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Sergio Vitor de Barros Bruno

Strategic risk management: A framework for renewable generation investment under uncertainty

Tese de Doutorado

Thesis presented to the Programa de Pós-Graduação Engenharia Elétrica of the Departamento de Engenharia Elétrica, PUC-Rio as partial fulfillment of the requirements for the degree of Doutor em Engenharia Elétrica.

Advisor: Prof. Alexandre Street de Aguiar

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Abstract

Bruno, Sergio Vitor de Barros; Aguiar, Alexandre Street de (Advisor). **Strategic risk management: A framework for renewable generation investment under uncertainty**. Rio de Janeiro, 2016, 142p. PhD Thesis – Departamento de Engenharia Elétrica, Pontifícia Universidade Católica do Rio de Janeiro.

Despite recent trend for investment in renewable energy, high volatility in short-term markets still may hinder some opportunities. Forwarding contracting is essential even in Over The Counter (OTC) markets such as the Brazilian Free Trading Environment. Forward contracts allow reducing revenue uncertainty, help ensure supply adequacy by signaling generation expansion and may also be required for project financing in new ventures. Still, renewable sources face the additional risk of uncertain generation, which, in low periods, combined with high spot prices, pose the hazardous price-quantity risk. Renewable investment may be fostered by applying risk management techniques such as forward contracting, diversification and optimal investment timing. By trading contracts and exploiting the seasonal complementarity of the renewable sources, it is possible to reduce risk exposure. The problem of investment in renewable energy plants may be seen as a multistage stochastic optimization model with integer variables, which is very hard to solve. The main approaches in the current literature simplify the problem by reducing the dimensionality of the scenario tree or by assuming simplifying hypothesis on the stochastic processes. Our objective is to introduce a renewable investment valuation framework, considering the main uncertainty sources and portfolio investment alternatives. The main contribution of this work is a method to solve, by applying decomposition techniques, the problem of optimal investment in seasonal complementary renewable plants in the Brazilian energy market. This is a multistage stochastic and non-convex problem. Our investment policies are devised using an algorithm based on Stochastic Dual Dynamic Programming (SDDP). Integrality constraints are considered in the forward step, where policies are evaluated, and relaxed in the backward step, where policies are built, to ensure convexity of the recourse functions. Numerical results show that it is not possible to assume stagewise independence of the price processes. We maintain the Markovian property of the stochastic processes by a discretization of the probability space, solvable by a known extension to the SDDP method. Performance evaluation is carried out using the original data, validating our heuristic. A forward energy price model is required in our framework. We apply the Schwartz-Smith model with spot and OTC data of the Brazilian market to build such a forward price curve. The framework is able to represent the characteristics of the Brazilian FTE and may be applied to similar markets. We incorporate risk aversion with coherent measures of risk and evaluate alternative strategies based on modern risk management concepts.

Keywords

Renewable energy investment planning; risk averse multistage stochastic programming; stochastic dual dynamic programming real options; Strategic risk management; RAROC; energy forward contracts.

Resumo

Bruno, Sergio Vitor de Barros; Aguiar, Alexandre Street de (Orientador). **Gestão de riscos estratégicos: um modelo para investimento em geração renovável sob incerteza**. Rio de Janeiro, 2016. 142p. Tese de Doutorado – Departamento de Engenharia Elétrica, Pontifícia Universidade Católica do Rio de Janeiro.

O investimento em fontes renováveis, apesar do crescimento recente, ainda é dificultado devido à volatilidade dos mercados de curto prazo. Contratos forward são essenciais mesmo em mercados de balcão como o Ambiente de Contratação Livre (ACL) Brasileiro. Contatos forward permitem a redução da incerteza sobre a receita, ajudam a garantir a adequação do fornecimento graças à sinalização de preços para a expansão e podem também ser obrigatórios para realização do project finance de novos empreendimentos. Apesar da oferta de contratos, as fontes renováveis ainda possuem o risco adicional em sua geração, o que pode, combinando-se altos preços spot em um momento de baixa geração, ocasionar uma exposição ao risco de preço-quantidade. Investimento em fontes renováveis pode ser incentivado através da aplicação de técnicas de gestão de riscos como contratação forward, diversificação e definição do momento ótimo de investimento. Através da negociação de contratos e aproveitando complementariedades sazonais entre as fontes, é possível minimizar a exposição aos riscos do mercado. O problema de investimento em centrais de energia renovável pode ser visto como um modelo de otimização estocástica multiestágio com variáveis inteiras, de difícil resolução. As principais soluções disponíveis na literatura simplificam o problema ao reduzir a dimensionalidade da árvore de cenários, ou assumindo hipóteses simplificadoras sobre os processos estocásticos. Nosso objetivo é apresentar um framework para valoração de investimentos em energia renovável, considerando as principais fontes de incerteza e alternativas para composição de uma carteira de investimentos. A principal contribuição desse trabalho é uma metodologia para resolver, utilizando técnicas de decomposição, o problema de investimento ótimo em centrais renováveis complementares no mercado elétrico brasileiro. Este é um problema estocástico multiestágio e não convexo. Nossas políticas de investimento são geradas através de um algoritmo baseado em Programação Dinâmica Dual Estocástica (SDDP). Restrições de integralidade são consideradas no passo forward, onde as políticas são avaliadas, e relaxados no passo backward, onde as políticas são geradas, para garantir a convexidade das funções de recurso. Os resultados numéricos mostram que não é possível assumir independência entre estágios dos processos estocásticos de preços. A estrutura Markoviana dos processos estocásticos é preservada usando uma discretização do espaço de probabilidade, que é resolvida utilizando uma conhecida extensão do SDDP. A avaliação da performance é feita utilizando os dados originais, validando nossa heurística. Nosso framework requer um modelo para o preço forward de energia. Nós aplicamos o modelo Schwartz-Smith usando dados do mercado spot e de balcão para construir a curva forward do mercado brasileiro. O framework contempla as particularidades do ACL no mercado brasileiro, mas também pode ser utilizado em mercados similares. Utilizando medidas coerentes de risco, incorporamos aversão a risco e avaliamos as estratégias concorrentes utilizando conceitos modernos de gestão de riscos.

Palavras-chave

Planejamento de investimentos em energia renovável; otimização estocástica multiestágio com aversão a risco; programação dinâmica estocástica dual; Opções Reais; gestão de riscos estratégicos; RAROC; contratos de energia elétrica.

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Guimarães Rosa, *Grande Sertão: Veredas.*

Notation

The following notation is used throughout this Thesis. We include here only notation used in more than one chapter, avoiding, as much as possible, duplicate meanings for any item.

J, j	Total number of investment projects, index for project
T, t, τ	Investment horizon (years), index for yearly periods, index for monthly periods
h_τ	Hours in month τ
G_τ^j	Energy (in MWh) generated in month τ for project j (random)
π_τ^k	Energy spot price (in \$/MWh) in month τ and market k (random)
f_t^k	Price (in \$/MWh) of forward contract in year t and market k (random)
ξ_t	Random data for period t
FEC^j, FEC	Physical guarantee (in average-MW) of project j, maximum power under contract $FEC = FEC^1 + \ldots, +FEC^J$
v_j	Present value (in \$) of investment cost of project j
α	Present value of an annuity with horizon equal to project lifetime
r_f	Annual risk free discount rate
r	Annual discount rate for risky cashflows
l	Project lifetime in years
b	Project build time in years
y_t	Binary decision variable whether to invest or not in year t
${x^{k}}_t^{sell}$	Nonnegative decision variable indicating amount of forward contract (as a fraction of maximum FEC) to sell in year t in market k
x_t^j	Nonnegative decision variable indicating fraction of project j to purchase in year t
x_t	$J + K$ dimensional vector of continuous decision variables $x_t := (x_t^1, \ldots, x_t^J, {x^1}_t^{sell}, \ldots, {x^K}_t^{sell})$ for period t
$g_t^F(x_t, \xi_t)$	fixed cash flow upon investment in period t
$g_t^{OPER}(x_t, \xi_t)$	market clearing cash flow for period t
z_t	Binary variable taking a value of one if an investment has already been made in or before period t and zero otherwise
X_t	Constraints on the continuous decisions variables for period t

1
Introduction

1.1
Motivation and Objective

Investments in renewable electricity generation have been gaining prominence as countries work to reduce the share of fossil fuels in their energy mix. Favored by public opinion, due to its environmental appeal, its growth is largely a consequence of the development of new technologies and increasing returns to scale, which have improved the economic viability of such investments. It is thus desirable to obtain frameworks to foster investment in renewable generation.

One of the advantages of such projects is that they take two to three years to be built, similar or even faster then a thermoelectric power station. Larger facilities, such as hydroelectric power stations, require an average of five or more years to be built and may face long licensing issues with environmental agencies. The short build time reduces uncertainty and enables a expedite operation, which all contribute to reduced financing costs.

In Brazil, mechanisms to increase the share of renewable sources in the Regulated Trading Environment (RTE - Ambiente de Contratação Regulada) have been successfully created since 2008. Renewable sources such as windpower, sugar cane bagasse and small hydro have enjoyed an unprecedented growth opportunity in such market. However, hurdles to the use of these sources in the Free Trading Environment (FTE - Ambiente de Contratação Livre) have yet to be overcome.

For example, if generation companies produce less energy than what was agreed on their contractual obligations, the remaining deficit must be paid at spot price. The Brazilian spot market price is typically low with occasional peaks. If generation deficits occur during these price peaks, generation companies run the risk of facing serious losses. This is known as price-quantity risk.

This generation uncertainty, and the risk of being exposed to price-quantity risk, are some of the biggest obstacles to the inclusion of renewable sources in the Free Trading Environment. The intrinsic variability in the amount of energy generated by renewable sources also hinders the negotiation of fixed-amount contracts, i.e. supply contracts in which the amount of energy delivered to the consumer is fixed and guaranteed, also known as forward contracts. Furthermore, it affects contracts' time periods, which typically last no more than a few years.

Also relevant to generation uncertainty is the fact that wind power and small hydros' seasonal variations are complementary. Entrepreneurs can therefore mitigate generation uncertainty by building project portfolios based on the fact that periods of low wind power output coincide with periods of increased hydropower output, and vice-versa. This strategy is explored in (107), minimizing exposure to price-quantity risk.

When generation companies sell supply contracts in the Free Trading Environment, they are exposed to the spot price market risk. Accurately modeling the future dynamics of spot prices is therefore an essential step in investment planning. The Brazilian spot market price is determined by running the DECOMP model, the centralized short-term planning model of the Brazilian Electric System National Operator (ONS) (101).

According to estimates provided by the Brazilian Electric Energy Trading Chamber (CCEE, (99)), Free Trading Environment contracts represent only 28% of the total of generation contracts, despite their potential for reaching 45% of the market. What is more, intermittent renewable contracts in the RTE auctions have paid lower prices than the FTE, a clear indication that risk aversion is inhibiting investments in the FTE.

In this way, it is worthwhile to foster the development of renewable sources investments in the Free Trading Environment. One way of doing this is by using a valuation methodology which takes into account the specificities of renewable energy investment portfolios in the Brazilian case. Investment strategies must protect investors from the risks inherent to this market. Energy spot and forward price and generation uncertainty dynamics have to be accurately modeled so that investment strategies can adequately mitigate those risks.

The recent trend of Enterprise Risk Management (ERM) adoption on companies has been accompanied by increased awareness in risk management. Still, as seen in (41), companies still struggle to tie their strategy and risk management planning in a Strategic Risk Management framework. We may define Strategic Risks as those risk factors that may inhibit a company from meeting its strategic goals or jeopardize its survival. As we will see, there are several efforts in the renewable investment literature to incorporate risk management in the investment strategy, but in a somewhat unstructured approach.

With these relevant challenges in mind, the objective of this Thesis is to provide a Strategic Risk Management Framework, tailored to assist renewable generation optimal timing and investment decision, using a multistage dynamic policy, in the FTE or a similar market. We suppose a marginal investor, with enough budget to invest in the portfolio under analysis.

We consider managerial flexibilities such as portfolio diversification, partnerships, postponement options and signing long term forward contracts. The investment policies are devised by solving the problem of joint investment in combined renewable power plants taking into account the deferral option, using multistage stochastic programming, and modeling risk aversion through the use of AV@R (Average value at risk, also known as CV@R (58)), a coherent risk measure (3). The uncertainties presented here derive from the spot price, the energy contract price, and the generation uncertainty inherent to renewable sources.

In order to apply our framework, we require technical and economic data from the projects, as well as data from the market itself. Starting from this data set, we develop the stochastic models for the market and project uncertainties. The proposed investment model allows creation and evaluation of investment policies with different risk-return profiles, allowing the application of a portfolio selection procedure. Figure 1.1 outlines the proposed framework.

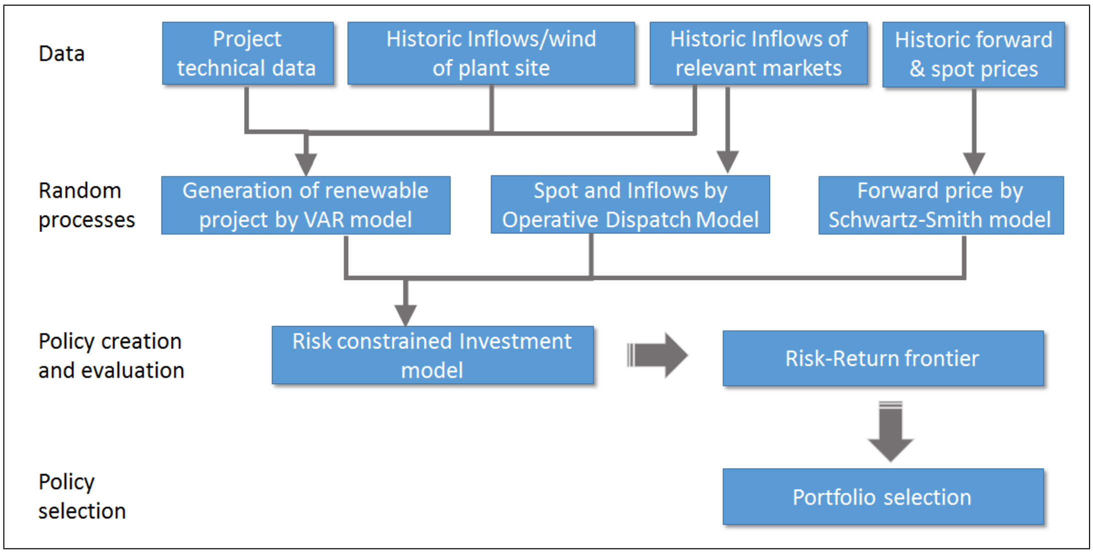

Figure 1.1: Outline of the proposed Strategic risk management framework.

