2
Literature review

2.1.
General introduction

Bamboo is the most important and abundant non-wood forest product, which grows in most tropical and sub-tropical zones around the world (Chaowana, 2013). This plant is a giant perennial grass with large woody stem, which belongs to the taxonomic family *Poaceae* and subfamily *Bambusoideae*. It encompasses about 1,800 species within 50 genera (Chapman, 1996). While bamboo attains maturity in 3-5 years, wood takes more than 20 years. It grows faster than any other plant, some *moso* bamboo species can achieve 20m in only 3 months, therefore, cutting down this timber substitute may not affect the ecological balance at all (Khalil et al, 2012).

It has a superior adaptability to most climatic and edaphic conditions than other fast-growing woods. Moreover, it has straight grain, smooth surface, toughness and excellent abrasion resistance (Chaowana, 2013). Due to its hollow section and circular configuration, bamboo is very light, handleable and easy to transport and store, allowing rapid construction of structures. Bamboo has diaphragm along the culm that makes it rigid and crack resistant when it bends; therefore, it has proved to be an ideal material for anti-seismic construction (Gonzales, 1999). However, bamboo presents some limitations, which until present time considerably restrict its widespread and large-scale use. Once it is cut, for example, insects, fungi and pests attack its structure weakening it. For this reason, untreated bamboo structures are viewed as temporary. Similarly, bamboo structures, as timber, must be fire proofed or protected from fire. Finally, it does not have a regular shape along its body and its variable width causes difficulties in the construction process. Due to this, the building industry does not fully regard engineering bamboo as a suitable, economically viable and green alternative for construction.

2.1.1.
Environmental context

Recently, energy awareness has caught the attention of people all around the globe, as the world is simply running out of fossil fuels. This fact will contribute to energy shortages and supplying problems to big industries, which has brought the attention of governments in many countries. In the aim of construction field to incorporate healthier methods, innovating building and preserving resources, bamboo surges as a large-scale alternative construction material. Particularly when improved building development and energy use, conform the high standards of environmental friendliness: to be lighter and stronger, to be efficient, to be cheaper, to be sustainable (Rittironk & Elnieiri, 2008).

It is estimated that by 2011, more than 1 billion people were living in informal settlements and over the next 25 years 2 billion people will be added to this number (United Nations Human Settlements Programme UN-HABITAT, 2011). Dickson 2002, estimates this number in 9 billion people by 2050, and adds that socio-economical gap between advanced and non-advanced societies will raise as well (Dickson, 2002). Besides that, Rand Corporation presented in a recent report (Silberglitt et al, 2006), an increasing technical-scientific gap between scientifically advanced, proficient, developing and lagging countries. Additionally, the report anticipates a similar widening gap between urban and rural populations throughout the globe. On the other hand, diminishing wood resources and restrictions imposed on natural forests, mainly in the tropic, have centered world attention on the need to identify new renewable, green and locally available materials (Sharma, 2010) in the same line of environmental friendliness standards. Due to the development of the world economy, and population explosion, the overall demand for wood and wood-based composites is rising sharply. Meanwhile the available wood supply will decrease due to the global biomass demands for green energy generation (Chaowana, 2013).

In rural areas, bamboo is called the poor man’s timber due to the entire aspects of bamboo utilization in human life (Chaowana, 2013). Bamboo grows in tropical and subtropical developing countries as can be seen on the Figure below.

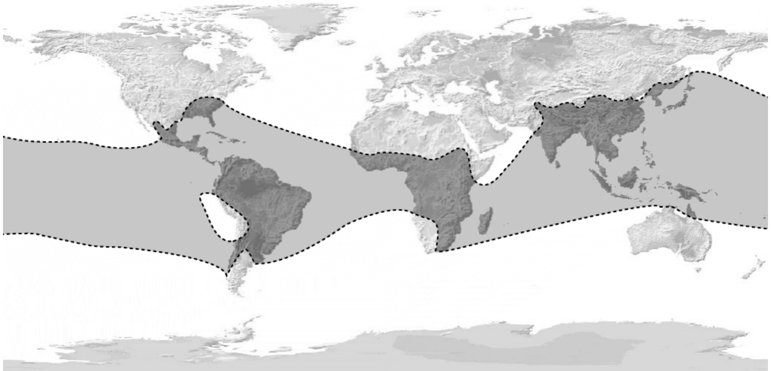


Figure 1 Global map of available bamboo species (Laroque, 2007).

Latin America shelters 6% of total world population (Latouche, 2006), so it turns its energy and resource problems more tractable comparing with super-populated and other developing regions in the world. Nevertheless, there is also a big need for adequate housing and infrastructure. The accelerated urbanization throughout the world is challenging provision of adequate dwelling, because it has forced many people to live in non-engineered or marginally engineered informal settlements (Richard, 2013). This makes reconsidering the real objectives of developing countries, which have followed the same relative consumption requirements as industrialized countries. However, by contextualizing local industries and giving way to other alternatives, some socio-economic problems and supply demands could be mitigated without excess production and pollution. Facing such energy, resource and social issues, bamboo growing in Latin America comes out as a resource to address those problems in the construction industry. Using these sustainable and friendly alternative solutions also helps to continue using traditional materials. Therefore, innovation and knowledge about local environment becomes a key factor to achieve a suitable balance between using sustainable and industrialized materials.

2.1.2.
Bamboo for construction

Structural use of bamboo offers potential advances on “reducing homeless rates, bridging the growing socio-economical gap and mitigating damage caused by natural disasters” (Richard, 2013). Bamboo is a fast-growing and renewable resource therefore; these features have turned bamboo culms into a suitable raw material used in building applications beyond housing, such as flooring, ceiling, walls, furniture, roofs, trusses and rafters. It is also used in construction as structural materials for bridges, water-transportation facilities and skyscraper scaffoldings. However, low-cost native materials like bamboo are often replaced for large-scale building materials due to lack of information about its implementation. As a result, it has been principally used in non-engineered, temporary and vernacular constructions (Richard, 2013).

