	1.454	1.374	1.472
Feb 04	1.08	1.4383	2.032	2.137	2.078	1.947	2.090	1.413	1.486	1.445	1.354	1.453
Mar 04	0.93	1.4516	2.031	2.129	2.040	1.925	2.056	1.399	1.466	1.405	1.326	1.416
Apr 04	1.15	1.4683	2.034	2.135	2.029	1.912	2.043	1.386	1.454	1.382	1.302	1.392
May 04	1.46	1.4898	2.042	2.208	2.135	1.985	2.058	1.371	1.482	1.433	1.332	1.381
Jun 04	1.29	1.5090	2.111	2.276	2.192	2.060	2.147	1.399	1.508	1.453	1.365	1.423
Jul 04	1.14	1.5262	2.163	2.332	2.225	2.107	2.188	1.417	1.528	1.458	1.381	1.434
Aug 04	1.31	1.5462	2.173	2.352	2.282	2.114	2.211	1.405	1.521	1.476	1.367	1.430
Sep 04	0.48	1.5536	2.173	2.366	2.291	2.126	2.205	1.399	1.523	1.475	1.368	1.419
Oct 04	0.53	1.5618	2.200	2.386	2.316	2.163	2.243	1.409	1.528	1.483	1.385	1.436
Nov 04	0.82	1.5747	2.255	2.379	2.315	2.141	2.278	1.432	1.511	1.470	1.360	1.447

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Dec 04	0.52	1.5828	2.361	2.478	2.407	2.222	2.374	1.492	1.566	1.521	1.404	1.500
Jan 05	0.33	1.5881	2.349	2.469	2.396	2.221	2.359	1.479	1.555	1.509	1.399	1.485
Feb 05	0.40	1.5944	2.311	2.466	2.401	2.217	2.340	1.449	1.547	1.506	1.390	1.468
Mar 05	0.99	1.6102	2.325	2.466	2.388	2.218	2.361	1.444	1.531	1.483	1.377	1.466
Apr 05	0.51	1.6184	2.323	2.465	2.392	2.217	2.419	1.435	1.523	1.478	1.370	1.495
May 05	-0.25	1.6144	2.327	2.465	2.354	2.202	2.386	1.441	1.527	1.458	1.364	1.478
Jun 05	-0.45	1.6071	2.316	2.458	2.336	2.175	2.350	1.441	1.529	1.454	1.353	1.462
Jul 05	-0.40	1.6007	2.320	2.452	2.362	2.180	2.381	1.449	1.532	1.476	1.362	1.487
Aug 05	-0.79	1.5880	2.318	2.450	2.381	2.186	2.385	1.460	1.543	1.499	1.377	1.502
Sep 05	-0.13	1.5860	2.476	2.575	2.451	2.315	2.518	1.561	1.624	1.545	1.460	1.588
Oct 05	0.63	1.5960	2.513	2.649	2.528	2.390	2.589	1.575	1.660	1.584	1.498	1.622
Nov 05	0.33	1.6012	2.530	2.701	2.568	2.390	2.587	1.580	1.687	1.604	1.493	1.616
Dec 05	0.07	1.6023	2.516	2.683	2.603	2.400	2.582	1.570	1.674	1.624	1.498	1.611
Jan 06	0.72	1.6139	2.546	2.667	2.639	2.438	2.585	1.578	1.653	1.635	1.511	1.602
Feb 06	-0.06	1.6129	2.603	2.662	2.659	2.441	2.590	1.614	1.650	1.649	1.513	1.606
Mar 06	-0.45	1.6057	2.672	2.689	2.659	2.520	2.652	1.664	1.675	1.656	1.569	1.652
Apr 06	0.02	1.6060	2.684	2.711	2.711	2.521	2.653	1.671	1.688	1.688	1.570	1.652
May 06	0.38	1.6121	2.681	2.702	2.673	2.504	2.651	1.663	1.676	1.658	1.553	1.644
Jun 06	0.67	1.6229	2.664	2.671	2.557	2.480	2.601	1.642	1.646	1.576	1.528	1.603
Jul 06	0.17	1.6256	2.668	2.673	2.638	2.485	2.608	1.641	1.644	1.623	1.529	1.604
Aug 06	0.41	1.6323	2.668	2.666	2.690	2.486	2.592	1.635	1.633	1.648	1.523	1.588
Sep 06	0.24	1.6362	2.685	2.665	2.682	2.480	2.574	1.641	1.629	1.639	1.516	1.573
Oct 06	0.81	1.6495	2.673	2.645	2.615	2.471	2.611	1.621	1.604	1.585	1.498	1.583
Nov 06	0.57	1.6589	2.633	2.621	2.676	2.457	2.609	1.587	1.580	1.613	1.481	1.573
Dec 06	0.26	1.6632	2.627	2.615	2.667	2.453	2.594	1.579	1.572	1.604	1.475	1.560
Jan 07	0.43	1.6703	2.635	2.598	2.583	2.461	2.567	1.578	1.555	1.546	1.473	1.537
Feb 07	0.23	1.6742	2.631	2.621	2.615	2.454	2.531	1.572	1.566	1.562	1.466	1.512
Mar 07	0.22	1.6779	2.645	2.614	2.602	2.459	2.525	1.576	1.558	1.551	1.466	1.505
Apr 07	0.14	1.6802	2.659	2.613	2.592	2.480	2.536	1.583	1.555	1.543	1.476	1.509
May 07	0.16	1.6829	2.642	2.638	2.678	2.491	2.506	1.570	1.568	1.591	1.480	1.489
Jun 07	0.26	1.6873	2.607	2.616	2.683	2.472	2.521	1.545	1.550	1.590	1.465	1.494
Jul 07	0.37	1.6935	2.589	2.591	2.654	2.451	2.510	1.529	1.530	1.567	1.447	1.482
Aug 07	1.39	1.7171	2.604	2.566	2.552	2.432	2.504	1.517	1.494	1.486	1.416	1.458
Sep 07	1.17	1.7372	2.588	2.568	2.544	2.417	2.477	1.490	1.478	1.464	1.391	1.426
Oct 07	0.75	1.7502	2.577	2.555	2.601	2.416	2.482	1.472	1.460	1.486	1.380	1.418
Nov 07	1.05	1.7686	2.557	2.577	2.625	2.429	2.505	1.446	1.457	1.484	1.373	1.416
Dec 07	1.47	1.7946	2.601	2.608	2.660	2.444	2.522	1.449	1.453	1.482	1.362	1.405
Jan 08	0.99	1.8123	2.602	2.626	2.643	2.443	2.506	1.436	1.449	1.458	1.348	1.383
Feb 08	0.38	1.8192	2.585	2.629	2.647	2.437	2.444	1.421	1.445	1.455	1.340	1.343
Mar 08	0.70	1.8319	2.570	2.648	2.572	2.441	2.501	1.403	1.445	1.404	1.332	1.365
Apr 08	1.12	1.8525	2.584	2.637	2.571	2.440	2.497	1.395	1.424	1.388	1.317	1.348
May 08	1.88	1.8873	2.590	2.634	2.560	2.444	2.492	1.372	1.396	1.356	1.295	1.320
Jun 08	1.89	1.9230	2.594	2.620	2.547	2.439	2.493	1.349	1.362	1.325	1.268	1.296
Jul 08	1.12	1.9445	2.611	2.628	2.549	2.437	2.511	1.343	1.352	1.311	1.253	1.291
Aug 08	-0.38	1.9371	2.626	2.641	2.556	2.443	2.504	1.356	1.363	1.319	1.261	1.293
Sep 08	0.36	1.9441	2.619	2.667	2.543	2.447	2.531	1.347	1.372	1.308	1.259	1.302

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Oct 08	1.09	1.9653	2.600	2.664	2.560	2.453	2.530	1.323	1.356	1.303	1.248	1.287
Nov 08	0.07	1.9666	2.583	2.669	2.611	2.454	2.536	1.313	1.357	1.328	1.248	1.290
Dec 08	-0.44	1.9580	2.590	2.699	2.656	2.455	2.527	1.323	1.378	1.356	1.254	1.291
Jan 09	0.01	1.9582	2.573	2.705	2.664	2.453	2.527	1.314	1.381	1.360	1.253	1.290
Feb 09	-0.13	1.9556	2.603	2.698	2.651	2.453	2.522	1.331	1.380	1.356	1.254	1.290
Mar 09	-0.84	1.9392	2.604	2.703	2.651	2.450	2.507	1.343	1.394	1.367	1.263	1.293
Apr 09	0.04	1.9400	2.544	2.699	2.648	2.438	2.517	1.311	1.391	1.365	1.257	1.297
May 09	0.18	1.9435	2.527	2.673	2.634	2.432	2.490	1.300	1.375	1.355	1.251	1.281
Jun 09	-0.32	1.9373	2.573	2.687	2.639	2.421	2.487	1.328	1.387	1.362	1.250	1.284
Jul 09	-0.64	1.9249	2.597	2.663	2.634	2.419	2.504	1.349	1.383	1.368	1.257	1.301
Aug 09	0.09	1.9266	2.600	2.665	2.638	2.424	2.516	1.350	1.383	1.369	1.258	1.306
Sep 09	0.25	1.9314	2.588	2.662	2.628	2.418	2.493	1.340	1.378	1.361	1.252	1.291
Oct 09	-0.04	1.9306	2.597	2.683	2.653	2.465	2.552	1.345	1.390	1.374	1.277	1.322
Nov 09	0.07	1.9320	2.590	2.729	2.693	2.490	2.577	1.341	1.413	1.394	1.289	1.334
Dec 09	-0.11	1.9299	2.590	2.732	2.699	2.498	2.575	1.342	1.416	1.399	1.294	1.334
Jan 10	1.01	1.9494	2.613	2.750	2.709	2.534	2.608	1.340	1.411	1.390	1.300	1.338
Feb 10	1.09	1.9706	2.631	2.796	2.716	2.562	2.633	1.335	1.419	1.378	1.300	1.336
Mar 10	0.63	1.9830	2.608	2.739	2.681	2.535	2.583	1.315	1.381	1.352	1.278	1.303
Apr 10	0.72	1.9973	2.630	2.764	2.626	2.505	2.549	1.317	1.384	1.315	1.254	1.276
May 10	1.57	2.0287	2.656	2.759	2.615	2.492	2.549	1.309	1.360	1.289	1.228	1.256
Jun 10	0.34	2.0356	2.651	2.745	2.584	2.480	2.516	1.302	1.349	1.269	1.218	1.236
Jul 10	0.22	2.0400	2.652	2.720	2.587	2.484	2.509	1.300	1.333	1.268	1.218	1.230
Aug 10	1.10	2.0625	2.643	2.720	2.609	2.487	2.544	1.281	1.319	1.265	1.206	1.233
Sep 10	1.10	2.0852	2.606	2.719	2.624	2.490	2.566	1.250	1.304	1.258	1.194	1.231
Oct 10	1.03	2.1066	2.628	2.725	2.688	2.517	2.588	1.247	1.294	1.276	1.195	1.229
Nov 10	1.58	2.1399	2.654	2.704	2.731	2.532	2.602	1.240	1.264	1.276	1.183	1.216
Dec 10	0.38	2.1481	2.658	2.775	2.733	2.546	2.608	1.237	1.292	1.272	1.185	1.214
Jan 11	0.98	2.1691	2.665	2.777	2.744	2.560	2.610	1.229	1.280	1.265	1.180	1.203
Feb 11	0.96	2.1899	2.679	2.775	2.759	2.570	2.617	1.223	1.267	1.260	1.174	1.195
Mar 11	0.61	2.2033	2.706	2.791	2.863	2.623	2.644	1.228	1.267	1.299	1.190	1.200
Apr 11	0.50	2.2143	2.797	2.916	2.948	2.797	2.831	1.263	1.317	1.331	1.263	1.279
May 11	0.01	2.2145	2.785	2.920	2.906	2.842	2.829	1.258	1.319	1.312	1.283	1.277
Jun 11	-0.13	2.2116	2.693	2.805	2.768	2.743	2.725	1.218	1.268	1.252	1.240	1.232
Jul 11	-0.05	2.2105	2.703	2.792	2.804	2.736	2.704	1.223	1.263	1.268	1.238	1.223
Aug 11	0.61	2.2240	2.712	2.834	2.812	2.733	2.696	1.219	1.274	1.264	1.229	1.212
Sep 11	0.75	2.2407	2.691	2.879	2.831	2.735	2.718	1.201	1.285	1.263	1.221	1.213
Oct 11	0.40	2.2497	2.673	2.885	2.838	2.739	2.759	1.188	1.282	1.262	1.218	1.226
Nov 11	0.43	2.2593	2.661	2.883	2.848	2.731	2.762	1.178	1.276	1.261	1.209	1.222
Dec 11	-0.16	2.2557	2.698	2.880	2.848	2.732	2.757	1.196	1.277	1.263	1.211	1.222
Jan 12	0.30	2.2625	2.687	2.883	2.842	2.724	2.748	1.188	1.274	1.256	1.204	1.215
Feb 12	0.07	2.2641	2.688	2.880	2.832	2.717	2.728	1.187	1.272	1.251	1.200	1.205
Mar 12	0.56	2.2768	2.700	2.878	2.834	2.726	2.726	1.186	1.264	1.245	1.197	1.197
Apr 12	1.02	2.3000	2.705	2.887	2.817	2.729	2.719	1.176	1.255	1.225	1.187	1.182
May 12	0.91	2.3209	2.702	2.886	2.810	2.720	2.720	1.164	1.243	1.211	1.172	1.172
Jun 12	0.69	2.3369	2.687	2.885	2.816	2.717	2.707	1.150	1.235	1.205	1.163	1.158
Jul 12	1.52	2.3724	2.676	2.883	2.821	2.713	2.714	1.128	1.215	1.189	1.144	1.144