We follow (107) to model the generation of the renewable projects, taking into account the correlation between generation and water inflow data of the market. This, consequently, also allows for a correlation with the market spot price, which will be useful in our strategy. Spot price and market inflows is derived from Monte Carlo simulations used in the planning models of the Brazilian integrated system, thus guaranteeing adherence to the market data. In order to evaluate the contracting strategies, we introduce a Forward contract pricing model. This tool, derived from the Schwartz-Smith two factor model, relies on market data that can be obtained from public and third party Forward Contract Benchmarking processes. This allows us to obtain forecasts of forward prices aligned with the spot price time-series from official planning scenarios.

We use an algorithm based on Stochastic Dual Dynamic Programming (SDDP) to generate investment policies. Two features of the problem make it non-convex, preventing the use of the regular SDDP decomposition technique: (i) the integrality of the investment decision, and (ii) the time dependence of uncertainty variables, namely the spot and forward prices.

In the proposed approach the challenge is overcome with (i) the relaxation of the integrality constraints in the backward step, taking advantage of the fact that relaxation solutions tend to be very similar to their integer counterparts, as shown in our results, and (ii) the use of a discretization of the random space, allowing the use of a known extension to SDDP method, which enables us to calculate the future cost function for each state. The resulting investment policy is evaluated on the forward pass and a gap measure is built in order to assess the quality of the solution. Our discretization is accomplished by Monte Carlo random sampling from the original time-series. This novel approach has the advantage that there is no need for assessing the probability of state transitions, since the transition probabilities are equiprobable by design. The framework suits the specificities of the Brazilian market Free Trading Environment but it can also be used in other markets.

The purpose of incorporating risk measures in our framework is two-fold: first, we avoid spurious strategies that try to exploit arbitrages in the forward market. Second, and more important, a strategy to invest in renewable generation must take into account the uncertainty of such projects. As will be motivated in the next chapter, incorporating risk management to one's strategy is crucial, due to the risks associated to the problem. The risk measures come into play to deploy risk management policies, used to shape the investment risk profile to the investor's appetite. Finally, we present a sound overall framework to decide over different investment strategies given the risk appetite of the investor.

In the next section, we will present a state-of-the-art literature review, as applied to the Brazilian case or to similar problems from other markets.

1.2 Literature Review and Contributions

Models for portfolio investment strategies in the Free Trading Environment and similar markets have been proposed in the past. The allocation problem for investors who wish to build portfolios using small hydros and sugar cane bagasse (biomass) in order to back up the sales of fixed-amount contracts in the Free Trading Environment was studied in (105). While very innovative, this portfolio strategy is adopted for static decisions and made through two-stage stochastic optimization, which hinders the evaluation of project deferral value.

A similar strategy was presented in (107) for small hydros and wind farms. The authors developed an energy generation time series model through a VAR (Vector Autoregressive) process. This allowed for a correlation between generation and water inflow data and, consequently, also a correlation with the spot price used in the planning models of the Brazilian integrated system.

The use of derivatives was exploited in (54). Their approach considers a strategic investor with a portfolio of investment and contract offerings. The dynamics of the interactions with other market players is represented using a equilibrium model. This is an extension of previous work in (53).

Properly incorporating the value of project deferral, a well-known concept from Real Options Theory, would require the use of a multistage stochastic programming model. This strategy, as applied to the European market, was adopted by (5) emphasizing the uncertainty concerning decreasing wind power investment costs. The framework was later extended in (6) to account for more uncertainty sources. Despite having a multistage structure, the investment problem has a property known as block-separable recourse, introduced by (61). Consequently, the model can be represented as a two-level problem, and be solved numerically using Bender's method. This numerical approach is the basis for the solution of the aforementioned articles. Risk aversion is represented by Average Value at Risk measure.

The disadvantage of the previously proposed models is the exponential growth in the number of scenarios needed for an accurate representation of the problem. As (94) pointed out, the number of scenarios needed to approximate the distribution of a problem through sampling, as in the Sample Average Approximation (SAA) method (55), in a multistage tree, grows exponentially with the number of decision stages. This effect severely limits the applicability of the proposed model of (5), especially when dealing with monthly cash flows. The lack of detailed monthly uncertainty scenarios is detrimental to an accurate characterization of price-quantity risk.

Scenario Reduction (SR) literature presents an alternative to the exponential growth of the SAA approach. The algorithms introduced by (49) allow scenario trees to be generated with a reduced number of branches by controlling approximation errors. The method was successfully applied to some two-stage and multistage problems in the energy field, as in the work of (32), (72) and (18).

There are disadvantages to the SR method. First, the criterion for tree reduction in based on a Lipschitz-type continuity between the scenarios and the optimal value of a stochastic optimization problem. A metric is then used to generate a theoretical distance limit between the optimal value and the optimal solution of the original problem and the approximate problem. In some instances, the approximate problem can be too different from the original problem for practical use.

The second disadvantage happens especially when uncertainty sources are multidimensional and of different measuring units. There is no clear criterion to define the weights between the dimensions of the random vector. The user needs to set a method for balancing the weight of each uncertainty source in the metrics. It is hard to balance in metrics, for example, the weight given to energy prices (measured in R\$/MWh), and the monthly generation (measured in Average MW). Some heuristics have been used, but there is no general rule of thumb.

Finally, it is important to highlight that reduced trees are obtained through heuristic techniques, since scenario reduction is an NP-complete problem. In the case of multistage problems, it is necessary to perform a forward (or backward) heuristic in which the scenarios are reduced stage-by-stage. As a result, SR solutions should be used with care and it is necessary to evaluate the quality of the solution in relation to the original problem.

Another major contribution to stochastic optimization literature is the Stochastic Dual Dynamic Programming method introduced by (78) and (77). This method requires random vectors to be stagewise independent and applies in particular to multistage linear optimization problems. The great advantage of this method is that it is very well suited to multistage problems, as long as the number of state variables is small. Extensions for investment valuation in an integrated system were proposed by (73). In her thesis, investments' integrality constraints were relaxed on the backward pass of the algorithm and recovered on the forward pass. As the process is heuristic, some metrics were proposed in order to assess the convergence.

The risk management of hiring a fleet of liquefied natural gas tankers was analyzed in (13). The authors developed an extension to SDDP through the use of a Markov Chain discretization to represent different states at each stage. The presence of several integer variables is treated with heuristics to allow for the inclusion of cuts.

Turning our attention to Real Options literature we can find, as expected, the representation of the value of postponing investment problems. Its limitation is due to the number of uncertainty sources and the capacity to model details of the problems' dynamics. Real Options problems rapidly fall under the curse of dimensionality, and their application is usually restricted to one or two random processes.

Real options methods based on financial options literature often apply only to assets whose values are expressed through simple linear operations. Analytical solutions, for instance, do not allow for options whose boundary conditions are expressed through highly complex calculations. Stochastic programming methods easily allow for a detailed description of the business rules of the problem, unlike the Real Options approach.

From a theoretical point of view, the lack of complete markets also imposes constraints. According to (46), since it is not possible to create perfect hedges, the use of utility functions is one of the main alternatives for contract valuation. A good literature review of option valuation may be seen in (16). As described in the illustrative work of (50), valuation of option-embedded cash flows should provide the same results for the three main alternatives: standard *risk neutral* valuation, risk-adjusted discounted cash flow or discounting certainty-equivalent cash flows with the riskless discount rate. Nevertheless, it requires proper handling and in general one should seek to employ the most appropriate method given the problem and available data at hand. As described in (104), there is an equivalence on using a risk measure such as AV@R to discounting certainty-equivalent cash flows.

An application to the Norwegian case can be found in (39). The authors use approximations which make it possible to model the real option of investing in small hydros and wind farms using the Black-Scholes analytical solution. The only uncertainty source that the authors take into account is the energy price. A similar application may be found in (12).

On the other hand, (75) approximates a binomial tree over the pricing dynamics of long-term fixed-amount contracts. The author explores the complementarity between small hydros and wind farms, as in (107), adding the deferral option value to the strategy. He assumes a main model with a contract pricing dynamics represented by the binomial tree. The clever concept explored is this work is that generation and spot prices uncertainties are represented by robust optimization subproblems at each tree node. The spot price uncertainty of each subproblem is represented as the solution of a robust optimization model proposed by (35), (36) and (37).

Other authors (51) use finite differences methods to solve partial differential equations numerically, but dimensionality remains a limiting factor when there are three or more uncertainty sources.

The current state of the art of high-dimensional American options valuation is based on the Least Squares Monte Carlo method (LSMC), proposed by (60). The method is based on the simulation of thousands of paths for the stock-price process and the use of least squares regression to generate (sub-optimal) exercise strategies. The performance of the method depends on the explanatory variables (basis) chosen for the regression. Applications of the LSMC method to renewable energy investments can be found in (14), and (20). The same hypotheses presented in (75) are made again in (20), but the LSMC method is used in the latter instead of the binomial tree of the former. The explanatory variables adopted were polynomials based on the contract price.

Almost all of the approaches studied consider the issue of forward contract pricing exogenous to the framework adopted. In the Brazilian case, where there is no futures market for energy contracts, a contract pricing strategy is essential to the investment analysis framework.

The literature on contract pricing is quite extensive. The two most widely adopted methods are: methods based on equilibrium price that results from market supply and demand, and time-series methods, which forecast price behavior as a function of its history.

The energy price forecasting techniques known as equilibrium methods (or fundamentalist approach) require detailed modeling of the whole energy system and the power supply and demand nodes, according to (48). Energy value arises as the energy market's equilibrium price, according to (30). These models provide close adherence to real prices in systems controlled by independent system operators because they establish the price in a way similar to that of the operators when defining the centralized dispatch. According to (30), despite the more realistic pricing, often the computational cost of equilibrium pricing is prohibitive because of the large number of scenarios that must be taken into account.

Time-series techniques derive from methods employed in the financial market and in other commodities. In some markets the only observable price series is the spot price, whereas in others there are forward contracts but the spot price is not observable. The historical series of the observable variable is typically used to model a stochastic process, and the remaining prices are derived through arbitrage arguments. Simpler models, such as the Geometric Brownian Motion, or mean reversion, cannot accurately model the term structure of the forward curve, as stated in (22). In other words, forward contracts with a long term to maturity are inaccurately priced. In this work we intend to price long-term contracts, and therefore we need to analyze some alternatives.

Two-factor models were developed by (43), (81), (89), among others. These models accurately represent long term forward contracts, as opposed to one factor models. A three-factor model was also proposed by (88) to represent varying interest rates. It should be noted that there are two main differences between forward and future contracts. One, is that the former are usually traded over the counter, while the latter are traded over future exchanges. Second, forwards are settled in the maturity, while futures have daily margin adjustments. If interest rates are dynamic, the different cash settlement dates can make their price diverge. Otherwise, forward contract prices should be equal to futures contract prices by arbitrage arguments. Differentiating between forward and futures pricing is unnecessary in most applications (this thesis included) and for that reason the three-factor model is not widely used.

A recurrent problem with these models, developed for commodities such as oil price, is that they may not capture the entire dynamics of electricity prices. There are some stylized facts of energy prices which are common to most markets, namely:

- relatively low prices as a rule, a feature of mean reversion, with short periods of very high prices (rare jumps or spikes)
- seasonality
- differences between weekdays and weekends in the short run
- intra-day fluctuation: highs and lows, such as price spikes and dips

Modified versions of the two-factor model which include, for example, seasonality and jumps were studied by (62). The aforementioned work analyzed the adherence of this extension to energy price in the Nordic market. Recently, multi-factor term structure models using the Heath, Jarrow & Morton (HJM) framework such as (24) and (56) have been receiving increasing interest. In some markets, while two factor models can account for only 70% of the uncertainty, HJM models may help explaining up to 95% of the term structure dynamics, as mentioned in (2) and (56).

Other models that have been proposed include error minimization, as in (38), (83), Extreme Value Theory models, as in (48), stochastic volatility (33), and regime-switching models. A good literature review is presented in (29).

As seen in our literature review, the approaches to renewable generation investment problems in stochastic programming literature lack applicability in multistage cases, rendering the analysis of the value of the deferral option infeasible. In special, the investment problem explored by (105), and extended to the multistage case by (5), needs careful treatment in terms of scenario generation so that its dimension remains tractable. The results obtained by (5) cannot usually be generalized to real world problems due to the number of scenarios involved.

Likewise, despite capturing the value of the deferral option, Real Options literature lacks methods for solving problems with multiple uncertainty dimensions while guaranteeing optimality or providing sufficient detailing of the problem.

Indeed, as described above, the most novel strategy in the literature is that presented by (75), who proposed an investment strategy which explores the complementarity of renewable sources and the value of deferral. That work represents uncertainty over contract pricing through a binomial tree, using a robust method to model the spot price endogenously. One of the main differences in the numerical approach in this thesis is that we represent the dynamics of sources of uncertainty through their stochastic processes. Also, (75) uses the risk neutral measure to model the price dynamics, while for risk management purposes the natural choice would be the real measure, as done in our work.

The dynamics of the Brazilian market spot price are obtained through a mid-term operation optimization model (DECOMP), and can be approximated by NEWAVE (66),(65), a model used by the Brazilian Electric System National Operator (ONS) whose results are available to all agents in the market. The results from NEWAVE simulations can be seen as forecasts of future months' supply and demand equilibrium. Thus, it is best for investors to use the fundamentalist approach and develop investment plans that are adherent to the market scenarios. The spot price scenarios and submarket inflows used as input data in our framework derive from Monte Carlo simulations generated by the NEWAVE model. Further details about NEWAVE scenarios will be presented in Section 4.1. The relation between generation uncertainty and spot prices will be represented analogously to (107). Accurately representing the correlation between generation and spot prices allow us to capture the price-quantity risk. Forward contract pricing is represented using the Schwartz-Smith two factor commodity pricing model. We are able to recover the forward curve of the market in order to price long term contracts.