Bamboo has similar properties to some timber and wood composites, therefore, Chaowana considered it an ecological viable substitute for them (Chaowana, 2013). After maturity, tensile strength of bamboo is comparable to mild steel (Correal et al, 2009). Moreover, the ratio of strength over density of bamboo pole, which indicates material efficiency, is 2,5 times higher than wood and 3 times than steel (Rittironk & Elnieiri, 2008). This shows how bamboo is extremely efficient due to its lightness and high strength. Figure 2. shows stress/strain diagram of some of the most used structural materials.

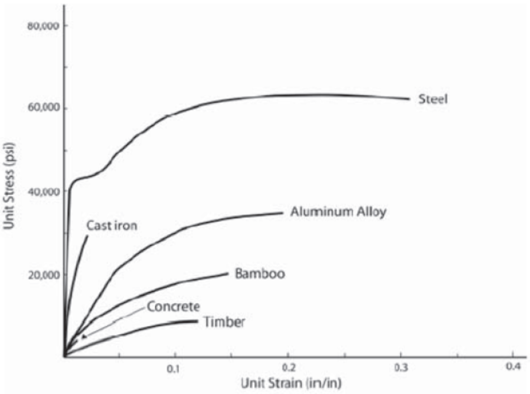


Figure 2 Stress/strain diagram comparing different structural materials (Rittironk & Elnieiri, 2008).

Ghavami reported that its strength and stiffness is suitable for construction and design of thinner structural elements than those made of wood (Ghavami et al, 2003). Its structural importance was seen, iamong others in Armenia, a city part of the Colombian Coffee Region devastated by 1999 earthquake. Bamboo came out rapidly as raw material for temporary shelters, permanent houses and public and institutional building reconstruction (Sharma, 2010).

The use of bamboo fibers as reinforcement in composite materials has increased sharply. “The amalgamation of matrix and natural fibers yield composite possessing best properties of each component” (Khalil et al, 2012). The exploitation of bamboo fibers in various applications has opened up new opportunities for both academicals as well as industrials to design a sustainable module for future use of bamboo fibers. However, in order to fully exploit the potential of bamboo as a construction material according Khalil (Khalil et al 2012), it results fundamental to increase knowledge on areas of preservation, joints, structural design and codification.

Khare reported an acceptable and feasible performance of bamboo used as a potential reinforcement in concrete structural members (Khare, 2005). Khare also concluded that bamboo is a potential substitute of steel reinforcement, even more in regions where availability of steel is limited and plain concrete members are commonly being used. Bamboo genre *Dendrocalamus* studied on this paper presents the better performance among other species groups with excellent bending properties and has great potential to be used as a load-carrying member (Korde, 2008). Scientists have already researched on bamboo-based composites but more research is still required to overcome potential challenges ahead. “These facts will make life easier for both urban as well as rural people who are more dependent on synthetic based composites in a big scale,” conclude Abdul (Abdul et al, 2012).

2.2.
Bamboo treatment

Bamboo strength is greater than most timber products, which is advantageous, but it has approximately half of steel tensile strength. Bamboo is easily accessible as it grows in almost every tropical and subtropical region. This fact reduces the cost of construction and increases the strength of the buildings that would otherwise be unreinforced. One major problem with bamboo is that it is more prone to insects than other trees and grasses because it has a high content of nutrients. On its raw state, it is vulnerable too, to fungi and plague attacks. In order to address this problem, it becomes necessary to treat bamboo to protect it from the environment. Steel does not have this problem but it also needs to be coated to protect it from rusting. In addition, bamboo is light in a strength-weight context compared to steel. Due to its low modulus of elasticity, bamboo can crack and deflect more than steel reinforcement under the same conditions. These properties, suggest that bamboo will make a fine addition to the current selection of materials, but it is necessary to be more familiar with its strengths and weaknesses.

For the current study, bamboo was not treated, but specimens and beams were tested three months after cutting the poles, which remained until outdoor assembling. Therefore, the effects of treated bamboo and its durability are not within the scope of this study.

2.2.1.
Weather, altitude and soil conditions

In order to get suitable bamboo culms there are some conditions to consider regarding its growing and harvesting. As it was previously said, the grass grows in tropical and subtropical regions and the ideal altitude ranges between 400 and 2000 MASL. Regarding weather conditions, temperature should be between 18 and 28 degrees Celsius with precipitations rainfall rates higher than 1200 mm. In terms of soils, it should be a well-drained and fertile clayey sandy loam. Soils must be moist, permeable and preferably rich in organic matter and protected from floods (Chara, 2014). They should not have obstacles, stones, old roots and undergrowth. It has had some indications about the certain period to cut bamboos, but some literature demonstrated that it has not significant factor (Ghavami & Marinho, 2003). They also suggested bamboo culms should be cut between three and six years after it has reached its highest level of maturation and culms are completely lignified.

2.2.2.
Curing process

After harvesting, green poles should be cured in order to protect them from plagues and fungi. Proper poles storage is a determinant fact to preserve its properties and keep the performance (Chiozzini, 2007). Moreover, there are plenty of methods to cure and seal poles however, some chemical treatments have presented similar results than natural ones (Chiozzini, 2007). Chiozzini also found that long-term treatments not always result more effective than short-term ones. Most of these are some curing methods that make use of a mixture of saline solutions (Chara, 2014). This study, in order hold a sustainable scope, addresses natural treatments implemented by local bamboo builders in Nariño, Colombia which are presented below:

Vertical: nodes are broken through the pole, except for the last one. Then, bamboo hollows are filled with immunizing during 3 to 5 days for liquid penetrates, and then it is rotated and refilled to the top according to the volume absorbed. After the 8th to 10th day, liquid is removed and poles are dried vertically. The solution requires, for 100 liters of water –preferably warmed-, 12 kg of borax and 12kg of boric acid. If possible, it is recommended to add 1 liter of salvia extract to improve the solution and red powdered pepper for fumigating it.