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Aug 12	1.29	2.4030	2.679	2.881	2.812	2.709	2.706	1.115	1.199	1.170	1.127	1.126
Sep 12	0.88	2.4242	2.678	2.881	2.813	2.709	2.699	1.105	1.188	1.160	1.117	1.113
Oct 12	-0.31	2.4167	2.716	2.886	2.812	2.711	2.699	1.124	1.194	1.164	1.122	1.117
Nov 12	0.25	2.4227	2.743	2.891	2.805	2.713	2.760	1.132	1.193	1.158	1.120	1.139
Dec 12	0.66	2.4387	2.735	2.894	2.816	2.722	2.774	1.121	1.187	1.155	1.116	1.137
Jan 13	0.31	2.4463	2.746	2.898	2.834	2.732	2.777	1.123	1.185	1.158	1.117	1.135
Feb 13	0.20	2.4512	2.885	3.012	2.972	2.846	2.912	1.177	1.229	1.212	1.161	1.188
Mar 13	0.31	2.4588	2.876	3.012	2.976	2.849	2.903	1.170	1.225	1.210	1.159	1.181
Apr 13	-0.06	2.4573	2.866	3.011	2.968	2.846	2.869	1.166	1.225	1.208	1.158	1.168
May 13	0.32	2.4651	2.852	3.012	2.956	2.834	2.843	1.157	1.222	1.199	1.150	1.153
Jun 13	0.76	2.4839	2.858	3.012	2.953	2.817	2.812	1.151	1.213	1.189	1.134	1.132
Jul 13	0.14	2.4874	2.845	3.015	2.945	2.803	2.814	1.144	1.212	1.184	1.127	1.131
Aug 13	0.46	2.4988	2.842	3.012	2.941	2.795	2.821	1.137	1.205	1.177	1.119	1.129
Sep 13	1.36	2.5328	2.840	3.010	2.945	2.793	2.821	1.121	1.188	1.163	1.103	1.114
Oct 13	0.63	2.5487	2.804	3.007	2.973	2.796	2.830	1.100	1.180	1.166	1.097	1.110
Nov 13	0.28	2.5559	2.805	3.006	2.975	2.799	2.861	1.097	1.176	1.164	1.095	1.119
Dec 13	0.69	2.5735	2.928	3.087	3.074	2.900	2.971	1.138	1.200	1.194	1.127	1.154
Jan 14	0.40	2.5838	2.934	3.095	3.088	2.911	2.981	1.136	1.198	1.195	1.127	1.154
Feb 14	0.85	2.6058	2.936	3.105	3.085	2.911	2.974	1.127	1.192	1.184	1.117	1.141
Mar 14	1.48	2.6443	2.944	3.120	3.106	2.949	2.982	1.113	1.180	1.175	1.115	1.128
Apr 14	0.45	2.6562	2.960	3.123	3.130	2.955	2.982	1.114	1.176	1.178	1.112	1.123
May 14	-0.45	2.6443	2.951	3.125	3.118	2.947	2.964	1.116	1.182	1.179	1.114	1.121
Jun 14	-0.63	2.6276	2.955	3.138	3.107	2.933	2.931	1.125	1.194	1.182	1.116	1.115
Jul 14	-0.55	2.6132	2.933	3.150	3.087	2.930	2.914	1.122	1.205	1.181	1.121	1.115
Aug 14	0.06	2.6147	2.928	3.156	3.096	2.929	2.933	1.120	1.207	1.184	1.120	1.122
Sep 14	0.02	2.6153	2.977	3.157	3.065	2.925	2.927	1.138	1.207	1.172	1.118	1.119
Oct 14	0.59	2.6307	2.975	3.157	3.080	2.918	2.925	1.131	1.200	1.171	1.109	1.112
Nov 14	1.14	2.6607	3.043	3.203	3.137	2.961	2.970	1.144	1.204	1.179	1.113	1.116
Dec 14	0.38	2.6708	3.048	3.231	3.170	2.983	3.001	1.141	1.210	1.187	1.117	1.124

Source: ANP (2014)

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Appendix B - Linear Regression Results

log-normal of Ethanol Deflated Price

Northeastern region

Regression Statistics	
Multiple R	0.163653
R Square	0.026782
Adjusted R Square	0.020662
Standard Error	0.027062
Observations	161

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.003204	0.003204	4.375597	0.038046
Residual	159	0.116441	0.000732		
Total	160	0.119645			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-0.00191	0.002158	-0.88552	0.377216
ln(DP)	-0.04866	0.023261	-2.09179	0.038046

Northern region

Regression Statistics	
Multiple R	0.130951
R Square	0.017148
Adjusted R Square	0.010967
Standard Error	0.026102
Observations	161

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.00189	0.00189	2.77412	0.097768375
Residual	159	0.108331	0.000681		
Total	160	0.110221			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.000883	0.002547	0.346848	0.729164
ln(DP)	-0.03094	0.018577	-1.66557	0.097768

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Central-Western region

Regression Statistics	
Multiple R	0.2027
R Square	0.041087
Adjusted R Square	0.035056
Standard Error	0.048162
Observations	161

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.015803	0.015803	6.812799	0.009915523
Residual	159	0.36882	0.00232		
Total	160	0.384623			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-0.00789	0.004446	-1.77453	0.077889
ln(DP)	-0.0807	0.030918	-2.61013	0.009916

Southeastern region

Regression Statistics	
Multiple R	0.23081
R Square	0.053273
Adjusted R Square	0.047319
Standard Error	0.052613
Observations	161

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.024767	0.024767	8.947074	0.003222154
Residual	159	0.44014	0.002768		
Total	160	0.464908			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-0.02389	0.008569	-2.78782	0.005954
ln(DP)	-0.103	0.034434	-2.99117	0.003222

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Southern region

Regression Statistics	
Multiple R	0.199023
R Square	0.03961
Adjusted R Square	0.03357
Standard Error	0.049307
Observations	161

ANOVA

	df	SS	MS	F	Significance F
Regression	1	0.015943	0.015943	6.55776	0.011373371
Residual	159	0.38655	0.002431		
Total	160	0.402493			

	Coefficients	Standard Error	t Stat	P-value
Intercept	-0.00966	0.004983	-1.93821	0.054369
ln(DP)	-0.0812	0.03171	-2.56081	0.011373

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log-normal of Gasoline Deflated Price

Northeastern region

Regression Statistics	
Multiple R	0.100296
R Square	0.010059
Adjusted R Square	0.003833
Standard Error	0.020576
Observations	161

ANOVA

	df	SS	MS	F	Significance F
Regression	1	0.000684	0.000684	1.615686	0.205550844
Residual	159	0.067319	0.000423		
Total	160	0.068003			

	Coefficients	Standard Error	t Stat	P-value
Intercept	0.002373	0.004191	0.566087	0.572133
ln(DP)	-0.01564	0.012306	-1.2711	0.205551

Northern region

Regression Statistics	
Multiple R	0.117899488
R Square	0.013900289
Adjusted R Square	0.007698404
Standard Error	0.019431152
Observations	161

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.000846	0.000846	2.241301	0.136350388
Residual	159	0.060034	0.000378		
Total	160	0.06088			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.003949318	0.004613	0.85618	0.393187
ln(DP)	-0.018147681	0.012122	-1.4971	0.13635

Central-Western region

Regression Statistics	
Multiple R	0.100329
R Square	0.010066
Adjusted R Square	0.00384
Standard Error	0.020914
Observations	161

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.000707	0.000707	1.616738	0.205403968
Residual	159	0.069548	0.000437		
Total	160	0.070255			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.003785	0.00496	0.763147	0.446506
ln(DP)	-0.01783	0.014025	-1.27151	0.205404

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Southeastern region

Regression Statistics	
Multiple R	0.113309
R Square	0.012839
Adjusted R Square	0.00663
Standard Error	0.019424
Observations	161