To the best of our knowledge, this Thesis' main contributions to the literature are:

- A framework for investment under uncertainty in renewable energy portfolios with risk management techniques. The problem has been partially addressed in some approaches the literature, but in an non-systematic fashion. Here, we review such risk management alternatives and propose a general model to address the investor problem, with a well motivated use of risk aversion. The model may be applied to markets with similar structure to the Brazilian FTE. Here, we differentiate ourselves from the most current approach, namely (75) where spot uncertainty is modeled by robust optimization, by representing the price stochastic processes, what requires a more involved solution method, using a type of approximate dynamic programming approach;
- A solution algorithm to the proposed model, which is the generalization to the multistage case of the problem proposed by (105), solved through the proposed decomposition methods. Usual decomposition methods do not apply to our problem because, as we will see, it is a non-convex stochastic optimization problem. We introduce a reformulation which linearizes the problem at hand, apart from investment integrality constraints, which we relax in the forward step. Our method differs from previous approaches in the SDDP literature mainly because, despite incorporating a Markov Chain discretization similar to (13), we do not need to provide state transitions probabilities. Policies generated by our heuristic are evaluated by Monte Carlo simulation over the original data. The solution algorithm may generate several alternative policies, which could leave the investor with another decision making problem. We use an evaluation criteria originated in the banking industry which may prove valuable to generation portfolio investors;
- In order to apply the contracting strategies of our investment model, we introduce a Forward contract pricing model. This tool may also be used in works such as (75) and (20) and by market participants for several applications, such as contract negotiation or to mark-to-market their positions.

In the next chapter, we will briefly describe the Brazilian energy market. We will then describe the Brazilian Free Trading Environment, to which the investment models developed in this thesis apply.

Following, Chapter 3 motivates the necessity of risk management for an investor in renewable projects. We will also present the most commonly used project valuation and risk management tools. In Chapter 4, we will discuss the framework for analyzing renewable energy investments. In the first section, we will provide a description of the uncertainties, and after that a model for investment under uncertainty. We will also describe an evaluation criterion for choosing among alternative strategies devised with the aid of the proposed model. Chapter 5 describes our model for the Brazilian forward curve. In Chapter 6 we will provide a numerical algorithm to the proposed model of the previous chapters. The numerical results from this methodology will be presented in a simplified case study in Chapter 7, along with the approach to evaluate the risk-return profile of the strategies. Finally, conclusions and future work are presented in Chapter 8.

2
The Brazilian Energy Market

The current regulation of the Brazilian electricity market started in 2004. In this model, consumers (distribution companies) should ensure that their entire energy demand is backed by contracts with generators to increase system security. These contracts may be celebrated in two environments: the Regulated Trading Environment (RTE) and the Free Trading Environment (FTE).

Any difference between the contracts and generated amounts are surplus or deficits which must be settled in the CCEE[1] at the spot price. Figure 2.1 below shows the main characteristics of the market.

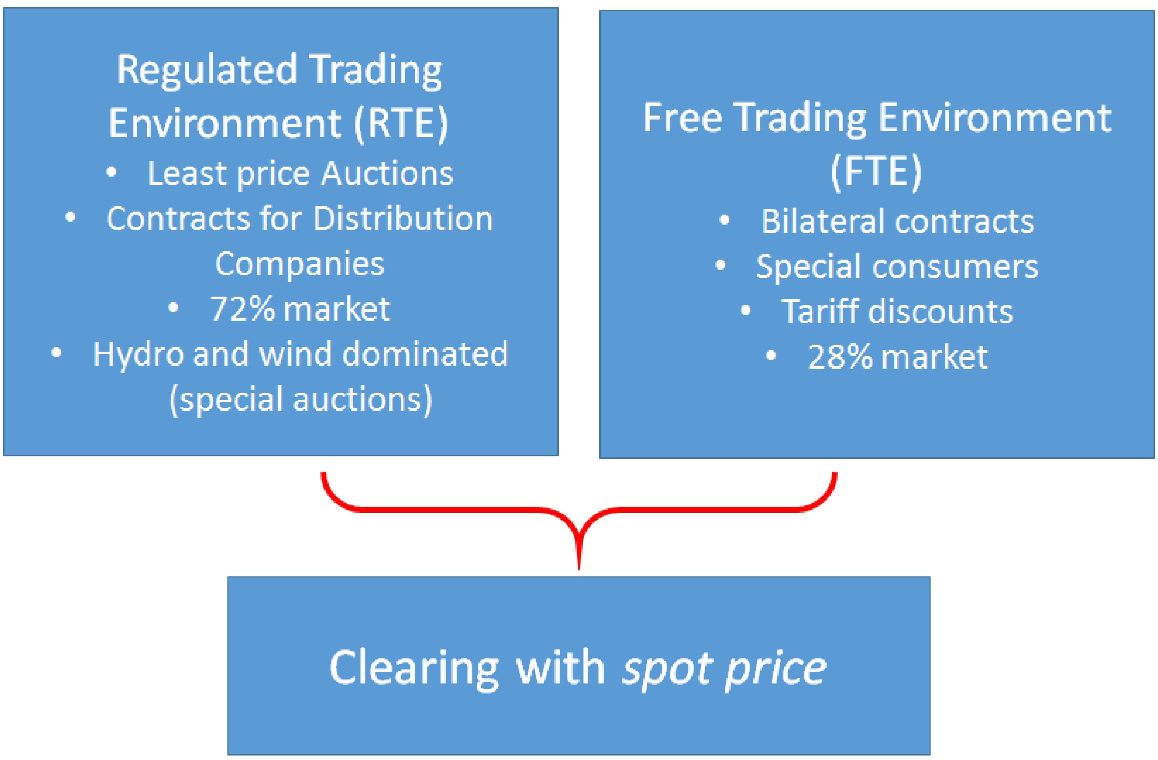

Figure 2.1: Main characteristics of the two trading environments in Brazil.

Renewable sources have the advantage of low environmental impact and the disadvantage of uncertainty in generation, as will be seen in Section 4.1.

[1]Câmara de Comercialização de Energia Elétrica, www.ccee.org.br

Since the renewable source availability is unpredictable, production at full capacity cannot be assured, and its energy contracting availability is limited. The regulation establishes the *physical guarantee* as the maximum marketable limit. This value is calculated by certification bodies considering the uncertainty profile of the generation project. The certified physical guarantee is denoted *Firm Energy Certificate* (FEC). The net contracted power of a generator must be equal or smaller than his FECs.

2.1
Regulated Trading Environment and Renewable Energy Auctions in Brazil

Distribution Companies (Discos) must deal all their demand contracts in the RTE. The goal of this model is to ensure *reasonable tariffs* to the consumers and to secure supply, since Discos can charge their customers for contracts representing up to 105% of the forecasted demand. As described by the Empresa de Pesquisa Energética (100), the main characteristics of this market are:

- Energy purchase through lower rate auctions;
- Joint bidding from a group of distributors, achieving economies of scale in the procurement of new generations projects, sharing risks and benefits and equalizing tariffs (unit costs are the same for all participating distributors);
- Procurement of existing and new plants is treated differently and done in distinct auctions;
- Reducing the business risks by signing contracts in regulated environment (Contrato de Comercialização de Energia Elétrica no Ambiente Regulado, CCEAR) for the (new) enterprises able to win the auctions.

Self-dealing, that is, a generator and a distributor within the same economic group negotiating contracts, is not allowed.

Auctions can be for the allocation of new consumer demand, to adjust short-term imbalances or even for power backup for system security. Typically, energy is procured in auctions for contracts with initial supply in a year $(A-1)$, three years $(A-3)$ and five years $(A-5)$. Existing energy auctions fall in category $A-1$ and the duration of the contracts is usually eight years. New energy auctions typically are traded in $A-3$ and $A-5$ auctions. This horizon also helps generators planning, so that they can begin building their plant only after signing the contract. To participate in an auction, a financial guarantee worth less than the amount of the investment is usually required from the bidding generators. The guaranteed profit associated with a CCEAR contract before the start of the construction of the project reduces the uncertainty of investors, allowing a reduction in the rates of return demanded by generation projects.

If the distributor underestimates its demand, there is the possibility of trading up to 1% of its demand in so-called adjustment auctions, held one year before delivery.

So far, the results of the auctions are characterized by the predominance of hydroelectric sources (38.7 GW) and Wind (9.6 GW). These sources also have the lowest rates, with an average of 121.44 R$/MWh and 136.26 R$/MWh, respectively. One reason for the success of renewables in the RTE is that special auctions with specific rules were created for this source. The so-called Alternative Sources Auctions (Leilões de Fontes Alternativas) guarantee the purchase of all production at a fixed price, with no penalties for seasonal variations. In the case of wind turbines, the generators receive the annual average and there is a four-year process to re-evaluate the contracted amount. The biomass plants have no obligation to supply in the period between harvests. The periods of these contracts may vary between ten and thirty years. Other auctions types also have clauses that benefit renewable sources, which they deem as *incentivized energy sources*.

This regulated environment is characterized by standardized contracts and high competitiveness in the auctions. For generators willing to participate in the FTE, there might be better opportunities, since there is greater contractual flexibility for consumers and generators.

2.2 Free Trading Environment

In the FTE energy is procured by bilateral contracts between generators and Free Consumers. Free Consumers are commercial and industrial agents with demand exceeding 3 MW which qualify to purchase energy directly from market participants. The so-called Special Consumers can even qualify to participate in the FTE with demand as low as 0.5 kW, but restricted to purchasing incentivized renewable energy. The main feature of FTE is the autonomy of participant consumers and generators to bilaterally trade non standardized contracts.

Renewable sources with installed capacity lower than 30MW receive transmission tariff incentives of 50% in the FTE, increasing its competitiveness. The consumer in the FTE can purchase energy contracts from generators or through a trading agent. Bilateral contracts are non standardized and parties may have full flexibility to decide on pricing, duration and quantity clauses. This allows consumers to develop an appropriate marketing strategy for their needs.

Contracts in the FTE have shorter durations than the RTE, usually up to five years. The absence of a longer-term contract makes it difficult to obtain financing for the project from financial institutions.

Bilateral contracts are typically divided into two types, described below.

2.2.1
Quantity contracts

In this type of contract the generator assumes the generation risks for a given energy amount procured by the consumer. The contracted energy amount may be constant or variable (following seasonal or other such pattern), as agreed with the consumer. The main feature of this contract type is that the unit cost of delivered energy is fixed, such as in financial forward contracts. It is common to refer to a contract amount as a *forward contract*. When selling a forward contract for a quantity Q (in average-MW)[2], the generator locks his revenue at a fixed price f (in \$/ MWh). The generated energy amount G_t (in MWh) of a given period may be greater or less than the contracted amount. The difference will be settled at the spot price π_t (in \$ / MWh), as illustrated in Figure 2.2.

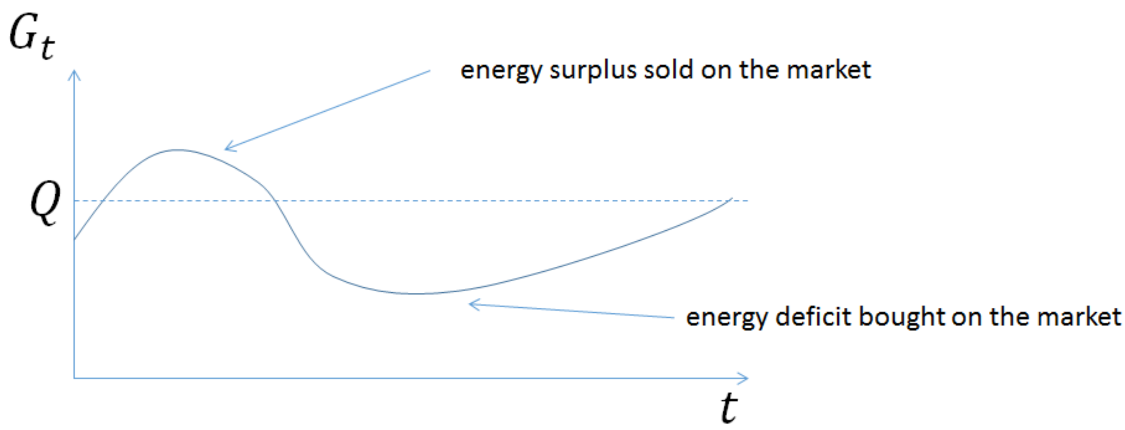

Figure 2.2: Clearing of surplus or deficit is done at spot price.

Consider a given period t, with a number of hours of operation h_t. A generator holding a forward contract with price f and quantity Q is entitled to a revenue R_t given by

[2]Average-MW, or avg-MW, is equivalent to the constant production of one MW during the (monthly) period.

$$R_t = fQh_t + \pi_t(G_t - Qh_t). \quad (2\text{-}1)$$

In the right hand side of this equation, the first term represents the fixed income of the contract and the second term represents a variable revenue (or loss) for differences settlement, which can be negative in case of deficit. The risk incurred by the generator of suffering deficit in a period of high spot price is denominated *price-quantity risk* . In Brazil, where hydro generation represents an important share of the market, it is common for price spikes to occur during drought periods. Hydraulic power plants, when generating below average during this period of higher prices, will incur at price-quantity risk.

Please note that the expected effect of a contract is to hedge (reduce) the volatility of the generator's cash flow. To illustrate this idea with a simple example, assume that the expected value of the spot price equals the price of the forward contract ($E[\pi_t] = f, \forall t$) and the variance of the spot is σ_π^2. Then, *ignoring uncertainties in generation*, we would have:

$$E[R_t] = fQh_t + E[\pi_t](G_t - Qh_t) = fQh_t + f(G_t - Qh_t) = fG_t, \quad (2\text{-}2)$$

which equals $E[\pi_t G_t]$, i.e., in our simplified example, *ceteris paribus*, average revenue of the hedged generator equals the revenue of a generator who sells all his energy in the spot market, with no contract. Variance, on the other hand, is

$$Var[R_t] = Var[\pi_t(G_t - Qh_t)] = \sigma_\pi^2(G_t - Qh_t)^2 \leq \sigma_\pi^2(G_t)^2 = Var[\pi_t G_t], \quad (2\text{-}3)$$

which is smaller than the variance of a generator that has no contracts ($Var[\pi_t G_t]$). We see that the cash flows under a quantity contract present lower volatility than the spot market.

The contracted amount Q is commonly represented as a percentage of physical guarantee FEC, since the FEC is the maximum possible contract amount. The resulting equation is

$$R_t = fFECx^{sell}h_t + \pi_t(G_t - FECx^{sell}h_t), \quad (2\text{-}4)$$

where x^{sell} is the percentage of the FEC sold under the quantity contract.

The cash flow volatility, which can cause major damage to the generator in case of exposure to price spikes (price spikes are common in energy markets, as described in the Introduction), motivates the interest of some generators in Availability Contracts.

2.2.2 Availability Contract

In this contract the consumer pays a fixed amount for all the generating capacity of the plant during the contracted period and may even decide what to do with the surplus production not consumed. In practice, the consumer rents equipment availability and assumes the generation risks. It is also possible that the contract is agreed upon a percentage of plant's capacity , leaving the remaining available energy to the generator.