Immersion: poles are submerged during 12 hours in a solution composed by 1kg of borax, 1kg of boric acid and 50 liters of water. In this case it is recommended to drill internally the nodes as in the vertical treatment.

Injection: due to results experienced by small producers, this method is no longer used, however it uses between 5 to 20 cm^3 of immersion solution which is injected in each bamboo conduit in a zig-zag pattern from its cross section.

Smoke dried: is performed by a poly-woody acid, produced by condensation of tar-saturated smoke. Poles must be kept within a tightly closed oven for a period not less than 3 weeks.

After curing process, bamboo is ready to be employed in construction process.

2.2.3.
Laminated bamboo

Laminated bamboo has become a way to standardize and foster its use, since it can be designed in many geometrical sections as required by structural applications. In wood composite manufactures, adhesives are required to bond wood elements together. Adhesives are not only a significant cost factor in wood composite production but also they are the key factor for some of the product properties. In bamboo, adhesive capacity is influenced by its surface properties, such as wettability, roughness, pH value, buffering capacity among others (Ahmad & Kamke, 2003). Availability of appropriate equipment for culm transformation into regular pieces is one of the principal limiting factors for the local production.

Laminated bamboo cannot replace entirely the use of traditional structural materials, but its use can lead to more suitable balance in many constructive aspects. Bamboo culm heterogeneity becomes a problem for standard housing processes even in developing countries in medium and big scale production. However, lamination comes out as a good opportunity to generate innovative solutions by people involved on the industry. Therefore, it is a challenge to young generations to achieve the balance between traditional industry and sustainable and decentralized production, even more in the Latin American context. In Brazil, many exotic bamboo species suitable for fabrication of Laminated Glued Bamboo grow naturally. Among them stand the *Dendrocalamus giganteus* and *Bambusa vulgaris* (Beraldo, Rivero, & Azzini, 2003). In this study, the wood-based composite assembled was a Laminated Glued Bamboo Lumber made from *Dendrocalamus giganteus* layers. Plies of bamboo were stacked in order to glue them and produce beams, aimed to continue the research on those materials.

At the same density level, strength properties of fiberboard increased with greater levels of resin content. Age had a significant effect on board properties. Fiberboard made with one-year-old bamboo at 8% resin content level had the highest modulus of rupture (MOR) and elasticity (MOE) among the bamboo panels, which is largely due to a higher percentage of larger fiber size. Fiberboard made with five-year-old bamboo at 8% resin level had the highest internal bond strength, which was largely attributed to the higher resin recovery on old bamboo fibers (Nugroho & Ando, 2001). Bamboo fiberboards showed comparable physical and mechanical properties with tallow wood fiberboards. The dimensional stability of bamboo fiberboard was not satisfactory. Wax was recommended to improve the dimensional stability. On the other hand, spread glue rates appeared to be a significant variable for the internal bond strength on two-ply. Moreover, orientation of glue line in the vertical direction demonstrated to maximize the ultimate strength of Laminated Bamboo Lumber LBL (Nugroho & Ando, 2001).

Chaowana stated that bamboo-based composites will become a highly competitive alternative to wood-based composites and will become an important forest based product in the future (Chaowana, 2013). However, in Brazil, Laminated Bamboo Lumber process is practically restricted to university research (Beraldo et al, 2003). Nevertheless, since the 20th century, bamboo has received increasing attention for industrial applications on regional markets, especially as a raw material for wood-based composites such as: particleboard (PB), medium density fiberboard (MDF), hard fiberboard (HB), laminate glued bamboo (LGB), plywood, oriented strand board (OSB), Glue-Laminated Lumber (GLL), laminated bamboo lumber (LBL), Laminated Veneer Lumber (LVL) and oriented strand lumber (OSL) (Chaowana, 2013).

2.3.
Bamboo as functionally graded material

Bamboo is a functionally graded composite material “constituted by long and aligned cellulose fiber embedded in a lignin matrix” (Ghavami et al, 2003). Fiber distribution is variable through its wall thickness, arising from center outwards (Ghavami et al, 2003). The variation on fiber distribution prevents the direct application of traditional solid mechanics equations, as they assume perfect bonding between fiber and matrix and uniform distribution of fibers along the wall thickness. This fact raised the need to establish how this variation occurs and therefore modified basic equations for composite materials. Ghavami states the importance of analyzing bamboo culms through DIA (Digital Image Analysis) aimed to establish the variation of the volume fraction of the cellulose fibers on the cross section. To the *naked eye*, it can be observed in Figure 3 how its constitution changes along its cross-section. In fact, this allows great application of rule of mixtures by adjusting variability of its fibers.

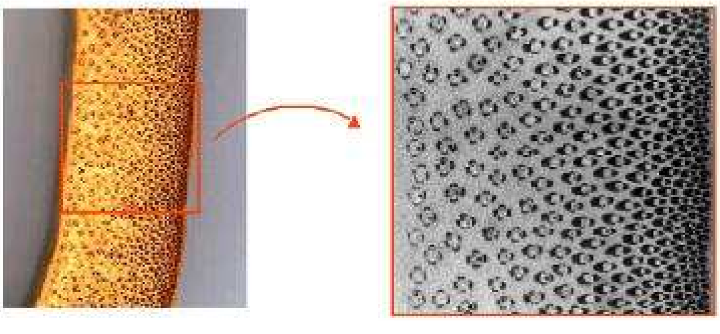


Figure 3 Variation of fiber volume fraction across bamboo wall (Ghavami & Marinho, 2003).