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.00078	0.00078	2.06795	0.152388847
Residual	159	0.059988	0.000377		
Total	160	0.060768			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.002826	0.00396	0.713576	0.476535
ln(DP)	-0.01884	0.013099	-1.43804	0.152389

Southern region

Regression Statistics	
Multiple R	0.085955
R Square	0.007388
Adjusted R Square	0.001145
Standard Error	0.020811
Observations	161

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.000513	0.000513	1.183479	0.278295198
Residual	159	0.068864	0.000433		
Total	160	0.069377			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.001644	0.004156	0.395585	0.692942
ln(DP)	-0.01337	0.012286	-1.08788	0.278295

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Appendix C - Cash Flows

VW GOL / Gasoline - regular cash flow by geographic region

Month	NE		N		CW		SE		S	
	Simulated price	Cash flow	Simulated price	Cash flow	Simulated price	Cash flow	Simulated price	Cash flow	Simulated price	Cash flow
1	2.99	354.71	3.16	375.16	3.10	368.28	2.92	346.22	2.95	349.95
2	2.93	347.75	3.09	367.01	3.04	360.45	2.85	338.56	2.90	343.75
3	2.87	340.96	3.02	359.11	2.97	352.86	2.79	331.15	2.84	337.67
4	2.82	334.35	2.96	351.46	2.91	345.48	2.73	323.98	2.79	331.70
5	2.76	327.90	2.90	344.05	2.85	338.33	2.67	317.05	2.74	325.86
6	2.71	321.61	2.84	336.88	2.79	331.38	2.61	310.33	2.70	320.13
7	2.66	315.48	2.78	329.93	2.73	324.64	2.56	303.85	2.65	314.52
8	2.61	309.52	2.72	323.20	2.68	318.10	2.51	297.57	2.60	309.02
9	2.56	303.69	2.67	316.68	2.63	311.75	2.46	291.49	2.56	303.64
10	2.51	298.03	2.61	310.36	2.57	305.59	2.41	285.60	2.51	298.38
11	2.46	292.50	2.56	304.23	2.52	299.61	2.36	279.91	2.47	293.22
12	2.42	287.12	2.51	298.30	2.47	293.80	2.31	274.40	2.43	288.17
13	2.37	281.88	2.46	292.56	2.43	288.16	2.27	269.06	2.39	283.22
14	2.33	276.78	2.42	286.98	2.38	282.69	2.22	263.90	2.34	278.39
15	2.29	271.80	2.37	281.59	2.34	277.38	2.18	258.90	2.31	273.65
16	2.25	266.96	2.33	276.35	2.29	272.23	2.14	254.06	2.27	269.03
17	2.21	262.25	2.29	271.28	2.25	267.23	2.10	249.37	2.23	264.51
18	2.17	257.66	2.24	266.36	2.21	262.38	2.06	244.84	2.19	260.08
19	2.13	253.18	2.20	261.59	2.17	257.67	2.03	240.44	2.15	255.75
20	2.10	248.82	2.16	256.97	2.13	253.10	1.99	236.18	2.12	251.51
21	2.06	244.57	2.13	252.49	2.09	248.67	1.95	232.05	2.08	247.37
22	2.03	240.44	2.09	248.14	2.06	244.36	1.92	228.06	2.05	243.32
23	1.99	236.41	2.05	243.92	2.02	240.17	1.89	224.18	2.02	239.36
24	1.96	232.49	2.02	239.83	1.99	236.11	1.86	220.42	1.98	235.49
25	1.93	228.67	1.99	235.87	1.96	232.16	1.83	216.79	1.95	231.70
26	1.89	224.94	1.95	232.02	1.92	228.34	1.80	213.26	1.92	228.00
27	1.86	221.32	1.92	228.29	1.89	224.61	1.77	209.84	1.89	224.39
28	1.83	217.80	1.89	224.66	1.86	221.00	1.74	206.52	1.86	220.86
29	1.81	214.36	1.86	221.14	1.83	217.49	1.71	203.30	1.83	217.39
30	1.78	211.00	1.83	217.72	1.80	214.08	1.69	200.17	1.80	214.01
31	1.75	207.74	1.81	214.40	1.78	210.77	1.66	197.14	1.77	210.70
32	1.72	204.56	1.78	211.18	1.75	207.55	1.64	194.21	1.75	207.47
33	1.70	201.46	1.75	208.06	1.72	204.42	1.61	191.36	1.72	204.31
34	1.67	198.45	1.73	205.03	1.70	201.38	1.59	188.60	1.70	201.23
35	1.65	195.51	1.70	202.08	1.67	198.42	1.57	185.92	1.67	198.22
36	1.62	192.64	1.68	199.22	1.65	195.55	1.54	183.31	1.64	195.27
37	1.60	189.85	1.65	196.44	1.62	192.76	1.52	180.78	1.62	192.40

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38	1.58	187.12	1.63	193.73	1.60	190.05	1.50	178.31	1.60	189.58
39	1.55	184.45	1.61	191.10	1.58	187.40	1.48	175.92	1.57	186.83
40	1.53	181.86	1.59	188.53	1.56	184.83	1.46	173.61	1.55	184.13
41	1.51	179.34	1.57	186.06	1.54	182.34	1.44	171.36	1.53	181.50
42	1.49	176.88	1.55	183.64	1.52	179.91	1.43	169.17	1.51	178.93
43	1.47	174.48	1.53	181.30	1.50	177.55	1.41	167.04	1.49	176.41
44	1.45	172.15	1.51	179.02	1.48	175.25	1.39	164.98	1.47	173.96
45	1.43	169.87	1.49	176.80	1.46	173.01	1.37	162.97	1.45	171.55
46	1.41	167.64	1.47	174.64	1.44	170.84	1.36	161.01	1.43	169.21
47	1.39	165.47	1.45	172.53	1.42	168.72	1.34	159.11	1.41	166.91
48	1.38	163.36	1.44	170.49	1.40	166.66	1.32	157.27	1.39	164.67
49	1.36	161.29	1.42	168.50	1.39	164.65	1.31	155.47	1.37	162.47
50	1.34	159.28	1.40	166.57	1.37	162.70	1.29	153.72	1.35	160.32
51	1.33	157.31	1.39	164.68	1.35	160.80	1.28	152.02	1.33	158.22
52	1.31	155.40	1.37	162.85	1.34	158.95	1.27	150.36	1.32	156.16
53	1.29	153.52	1.36	161.07	1.32	157.13	1.25	148.76	1.30	154.16
54	1.28	151.70	1.34	159.33	1.31	155.37	1.24	147.19	1.28	152.19
55	1.26	149.92	1.33	157.63	1.29	153.66	1.23	145.67	1.27	150.27
56	1.25	148.18	1.31	155.99	1.28	151.99	1.21	144.19	1.25	148.39
57	1.23	146.48	1.30	154.38	1.27	150.37	1.20	142.75	1.23	146.55
58	1.22	144.82	1.29	152.81	1.25	148.79	1.19	141.34	1.22	144.76
59	1.21	143.21	1.27	151.29	1.24	147.24	1.18	139.97	1.20	142.99
60	1.19	141.63	1.26	149.80	1.23	145.74	1.17	138.64	1.19	141.27

VW GOL / Ethanol - regular cash flow by geographic region

Month	NE		N		CW		SE		S	
	Simulated price	Cash flow	Simulated price	Cash flow	Simulated price	Cash flow	Simulated price	Cash flow	Simulated price	Cash flow
1	2.34	409.81	2.51	440.57	2.00	351.00	1.79	313.43	1.93	337.84
2	2.23	390.36	2.43	426.24	1.87	327.12	1.63	286.29	1.80	315.14
3	2.13	372.60	2.35	412.69	1.75	306.34	1.50	263.71	1.68	295.36
4	2.03	356.35	2.28	399.85	1.64	288.20	1.40	244.79	1.59	278.08
5	1.95	341.47	2.21	387.68	1.55	272.31	1.31	228.84	1.50	262.93
6	1.87	327.82	2.15	376.15	1.47	258.35	1.23	215.32	1.42	249.60
7	1.80	315.28	2.08	365.22	1.40	246.03	1.16	203.80	1.36	237.85
8	1.73	303.73	2.02	354.85	1.34	235.15	1.11	193.92	1.30	227.45
9	1.67	293.09	1.97	345.01	1.29	225.50	1.06	185.42	1.24	218.23
10	1.62	283.27	1.91	335.67	1.24	216.92	1.02	178.08	1.20	210.03
11	1.56	274.20	1.86	326.80	1.19	209.27	0.98	171.72	1.16	202.72
12	1.52	265.80	1.82	318.38	1.15	202.44	0.95	166.18	1.12	196.19
13	1.47	258.03	1.77	310.37	1.12	196.32	0.92	161.35	1.09	190.34
14	1.43	250.82	1.73	302.76	1.09	190.84	0.90	157.12	1.06	185.10
15	1.39	244.12	1.69	295.52	1.06	185.91	0.87	153.42	1.03	180.39
16	1.36	237.90	1.65	288.63	1.04	181.48	0.86	150.16	1.00	176.16