The revenue of a generator under this type of contract is given by:

$$R_t = Ph_tFECx, \tag{2-5}$$

where P is the energy price under the availability contract, in R\$/MWh and x is the percentage of the plant capacity under contract.

Since the consumer assumes all generation risks, typically there is a large risk premium in this kind of contract. Availability contracts usually are procured at a much lower price than forward contracts.

All contracts in the FTE must be reported to CCEE. (67) makes an in-depth analysis of the contract mechanisms.

3
Investment Valuation and Portfolio Risk Management

Despite a thriving expansion of the renewable market over the last decade in Brazil, the author of this Thesis has no knowledge of any business being developed without previously signing a forward contract. Not only investors are unwilling to do it, but also banks are unwilling to lend without some assurance that their customer will not default the payments. This is evidence that there is some degree of risk aversion in this market, which should be better understood.

In this chapter, we see the most commonly used investment valuation tools, then, given potential investment portfolios, why and how should companies manage strategic risks. We will present how valuation techniques evolved from Discounted Cash Flow analysis to approaches that account for uncertainty and managerial flexibilities. Then, we turn to the modern portfolio theory and the mean variance model of Harry Markowitz and review the subsequent approaches that were proposed, up to the state of the art. Most of the results in this chapter follow references in (15) and (27).

3.1
Valuation of Capital Projects under uncertainty

We will begin our discussion of valuation techniques with the traditional Discounted Cash Flow method.

3.1.1
Discounted Cash Flow

The Discounted Cash Flow (DCF) method requires the evaluation of the revenues, expenses and investment over the lifetime of the project. Considering the time value of money, we valuate the Net Present Value (NPV) of the project. The main idea in the DCF method is that there exists an appropriate rate that allows the investor to discount future free cash flows to the present time. Given a discount rate r for projects with similar characteristics, we evaluate the NPV of the project considering the sequence of free (either positive or negative) cash flows CF_t for each period t, by

$$NPV = \sum_{t=1}^{l} \frac{CF_t}{(1+r)^{t-1}}, \tag{3-1}$$

where l is the project lifetime. Usually, a firm investing in similar projects will have a cost of capital equivalent to its discount rate. If the NPV is positive, then the project creates value to the firm and should be undertaken. More details in (63). It is often necessary to compare several projects under evaluation. In those cases, usually one resorts to additional criteria. The Profitability Index (PI) (see (15)) is given by

$$PI = \frac{\mathbb{E}[NPV(X)]}{C(X)}, \tag{3-2}$$

where $C(X)$ is the planned investment cost, discounted as in the NPV.

PI is often used for ranking projects, when one does not resort to a portfolio management process, since it helps maximizing the value created by unit of capital expenditures. Despite its appeal, under circumstances such as when one is faced with mutually exclusive projects or under resource constraints, decisions based solely on the PI may not lead to the best investment alternatives.

Suppose now that a generator company is willing to invest in a new venture immediately, and operations begin in the next period. This investor will spend in the current period $t = 1$ a capital expenditure v. If the investor does not sign any contracts, his revenues are given by the spot market during the whole lifetime l of the project, as illustrated in Figure 3.1.

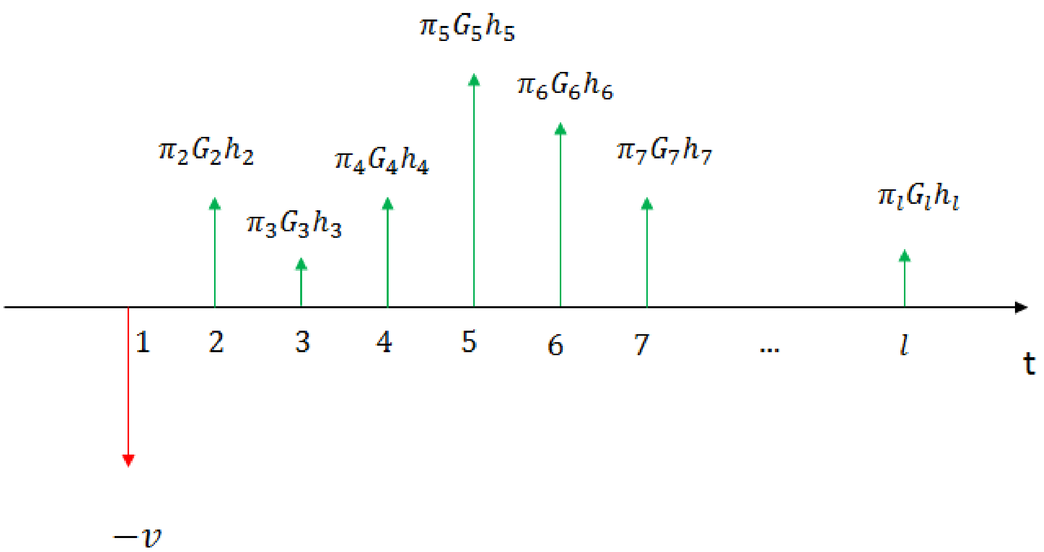

Figure 3.1: Example of cash flow of generator in FTE without contract.

If we disregard (very low) operational costs, the NPV of this project is:

$$NPV = \sum_{t=1}^{l} \frac{CF_t}{(1+r)^{t-1}} = -v + \frac{\pi_2 G_2 h_2}{(1+r)} + \frac{\pi_3 G_3 h_3}{(1+r)^2} + \ldots + \frac{\pi_l G_l h_l}{(1+r)^{l-1}} \quad (3\text{-}3)$$

The projects should be pursued if the NPV is positive. Since the spot prices and generation amounts are uncertain, there may be a positive probability of a negative cash flow, i.e., there is risk of losses. This risk may be diminished by acquiring contracts.

A forward contract may protect the project cash flows from very low spot prices. If the investor sells forward contracts, then his revenue is given by equation (2-1). This results in cash flows as illustrated in Figure 3.2.

Notice that, despite reduction of cash flow volatility, as seen in Chapter 2, if the generation is lower than the contract amount, high spot prices may lead to negative free cash flows, incurring in the price-quantity risk. It is then clear that it may not be optimal to acquire as much forward contracts as possible.

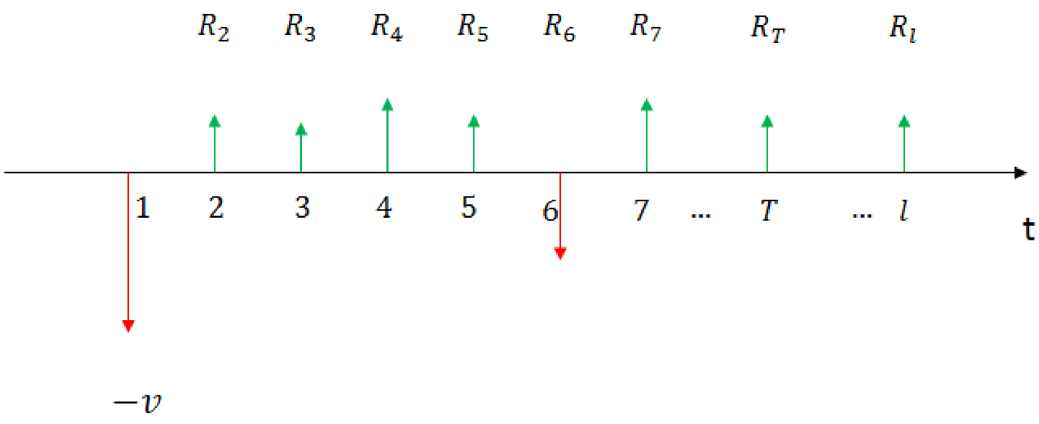

Figure 3.2: Example of cash flow of generator in FTE with a quantity contract.

We will follow here the convention that all flows occur at the beginning of the period.

For monthly revenues in a yearly period, we will discount all flows at the beginning of the year. For instance, let annual periods $t = 1, \ldots, T$ and monthly periods be denoted τ. The revenue R_t of a generator, in each yearly period, is

$$R_t = \sum_{\tau=12(t-1)+1}^{12t} fFECx^{sell} h_\tau + \pi_\tau (G_\tau - FECx^{sell} h_\tau) =$$
$$fFECx^{sell} \sum_{\tau=12(t-1)+1}^{12t} h_\tau + \sum_{\tau=12(t-1)+1}^{12t} \pi_\tau (G_\tau - FECx^{sell} h_\tau). \quad (3\text{-}4)$$

With a slight approximation, say the number of hours in a year equals $h = \sum_{\tau=1}^{12} h_\tau$, disregarding leap year differences. Then we see that the forward contract cash flow is equal every year. This steady cash flow is denoted Annuity[1] and may be discounted to its present value, as in Figure 3.3. The annuity factor α is a function of the cash flow duration (usually equal to lifetime l), and gives us relation

$$\alpha fFECx^{sell}h = \frac{fFECx^{sell}h}{(1+r)} + \frac{fFECx^{sell}h}{(1+r)^2} + \ldots + \frac{fFECx^{sell}h}{(1+r)^l}. \qquad (3\text{-}5)$$

Using the relation in equation (3-5) for α and considering the capital expenditures v, we may represent the fixed part of equation (3-3) by

$$\alpha fFECx^{sell}h - v \qquad (3\text{-}6)$$

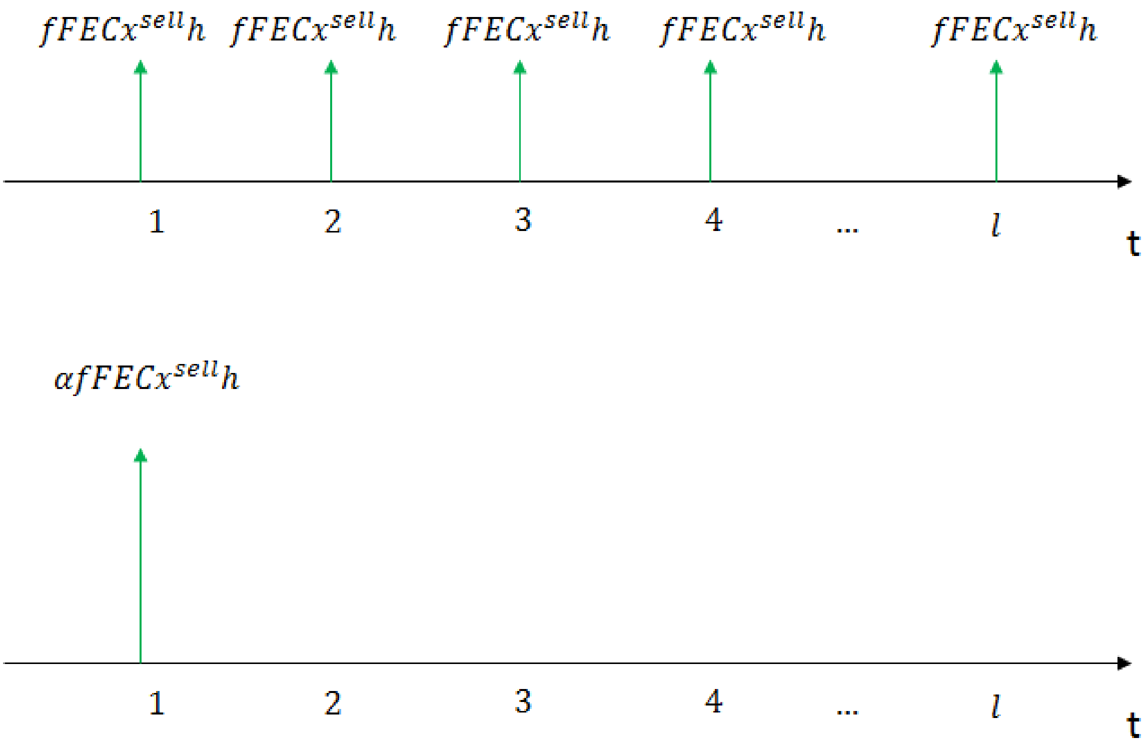

Figure 3.3: Equivalence of the Annuity and the net present value of its cash flows. The NPV is the same for both displayed cash flows.

It is worth reminding that, when cash flows are obtained by simulation procedures, since we still evaluate the expected free cash flows of each period, we must use the proper discounting rate of similar projects. As explained in

[1] an Annuity paying 1\$ for n periods has a present value of $\frac{1-(1+r)^{-n}}{r}$\$ for a given discount rate r.

(27), simulation may help improve the accuracy of the DCF analysis, but it does not price the risk, so one may not discount the cash flows using the risk free rate, we must still use the risk-adjusted rate of the traditional DCF method.

3.1.2 Decision Trees

In several investment projects we may exploit managerial flexibilities that occur during the project execution and operation, improving the opportunity's value. In those cases, the DCF analysis may be augmented with decision trees. While in DCF analysis we assume a unique expected cash flow during the future periods, in practice, conditional on information that reveals through time, a decision maker will take decisions that will lead to alternative cash flow possibilities.

In decision making under uncertainty, there can be more than one decision stage. In this situation, there is a first decision stage, before any information is revealed. After some information gets revealed, the second decision stage happens, and so on. This process may repeat for several stages.

A decision tree is a illustrative way to represent this dynamics. In the tree, there are information nodes (usually represented by a circle), were new information may be revealed, and decision nodes (usually portrayed as a square), were decision stages are represented.

In the information nodes there can be as many branches as necessary to represent different scenarios, and we associate probabilities to them. In the decision nodes each branch represent a different decision alternative. Its is then clear that decision trees are usually applied to problems with discrete number of scenarios and decisions.

In a dynamic decision making environment, information is revealed through time. In this context, it is usual to define a sequence of σ-algebras, $\mathfrak{F}_1 \subseteq \mathfrak{F}_2 \subseteq \ldots \subseteq \mathfrak{F}_n$, where every σ-algebra in the sequence contains the sets of the previous σ-algebra. We call this sequence a filtration. The filtration records the information available at each moment t, by the σ-algebra $\mathfrak{F}_t$, analogous to information nodes in the decision tree. For more details, please refer to Appendix A.