Rule of mixtures achieves a preliminary assessment of the mechanical behavior of composite materials in elastic range. This rule is a group of equations that assign values of mechanical properties to composites based on individual mechanical properties and volume fraction. In this way, Ghavami put forward an example of how to establish the composite modulus of elasticity knowing the modulus of elasticity and volume fraction of both components.

$$E_c = E_f V_f + E_m V_m = E_f V_f + E_m (1 - V_f) \qquad (2.1)$$

Where E_c is the composite modulus of elasticity

E_f and V_f are fiber modulus of elasticity and volume fraction respectively.

E_m and V_m are matrix modulus of elasticity and volume fraction respectively.

However, this procedure assumes perfect bonding between components, thus a more suitable approach would be necessary to obtain bamboo components properties. Furthermore, Nogata & Takahashi suggested investing more time and resources on developing functionally graded materials, which are governed by uniform strength as could be bamboo, rather than developing materials with high-stiffness (Nogata & Takahashi, 1995).

For instance, bamboo wall is composite by bundles of more than one hundred elementary fibers. Elementary fibers consist of layers of crystalized cellulose nano-fibrils aligned with many angles with respect to the fiber on the longitudinal direction and are embedded with hemi-cellulose and lignin (Fuentes et al, 2011).

2.4.
General Mechanical and Physical Properties

Taking Bamboo as giant timber and a potential composite of layered structural material requires more knowledge about its mechanical and physical properties. Despite the bamboo culm presents some excellent features such as rapid growth rate, short rotation age, excellent flexibility and high tensile strength, it also has some drawbacks. These disadvantages refer mainly to its natural composition and structure, and this means that it is fundamental to understand its physical and mechanical properties. As previous studies have demonstrated, properties vary depending on the position in the culm, therefore, obtaining and comparing these properties at different positions along the culms is a good start point for the analysis.

In the first place, a micro and macro mechanical approach is presented, then, the procedure to analyze laminated beams by composite materials theory and elementary solid mechanics of materials is explained. Finally, a section covering rupture mechanics is presented.

2.4.1.
Micro-mechanical Analysis

Formulations for micro-mechanical analysis based on mechanic of materials could be used in bamboo modeling as a natural composite material with aligned elongated fibers. This is supported by studies of graded functionality of bamboo at micro-structural level (Ghavami et al, 2003).

As bamboo fibers are not distributed uniformly throughout the thickness, engineering constants cannot be obtained by the rule-of-mixtures, commonly used for composites as shown below.

$$E_1 = E_f V_f + E_m V_m \tag{2.2}$$

$$E_2 = \frac{E_f E_m}{E_f V_m + E_m V_f} \tag{2.3}$$

$$G_{12} = \frac{G_f G_m}{G_f V_m + G_m V_f} \tag{2.4}$$

Where G_{12} is the composite shear modulus, G_m and G_f are the shear modulus of matrix and fiber respectively

$$\upsilon_{12} = \upsilon_f V_f + \upsilon_m V_m \tag{2.5}$$

And ν_{12} is the composite Poisson coefficient, being ν_m and ν_f this coefficient for matrix and fiber respectively.

To apply rule-of-mixtures in bamboo lumbers, it must determined a function of volume fraction of fibers and thickness, assuming that fiber distribution is symmetric to radial axis (arising from the inner wall outwards). Thereby, the young modulus at main (longitudinal) axis E_1 could be indicated in a simplified form in the following equation.

$$E_1 = E_{f1}V_f(x) + E_m(1 - V_f(x)) \tag{2.6}$$

To determine $V_f(x)$ it is necessary to implement digital images processing. To achieve this, cross sections are digitalized and divided in little segments with uniform fiber distributions. The function is defined after processing quantity of fibers in each segment from the curve of volume fraction against the position on *x*-axis. In this way, a regression of function $V_f(x)$ for *Dendrocalamus Giganteus* is obtained this is shown on Figure 4.

This function allows the use of rule-of-mixtures and to calculate bamboo mechanical properties as a function of fiber matrix distribution. As it can be seen, fiber and matrix form a unified compound; therefore, a composite analysis to determine modulus of elasticity is done to assess the properties of these elements.

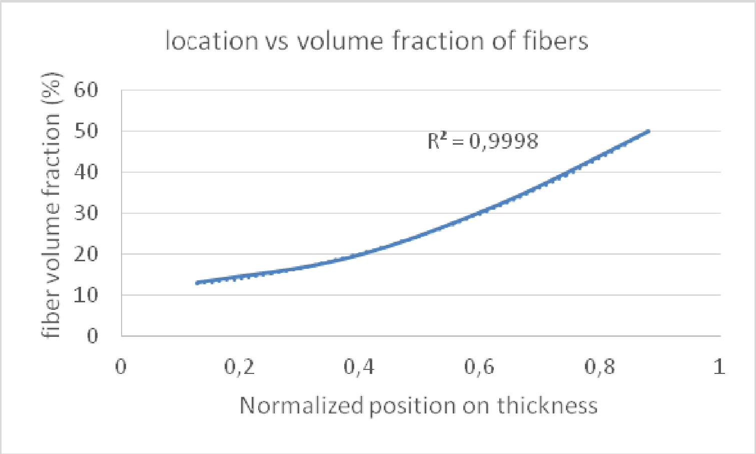


Figure 4 Relationship between volume fraction and location on bamboo wall thickness $\boldsymbol{V_f(x) = 46,27x^2 + 1,60x + 11,85}$ (Ghavami et al, 2003).

The parabolic trend plotted shows a correlation index of 0,998 and evidence the non-linear trend of fiber distribution. This parabolic behavior could allow defining a trend between fiber and matrix concentration and issue about need to use functional graded material methods instead of simpler rule-of-mixtures. Figure 5 shows wall thickness division (a) and (b) in segments with uniform fiber distribution (c). Modulus of elasticity is determined for each individual segment by tensile tests.