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17	1.32	232.11	1.61	282.07	1.01	177.48	0.84	147.29	0.98	172.34
18	1.29	226.71	1.57	275.82	0.99	173.88	0.83	144.76	0.96	168.89
19	1.26	221.69	1.54	269.87	0.97	170.62	0.81	142.52	0.95	165.77
20	1.24	216.99	1.51	264.19	0.96	167.67	0.80	140.53	0.93	162.95
21	1.21	212.61	1.48	258.79	0.94	164.99	0.79	138.78	0.91	160.39
22	1.19	208.51	1.45	253.63	0.93	162.56	0.78	137.22	0.90	158.07
23	1.17	204.68	1.42	248.71	0.91	160.36	0.77	135.83	0.89	155.97
24	1.15	201.09	1.39	244.01	0.90	158.35	0.77	134.60	0.88	154.05
25	1.13	197.73	1.37	239.52	0.89	156.53	0.76	133.50	0.87	152.31
26	1.11	194.58	1.34	235.24	0.88	154.87	0.76	132.53	0.86	150.72
27	1.09	191.62	1.32	231.14	0.87	153.35	0.75	131.65	0.85	149.28
28	1.08	188.85	1.30	227.23	0.87	151.97	0.75	130.88	0.84	147.96
29	1.06	186.24	1.27	223.48	0.86	150.71	0.74	130.18	0.84	146.76
30	1.05	183.79	1.25	219.90	0.85	149.57	0.74	129.56	0.83	145.66
31	1.04	181.48	1.23	216.47	0.85	148.52	0.74	129.01	0.83	144.65
32	1.02	179.31	1.22	213.19	0.84	147.56	0.73	128.52	0.82	143.74
33	1.01	177.27	1.20	210.05	0.84	146.68	0.73	128.08	0.82	142.90
34	1.00	175.35	1.18	207.05	0.83	145.87	0.73	127.68	0.81	142.14
35	0.99	173.53	1.16	204.17	0.83	145.14	0.73	127.33	0.81	141.44
36	0.98	171.83	1.15	201.40	0.82	144.47	0.72	127.01	0.80	140.81
37	0.97	170.21	1.13	198.76	0.82	143.85	0.72	126.73	0.80	140.22
38	0.96	168.69	1.12	196.21	0.82	143.28	0.72	126.47	0.80	139.68
39	0.95	167.25	1.11	193.78	0.81	142.77	0.72	126.25	0.79	139.19
40	0.95	165.90	1.09	191.44	0.81	142.29	0.72	126.04	0.79	138.74
41	0.94	164.61	1.08	189.20	0.81	141.85	0.72	125.86	0.79	138.33
42	0.93	163.40	1.07	187.04	0.81	141.46	0.72	125.70	0.79	137.95
43	0.93	162.26	1.05	184.97	0.80	141.09	0.72	125.55	0.78	137.60
44	0.92	161.18	1.04	182.99	0.80	140.76	0.72	125.42	0.78	137.28
45	0.91	160.16	1.03	181.08	0.80	140.45	0.71	125.30	0.78	136.99
46	0.91	159.19	1.02	179.24	0.80	140.17	0.71	125.19	0.78	136.73
47	0.90	158.27	1.01	177.48	0.80	139.91	0.71	125.10	0.78	136.48
48	0.90	157.40	1.00	175.78	0.80	139.67	0.71	125.01	0.78	136.26
49	0.89	156.58	0.99	174.15	0.80	139.45	0.71	124.94	0.78	136.05
50	0.89	155.80	0.98	172.59	0.79	139.25	0.71	124.87	0.77	135.86
51	0.88	155.07	0.98	171.08	0.79	139.07	0.71	124.81	0.77	135.68
52	0.88	154.37	0.97	169.63	0.79	138.90	0.71	124.75	0.77	135.53
53	0.88	153.71	0.96	168.23	0.79	138.75	0.71	124.70	0.77	135.38
54	0.87	153.08	0.95	166.88	0.79	138.60	0.71	124.66	0.77	135.25
55	0.87	152.49	0.94	165.59	0.79	138.47	0.71	124.62	0.77	135.12
56	0.87	151.92	0.94	164.34	0.79	138.35	0.71	124.59	0.77	135.01
57	0.86	151.39	0.93	163.14	0.79	138.24	0.71	124.56	0.77	134.91
58	0.86	150.88	0.92	161.98	0.79	138.14	0.71	124.52	0.77	134.81
59	0.86	150.40	0.92	160.86	0.79	138.05	0.71	124.50	0.77	134.72
60	0.86	149.95	0.91	159.79	0.79	137.96	0.71	124.48	0.77	134.64

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Fiat UNO / Gasoline - regular cash flow by geographic region

Month	NE		N		CW		SE		S	
	Simulated price	Cash flow	Simulated price	Cash flow	Simulated price	Cash flow	Simulated price	Cash flow	Simulated price	Cash flow
1	2.99	338.51	3.16	358.02	3.10	351.45	2.92	330.40	2.95	333.96
2	2.93	331.86	3.09	350.24	3.04	343.98	2.85	323.09	2.90	328.04
3	2.87	325.38	3.02	342.70	2.97	336.73	2.79	316.02	2.84	322.24
4	2.82	319.07	2.96	335.40	2.91	329.70	2.73	309.18	2.79	316.54
5	2.76	312.91	2.90	328.33	2.85	322.87	2.67	302.56	2.74	310.97
6	2.71	306.91	2.84	321.48	2.79	316.24	2.61	296.15	2.70	305.50
7	2.66	301.07	2.78	314.85	2.73	309.81	2.56	289.96	2.65	300.14
8	2.61	295.37	2.72	308.43	2.68	303.57	2.51	283.97	2.60	294.90
9	2.56	289.82	2.67	302.20	2.63	297.51	2.46	278.17	2.56	289.77
10	2.51	284.41	2.61	296.17	2.57	291.63	2.41	272.55	2.51	284.74
11	2.46	279.14	2.56	290.33	2.52	285.92	2.36	267.12	2.47	279.82
12	2.42	274.00	2.51	284.67	2.47	280.38	2.31	261.86	2.43	275.00
13	2.37	269.00	2.46	279.19	2.43	274.99	2.27	256.77	2.39	270.28
14	2.33	264.13	2.42	273.87	2.38	269.78	2.22	251.84	2.34	265.67
15	2.29	259.38	2.37	268.72	2.34	264.70	2.18	247.07	2.31	261.15
16	2.25	254.76	2.33	263.72	2.29	259.79	2.14	242.46	2.27	256.74
17	2.21	250.27	2.29	258.89	2.25	255.02	2.10	237.98	2.23	252.43
18	2.17	245.89	2.24	254.19	2.21	250.39	2.06	233.65	2.19	248.19
19	2.13	241.61	2.20	249.64	2.17	245.90	2.03	229.45	2.15	244.06
20	2.10	237.45	2.16	245.23	2.13	241.54	1.99	225.39	2.12	240.02
21	2.06	233.39	2.13	240.95	2.09	237.30	1.95	221.45	2.08	236.07
22	2.03	229.45	2.09	236.80	2.06	233.19	1.92	217.64	2.05	232.20
23	1.99	225.61	2.05	232.78	2.02	229.20	1.89	213.93	2.02	228.42
24	1.96	221.86	2.02	228.87	1.99	225.32	1.86	210.35	1.98	224.73
25	1.93	218.22	1.99	225.09	1.96	221.56	1.83	206.88	1.95	221.11
26	1.89	214.66	1.95	221.42	1.92	217.90	1.80	203.51	1.92	217.58
27	1.86	211.21	1.92	217.86	1.89	214.35	1.77	200.25	1.89	214.13
28	1.83	207.84	1.89	214.39	1.86	210.90	1.74	197.08	1.86	210.76
29	1.81	204.56	1.86	211.03	1.83	207.55	1.71	194.01	1.83	207.46
30	1.78	201.36	1.83	207.77	1.80	204.29	1.69	191.03	1.80	204.23
31	1.75	198.25	1.81	204.60	1.78	201.13	1.66	188.13	1.77	201.07
32	1.72	195.21	1.78	201.53	1.75	198.06	1.64	185.34	1.75	197.99
33	1.70	192.26	1.75	198.55	1.72	195.08	1.61	182.62	1.72	194.98
34	1.67	189.38	1.73	195.66	1.70	192.18	1.59	179.98	1.70	192.04
35	1.65	186.57	1.70	192.85	1.67	189.35	1.57	177.42	1.67	189.16
36	1.62	183.84	1.68	190.12	1.65	186.62	1.54	174.93	1.64	186.35
37	1.60	181.17	1.65	187.46	1.62	183.95	1.52	172.52	1.62	183.61
38	1.58	178.57	1.63	184.88	1.60	181.36	1.50	170.17	1.60	180.92
39	1.55	176.03	1.61	182.36	1.58	178.84	1.48	167.89	1.57	178.29
40	1.53	173.55	1.59	179.92	1.56	176.39	1.46	165.68	1.55	175.72

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41	1.51	171.14	1.57	177.56	1.54	174.00	1.44	163.53	1.53	173.20
42	1.49	168.80	1.55	175.25	1.52	171.69	1.43	161.44	1.51	170.75
43	1.47	166.51	1.53	173.01	1.50	169.44	1.41	159.41	1.49	168.35
44	1.45	164.28	1.51	170.84	1.48	167.24	1.39	157.44	1.47	166.01
45	1.43	162.11	1.49	168.72	1.46	165.11	1.37	155.52	1.45	163.71
46	1.41	159.98	1.47	166.66	1.44	163.03	1.36	153.66	1.43	161.48
47	1.39	157.91	1.45	164.64	1.42	161.01	1.34	151.84	1.41	159.29
48	1.38	155.89	1.44	162.70	1.40	159.05	1.32	150.08	1.39	157.14
49	1.36	153.92	1.42	160.80	1.39	157.13	1.31	148.36	1.37	155.05
50	1.34	152.00	1.40	158.96	1.37	155.26	1.29	146.69	1.35	152.99
51	1.33	150.13	1.39	157.16	1.35	153.45	1.28	145.07	1.33	150.99
52	1.31	148.30	1.37	155.41	1.34	151.68	1.27	143.49	1.32	149.03
53	1.29	146.51	1.36	153.71	1.32	149.95	1.25	141.96	1.30	147.11
54	1.28	144.77	1.34	152.05	1.31	148.27	1.24	140.47	1.28	145.23
55	1.26	143.07	1.33	150.43	1.29	146.64	1.23	139.02	1.27	143.40
56	1.25	141.41	1.31	148.86	1.28	145.05	1.21	137.60	1.25	141.61
57	1.23	139.79	1.30	147.33	1.27	143.50	1.20	136.22	1.23	139.86
58	1.22	138.21	1.29	145.83	1.25	141.99	1.19	134.88	1.22	138.14
59	1.21	136.66	1.27	144.37	1.24	140.51	1.18	133.57	1.20	136.46
60	1.19	135.16	1.26	142.96	1.23	139.08	1.17	132.30	1.19	134.82

PUC-Rio - Certificação Digital Nº 1312423/CA

Fiat UNO / Ethanol - regular cash flow by geographic region

Month	NE		N		CW		SE		S	
	Simulated price	Cash flow	Simulated price	Cash flow	Simulated price	Cash flow	Simulated price	Cash flow	Simulated price	Cash flow
1	2.34	381.49	2.51	410.12	2.00	326.75	1.79	291.77	1.93	314.50
2	2.23	363.39	2.43	396.79	1.87	304.52	1.63	266.51	1.80	293.36
3	2.13	346.85	2.35	384.17	1.75	285.17	1.50	245.49	1.68	274.95
4	2.03	331.73	2.28	372.22	1.64	268.29	1.40	227.88	1.59	258.87
5	1.95	317.88	2.21	360.89	1.55	253.50	1.31	213.03	1.50	244.76
6	1.87	305.17	2.15	350.16	1.47	240.50	1.23	200.44	1.42	232.36
7	1.80	293.49	2.08	339.98	1.40	229.03	1.16	189.71	1.36	221.41
8	1.73	282.74	2.02	330.33	1.34	218.90	1.11	180.52	1.30	211.73
9	1.67	272.84	1.97	321.17	1.29	209.92	1.06	172.61	1.24	203.15
10	1.62	263.70	1.91	312.48	1.24	201.93	1.02	165.78	1.20	195.52
11	1.56	255.25	1.86	304.22	1.19	194.81	0.98	159.85	1.16	188.71
12	1.52	247.44	1.82	296.38	1.15	188.45	0.95	154.70	1.12	182.63
13	1.47	240.20	1.77	288.93	1.12	182.76	0.92	150.20	1.09	177.19
14	1.43	233.49	1.73	281.84	1.09	177.65	0.90	146.27	1.06	172.31
15	1.39	227.25	1.69	275.10	1.06	173.07	0.87	142.82	1.03	167.93
16	1.36	221.46	1.65	268.68	1.04	168.94	0.86	139.79	1.00	163.99
17	1.32	216.07	1.61	262.58	1.01	165.22	0.84	137.11	0.98	160.43
18	1.29	211.05	1.57	256.76	0.99	161.86	0.83	134.76	0.96	157.22
19	1.26	206.37	1.54	251.22	0.97	158.83	0.81	132.67	0.95	154.32