Let's take as an example a three period (or stage) decision problem. In this example, the uncertainty sources are the rain volume (High or Low) and the amount of available generation energy (High or Low). This uncertainty is represented by four scenarios:

1. ω_1: High rain volume, High generation

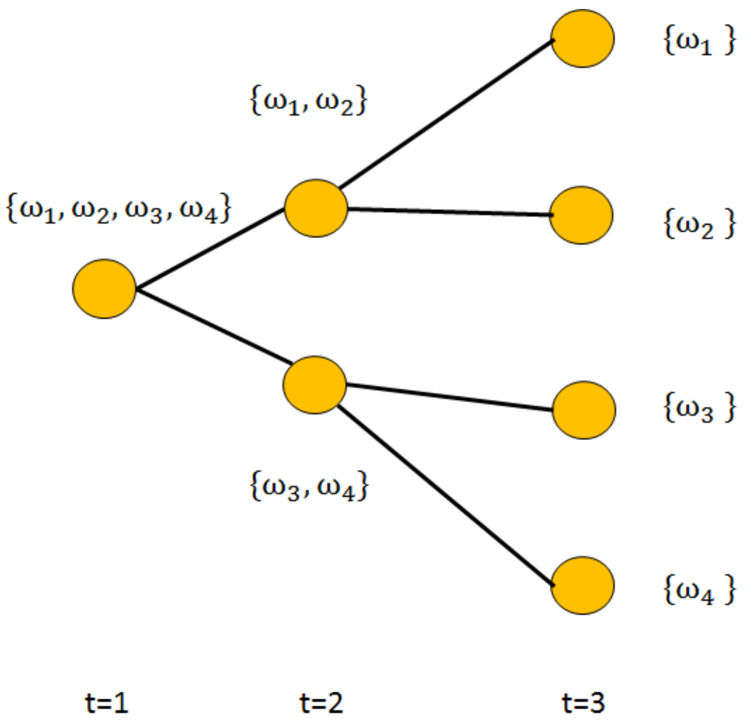

Figure 3.4: Scenario tree.

2. ω_2: High rain volume, Low generation

3. ω_3: Low rain volume, High generation

4. ω_4: Low rain volume, Low generation

In the second period uncertainty about rain is revealed, and generation uncertainty is revealed in the third period. Figure 3.4 represents the associated scenario tree. The filtration composed by σ-algebras $\mathfrak{F}_1 := \{\emptyset, \Omega\}, \mathfrak{F}_2 := \{\emptyset, \Omega, \{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}\}$ and $\mathfrak{F}_3 := 2^\Omega$ represents information available at each moment. In the second period, given probability measure $P : \mathfrak{F}_2 \to [0, 1]$, it is possible to measure the probability of low rain. Given a functional $X : \Omega \to R$ $\mathfrak{F}_2$-measurable, then $X(\omega_1) = X(\omega_2)$ and $X(\omega_3) = X(\omega_4)$.

Decision trees allow us to visually represent this formal information structure. We present in Figure 3.5 a sample renewable investment problem.

In this example, one must decide between investing today or waiting for better contract prices. If one decides to invest immediately, there is 70% probability of low spot prices during the project lifetime, with profit \$2.000,00. If, on the other hand, high spot prices occur, the investor will incur in a loss of \$1.000,00. By waiting, there is 80% probability of higher contract prices. In this case, since spot and contract prices are correlated, there is only 60% of *conditional probability* of low spot prices during the lifetime of the deferred project, with profit $1.000,00. In case spot prices turn up high, losses will be $3.000,00. If, on the other hand, contract prices fall, investment has 80% probability of profiting $4.000,00, from lower spot prices and 20% probability of $2.000,00 loss by high spot prices. If he waits, the decision maker may decide to abandon the project with zero costs.

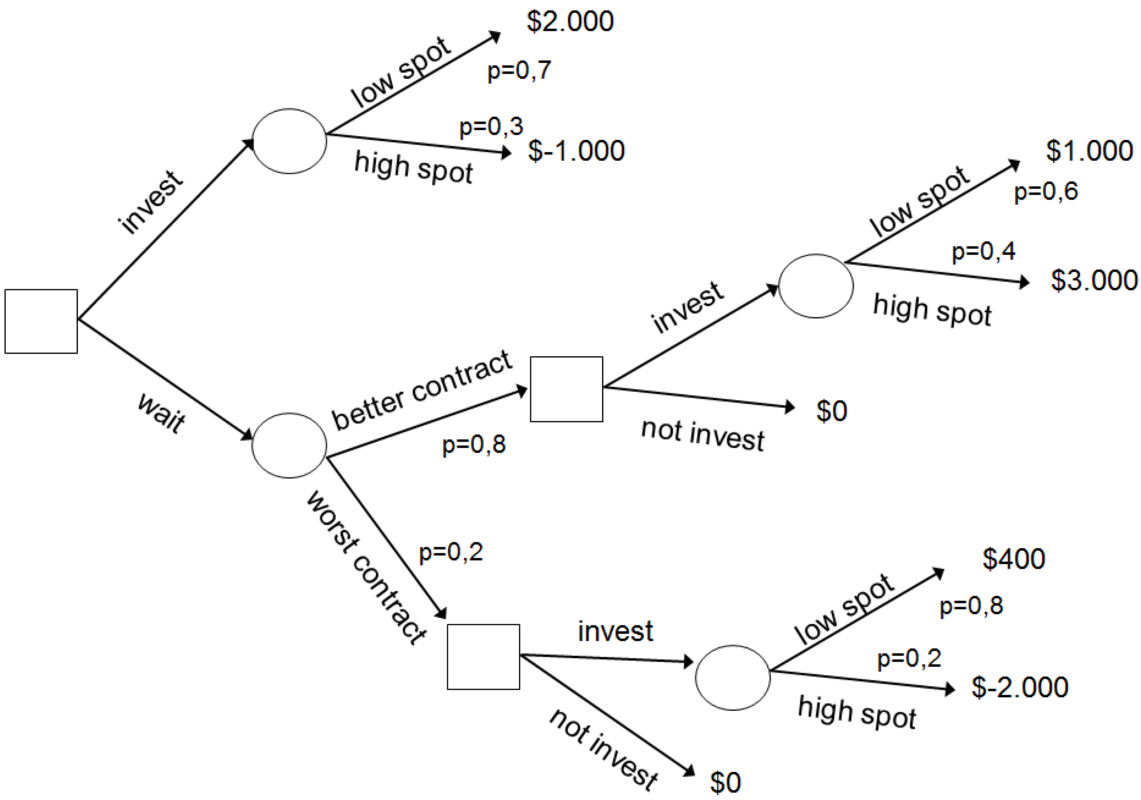

Figure 3.5: Decision tree example.

We solve this by dynamic programming, evaluating terminal nodes in each alternative. Nodes are valued by their expected value and we choose the higher valued option in each decision node. We proceed to the nodes in the previous stages, recursively. An optimal strategy is given by the alternatives with higher value in each decision node. In Figure 3.6 we present the optimal strategy in red.

This strategy applies to a risk neutral investor. We will see that we can incorporate risk aversion to the problem.

Next, we see how to solve large decision under uncertainty problems.

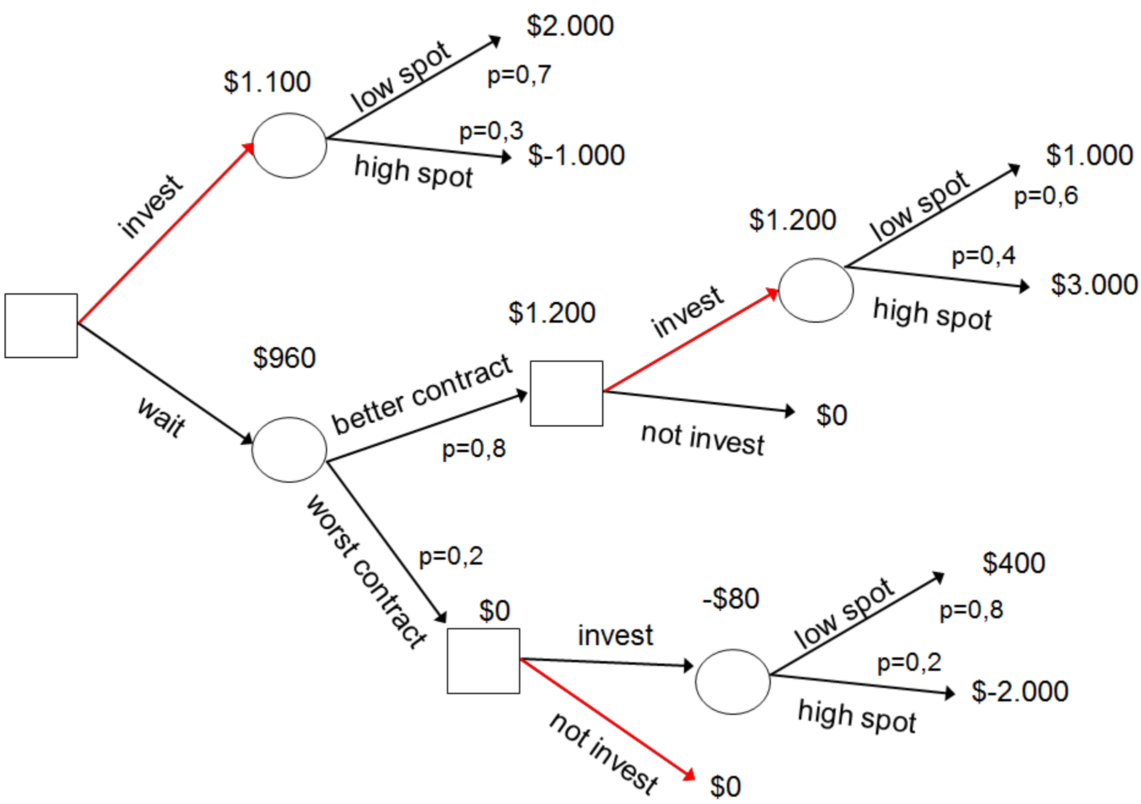

Figure 3.6: Solution of decision tree example.

3.2 Multistage stochastic problems

Multistage stochastic programming allows us to define and solve larger decision problems under uncertainty. Following (96) and (1), we define a (linear) multistage stochastic problem as

$$\underset{\substack{A_1 x_1 = b_1 \\ x_1 \geq 0}}{\text{Min}} \; c_1^{\mathsf{T}} x_1 + \mathbb{E}_{\xi_2}\left[\underset{\substack{B_2 x_1 + A_2 x_2 = b_2 \\ x_2 \geq 0}}{\min} c_2^{\mathsf{T}} x_2 + \mathbb{E}_{\xi_3|\xi_2}\left[\cdots + \mathbb{E}_{\xi_T|\xi_{T-1}}\left[\underset{\substack{B_T x_{T-1} + A_T x_T = b_T \\ x_T \geq 0}}{\min} c_T^{\mathsf{T}} x_T \right]\right]\right] \tag{3-7}$$

where vectors c_t, b_t and matrices A_t, B_t are random variables from the stochastic data process $\xi_t = (c_t, A_t, B_t, b_t)$, $t = 2, ..., T$, with $\mathbb{E}_{\xi_t|\xi_{t-1}}$ denoting expectation operation in stage t conditional to information available up to stage $t-1$ and $\xi_1 = (c_1, A_1, b_1)$ deterministic and a usually finite number of scenarios.

While small sized problems may be solved by linear programming, in general this is not the case. There are several methods in the literature to try to decompose the problem based in its structure (61), (10), (21).

An interesting example of special structure is when multistage problems have the block-separable recourse property (61). In (11), this property is studied in capacity expansion problems where decision to expand capacity can be made at the beginning and then the future only involves reactions to these outcomes. This allows representing the model as a two level problem: a first stage with aggregate level decisions with all investment decisions, and a second stage composed of detailed level decisions with the operations.

In this type of multistage problem, decision vectors x_t may be divided in x_t^a, representing aggregate level (investment) decisions, and detailed level decisions (operation) x_t^d. Also, it is required that, for all t:

- cost vectors c_t can be partitioned such that $c_t^\intercal x_t = c_t^{a\intercal} x_t^a + c_t^{d\intercal} x_t^d$.
- matrices A_t are block diagonal:

$$A_t = \begin{bmatrix} A_t^a & 0 \\ 0 & A_t^d \end{bmatrix}. \tag{3-8}$$

- B_t and b_t can be partitioned such that:

$$B_t = \begin{bmatrix} B_t^a & 0 \\ B_t^d & 0 \end{bmatrix} \text{ and } b_t = \begin{bmatrix} b_t^a \\ b_t^d \end{bmatrix}.$$

This bilevel decomposition allows for the usage of several efficient decomposition methods, such as bundle methods (74).

Notice that problem 3-7 may be decomposed into the main problem

$$\begin{aligned} \underset{x_1 \geq 0}{\text{Min}} \quad & c_1^\intercal x_1 + \mathbb{E}_{\xi_2}\left[Q_2(x_1, \xi_2)\right] \\ \text{s.t.} \quad & A_1 x_1 = b_1, \end{aligned} \tag{3-9}$$

where the cost-to-go functions $Q_t(x_{t-1}, \xi_t)$ for stages $t = 2, ..., T$ are defined by

$$\begin{aligned} Q_t(x_{t-1}, \xi_t) := \underset{x_t \geq 0}{\text{Min}} \quad & c_t^\intercal x_t + \mathbb{E}_{\xi_{t+1}|\xi_t}\left[Q_t(x_t, \xi_{t+1})\right] \\ \text{s.t.} \qquad\qquad & A_t x_t = b_t - B_{t-1} x_{t-1} \end{aligned} \tag{3-10}$$

and $Q_{T+1}(x_{t-1}, \xi_t) := 0$.

Solutions to problem (3-10) are functions of previous stage solution and the realized data process. Such solutions $x_t(x_{t-1}, \xi_t)$ are denominated *policies.*

When the data process is stagewise independent, conditional expectation in problem (3-10) becomes the usual (unconditional) expectation and may be solved by the SDDP method, introduced by (78) and (77). The SDDP is an *approximate dynamic programming* method. The basic idea of the SDDP algorithm is to approximate the cost-to-go functions by piecewise linear functions $\mathfrak{Q}_t^k(x_{t-1}, \xi_t)$ going backward and forward in each iteration k of the algorithm. With the improvement of the cost-to-go functions, one should expect to obtain better policies with each iteration of the method. More details in (91).

The forward step performs two functions: creating candidate solutions for the next backward step iteration and evaluation of the policy value by statistical sampling.

The backward step evaluates candidate solutions obtained in the forward step and includes new cuts to the piecewise linear approximation of the cost-to-go function. Since the problem is stagewise independent, evaluating the policy for a sample $\xi_t^n, n = 1, \ldots, N_t$ allows us to compute new cuts using the dual solutions $\widetilde{\Pi}_{tn}$.

In the SDDP method, the backward step is responsible for improvement of the policies quality. The forward step allows us to evaluate the quality of the policies obtained so far and most stopping criteria are based in in the forward step results.