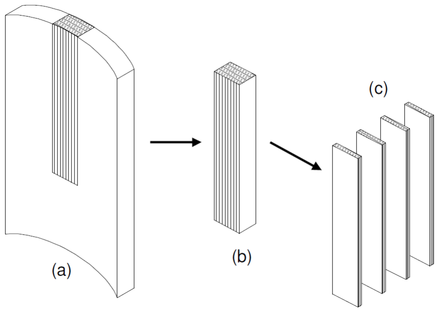


Figure 5 Layering segmentation on bamboo wall thickness.

After that, each segment is analyzed through digital image process method to calculate its volume fraction. It allows plotting a curve of volume fraction against modulus of elasticity. Using a lineal regression it is possible to define values for matrix and fiber modulus by extrapolating values from 0% and 100% respectively as shown in Figure 6.

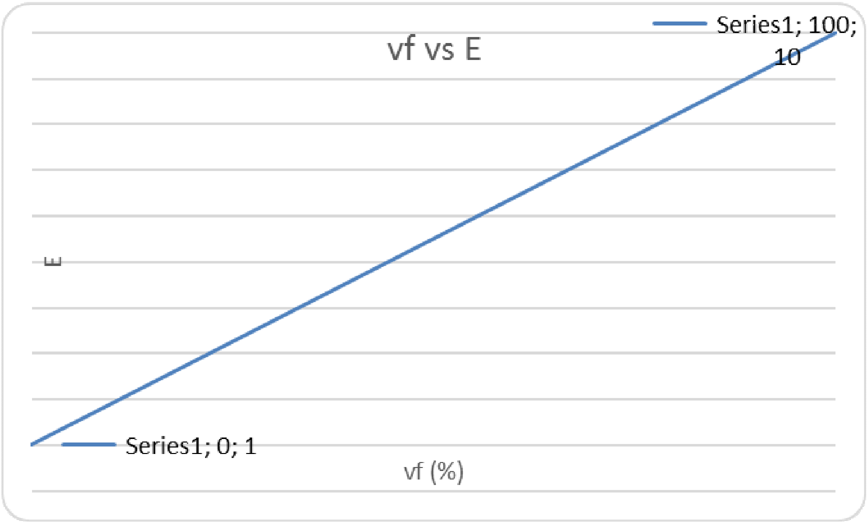


Figure 6. Volume fraction vs. normalized modulus of elasticity for E_m and E_{f1} calculus (Ghavami, 1990).

This methodology could be implemented to determine other elastic properties and to establish separate properties of components, matrix and fibers.

2.4.2.
Macro-mechanical Analysis

To characterize mechanical properties of bamboo is to analyze the structural composite of whole. In this case, matrix and fiber properties are state in terms of a new homogenous equivalent material. Average stresses replace real stresses.

As bamboo is a composite material with aligned fibers, whose wall thickness is smaller compared to its diameter and length. It can be assumed as a unidirectional lamina especially orthotropic under a plane stress state. This means that there is not any stress in the *z* direction and there is only one in the plane shear. Constitutive relation for these approach is shown on equations 2.7 – 2.18.

$$\varepsilon_1 = \frac{1}{E_1}\sigma_1 - \frac{V_{21}}{E_2}\sigma_2 \tag{2.7}$$

$$\varepsilon_2 = \frac{1}{E_2}\sigma_2 - \frac{V_{12}}{E_1}\sigma_1 \tag{2.8}$$

$$\gamma_{12} = \frac{1}{G_{12}}\tau_{12} \tag{2.9}$$

Where ε_1 and ε_2 are the components 1 and 2 deformations respectively. σ_1 and σ_2 are the 1 and 2 component tensile strengths. ν_{12} and Υ_{12} the composite modulus of Poisson and τ_{12} and G_{12} the composite shear strength and modulus.

Stress-deformation relation could be defined in a matrix form from flexibility matrices and then, engineer constants can be defined as follows:

$$S_{11} = \frac{1}{E_1} \qquad S_{22} = \frac{1}{E_2} \tag{2.10}$$

$$S_{12} = S_{21} = -\frac{V_{21}}{E_2} = -\frac{V_{12}}{E_1} \qquad S_{66} = \frac{1}{G_{12}} \tag{2.11}$$

Thus, the matrix form is defined by:

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ & & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \tag{2.12}$$

Stresses on the lamina could also be expressed in terms of strain tensor, where Q_{ij} corresponds to the stiffness matrix elements of the lamina that is the inverse of the flexibility matrix.

$$[Q] = [S]^{-1} \tag{2.13}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q & 0 \\ Q_{21} & Q_{22} & 0 \\ & & 2Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{\gamma_{12}}{2} \end{Bmatrix} \tag{2.14}$$

Where

$$Q_{11} = \frac{S_{22}}{S_{11}S_{22} - S_{12}^2} = \frac{E_1}{1 - V_{12}V_{21}} \tag{2.15}$$

$$Q_{12} = \frac{S_{12}}{S_{11}S_{22} - S_{12}^2} = \frac{V_{12}E_2}{1 - V_{12}V_{21}} = Q_{21} \tag{2.16}$$

$$Q_{22} = \frac{S_{11}}{S_{11}S_{22} - S_{12}^2} = \frac{E_2}{1 - V_{12}V_{12}} \tag{2.17}$$

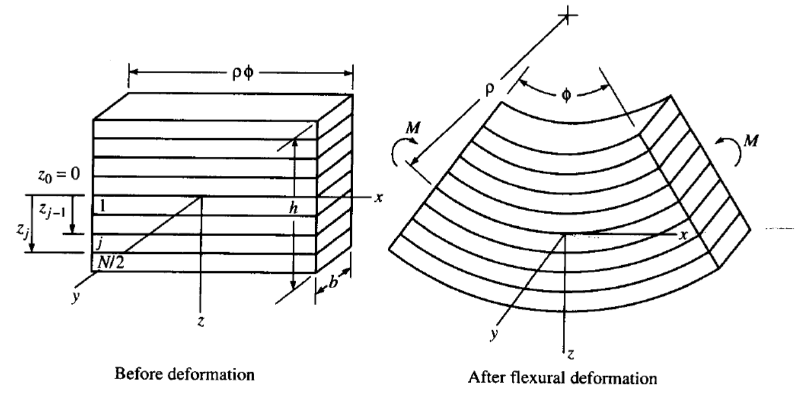
$$Q_{66} = \frac{1}{S_{66}} = G_{12} \tag{2.18}$$

2.4.3.
Beam analysis by mechanics of materials

A theory of laminated beams in pure bending was developed from Bernoulli-Euler theory of elementary mechanics of materials on Principles of Composite Materials (Gibson, 1994). Although the application of this theory is quite restricted, it provides considerable insight into the analysis of laminated structures and introduces the general lamination theory.