20	1.24	202.00	1.51	245.94	0.96	156.08	0.80	130.82	0.93	151.69
21	1.21	197.92	1.48	240.91	0.94	153.59	0.79	129.19	0.91	149.31
22	1.19	194.11	1.45	236.10	0.93	151.33	0.78	127.73	0.90	147.15
23	1.17	190.54	1.42	231.52	0.91	149.28	0.77	126.44	0.89	145.19
24	1.15	187.20	1.39	227.15	0.90	147.41	0.77	125.30	0.88	143.41
25	1.13	184.07	1.37	222.97	0.89	145.71	0.76	124.28	0.87	141.78
26	1.11	181.13	1.34	218.98	0.88	144.17	0.76	123.37	0.86	140.31
27	1.09	178.38	1.32	215.17	0.87	142.76	0.75	122.56	0.85	138.97
28	1.08	175.80	1.30	211.53	0.87	141.47	0.75	121.83	0.84	137.74
29	1.06	173.37	1.27	208.04	0.86	140.30	0.74	121.19	0.84	136.62
30	1.05	171.09	1.25	204.71	0.85	139.23	0.74	120.61	0.83	135.59
31	1.04	168.94	1.23	201.52	0.85	138.25	0.74	120.09	0.83	134.66
32	1.02	166.92	1.22	198.46	0.84	137.36	0.73	119.64	0.82	133.81
33	1.01	165.02	1.20	195.54	0.84	136.54	0.73	119.23	0.82	133.03
34	1.00	163.23	1.18	192.74	0.83	135.79	0.73	118.86	0.81	132.32
35	0.99	161.54	1.16	190.06	0.83	135.11	0.73	118.53	0.81	131.67
36	0.98	159.95	1.15	187.49	0.82	134.48	0.72	118.23	0.80	131.08
37	0.97	158.45	1.13	185.02	0.82	133.91	0.72	117.97	0.80	130.53
38	0.96	157.03	1.12	182.66	0.82	133.38	0.72	117.73	0.80	130.03
39	0.95	155.70	1.11	180.39	0.81	132.90	0.72	117.52	0.79	129.58
40	0.95	154.43	1.09	178.21	0.81	132.46	0.72	117.33	0.79	129.16
41	0.94	153.24	1.08	176.12	0.81	132.05	0.72	117.16	0.79	128.77
42	0.93	152.11	1.07	174.12	0.81	131.68	0.72	117.01	0.79	128.42
43	0.93	151.05	1.05	172.19	0.80	131.34	0.72	116.87	0.78	128.09
44	0.92	150.04	1.04	170.34	0.80	131.03	0.72	116.75	0.78	127.80
45	0.91	149.09	1.03	168.57	0.80	130.74	0.71	116.64	0.78	127.53
46	0.91	148.19	1.02	166.86	0.80	130.48	0.71	116.54	0.78	127.28
47	0.90	147.33	1.01	165.21	0.80	130.24	0.71	116.45	0.78	127.05
48	0.90	146.53	1.00	163.64	0.80	130.02	0.71	116.38	0.78	126.84
49	0.89	145.76	0.99	162.12	0.80	129.82	0.71	116.30	0.78	126.65
50	0.89	145.04	0.98	160.66	0.79	129.63	0.71	116.24	0.77	126.47
51	0.88	144.35	0.98	159.26	0.79	129.46	0.71	116.18	0.77	126.31
52	0.88	143.70	0.97	157.90	0.79	129.31	0.71	116.13	0.77	126.16
53	0.88	143.09	0.96	156.60	0.79	129.16	0.71	116.09	0.77	126.03
54	0.87	142.50	0.95	155.35	0.79	129.02	0.71	116.05	0.77	125.90
55	0.87	141.95	0.94	154.14	0.79	128.90	0.71	116.01	0.77	125.79
56	0.87	141.43	0.94	152.98	0.79	128.79	0.71	115.98	0.77	125.68
57	0.86	140.93	0.93	151.86	0.79	128.69	0.71	115.95	0.77	125.59
58	0.86	140.46	0.92	150.79	0.79	128.60	0.71	115.92	0.77	125.50
59	0.86	140.01	0.92	149.75	0.79	128.51	0.71	115.90	0.77	125.41
60	0.86	139.58	0.91	148.75	0.79	128.43	0.71	115.88	0.77	125.34

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VW GOL / Gasoline - Fuzzy cash flow (considering USS and TC impact) by geographic region

Month	NE			N			CW			SE			S		
	Lower limit	Most likely	Upper limit	Lower limit	Most likely	Upper limit	Lower limit	Most likely	Upper limit	Lower limit	Most likely	Upper limit	Lower limit	Most likely	Upper limit
1	254.71	393.14	574.79	292.17	429.53	594.77	286.86	425.81	595.75	252.28	389.04	568.19	257.55	381.47	532.46
2	249.71	385.43	563.50	285.82	420.19	581.85	280.76	416.76	583.09	246.70	380.44	555.62	252.98	374.71	523.02
3	244.84	377.90	552.50	279.67	411.15	569.33	274.84	407.97	570.80	241.30	372.11	543.47	248.51	368.08	513.77
4	240.09	370.57	541.78	273.72	402.40	557.20	269.10	399.45	558.87	236.08	364.06	531.70	244.12	361.58	504.69
5	235.46	363.42	531.33	267.95	393.91	545.46	263.53	391.18	547.30	231.03	356.26	520.32	239.82	355.20	495.80
6	230.94	356.45	521.14	262.36	385.70	534.08	258.12	383.15	536.06	226.14	348.72	509.31	235.60	348.96	487.09
7	226.54	349.66	511.22	256.95	377.74	523.06	252.87	375.35	525.16	221.41	341.43	498.66	231.47	342.84	478.55
8	222.26	343.05	501.55	251.71	370.04	512.39	247.77	367.79	514.58	216.83	334.38	488.35	227.43	336.86	470.19
9	218.08	336.60	492.11	246.63	362.57	502.05	242.83	360.45	504.31	212.40	327.54	478.37	223.47	330.99	462.00
10	214.01	330.32	482.93	241.71	355.33	492.03	238.03	353.33	494.35	208.11	320.93	468.71	219.59	325.25	453.99
11	210.04	324.19	473.98	236.94	348.32	482.33	233.37	346.41	484.67	203.96	314.53	459.37	215.80	319.63	446.14
12	206.17	318.22	465.25	232.32	341.53	472.92	228.85	339.70	475.27	199.95	308.34	450.33	212.08	314.12	438.45
13	202.41	312.42	456.76	227.84	334.95	463.81	224.45	333.18	466.15	196.06	302.34	441.57	208.44	308.73	430.93
14	198.75	306.76	448.50	223.50	328.57	454.98	220.19	326.85	457.30	192.30	296.54	433.09	204.88	303.46	423.57
15	195.18	301.25	440.44	219.30	322.40	446.42	216.06	320.71	448.70	188.66	290.92	424.89	201.40	298.30	416.37
16	191.70	295.88	432.59	215.22	316.40	438.13	212.04	314.75	440.37	185.13	285.49	416.96	198.00	293.26	409.34
17	188.32	290.66	424.96	211.28	310.60	430.09	208.15	308.97	432.28	181.71	280.22	409.26	194.67	288.34	402.46
18	185.02	285.57	417.52	207.44	304.96	422.29	204.37	303.36	424.44	178.41	275.12	401.82	191.41	283.50	395.71
19	181.80	280.61	410.26	203.73	299.50	414.72	200.70	297.92	416.82	175.20	270.18	394.60	188.22	278.78	389.13
20	178.67	275.78	403.19	200.13	294.21	407.40	197.15	292.64	409.43	172.10	265.40	387.61	185.10	274.16	382.68
21	175.62	271.07	396.31	196.64	289.08	400.29	193.69	287.51	402.26	169.10	260.76	380.84	182.06	269.65	376.39
22	172.66	266.49	389.61	193.25	284.10	393.40	190.33	282.53	395.28	166.18	256.27	374.28	179.07	265.24	370.22
23	169.76	262.02	383.08	189.97	279.27	386.71	187.07	277.69	388.51	163.36	251.91	367.91	176.16	260.92	364.19