In Algorithm 1 we outline the SDDP method, as done in (96) and (1). As we will see in the following sections, we may use additional criteria to choose between different policies evaluated in the forward step, particularly in a risk averse setting, when there should be different policies to choose from.

3.2.1 Discounted Cash Flow with Risk Measure

Increased risk management usage by the industry led to a demand for better risk measurement and management tools. This necessity has driven the development of several risk measures by the academy over the past two decades. The seminal work of (4) defined the axiomatic concept of coherent measures of risk. Formally, let $\mathcal{X}$ denote a linear space of financial positions $X : \Omega \to \mathbb{R}$.

A function $\varrho : \mathcal{X} \to \mathbb{R} \cup \{\infty\}$ is called a coherent risk measure when:

a) Subadditivity : $\varrho(X + Y) \leq \varrho(X) + \varrho(Y)$

b) Positive Homogeneity: If $\lambda \geq 0$, then $\varrho(\lambda X) = \lambda \varrho(X)$

c) Monotonicity: If $X \leq Y$ (that is, if $X(\omega) \leq Y(\omega), \forall \omega \in \Omega$), then $\varrho(X) \geq \varrho(Y)$

d) Translation Invariance: If $m \in R$, then $\varrho(X + m) = \varrho(X) - m$

An example of a coherent risk measure is the Average Value at Risk (or Conditional Value at Risk[2]), usually denoted AV@R, defined as

[2]we avoid the Conditional Value at Risk terminology because it can be confusing when dealing with conditional expectations.

Algorithm 1 SDDP algorithm

Require: $\{\mathfrak{Q}_t^0\}_{t=2,\ldots,T+1}$(initial approximations) and $\epsilon > 0$

Initialize: $i \leftarrow 0$, $\bar{z} = \infty$ (Upper bound), $\underline{z} = -\infty$ (Lower bound)

while $\bar{z} - \underline{z} > \epsilon$ **do**

  Sample M scenarios: $\left\{\{c_{tk}, A_{tk}, B_{tk}, b_{tk}\}_{2\le t\le T}\right\}_{1\le k\le M}$

  Forward step:

  for $k = 1, \ldots, M$ **do**

    for $t = 1, \ldots, T$ **do**

      $\bar{x}_t^k \leftarrow \arg\min_{x_t \ge 0} \left\{ \begin{array}{l} c_{tk}^\top x_t + \mathfrak{Q}_{t+1}^i(x_t) : \\ A_{tk}x_t = b_{tk} - B_{tk}x_{t-1} \end{array} \right\}$

    end for

    $\vartheta_k \leftarrow \sum_{t=1}^{T} c_{tk}^\intercal \bar{x}_t^k$

  end for

  Upper bound update:

  Let $\tilde{\vartheta}_M = \frac{1}{M}\sum_{k=1}^{M}\vartheta_k$ and $\tilde{\sigma}_M = \frac{1}{M-1}\sum_{k=1}^{M}(\vartheta_k - \tilde{\vartheta}_M)^2$

  $\overline{z} \leftarrow \tilde{\vartheta}_M + z_{\alpha/2}\frac{\tilde{\sigma}_M}{\sqrt{M}}$

  Backward step:

  for $k = 1, \ldots, M$ **do**

    for $t = T, \ldots, 2$ **do**

      for $n = 1, \ldots, N_t$ **do**

        $\left[\widetilde{Q}_{tn}(\bar{x}_{t-1}^k), \widetilde{\Pi}_{tn}^k\right] \leftarrow \min_{x_t\ge 0} \left\{ \begin{array}{l} c_{tn}^\top x_t + \mathfrak{Q}_{t+1}^i(x_t) : \\ A_{tn}x_t = b_{tn} - B_{tn}\bar{x}_{t-1} \end{array} \right\}$

      end for

      $\widetilde{\mathcal{Q}}_t(\bar{x}_{t-1}^k) := \frac{1}{N_t}\sum_{n=1}^{N_t}\widetilde{Q}_{t,n}(\bar{x}_{t-1}^k)$; $\tilde{g}_t^k := -\frac{1}{N_t}\sum_{n=1}^{N_t}\tilde{\Pi}_{tn}^k \widetilde{B}_{t,n}$

      $\mathfrak{Q}_t^{i+1} \leftarrow \{x_{t-1} \in \mathfrak{Q}_t^i : -\tilde{g}_t^k x_{t-1} \ge \widetilde{\mathcal{Q}}_t(\bar{x}_{t-1}^k) - \tilde{g}_t^k \bar{x}_{t-1}^k\}$

    end for

  end for

  Lower bound update:

  $\underline{z} \leftarrow \min_{x_1\ge 0}\left\{c_1^\top x_1 + \mathfrak{Q}_2(x_1) : A_1x_1 = b_1\right\}$

  $i \leftarrow i + 1$

end while

$$AV@R_\beta(X) = \frac{1}{1-\beta}\int_0^{1-\beta} V@R_s(X)ds \tag{3-11}$$

for $\beta \in (0,1)$, where $V@R_\beta[X] := \inf\{t : F_X(t) \ge 1-\beta\}$, with $F_X(\cdot)$ being the cumulative distribution function of the random (profit) variable X. It can be equivalently represented by

$$AV@R_\beta[X] := V@R_\beta[X] + \beta^{-1}\mathbb{E}\big[X - V@R_\beta[X]\big]_-. \tag{3-12}$$

Here $[a]_- = \min\{0, a\}$. This equivalent formulation holds under mild conditions as introduced in (86). For a strict treatment of risk measures, see (40).

AV@R has drawn a lot of attention in the literature not only because it is a simple and intuitive example of a coherent risk measure but also because it is easily represented in linear optimization problems. An investor using AV@R in a discounted cash flow valuation will be indifferent between the uncertain cash flows and its AV@R certainty equivalent on any period. Recursive valuation with AV@R will result in a composition of AV@R, what guarantees the time consistency of the decisions, as seen in (87), (90). Notice that, since we are evaluating certainty equivalents with the risk measure, we must discount the cash flows in the risk neutral measure (50).

Let's consider again the example decision tree in Figure 3.6. Now, we will solve it as a decision maker that valuates with AV@R at $\beta = 0.9$. At each node, we will value the AV@R *conditional* to the information revealed so far. As in the expected value case, we will use the conditional probabilities of the node to evaluate the conditional AV@R. Notice that, in this instance, a node will be valuated by the worst scenario whenever it has probability higher than 0.1. Every subsequent decision node is evaluated by another conditional AV@R, so in practice we are using a *nested AV@R* evaluation criterion. The optimal strategy is displayed in Figure 3.7. Notice that risk aversion induces the investor to wait for better contract opportunities in this example.

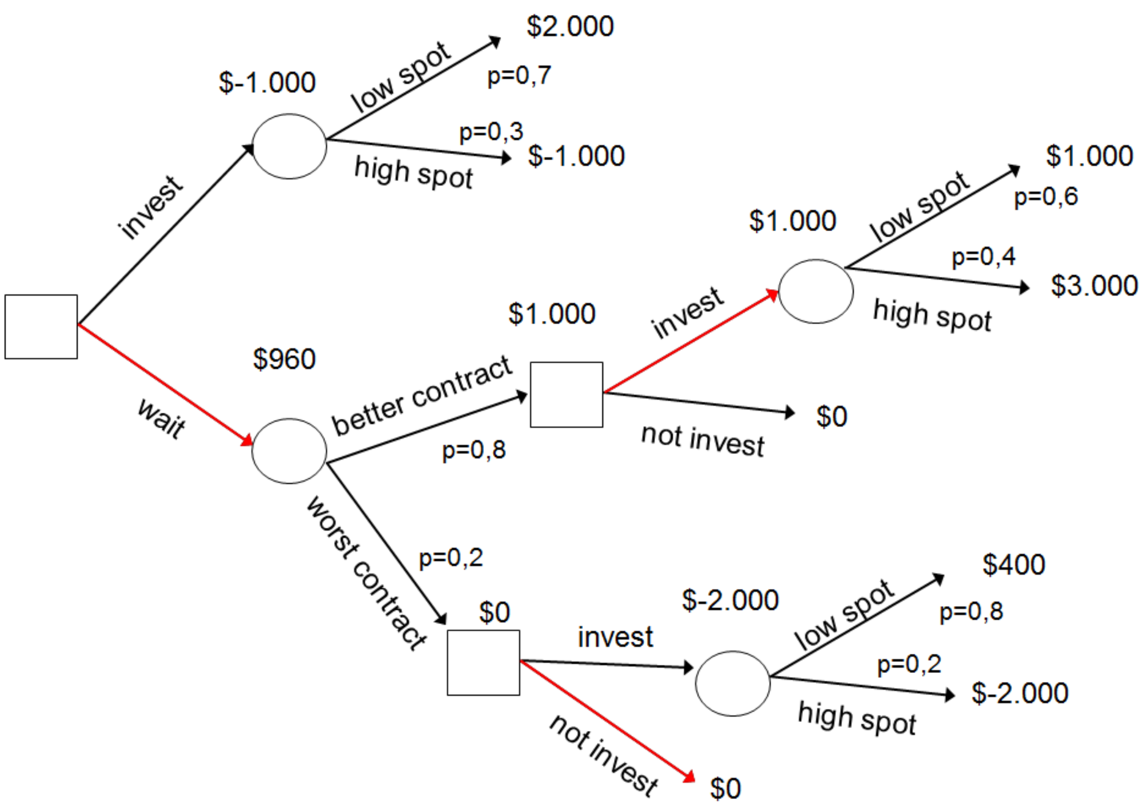

Figure 3.7: Solution of scenario tree example for risk averse decision maker, considering a AV@R function with $\beta = 0, 9$.

Other valuation methods, such as relative valuation, may be found in (27).

Next, we will see how portfolio and risk management tools may help defining the company's strategy.

3.3 The Mean Variance Portfolio Model

Modern Portfolio Theory began with the seminal work of Harry Markowitz, in (68, 69). So far, there were no asset allocation models to help investors defining a stock portfolio accounting both to expected returns and risk of the assets.

Markowitz proposed a simple yet insightful improvement over the existing approaches by assuming that asset (linear) returns followed a multivariate normal distribution. In this context, the return of any portfolio composed by a basket of such stocks would also follow a normal distribution. Those portfolios could be distinguished by two return metrics, mean and variance (or standard deviation).

Clearly, two portfolios could present the same expected return but different variance. This dispersion measure was used as a *proxy* for the portfolio risk, since larger variance means larger uncertainty (or volatility) of the returns.

In the framework proposed by Markowitz, a decision maker must define $\hat{r}$, his desired level of portfolio expected return. Considering M assets with multivariate normal returns $r \sim N(\mu, \Sigma)$, the weights w of the assets in the portfolio are given by the mean-variance portfolio optimization problem

$$\underset{w}{\text{Min}}\, w^T \Sigma w \tag{3-13}$$

$$e^{\mathsf{T}} w = 1 \tag{3-14}$$

$$\mu^{\mathsf{T}} w = \hat{r}. \tag{3-15}$$

The portfolios w that solve problem (3-13)-(3-15) have minimum risk (as measured by variance) for a given expected return. Notice that this problem allows short-selling (negative w_i) for some asset $i \in M$. Alternative formulations might restrict short-selling, introduce budget constraints, among others.

One of the great advantages of this simple approach is that this model can be solved analytically. Taking variance as a function of expected returns, solutions lie on the boundary of a hyperbolic feasible set.

By changing the required expected return $\hat{r}$, we obtain the set of portfolios known as *efficient frontier*. Any feasible portfolio that is not in the efficient frontier has more risk than an efficient portfolio with same expected return, thus, it is Pareto dominated. A rational decision maker would only choose a portfolio that lies in the efficient frontier. The set of feasible portfolios and the efficient frontier are presented in Figure 3.8.

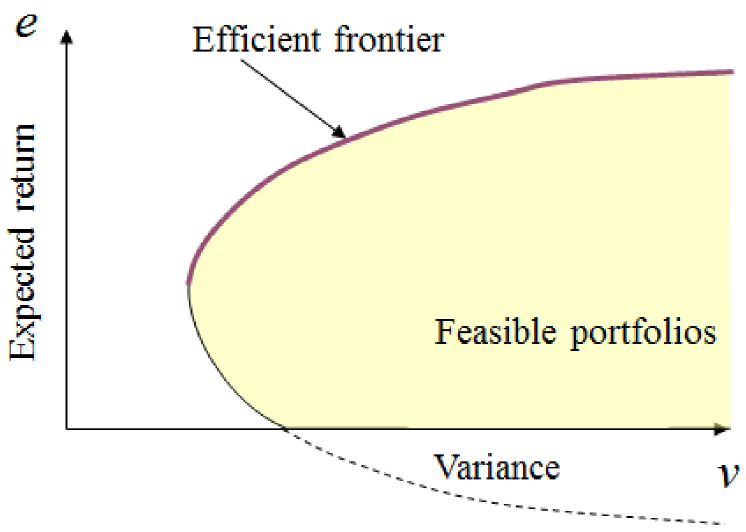

Figure 3.8: Mean-variance efficient frontier

The framework devised by Markowitz has great relevance, since it defined the main notion of modern portfolio theory: a rational decision maker must balance how much risk he is willing to take in order to have additional return. That is, in order to have higher return that a given efficient portfolio, one must incur in additional risk.

The highest return might be achieved by investing all the resources in the asset with higher expected return. This would also be the efficient portfolio with highest risk. On the other hand, the minimum variance portfolio usually is composed of several assets, because of the reduced portfolio variance that can be enjoyed by exploiting the (possibly negative) correlation of the assets. Thus, we also learn from the mean-variance model that *diversification is a key aspect to reducing risk.*

As an example, consider N assets with the same distribution $x_i \sim N(\mu, \sigma), i = 1, ...N$, having the same covariance *cov* for every pair of different assets. If one composes a portfolio P of equally weighted assets, the average of the return r_p of this portfolio is

$$\mathbb{E}[r_p] = \mathbb{E}\left[\sum_{i=1}^{N} \frac{1}{N} r_i\right] = \frac{1}{N}\left[\sum_{i=1}^{N} \mathbb{E}[r_i]\right] = \frac{N}{N}\mu = \mu, \quad (3\text{-}16)$$

with no benefits to expected returns. Portfolio variance σ_p^2, on the other hand, is given by

$$\sigma_p^2 = \frac{1}{N^2}\sum_{i=1}^{N}\sigma^2 + \sum_{i=1}^{N}\frac{1}{N^2}\sum_{j=1,j\neq i}^{N} cov = \frac{1}{N}\sigma^2 + \frac{N-1}{N}cov, \tag{3-17}$$

leading to a reduction in the portfolio variance as long as the assets are not perfectly correlated. As the number of assets in the portfolio increase, the relevance of volatility of the individual assets decreases and portfolio volatility becomes mostly a function of the assets correlation.The portfolio risk can only be eliminated if there are negative perfectly correlated assets. The part of the risk that cannot be eliminated by diversification is called *systematic risk*. Figure 3.9 below presents the effect of diversification in our example.