Theory based on the analysis of Pagano (Pagano, 1967) for bidirectional composites used the following assumptions:

1. Plane sections, which are initially normal to the longitudinal axis of the beam, remain plane and normal during flexure.
2. The beam has both geometric and material property symmetry about the neutral surface (the plies are symmetrically arranged over the *xy* plane).
3. Each ply is linearly elastic with no shear coupling (ply orientations are either 0° or 90°).
4. The plies are perfectly bonded together, so that no slip occurs at ply interfaces.
5. The only stress component present are σ_x and τ_{xz}



Before deformation

After flexural deformation

Figure 7 Element of a laminated beam before and after the application of a bending moment (Gibson, 1994).

Longitudinal normal strain at a distance z from the neutral surface is given by the familiar equation from assumption 1.

$$\epsilon_x = \frac{(\rho+z)\phi-\rho\phi}{\rho\phi} = \frac{z}{\rho} \tag{2.19}$$

Where ρ = radius of curvature of the neutral surface during flexure

ϕ = angle as defined on Figure 7

z = distance from neutral surface defined by the *xy* plane.

From assumption 3 the longitudinal stress in the *j*th ply is given by.

$$(\sigma_x)_j = (E_x)_j(\epsilon_x)_j \tag{2.20}$$

Static equilibrium requires that the applied bending moment M must be related to the longitudinal stresses by

$$M = 2\int_0^{h/2} \sigma_x \, zbdz \tag{2.21}$$

Recall that for a homogeneous, isotropic beam the moment-curvature relation is given by

$$M = \frac{E_f I_{yy}}{\rho} = \frac{E_f bh^3}{12\rho} \tag{2.22}$$

Where I_{yy} is the moment of inertia of cross section.

The effective flexural modulus of the laminated beam can be expressed

$$E_f = \frac{8}{h^3}\sum_{j=1}^{N/2}(E_x)_j(z_j^3 - z_{j-1}^3) \tag{2.23}$$

or for an even number of plies

$$E_f = \frac{8}{h^3}\sum_{j=1}^{N/2}(E_x)_j(3j^2 - 3j + 1) \tag{2.24}$$

Thus the bending modulus of laminated beam, unlike the modulus of elasticity of the homogeneous isotropic beam, depends on the ply stacking sequence and the ply moduli. “That is, if the properties do not change through the thickness of a beam, the flexural modulus is the same as the Young’s modulus” (Gibson, 1994). Due to lamination orientation and previous assumptions made by Pagano (Pagano, 1967), an analysis by elementary solid mechanic could be carried out. Considering LGB as a homogeneous and isotropic beam with ply stacking unidirectionally. Elementary equation (2.25) for inertia moment of beam cross-section (Hibbeler, 1997) was used to relate strain and moment of the beams tests, and then solve for E_t and E_c.

$$M = \frac{\sigma_{max}}{c}\int_A y^2 dA \tag{2.25}$$

and then solving for σ_{max}

$$\sigma_{max} = \frac{Mc}{I} \tag{2.26}$$

$$and \quad \sigma_t = E_t\epsilon_t,\ \sigma_c = E_c\epsilon_c \tag{2.27}$$

where

σ_{max} = maximum normal strength that occurs at the furthest point of the cross section of the neutral axis

M = resulting internal moment determined by sections method and equilibrium equations, it is calculated regarding neutral axis of cross section.

I = moment of inertia of the cross section calculated regarding neutral axis.

c = perpendicular distance from neutral axis to the furthest point of *y*-axis where σ_{max} acts.

In most problems, bending stiffness remains constant along the beam. Consequently, following equations for beam elastic curve was used to relate load, shear and moment with E.

$$\frac{EId^4v}{dx^4} = -w(x) \tag{2.28}$$

$$\frac{EId^3v}{dx^3} = V(x) \tag{2.29}$$

$$\frac{EId^2v}{dx^2} = M(x) \tag{2.30}$$

These equations are integrated to obtain deflection v of the elastic curve. Each integration introduces an integration constant, which are solved by boundary conditions and provides a unique solution for a particular problem.

2.4.4.
Failure analysis

“The rupture of an element is separation or fragmentation due to external loads, as result of process of creation of new rupture surfaces, which can origin from a fissure existent” (Gonzales J. L., 2004). Gonzales also stated that fracture process generally happens in little regions with strengths smaller than maximums and he characterize them as a sudden, unexpected and catastrophic action.

Under behavior of materials standpoint, fracture divides in two sorts depending on quantity of plastic deformation prior to failure. Figure 8 shows a diagram of fragile and ductile fracture.

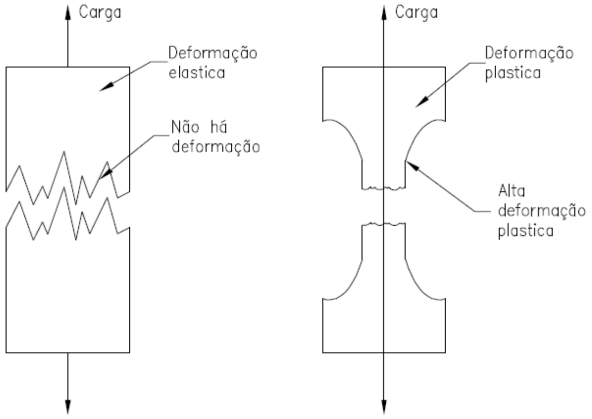


Figure 8 Types of fracture fragile fracture and ductile fracture (Rusinque Guatibonza, 2009).