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24	166.95	257.67	376.73	186.78	274.59	380.23	183.91	272.99	381.94	160.62	247.69	361.75	173.31	256.70	358.30
25	164.20	253.44	370.54	183.70	270.05	373.95	180.84	268.43	375.56	157.97	243.60	355.78	170.52	252.57	352.54
26	161.53	249.31	364.50	180.70	265.65	367.84	177.86	264.00	369.37	155.40	239.64	349.99	167.80	248.54	346.91
27	158.93	245.30	358.63	177.79	261.37	361.92	174.95	259.70	363.34	152.91	235.79	344.38	165.14	244.60	341.41
28	156.40	241.39	352.92	174.96	257.22	356.17	172.14	255.52	357.50	150.49	232.07	338.93	162.54	240.75	336.04
29	153.93	237.58	347.35	172.22	253.19	350.59	169.40	251.46	351.82	148.14	228.45	333.64	159.99	236.97	330.77
30	151.52	233.86	341.92	169.56	249.27	345.17	166.75	247.52	346.30	145.86	224.93	328.51	157.50	233.28	325.62
31	149.18	230.25	336.63	166.98	245.47	339.91	164.17	243.69	340.95	143.66	221.53	323.54	155.07	229.68	320.59
32	146.89	226.72	331.47	164.47	241.79	334.81	161.66	239.97	335.74	141.52	218.23	318.73	152.69	226.16	315.68
33	144.67	223.29	326.46	162.04	238.21	329.85	159.23	236.35	330.68	139.44	215.03	314.05	150.37	222.71	310.87
34	142.50	219.95	321.57	159.67	234.74	325.05	156.86	232.84	325.76	137.43	211.93	309.52	148.10	219.36	306.18
35	140.39	216.69	316.81	157.38	231.37	320.38	154.55	229.42	320.98	135.47	208.91	305.12	145.88	216.07	301.59
36	138.34	213.52	312.17	155.15	228.09	315.84	152.32	226.10	316.34	133.58	205.98	300.84	143.71	212.86	297.12
37	136.32	210.41	307.63	152.98	224.90	311.43	150.14	222.87	311.82	131.73	203.14	296.68	141.60	209.73	292.75
38	134.37	207.39	303.21	150.88	221.80	307.14	148.03	219.73	307.43	129.94	200.37	292.64	139.52	206.66	288.45
39	132.45	204.44	298.89	148.83	218.79	302.96	145.97	216.68	303.16	128.19	197.69	288.72	137.50	203.65	284.26
40	130.59	201.57	294.70	146.83	215.86	298.90	143.97	213.71	299.00	126.51	195.09	284.92	135.51	200.72	280.16
41	128.78	198.77	290.61	144.90	213.02	294.97	142.02	210.82	294.96	124.87	192.56	281.23	133.58	197.84	276.15
42	127.02	196.05	286.63	143.02	210.26	291.15	140.13	208.01	291.03	123.28	190.10	277.64	131.68	195.04	272.24
43	125.29	193.39	282.74	141.19	207.57	287.43	138.30	205.29	287.22	121.72	187.71	274.14	129.83	192.30	268.42
44	123.62	190.80	278.95	139.42	204.97	283.82	136.51	202.63	283.50	120.22	185.38	270.75	128.02	189.62	264.68
45	121.98	188.27	275.26	137.69	202.42	280.30	134.76	200.04	279.88	118.75	183.13	267.45	126.26	187.00	261.02
46	120.38	185.80	271.64	136.01	199.94	276.87	133.07	197.53	276.36	117.33	180.93	264.25	124.53	184.45	257.45
47	118.82	183.40	268.14	134.36	197.53	273.52	131.42	195.08	272.93	115.94	178.79	261.12	122.84	181.95	253.96
48	117.30	181.05	264.71	132.78	195.20	270.29	129.82	192.70	269.60	114.60	176.72	258.10	121.19	179.50	250.55
49	115.82	178.77	261.36	131.23	192.92	267.14	128.25	190.37	266.35	113.29	174.70	255.15	119.57	177.10	247.20
50	114.37	176.53	258.10	129.72	190.71	264.07	126.73	188.11	263.19	112.01	172.73	252.28	117.99	174.76	243.93
51	112.96	174.36	254.91	128.26	188.55	261.09	125.25	185.91	260.11	110.77	170.82	249.48	116.44	172.47	240.73

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52	111.59	172.23	251.81	126.83	186.45	258.18	123.81	183.77	257.12	109.57	168.96	246.77	114.93	170.23	237.61
53	110.24	170.15	248.77	125.44	184.41	255.35	122.39	181.68	254.19	108.40	167.16	244.13	113.45	168.04	234.55
54	108.93	168.13	245.82	124.08	182.42	252.59	121.02	179.64	251.34	107.26	165.40	241.57	112.01	165.90	231.56
55	107.65	166.16	242.93	122.76	180.48	249.91	119.69	177.66	248.56	106.15	163.69	239.07	110.59	163.80	228.64
56	106.41	164.23	240.12	121.48	178.59	247.30	118.39	175.73	245.87	105.07	162.03	236.64	109.21	161.75	225.78
57	105.19	162.35	237.36	120.23	176.75	244.75	117.13	173.86	243.25	104.02	160.40	234.27	107.86	159.75	222.98
58	104.00	160.51	234.68	119.01	174.96	242.27	115.89	172.03	240.68	102.99	158.82	231.96	106.54	157.80	220.25
59	102.83	158.72	232.05	117.82	173.21	239.85	114.69	170.24	238.19	101.99	157.28	229.71	105.24	155.87	217.57
60	101.70	156.97	229.50	116.67	171.51	237.49	113.52	168.50	235.75	101.02	155.79	227.52	103.97	154.00	214.95

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VW GOL / Ethanol - Fuzzy cash flow (considering USS and TC impact) by geographic region

Month	NE			N			CW			SE			S		
	Lower limit	Most likely	Upper limit	Lower limit	Most likely	Upper limit	Lower limit	Most likely	Upper limit	Lower limit	Most likely	Upper limit	Lower limit	Most likely	Upper limit
1	288.50	453.40	677.36	336.38	504.02	712.46	268.03	405.44	579.17	223.91	351.58	524.69	243.75	367.93	524.33
2	274.81	431.87	645.21	325.44	487.64	689.30	249.80	377.86	539.77	204.52	321.13	479.25	227.37	343.20	489.09
3	262.30	412.22	615.86	315.09	472.13	667.37	233.93	353.85	505.48	188.39	295.81	441.46	213.11	321.67	458.40
4	250.87	394.25	589.01	305.28	457.44	646.60	220.08	332.90	475.55	174.88	274.59	409.79	200.64	302.85	431.58
5	240.39	377.79	564.41	296.00	443.52	626.93	207.94	314.55	449.33	163.48	256.70	383.09	189.71	286.34	408.06
6	230.78	362.69	541.84	287.19	430.33	608.28	197.28	298.41	426.29	153.82	241.53	360.45	180.09	271.83	387.38
7	221.95	348.81	521.11	278.85	417.82	590.61	187.88	284.19	405.97	145.59	228.60	341.16	171.61	259.03	369.14
8	213.82	336.03	502.03	270.93	405.96	573.84	179.56	271.62	388.01	138.53	217.52	324.63	164.11	247.71	353.00
9	206.33	324.26	484.44	263.42	394.71	557.93	172.19	260.47	372.08	132.46	207.99	310.40	157.45	237.66	338.69
10	199.42	313.40	468.21	256.29	384.02	542.83	165.64	250.56	357.92	127.22	199.76	298.11	151.54	228.73	325.97
11	193.03	303.36	453.22	249.52	373.88	528.49	159.80	241.72	345.30	122.67	192.62	287.46	146.26	220.77	314.62
12	187.12	294.07	439.34	243.08	364.24	514.86	154.59	233.83	334.03	118.72	186.41	278.20	141.55	213.66	304.49
13	181.65	285.47	426.49	236.97	355.08	501.91	149.92	226.77	323.94	115.27	180.99	270.11	137.34	207.30	295.41
14	176.57	277.49	414.57	231.16	346.37	489.60	145.73	220.44	314.90	112.25	176.25	263.03	133.55	201.59	287.28
15	171.86	270.08	403.50	225.63	338.08	477.89	141.97	214.74	306.76	109.60	172.09	256.83	130.15	196.46	279.97
16	167.48	263.20	393.21	220.37	330.20	466.75	138.58	209.62	299.45	107.27	168.44	251.38	127.10	191.85	273.40
17	163.40	256.79	383.65	215.36	322.70	456.14	135.53	205.01	292.86	105.22	165.21	246.56	124.35	187.69	267.47
18	159.60	250.83	374.73	210.59	315.55	446.04	132.78	200.84	286.91	103.41	162.38	242.33	121.86	183.93	262.12
19	156.06	245.26	366.42	206.04	308.74	436.41	130.29	197.08	281.53	101.81	159.86	238.58	119.61	180.53	257.28
20	152.76	240.07	358.66	201.71	302.25	427.24	128.04	193.67	276.66	100.39	157.64	235.26	117.57	177.46	252.90
21	149.67	235.22	351.42	197.59	296.06	418.49	125.99	190.58	272.25	99.14	155.67	232.31	115.73	174.68	248.93
22	146.79	230.69	344.65	193.65	290.16	410.15	124.14	187.78	268.24	98.02	153.92	229.70	114.05	172.15	245.33
23	144.09	226.45	338.31	189.89	284.53	402.19	122.45	185.23	264.60	97.03	152.36	227.38	112.53	169.86	242.06
24	141.57	222.48	332.38	186.30	279.15	394.59	120.92	182.91	261.29	96.15	150.98	225.32	111.15	167.77	239.09
25	139.20	218.76	326.82	182.88	274.02	387.34	119.53	180.81	258.28	95.37	149.75	223.48	109.89	165.87	236.38

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26	136.98	215.27	321.61	179.61	269.12	380.41	118.26	178.89	255.54	94.67	148.66	221.85	108.75	164.15	233.92
27	134.90	212.00	316.73	176.48	264.44	373.79	117.10	177.14	253.04	94.05	147.68	220.39	107.71	162.58	231.68
28	132.94	208.93	312.14	173.49	259.96	367.46	116.05	175.54	250.76	93.50	146.81	219.09	106.76	161.14	229.64
29	131.11	206.05	307.83	170.63	255.67	361.40	115.09	174.09	248.69	93.00	146.03	217.93	105.89	159.83	227.77
30	129.38	203.33	303.78	167.90	251.57	355.61	114.21	172.76	246.79	92.56	145.33	216.89	105.09	158.63	226.06
31	127.76	200.78	299.97	165.28	247.65	350.07	113.41	171.55	245.06	92.16	144.71	215.96	104.37	157.54	224.50
32	126.23	198.38	296.38	162.78	243.90	344.76	112.68	170.44	243.47	91.81	144.16	215.14	103.71	156.54	223.09
33	124.79	196.12	293.00	160.38	240.31	339.69	112.01	169.43	242.03	91.50	143.66	214.40	103.11	155.63	221.79
34	123.44	194.00	289.83	158.08	236.87	334.82	111.39	168.50	240.70	91.21	143.22	213.74	102.56	154.80	220.61
35	122.16	191.99	286.83	155.88	233.57	330.16	110.83	167.65	239.49	90.96	142.82	213.15	102.05	154.04	219.52
36	120.96	190.10	284.01	153.77	230.41	325.70	110.32	166.87	238.38	90.73	142.47	212.62	101.59	153.35	218.53
37	119.83	188.31	281.34	151.75	227.38	321.41	109.85	166.16	237.36	90.53	142.15	212.14	101.17	152.71	217.63
38	118.75	186.63	278.82	149.81	224.48	317.31	109.42	165.51	236.43	90.35	141.86	211.71	100.78	152.13	216.79
39	117.74	185.04	276.45	147.95	221.69	313.36	109.02	164.91	235.57	90.19	141.61	211.34	100.43	151.59	216.03
40	116.79	183.54	274.20	146.16	219.01	309.58	108.65	164.36	234.78	90.04	141.38	211.00	100.10	151.10	215.33
41	115.89	182.12	272.09	144.45	216.45	305.96	108.32	163.85	234.06	89.91	141.18	210.69	99.81	150.65	214.68
42	115.03	180.78	270.09	142.81	213.98	302.47	108.02	163.40	233.41	89.80	141.00	210.42	99.53	150.23	214.10
43	114.23	179.52	268.19	141.23	211.62	299.13	107.74	162.97	232.81	89.69	140.83	210.17	99.28	149.86	213.56
44	113.47	178.32	266.41	139.71	209.35	295.92	107.49	162.59	232.26	89.60	140.68	209.95	99.05	149.51	213.07
45	112.75	177.19	264.72	138.25	207.16	292.83	107.25	162.23	231.75	89.51	140.55	209.75	98.84	149.19	212.61
46	112.06	176.12	263.11	136.85	205.06	289.86	107.04	161.91	231.29	89.44	140.43	209.58	98.65	148.91	212.20
47	111.42	175.10	261.60	135.51	203.04	287.00	106.84	161.61	230.85	89.37	140.32	209.41	98.47	148.64	211.82
48	110.81	174.14	260.17	134.21	201.10	284.27	106.66	161.33	230.46	89.31	140.23	209.28	98.31	148.39	211.47
49	110.23	173.24	258.81	132.97	199.24	281.63	106.49	161.08	230.10	89.25	140.14	209.15	98.16	148.17	211.15
50	109.68	172.37	257.52	131.77	197.44	279.09	106.34	160.85	229.77	89.20	140.07	209.03	98.03	147.96	210.86
51	109.16	171.56	256.31	130.62	195.72	276.65	106.20	160.64	229.47	89.16	140.00	208.93	97.90	147.77	210.58
52	108.67	170.79	255.15	129.51	194.06	274.31	106.07	160.45	229.20	89.12	139.94	208.84	97.79	147.60	210.34
53	108.21	170.05	254.06	128.44	192.46	272.05	105.95	160.26	228.94	89.08	139.88	208.75	97.68	147.44	210.11