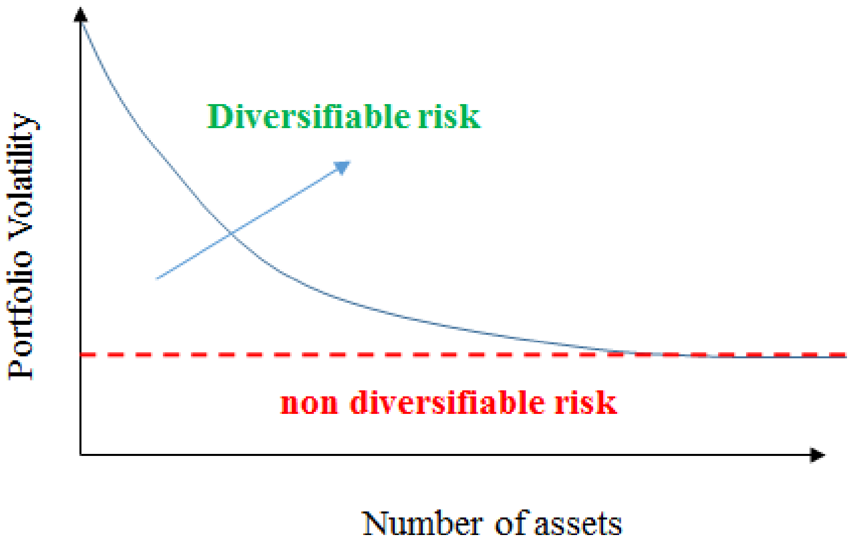

Figure 3.9: Effect of diversification as a function of the number of assets in a portfolio

In practice, Markowitz's model has several drawbacks. First, asset returns present stylized facts that depart from the normal distribution assumption. Fat tails exists for most assets and asymmetry is expected, for instance, if dealing with derivatives. Variance includes both positive and negative risk and would penalize asset's upsides. Markowitz himself acknowledged that problem and later proposed an enhancement, using semi-variance (semi-standard deviation) as the risk proxy.

Estimation of asset expected returns is also one of the major pitfalls of the Markowitz approach. It turns out that the point estimator of the returns is very unstable, often failing the null hypothesis of being different from zero on a test. Lack of appropriate estimates of the expected returns results in over concentration in a few assets. Also, when reestimating the model's parameters in a future period, very different estimates are expected, leading to aggressive rebalancing of the portfolio. Recently, there has been plenty of research in the Robust Optimization field, with models that circumvent the expected returns volatility by defining a so-called uncertainty region.

The Capital Asset Pricing Model, detailed in Appendix B, manages to bypass these estimation problems. It introduces the Security Market Line (SML), a equation that provides a relation of the expected returns $\mathbb{E}[r_A]$ of any portfolio or asset A to the systematic risk of the market.

One advantage over the mean-variance framework is that its main results do not depend on estimating expected returns over historical data, so expected returns obtained by the SML are more reliable than those obtained using past data. Also, it makes clear that investors are only rewarded with higher expected returns by their exposure to systematic (or market) risk. Diversifiable risk is not rewarded by investors.

An important contribution of this model is that investors may manage the level of risk in their portfolios, thus deciding the risk exposure they want to assume by changing the weights of the assets they hold, independently of any risk management efforts made by the company itself.

Despite the fact that investors may manage their portfolio risk, there is still value for risk management efforts of the firm. More details in Appendix C.

Given that we understand the value of risk management, what would be an appropriate market investment strategy for an investor? First of all, he would probably try to choose an efficient portfolio, in order to avoid any unnecessary risk taking. There is still the question of which portfolio in the frontier to choose. An investor who is willing to choose a portfolio on the CML might choose the portfolio with highest expected return that respects his *risk appetite*, since by this approach he maximizes his return on a diversified portfolio while staying solvent given some confidence level.

Risk appetite is defined in (23) as *the degree of risk, on a broad-based level, that a business is willing to accept in pursuit of its objectives*[3]. Defining the risk appetite is a non trivial question that encompasses the Enterprise Risk Management framework, that we discuss in Section 3.5.

For firms determining their investment portfolios, an efficient frontier from market assets might not be available, since several of those investment opportunities represent unique projects that are particular to this firm (what

[3]This should not be confused with a risk seeking investor, i.e., an investor who would prefer an uncertain cash flow rather than its expected value, or, equivalently, has a convex utility function.

also allows them to exercise real options over those opportunities). In this context, a usual approach to compare assets or portfolios is the use of performance measures.

3.4 Risk-Adjusted Performance Measurement

The measures in this section follow (52).

The family of Risk-adjusted performance measures (RAPM) is usually defined as a measure of *Profit*, in monetary unit, divided by a measure of *Risk*, usually also denoted in some monetary risk measure,

$$RAPM = \frac{Profit}{Risk}. \tag{3-18}$$

There are a few exceptions, such as the Jensen's alpha, which we will not explore. RAPM may be used to evaluate a portfolio, an asset, a company or its business units. We will, without loss of generality, assume that we are measuring the performance of a portfolio P with uncertain return R_p.

Figure 3.10 illustrates the benefits of using a RAPM. On the left hand side it is rather straightforward to pick a best portfolio: portfolio A dominates (by second order stochastic dominance) portfolio B. Nonetheless, on the right hand side, even tough both distributions are symmetric, there is no clear consensus over which portfolio should be chosen.

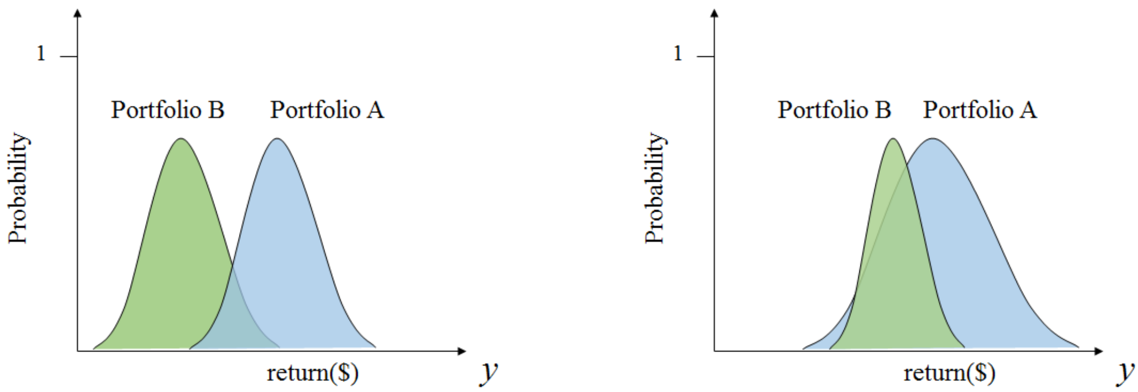

Figure 3.10: In left hand side portfolio A presents stochastic dominance over portfolio B. In right hand side no such feature is present.

3.4.1 Sharpe Ratio (SR)

The Sharpe ratio (SR) measures the ratio between average return of the portfolio, in excess of the risk-free rate, and the volatility of the portfolio. It is defined as

$$SR = \frac{\mathbb{E}[R_p - r_f]}{\sigma(R_p)}, \tag{3-19}$$

where r_f is the risk-free rate of return and σ_p the portfolio volatility, measured by standard deviation. It was developed by Sharpe in 1966 (52) to aid in portfolio evaluation.

In the mean-variance or CAPM framework, the tangent or market portfolio is the portfolio that presents the maximum Sharpe Ratio.

In 1994 Sharpe reviewed the SR and suggested the generalization

$$SR = \frac{\mathbb{E}[R_p - R_X]}{STD(R_p - R_X)}, \tag{3-20}$$

where $STD(\cdot)$ stands for the standard deviation operator and X is some benchmark portfolio. The SR is unchanged if the benchmark portfolio is the risk free asset.

3.4.2 Sortino Ratio (SO)

A rather obvious improvement to the SR is to consider only the downside of volatility. The Sortino Ratio (SO) replaces standard deviation by the semi standard deviation,

$$SO = \frac{\mathbb{E}[R_p - r_f]}{\sigma_-(R_p)}, \tag{3-21}$$

where $\sigma_-(\cdot)$ denotes the semi standard deviation. It presents a clear advantage in case of asymmetric returns.

3.4.3 Treynor Ratio (TR)

The previous risk measure accounts only for losses, but still prices diversifiable risk. Since the CAPM shows that the market only prices systematic risk, it is relevant to consider only those risk sources. The Treynor Ratio is defined as

$$TR = \frac{\mathbb{E}[R_p - r_f]}{\beta_p}, \tag{3-22}$$

where β_p is the beta of the portfolio. If the CAPM equation holds, then all portfolios should present the same TR. Then, the TR is a better risk measure for a well diversified portfolio.

3.4.4
Risk Adjusted Return on Capital (RAROC)

RAROC was developed in Banker's Trust between late 1970's and early 1980's. The idea was to have an unified measure to evaluate the risks of the different business units. RAROC is originally defined for an asset X as

$$RAROC = \frac{\mathbb{E}[Revenues(X) - Cost(X) - Losses(X)]}{EC(X)}, \tag{3-23}$$

where $EC(X)$ is the Economic Capital, also known as risk capital, associated with X.

Economic Capital is defined by (52) as the largest acceptable loss a firm is willing to suffer over a specified period and at a specified confidence level. This may be estimated by a V@R or AV@R type measure. The level should be defined by senior management and is directly tied to the risk appetite of a company. In practice, economic capital is usually inferior to the equity of the company, but superior to regulatory capital. Regulatory capital exists especially in the banking industry, where there are enforced regulatory constraints that demand the company to assure enough capital to unexpected losses.

The RAROC may be in general interpreted as a risk adjusted version of Return on Capital (ROC).

One of the key benefits of using RAPM such as the RAROC is that these measures can be used across divisional level in an organization and then aggregated all the way to a company total. This allows management to:

– measure the risk adjusted profitability of different business segments;
– calculate the aggregate employed risk capital and redistribute it, if necessary; and
– motivate different units to search for in-company natural hedges. Since their performance is measured by the RAPM, their results can be enhanced by reducing the risk capital needs of their business. Combined risk capital of different segments should be smaller than the sum of the parts if there are natural hedges (in this case, it is recommended to use a subadditive risk measure).

The focus of the RAROC traditional definition is on the banking industry. An application of RAROC to project capital budgeting may be obtained by applying the following alternate definition: for a given project X, we define

$$RAROC = \frac{\mathbb{E}[NPV(X)]}{EC(X)}, \tag{3-24}$$

thus, RAROC here is the expected Net present value of the project divided by its associated Economic Capital (EC). In this case, economic capital is the sum of the planned investment and the net present Value at Risk (alternatively, a AV@R measure could be used). Here, Value at Risk is defined as the difference between expected value and a quantile associated with a confidence level.

The ideas presented so far are some of the key aspects of *Enterprise Risk Management* (ERM), or *Enterprise Wide Risk Management*, which has become very popular in the last decade.

3.5 Enterprise Risk Management

ERM is a holistic approach that focuses not only on strategic risk management, but in addressing risk management in every process of the firm. One of its goals is to avoid risk decisions being treated in business unit silos, with no central coordination, which might destroy value. ERM may be defined (23) as "a comprehensive and integrated framework for managing company-wide risk in order to maximize a company's value". Some of the benefits enlisted by the author are:

- increase the likelihood of a company realizing its objectives;
- Build confidence in stakeholders and in the investment community;
- Comply with relevant legal and regulatory requirements;
- Align risk appetite and strategy;
- improve organizational resilience;
- enhance corporate governance
- embed the risk process in the organization;
- enhance risk response decisions;
- optimize allocation of resources;
- identify and manage cross-enterprise risks;
- link growth, risk and return.

These statements are similar to the ones in COSO's Integrated Framework (25).

Enterprise Risk Management appeared as response to several risk exposure problems. Numerous examples of troubled companies due to incorrect use of derivatives, or bad risk and financial governance, led to several efforts to avoid such cases to repeat. ERM encompasses Strategic Risk Management, Credit Risk Management, Operational Risk Management, and others, in order to implement value creation using some framework.

The most known and applied framework is the COSO ERM Framework, by the Committee of Sponsoring Organizations of the Treadway Commission (COSO) (25). In practice there are several other frameworks, and usually companies will try to handpick some of the practices of more than one of them, according to their specific needs.

A value creating ERM process is Liquidity Risk Management. *Funding Liquidity* risk is defined by the Committee of European Banking Supervisors (108) as *the current or prospective risk arising from an institution's inability to meet its liabilities and obligations as they come due without incurring unacceptable losses* and also by the Basel Committee as *the risk that the firm will not be able to meet efficiently both expected and unexpected current and future cash flow and collateral needs without affecting either daily operations or the financial condition of the firm* (7).

An example of such a solution is the Liquidity Risk management framework (42). The authors show that by using standard Cash flow at risk models, it is possible to assess and mitigate the liquidity risk of an energy company.

It is beyond the scope of this work to present a complete overview of ERM, but we would like to focus on a key aspect of ERM: defining the company's risk appetite.

A common approach to support defining the risk appetite is measuring the company's Economic Capital.

Economic Capital is in practice an additional constraint to a company investment and operation decision. It may be seen as a risk version of the budget constraints that investors and companies are usually faced with. The process of distributing the available economic capital in a portfolio of assets is known as *risk budgeting.*

The firm's risk appetite and Economic Capital might be derived from a Cash flow at Risk or Liquidity Model (42).

It must be stressed that the firm should never try to hedge away all its risks, the excessive hedging would certainly lead to value destruction. Equivalently, it should know on which risks the company presents some competitive advantage and keep those risks, since it is the natural owner of the risk, i.e., other party would charge higher to accept this risk transfer.

According to (52), firms should use enterprise wide integrated risk management to take core, strategic risks, and any additional business risks where it can explore some value creation given differential skill possessed by the company. Remaining risks that steer the company away from its strategic objectives may be mitigated by some technique.

3.6 Risk Management tools

We now explain some of the most popular techniques to manage risk. Each one should be used according to the situation. A risk manager will probably make use of a mix of some of the options below.