Fragile fracture happens when deformation of the most of the body is elastic, so that after fracture under small deformations, element fragments can be jointed without big changes on geometric piece. On the other hand, ductile fracture happens after a considerable plastic deformation and a stable propagation of cracks. However, for static bending tests, ASTM classifies them in accordance with the appearance of the fractured surface and the manner in which the failure develops (ASTM D143, 2014). Those fracture surfaces may be roughly divided into brash and fibrous regarding fragile and ductile types shown on Figure 8.

Figures 8 to 14 below present ASTM failure classifications that was used to classify failures in present study.

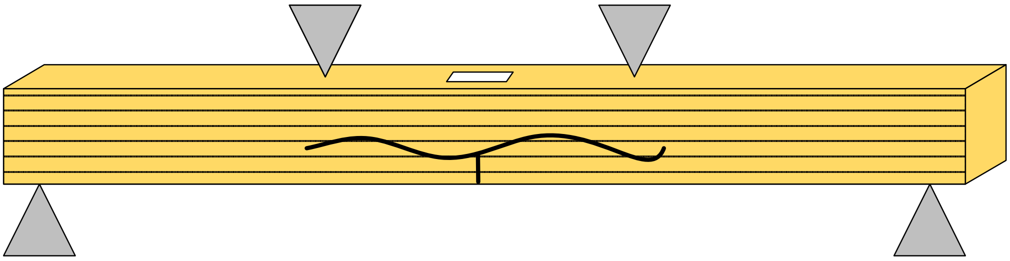


Figure 9 Simple tension failure.

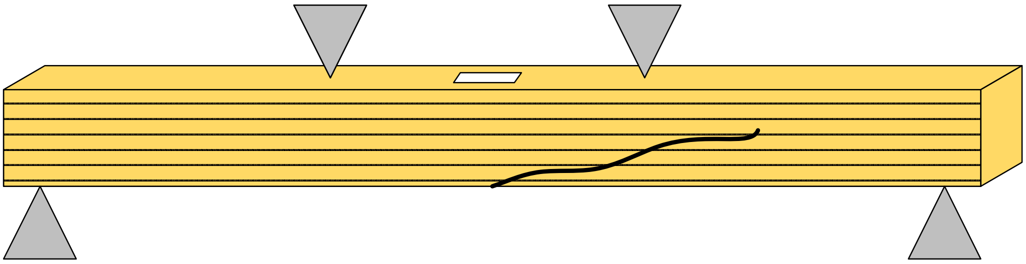


Figure 10 Cross-grain tension failure.

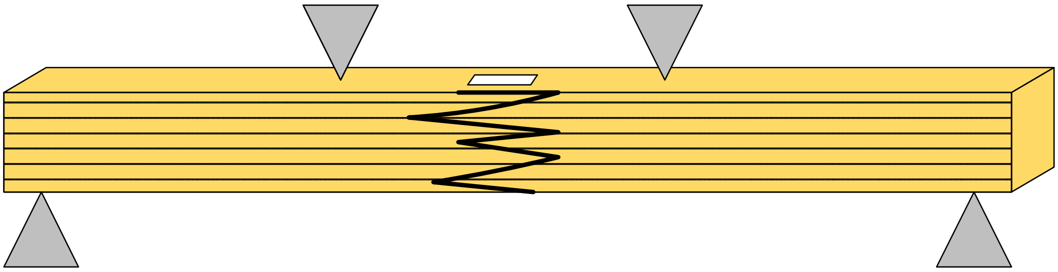


Figure 11 Splintering tension failure.

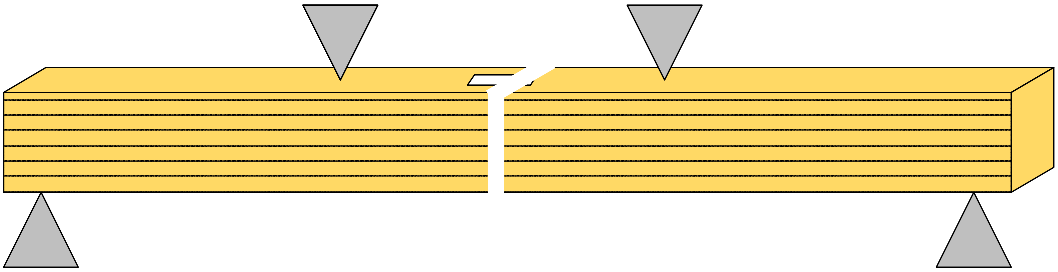


Figure 12 Brash tension failure.

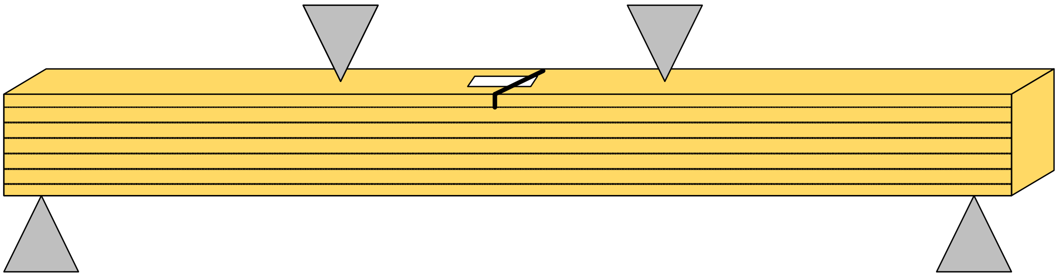


Figure 13 Compression failure.