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54	107.77	169.36	253.02	127.42	190.92	269.87	105.84	160.10	228.70	89.06	139.83	208.69	97.58	147.29	209.90
55	107.35	168.70	252.04	126.43	189.44	267.78	105.74	159.94	228.48	89.03	139.79	208.62	97.49	147.16	209.71
56	106.95	168.08	251.11	125.47	188.01	265.76	105.65	159.81	228.29	89.00	139.75	208.56	97.41	147.04	209.54
57	106.58	167.49	250.23	124.56	186.64	263.82	105.57	159.68	228.11	88.98	139.72	208.51	97.34	146.92	209.38
58	106.22	166.93	249.39	123.67	185.31	261.94	105.49	159.57	227.94	88.96	139.68	208.46	97.27	146.82	209.23
59	105.88	166.40	248.59	122.82	184.03	260.14	105.42	159.46	227.79	88.94	139.65	208.41	97.20	146.72	209.09
60	105.56	165.89	247.84	122.00	182.80	258.40	105.35	159.36	227.64	88.93	139.63	208.38	97.14	146.63	208.96

PUC-Rio - Certificação Digital Nº 1312423/CA

Fiat UNO / Gasoline - Fuzzy cash flow (considering USS and TC impact) by geographic region

Month	NE			N			CW			SE			S		
	Lower limit	Most likely	Upper limit	Lower limit	Most likely	Upper limit	Lower limit	Most likely	Upper limit	Lower limit	Most likely	Upper limit	Lower limit	Most likely	Upper limit
1	236.04	374.18	565.04	270.75	409.43	584.69	265.82	405.76	585.65	233.78	370.29	558.56	238.66	363.53	523.43
2	231.40	366.84	553.95	264.86	400.53	571.98	260.17	397.14	573.21	228.61	362.10	546.21	234.43	357.09	514.15
3	226.89	359.67	543.14	259.16	391.91	559.68	254.69	388.77	561.12	223.61	354.17	534.26	230.28	350.77	505.06
4	222.48	352.69	532.60	253.65	383.57	547.76	249.37	380.65	549.40	218.77	346.51	522.69	226.22	344.58	496.14
5	218.19	345.89	522.33	248.30	375.48	536.21	244.20	372.76	538.02	214.09	339.09	511.50	222.23	338.51	487.40
6	214.01	339.26	512.31	243.12	367.65	525.03	239.19	365.11	526.98	209.55	331.91	500.67	218.33	332.56	478.83
7	209.93	332.80	502.55	238.10	360.06	514.20	234.33	357.69	516.26	205.17	324.97	490.21	214.50	326.73	470.43
8	205.96	326.50	493.05	233.25	352.72	503.71	229.60	350.48	505.86	200.93	318.26	480.08	210.75	321.02	462.22
9	202.09	320.36	483.77	228.54	345.60	493.54	225.02	343.49	495.76	196.83	311.75	470.26	207.08	315.43	454.17
10	198.32	314.38	474.75	223.98	338.71	483.69	220.58	336.70	485.97	192.85	305.46	460.77	203.49	309.96	446.29
11	194.64	308.55	465.94	219.56	332.02	474.15	216.26	330.10	476.45	189.01	299.37	451.58	199.97	304.60	438.58
12	191.05	302.87	457.36	215.28	325.55	464.91	212.06	323.71	467.21	185.29	293.48	442.70	196.53	299.35	431.02
13	187.57	297.35	449.02	211.13	319.28	455.95	207.99	317.49	458.25	181.68	287.77	434.09	193.16	294.22	423.63
14	184.17	291.97	440.89	207.11	313.20	447.27	204.05	311.47	449.55	178.20	282.24	425.75	189.86	289.19	416.39
15	180.87	286.72	432.97	203.22	307.31	438.86	200.21	305.61	441.10	174.82	276.90	417.69	186.63	284.27	409.31
16	177.64	281.61	425.26	199.44	301.60	430.70	196.49	299.93	432.91	171.56	271.73	409.89	183.48	279.48	402.40
17	174.51	276.64	417.76	195.78	296.06	422.80	192.88	294.43	424.96	168.39	266.71	402.32	180.40	274.78	395.64
18	171.45	271.80	410.44	192.23	290.69	415.13	189.38	289.08	417.24	165.33	261.86	395.01	177.37	270.17	389.01
19	168.47	267.07	403.31	188.79	285.49	407.69	185.99	283.90	409.76	162.36	257.16	387.91	174.42	265.68	382.53
20	165.57	262.47	396.36	185.45	280.44	400.49	182.69	278.86	402.49	159.48	252.60	381.04	171.53	261.28	376.20
21	162.74	257.99	389.59	182.22	275.55	393.51	179.49	273.98	395.44	156.69	248.19	374.38	168.71	256.98	370.01
22	159.99	253.63	383.01	179.08	270.81	386.73	176.38	269.23	388.59	154.00	243.92	367.94	165.94	252.77	363.94
23	157.31	249.38	376.59	176.04	266.21	380.16	173.35	264.62	381.93	151.38	239.76	361.67	163.24	248.65	358.02
24	154.70	245.25	370.34	173.08	261.74	373.78	170.42	260.14	375.46	148.84	235.75	355.62	160.60	244.63	352.23
25	152.16	241.22	364.26	170.22	257.42	367.61	167.58	255.80	369.20	146.39	231.86	349.75	158.02	240.69	346.56

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26	149.68	237.29	358.32	167.45	253.22	361.61	164.81	251.58	363.11	144.00	228.09	344.06	155.50	236.85	341.03
27	147.27	233.47	352.56	164.75	249.14	355.79	162.12	247.47	357.19	141.69	224.43	338.54	153.03	233.10	335.63
28	144.93	229.75	346.94	162.13	245.18	350.13	159.51	243.49	351.44	139.45	220.88	333.19	150.62	229.43	330.34
29	142.64	226.12	341.46	159.59	241.34	344.65	156.98	239.62	345.85	137.28	217.43	327.99	148.26	225.83	325.16
30	140.41	222.58	336.12	157.13	237.61	339.32	154.52	235.87	340.43	135.17	214.09	322.94	145.95	222.32	320.10
31	138.24	219.14	330.92	154.73	233.99	334.15	152.13	232.22	335.17	133.12	210.85	318.06	143.70	218.88	315.16
32	136.12	215.79	325.86	152.41	230.47	329.13	149.81	228.67	330.05	131.14	207.71	313.33	141.49	215.53	310.33
33	134.06	212.52	320.92	150.15	227.07	324.26	147.55	225.23	325.08	129.22	204.67	308.73	139.34	212.25	305.60
34	132.05	209.34	316.12	147.96	223.75	319.54	145.35	221.88	320.24	127.35	201.71	304.28	137.24	209.05	300.99
35	130.10	206.24	311.44	145.84	220.54	314.95	143.22	218.62	315.54	125.54	198.84	299.94	135.18	205.91	296.48
36	128.19	203.22	306.87	143.77	217.42	310.49	141.15	215.46	310.98	123.78	196.05	295.74	133.18	202.86	292.08
37	126.33	200.26	302.42	141.77	214.38	306.15	139.13	212.38	306.53	122.07	193.35	291.65	131.22	199.87	287.78
38	124.51	197.39	298.07	139.81	211.43	301.93	137.17	209.39	302.22	120.41	190.71	287.68	129.29	196.94	283.56
39	122.74	194.58	293.83	137.91	208.55	297.83	135.27	206.48	298.02	118.79	188.16	283.82	127.41	194.08	279.44
40	121.02	191.84	289.70	136.06	205.76	293.83	133.41	203.65	293.93	117.23	185.68	280.09	125.58	191.28	275.41
41	119.34	189.18	285.68	134.28	203.05	289.97	131.61	200.89	289.96	115.71	183.28	276.46	123.78	188.54	271.47
42	117.70	186.59	281.77	132.53	200.42	286.21	129.86	198.22	286.10	114.23	180.94	272.93	122.03	185.87	267.63
43	116.11	184.06	277.94	130.84	197.86	282.55	128.16	195.62	282.35	112.80	178.66	269.50	120.31	183.26	263.87
44	114.55	181.59	274.22	129.20	195.37	279.01	126.50	193.09	278.69	111.40	176.45	266.16	118.64	180.71	260.19
45	113.03	179.19	270.59	127.59	192.95	275.55	124.88	190.62	275.13	110.04	174.30	262.92	117.00	178.21	256.60
46	111.55	176.84	267.04	126.03	190.59	272.17	123.31	188.23	271.68	108.72	172.21	259.77	115.40	175.78	253.09
47	110.11	174.55	263.59	124.51	188.29	268.89	121.78	185.89	268.31	107.44	170.17	256.70	113.83	173.39	249.66
48	108.70	172.32	260.22	123.04	186.06	265.71	120.30	183.62	265.03	106.19	168.20	253.72	112.30	171.06	246.30
49	107.33	170.14	256.93	121.61	183.90	262.62	118.84	181.41	261.83	104.98	166.28	250.82	110.80	168.78	243.02
50	105.99	168.02	253.72	120.21	181.78	259.60	117.43	179.26	258.73	103.80	164.41	248.00	109.34	166.54	239.80
51	104.68	165.95	250.59	118.85	179.73	256.66	116.06	177.16	255.70	102.65	162.59	245.26	107.90	164.36	236.65
52	103.41	163.93	247.54	117.53	177.73	253.81	114.73	175.12	252.76	101.53	160.82	242.59	106.50	162.23	233.58
53	102.16	161.95	244.55	116.24	175.78	251.02	113.42	173.13	249.88	100.45	159.10	239.99	105.13	160.14	230.58

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54	100.94	160.02	241.65	114.98	173.88	248.31	112.15	171.19	247.08	99.39	157.43	237.47	103.79	158.10	227.63
55	99.76	158.14	238.81	113.76	172.03	245.67	110.91	169.30	244.35	98.36	155.80	235.02	102.48	156.10	224.76
56	98.60	156.31	236.05	112.57	170.23	243.11	109.71	167.46	241.70	97.37	154.22	232.63	101.20	154.15	221.95
57	97.47	154.52	233.34	111.41	168.48	240.60	108.54	165.68	239.12	96.39	152.67	230.30	99.95	152.24	219.21
58	96.37	152.77	230.70	110.28	166.77	238.16	107.39	163.93	236.60	95.44	151.17	228.03	98.72	150.38	216.52
59	95.29	151.06	228.12	109.18	165.11	235.78	106.28	162.23	234.15	94.51	149.70	225.82	97.52	148.55	213.88
60	94.24	149.40	225.61	108.11	163.49	233.47	105.19	160.57	231.76	93.61	148.28	223.67	96.35	146.76	211.31

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Fiat UNO / Ethanol - Fuzzy cash flow (considering USS and TC impact) by geographic region

Month	NE			N			CW			SE			S		
	Lower limit	Most likely	Upper limit	Lower limit	Most likely	Upper limit	Lower limit	Most likely	Upper limit	Lower limit	Most likely	Upper limit	Lower limit	Most likely	Upper limit
1	266.31	421.74	636.06	310.50	469.04	669.01	247.42	377.27	543.86	206.68	327.03	492.69	225.00	342.37	492.36
2	253.67	401.72	605.87	300.41	453.79	647.27	230.58	351.60	506.86	188.79	298.72	450.03	209.88	319.36	459.27
3	242.13	383.44	578.30	290.85	439.36	626.68	215.94	329.26	474.66	173.90	275.16	414.54	196.72	299.32	430.45
4	231.57	366.73	553.09	281.80	425.69	607.18	203.15	309.77	446.55	161.42	255.42	384.80	185.20	281.81	405.27
5	221.90	351.41	530.00	273.23	412.74	588.70	191.95	292.69	421.93	150.91	238.78	359.73	175.11	266.45	383.18
6	213.03	337.36	508.81	265.10	400.46	571.19	182.10	277.68	400.29	141.99	224.67	338.47	166.24	252.95	363.76
7	204.88	324.45	489.34	257.40	388.82	554.59	173.43	264.44	381.22	134.39	212.64	320.36	158.41	241.03	346.63
8	197.37	312.57	471.42	250.09	377.79	538.85	165.75	252.74	364.35	127.88	202.34	304.83	151.48	230.50	331.48
9	190.46	301.62	454.90	243.16	367.31	523.91	158.95	242.37	349.39	122.27	193.47	291.47	145.34	221.15	318.04
10	184.08	291.52	439.66	236.57	357.37	509.73	152.90	233.15	336.10	117.43	185.81	279.93	139.88	212.84	306.09
11	178.18	282.18	425.58	230.32	347.93	496.26	147.51	224.93	324.25	113.24	179.17	269.93	135.01	205.44	295.44
12	172.73	273.54	412.55	224.39	338.96	483.47	142.69	217.58	313.66	109.59	173.40	261.23	130.66	198.82	285.92
13	167.67	265.54	400.48	218.74	330.43	471.31	138.38	211.01	304.19	106.40	168.36	253.64	126.77	192.89	277.40
14	162.99	258.12	389.29	213.38	322.33	459.75	134.52	205.12	295.69	103.61	163.94	246.99	123.28	187.58	269.76
15	158.64	251.23	378.90	208.27	314.62	448.75	131.05	199.82	288.06	101.17	160.08	241.17	120.14	182.81	262.90
16	154.59	244.82	369.24	203.42	307.28	438.29	127.92	195.06	281.19	99.02	156.68	236.05	117.32	178.52	256.73
17	150.83	238.87	360.25	198.79	300.30	428.33	125.10	190.76	275.00	97.13	153.68	231.53	114.78	174.65	251.17
18	147.33	233.32	351.88	194.39	293.65	418.84	122.56	186.89	269.41	95.46	151.04	227.55	112.48	171.15	246.13
19	144.06	228.14	344.08	190.20	287.31	409.80	120.27	183.38	264.36	93.98	148.70	224.03	110.41	167.99	241.59
20	141.01	223.31	336.79	186.20	281.27	401.19	118.19	180.21	259.79	92.67	146.63	220.91	108.53	165.13	237.48
21	138.16	218.80	329.99	182.39	275.51	392.98	116.30	177.34	255.65	91.51	144.80	218.15	106.82	162.54	233.75
22	135.50	214.58	323.63	178.75	270.02	385.14	114.59	174.73	251.88	90.48	143.17	215.70	105.28	160.19	230.37
23	133.01	210.64	317.68	175.28	264.78	377.67	113.03	172.36	248.47	89.57	141.72	213.51	103.87	158.06	227.30
24	130.68	206.95	312.11	171.97	259.78	370.53	111.62	170.20	245.36	88.76	140.44	211.58	102.60	156.11	224.51
25	128.49	203.49	306.90	168.81	255.00	363.72	110.34	168.24	242.53	88.03	139.30	209.86	101.44	154.35	221.97

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26	126.44	200.24	302.00	165.79	250.44	357.22	109.17	166.46	239.96	87.39	138.28	208.32	100.38	152.74	219.66
27	124.52	197.20	297.41	162.90	246.08	351.00	108.10	164.83	237.61	86.82	137.37	206.95	99.42	151.28	217.56
28	122.72	194.34	293.11	160.14	241.91	345.05	107.12	163.35	235.47	86.30	136.56	205.73	98.55	149.95	215.64
29	121.02	191.66	289.06	157.51	237.93	339.37	106.24	161.99	233.52	85.85	135.83	204.64	97.74	148.73	213.88
30	119.43	189.14	285.25	154.98	234.11	333.93	105.43	160.76	231.74	85.44	135.18	203.66	97.01	147.61	212.28
31	117.93	186.77	281.68	152.57	230.46	328.72	104.69	159.63	230.12	85.07	134.61	202.79	96.34	146.59	210.82
32	116.52	184.53	278.31	150.25	226.97	323.74	104.01	158.60	228.63	84.75	134.09	202.02	95.73	145.67	209.48
33	115.20	182.43	275.14	148.04	223.63	318.97	103.39	157.65	227.27	84.46	133.64	201.33	95.18	144.82	208.27
34	113.95	180.45	272.15	145.92	220.43	314.41	102.82	156.79	226.02	84.20	133.22	200.71	94.67	144.05	207.16
35	112.77	178.59	269.34	143.89	217.36	310.03	102.31	156.00	224.88	83.96	132.85	200.15	94.20	143.34	206.13
36	111.66	176.83	266.69	141.94	214.42	305.84	101.83	155.28	223.84	83.75	132.52	199.65	93.78	142.69	205.21
37	110.61	175.17	264.18	140.08	211.60	301.82	101.40	154.61	222.88	83.57	132.23	199.21	93.39	142.10	204.36
38	109.62	173.60	261.82	138.29	208.90	297.96	101.00	154.01	222.01	83.40	131.96	198.81	93.03	141.56	203.57
39	108.69	172.12	259.59	136.57	206.30	294.26	100.63	153.45	221.21	83.25	131.73	198.45	92.70	141.06	202.86
40	107.80	170.73	257.48	134.92	203.81	290.70	100.30	152.94	220.47	83.12	131.51	198.13	92.40	140.60	202.20
41	106.97	169.41	255.49	133.34	201.42	287.30	99.99	152.47	219.79	83.00	131.32	197.84	92.13	140.18	201.59
42	106.19	168.16	253.62	131.82	199.13	284.03	99.71	152.04	219.18	82.89	131.15	197.59	91.88	139.80	201.04
43	105.44	166.98	251.84	130.36	196.93	280.89	99.45	151.65	218.61	82.79	131.00	197.36	91.64	139.45	200.54
44	104.74	165.87	250.16	128.97	194.82	277.87	99.22	151.29	218.09	82.70	130.86	197.15	91.43	139.12	200.07
45	104.07	164.82	248.58	127.62	192.78	274.97	99.00	150.96	217.62	82.63	130.74	196.96	91.24	138.83	199.65
46	103.44	163.82	247.07	126.33	190.83	272.19	98.80	150.66	217.18	82.56	130.63	196.80	91.06	138.56	199.26
47	102.85	162.88	245.65	125.08	188.95	269.50	98.62	150.38	216.78	82.49	130.53	196.64	90.90	138.31	198.91
48	102.29	161.99	244.30	123.89	187.14	266.93	98.45	150.12	216.41	82.44	130.44	196.52	90.75	138.09	198.58
49	101.75	161.14	243.03	122.74	185.41	264.46	98.30	149.89	216.07	82.39	130.36	196.39	90.61	137.87	198.28
50	101.25	160.34	241.82	121.63	183.74	262.08	98.16	149.67	215.76	82.34	130.29	196.29	90.48	137.68	198.00
51	100.77	159.58	240.68	120.57	182.13	259.79	98.03	149.47	215.48	82.30	130.22	196.19	90.37	137.50	197.74
52	100.31	158.86	239.59	119.55	180.59	257.58	97.91	149.30	215.22	82.26	130.17	196.10	90.26	137.35	197.52
53	99.88	158.18	238.57	118.56	179.10	255.46	97.80	149.13	214.98	82.23	130.11	196.02	90.17	137.20	197.30

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54	99.48	157.54	237.59	117.61	177.67	253.42	97.70	148.97	214.75	82.21	130.07	195.96	90.08	137.06	197.11
55	99.09	156.93	236.67	116.70	176.29	251.45	97.60	148.83	214.55	82.18	130.03	195.90	89.99	136.94	196.93
56	98.72	156.35	235.80	115.82	174.96	249.55	97.52	148.70	214.37	82.16	130.00	195.85	89.92	136.82	196.76
57	98.38	155.80	234.97	114.98	173.68	247.73	97.45	148.59	214.20	82.14	129.96	195.80	89.85	136.72	196.61
58	98.05	155.27	234.18	114.16	172.45	245.97	97.37	148.48	214.04	82.12	129.93	195.75	89.79	136.62	196.47
59	97.74	154.78	233.44	113.37	171.26	244.28	97.31	148.38	213.90	82.10	129.90	195.71	89.73	136.53	196.34
60	97.44	154.31	232.73	112.61	170.12	242.64	97.25	148.28	213.76	82.08	129.88	195.67	89.67	136.44	196.22