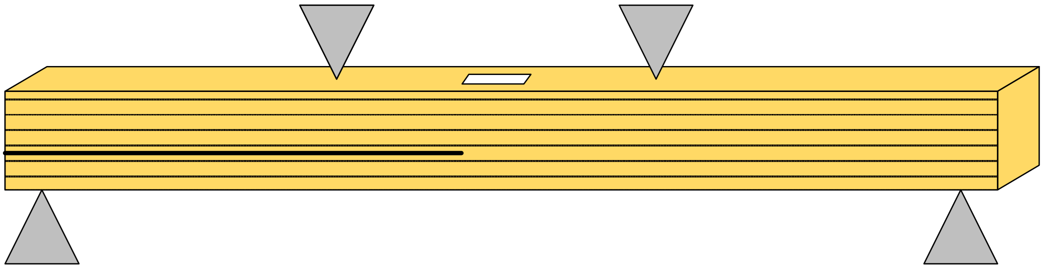


Figure 14 Horizontal shear failure.

2.5.
Experimental procedures

Studies aimed to characterize bamboo properties in terms of location both along its wall thickness and its culm as specific gravity, relative density, modulus of elasticity and volume fraction are addressed in this section. Specific gravity and bending properties of bamboo vary with age and height location as well as cross the layer. They all increase from one-year-old bamboo to five-year-old bamboo, as mentioned Ghavami referring to the age for cutting (Ghavami & Marinho, 2003). “The bamboo culm comprises about 50% parenchyma, 40% fibers and 10% vessels and sieve tubes” (Liese, 1985). The fibers contribute 60-70% of the weight of the total culm tissue. They are long and tapered at their ends. Li stated that outer layer had significantly higher SG and bending properties than the inner layer, ratified by rising on E as shown on Figure 4 and 6 above. This because, outer layer supports bamboo more than the inner layer. Bending strength had a strong positive correlation with SG (Li, 2004) and so that with volume fraction. Furthermore, height location of culms affects its physical and mechanical properties (Lee, Bai, & Peralta, 1994) other studies about variation in mechanical properties of moso bamboo established an equation for predicting the tensile strength and modulus of elasticity from the position on the wall thickness (Xian, Shen, & Ye, 1995) similar with interpolation established by Ghavami.

Yu et al found volume fraction of bamboo fibers denser in outer region (60–65%), sparse (15–20%) in the inner region and increases linearly with height by about 20–40%. For this reason, these studies focused in mechanical properties of bamboo culms along and across the fiber direction. Experimental results indicate that stiffness and strength under tensile loading of bamboo laminas is higher in outer region and lower in inner region. To understand variations along the bamboo wall, Yu et al conducted tests focused on that classificatory parameter. On their work, bamboo specie was *Phyllostachys pubescenes*, which presented a relative density ranging from 0,553 g/cm3 at internal edge to 1,006 g/cm3 at external one (Yu, Jiang, Hse, & Shupe, 2008). The mean longitudinal tensile MOE ranged from 8,987 to 27,397 GPa and mean longitudinal tensile strength ranged from 115.349 to 309.322 MPa, both from inner wall outwards. Relative density decreases significantly from outer layer to the middle layer and the difference in relative density between the layers toward the inner surface was not significant. (Yu, Jiang, Hse, & Shupe, 2008)

Layer and height position have a significant effect on all of those studied properties except for tensile strength. Relative density, tangential shrinkage, tensile MOE and tensile strength of bamboo increase greatly from inner layer outwards. Longitudinal shrinkage decreased greatly from the inner layer outwards, relative density, tangential shrinkage and tensile MOE at 1,3m were less than those are at 4,0m height (Li, 2004). One year old fibers showed a higher percentage of larger fiber size, less percentage of fine fibers retained on sieves higher than 60 meshes, and less lumpy fiber clumps than three and five year old bamboo fibers due to the refinement process. Compression properties parallel to the longitudinal direction are significantly higher than perpendicular to the longitudinal direction therefore, it makes bamboo aligned longitudinally an optimal structural material for compression strengths (Li, 2004).

Ghavami and Solorzano proposed first split bamboo wall in two ranges. Both have a fiber distribution relatively uniform with 40 to 90% at outer face and a 15 to 30% at inner face (Ghavami & Solorzano, 1995). Furthermore, Verma and Chariar 2012, carried out a layered laminate bamboo composite study, analyzing mechanical properties in segmented sector along the culm and from the center outwards. Table 4 presents tensile properties of laminas of *Dendrocalamus strictus.* It can be seen how stiffness and strength increase from lower internodes to top and a considerable difference between inner and outer regions. Middle regions in some cases have higher stiffness than outer region, but strength always increase from center outwards. This presents orthotropic character of bamboo. Nevertheless, it also lets assume that an assemblig based in segment location could improve use of beams according applying loads and homogenize somehow sections properties. Verma & Chariar, 2012 concludes that all laminas presented a bi-linear stress-train curves and tensile strength and modulus of elasticity increase from inner to outer region across any cross section and from bottom to top of culms.

Specimen No.		Bottom		Middle		Top	
		1	4	8	11	14	17
Outer Region	MOE [GPa]	4,6	5,9	6,4	5,23	6,93	8,9
	Tensile Strength [MPa]	240	257	250	281	298	302
Middle region	MOE [GPa]	4,63	6,2	6,3	6,6	6,3	7,56
	Tensile Strength [MPa]	175	173	204	226	230	276
Inner region	MOE [GPa]	2,1	2,5	2,7	3,63	3,7	4,66
	Tensile Strength [MPa]	101	104	169	172	217	212

Table 4 Tensile properties of laminae of *Dendrocalamus strictus* (Verma & Chariar, 2013).

Verma and Chariar, 2013 found on their experimental results good agreements between the estimated and predicted value for modulus of elasticity and tensile strength, which are satisfactory agreement for initial design purposes (Verma & Chariar, 2013).