3.6.1 Derivatives

There is a large number of derivative contracts available nowadays. With the increasing securitization of commodities and other markets, there is availability of calls, puts, collars, swaps, futures, and many more standardized contracts, as well as banking services that supply customized contract opportunities for a fee. Derivatives are mostly used to hedge short to midterm cash flow operations. Most of the derivatives used by regular companies are related to hedging foreign currency and interest rate risk. The remaining large markets for derivatives are related to commodities, such as oil, energy, gas and metals, such as gold. Currency derivatives have a wide interest of companies in general because of global markets, the companies do not want to allow volatility in exchange rates to jeopardize their strategies. Care must be taken with derivatives: larger exposition to derivatives may require substantial deposit margins. If the derivative price has high volatility, fluctuations might require immediate margin calls, which can be a severe unexpected cash demand. This has been the cause of some well-known financial disasters such as the Metallgesellschaft case (52).

3.6.2 (long term) forwards

Forwards are usually non standardized contracts traded Over the Counter. They are superior to futures for long term purposes because margin deposits can be avoided and longer periods can be arranged. An owner of a forward contract still is exposed to counterparty risk, the risk of the contract not being honored by the other party. Counterparty risk can be mitigated by diversification, signing several smaller contracts with different counterparties.

3.6.3
Diversification

Diversification, as mentioned before, is one of the most common and effective ways to hedge. As long as the assets do not have perfect (positive) correlation, some level of diversification benefit is possible. Diversification is also typically cheap, so there are few reasons not to pursue diversification, unless there is some constraint, such as legal requirements. Diversification can refer to pursuing different businesses but also geographical regions, dealing contracts with different counterparts,

3.6.4
Insurance

Insurance is one of the oldest known risk management tools. Typically a company will try to insure against risks identified with low probability of occurrence but with a high impact that might severely compromise its operations. Since an insurance company is able to diversify its portfolio of contracts, it can offer a reasonable price for taking this risk. Also, insurance companies have expertise in preventing losses, so they can provide valuable guidance on how to prevent loss events. It is worth noticing that large companies may prefer to practice self-insurance in some cases. If a large logistics operator decides not to insure is own fleet, he will probably spend less money with the expected losses in case of an event than the premium charged by insurers.

3.6.5
Partnerships

Partnerships are usually resorted to as a diversification technique. There are also some special benefits of partnership, for instance, when developing a project with a partner that has some technical expertise in some critical aspect of the project. In this case, the partnership will also mitigate a project risk.

3.6.6
Real Options

Real Options have been extensively studied as one of the most important managerial tools in project valuation over the past few decades, but they also may be seen as risk management tools. Since a company frequently has a portfolio of assets, under study and ongoing projects, there are several options that may be exploited in order to avoid exposition to market risks , such as

- postpone: most projects under consideration and some ongoing projects may be postponed to a later date. This may allow some uncertainty to be revealed, such as a regulatory change or a technological improvement, but also, in case the project is marginally profitable and subject to price uncertainty, may help the company decide whether go on with the project or abandon it.
- fast track: if the project is considered *deep in the money*, with high returns, it may be worth spending some additional money to hurry the investment phase and anticipate the delivery, so that early returns can be cashed in.
- abandon: whenever an asset or project becomes irremediably incapable of returning positive net cash flows, it may be best to stop the enterprise and assume some losses than to stick to the plan and endure years of negative results.
- hibernate: some assets, such as refineries, may have volatile margins, and during some low margin periods it may be wiser to stop operations until better market conditions present themselves. It must account for the costs of startup and maintenance during the hibernation period.
- switch: several industrial processes may enjoy from switching options. A very common option is switching fuels in generators, according to the fuel prices and efficiency of the machinery.
- expand: Some business can be expanded if market conditions seem favorable. A wind power plant can expand its capacity by adding a few more units to the park, using the existing infrastructure.

there are several other options available, each of them having different appeal according to the business in question.

In the next chapters, we will show how to use the aforementioned valuation and risk management instruments to build sound long term investment policies in renewable energy markets, specifically in the Brazilian regulatory context.

4
Strategic Risk Management Framework for Renewable Investment

In this Chapter we will formally define and model the main uncertainty sources associated to the investment in renewable energy. Then we will outline the proposed framework, explaining the alternatives that we will pursue in our investment policies. Finally, we will describe the suggested approach to compare the alternative policies.

The strategies might contain the possibility to postpone investment decision, in order to exploit new available information. The solution approach must incorporate such options in its model. Such multistage feature of the decision process is incompatible with the available data, provided as a deck of Monte Carlo simulations. Since the simulations do not include a filtration structure (as exemplified in Figure 4.1), we will have to try and recover or reproduce the filtration information in our solution approach.

4.1
Uncertainties in the Renewable Energy investment planning problem

Renewable energy investment projects have the advantage of not being subject to risk of market prices for their inputs, as, for example, it would be the case for a thermoelectric plant. On the other hand, the generator has no control over the renewable source and is exposed to its availability. A hydroelectric plant could use is reservoir to adapt its production profile to the demand, but this is not the case of run-of-the-river hydroelectric plants, nor wind generation.

The generator is also subject to sell energy in a market whose price has great variability. We will now describe in more detail the uncertainty sources of the problem and define models to the sources of uncertainty.

4.1.1
Spot prices

As mentioned in the introduction, the ONS publishes the results of its long-term model, NEWAVE (66), to market participants. The model gives

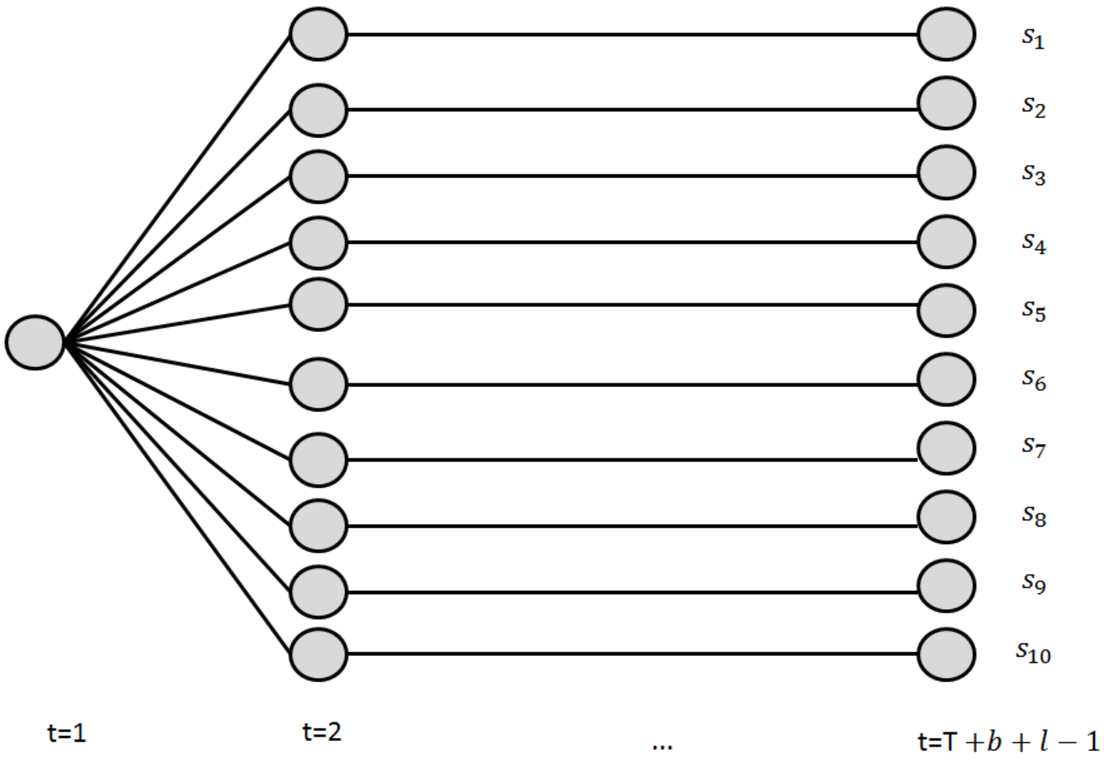

Figure 4.1: Monte Carlo samples do not reflect the filtration information.

a proxy of the energy spot price forecasts by balancing supply and demand during the planning horizon.

The NEWAVE model is a tool for long-term scenario modeling used by official Brazilian agencies. In the Brazilian regulatory model, NEWAVE is used to support long term planning studies, such as network expansion planing, more details in (65). The great advantage of using scenarios derived from the NEWAVE model is to incorporate the market dynamics of the future spot price, taking into consideration the current system structure, demand growth curves, and planned investment on the generation and transmission network.

As planned investments in this study are considered small in relation to the market at hand, we will assume that the entry of these investments does not influence the market price, being deemed a marginal generation. Large scale investments might influence the market balance. In this case, it would be more appropriate to apply a methodology based in equilibrium models or Game Theory, as discussed in (64).

Uncertainty of water inflows is represented in NEWAVE model by aggregating submarkets. Each submarket represents a region of the country in terms of hydrology and the transmission network. The market is currently divided into Northeast (NE), North (N), South (S) and Southeast (SE) submarkets. Spot prices resulting from marginal operating costs are also reported for each submarket, informed in monthly periods. The result of a NEWAVE optimization is accompanied by 2,000 Monte Carlo simulated series. For each simulated series s, we may obtain the inflows $I^k_{\tau,s}$ and spot prices $\pi^k_{\tau,s}$ of each submarket in k and month τ.

These stochastic processes $I^k_{\tau,s}$ and $\pi^k_{\tau,s}$ are considered in our model and we will use the time series data from the NEWAVE simulation results.

4.1.2
Forward prices

The forward contract price is another significant uncertainty. Following (26), future prices can be represented by the expected spot price plus a market risk premium. Thus, given our deterministic interest rate, the forward price $F_{t,T}$ at time t with maturity T may be represented by

$$F_{t,T} := \mathbb{E}_t\left[\pi_T\right] + \Lambda_t(T), \qquad (4\text{-}1)$$

where $\mathbb{E}_t[\cdot]$ denotes conditional expectation given information available in time t and $\Lambda_t(T)$ is a *risk premium* associated with the contract $F_{t,T}$.

We are not interested in contracts for a given maturity, but rather in contracts that guarantee a fixed price over the whole period of plant operation. Following (19), this contract f_t, with monthly deliveries from periods T_1 to T_n, may be priced by

$$f_t = \frac{\sum_{i=1}^{n} \frac{F_{t,T_i}}{(1+r_f)^{(T_i-t)}}}{\sum_{i=1}^{n} \frac{1}{(1+r_f)^{(T_i-t)}}}, \qquad (4\text{-}2)$$

where r_f is the risk free interest rate and $F_{t,T_1}, ..., F_{t,T_n}$ is a series of contracts with maturities $T_1, T_2, \ldots, T_n$.

The Brazilian energy market lacks a formal future market. Agents may define a forward curve based on their expectations (possibly using a service such as forward curve benchmarking by DCIDE[1]) or with the aid of a trading platform[2]. There is currently legislation in study to disclose prices of FTE contracts.

In Figure 4.2, we present the historic time series of the spot price and a forward contract in the Nordpool market. As can be seen, the spot price shows a much higher volatility, with some spikes. This behavior is similar in the Brazilian Market.

More details on the subject will be given in Chapter 5, where we develop our model to the forward prices.

[1] www.dcide.combr

[2] BRIX at www.brix.com.br, BBCE at http://www.bbce.com.br/

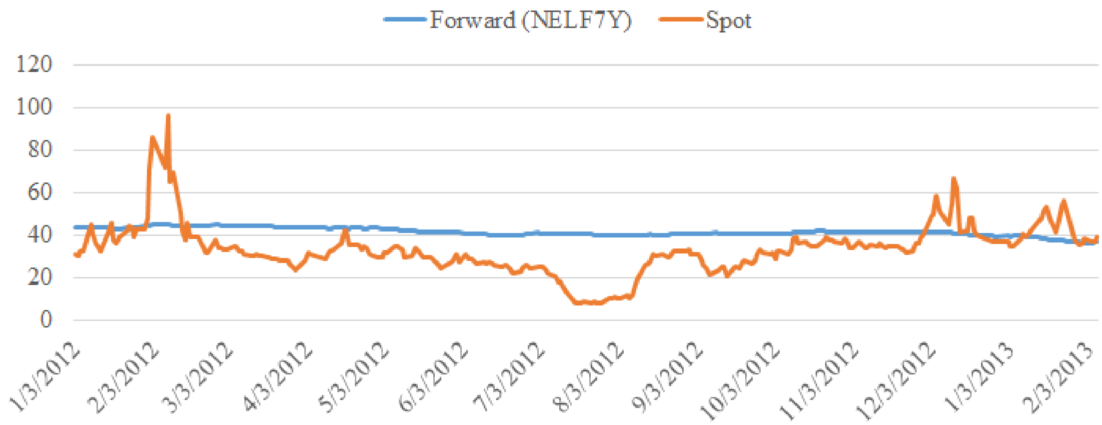

Figure 4.2: Spot and forward prices in the Nordpool market.

4.1.3 Renewable Generation

Generation of renewable energy sources is another important uncertainty to be modeled. The energy G^j_τ generated in the month t by a windpower renewable source j is given by:

$$G^j_\tau := h_\tau \left[C_j W^j_\tau \right]_+, \tag{4-3}$$

where $[a]_+ = \max\{a, 0\}$ for $a \in \mathbb{R}$, h_τ is the number of hours in period τ, C_j the nominal capacity and W^j_τ is the capacity factor of the plant j in period τ, a function of wind availability in the plant site. Monthly capacity factor may be obtained by evaluating the wind profile with the turbine power curve.

For a small hydro, G^j_τ is given by:

$$G^j_\tau := h_\tau \left[\min\{C_j, \kappa_j \zeta_j g W^j_\tau\} \right]_+, \tag{4-4}$$

where κ_j is the efficiency factor of the plant, ζ_j its falling height, g acceleration of gravity and W^j_τ is water inflow. We assume the small hydro has a small or no reservoir, so that energy output is directly defined by the river's inflow and there is no decision on the generation output profile.

Equations (4-4) and (4-3) are deterministic functions of the stochastic process W^j_τ. We will represent this process as a Vector Auto Regressive (VAR) model, where each process depends on itself and the submarket inflows. Since prices calculated by the NEWAVE model are also inflow dependent, any correlations between generation and spot prices would be represented, as done in (107).

For p autoregressive lags and q explanatory (inflow) variable lags, the model is defined for a given plant j as

$$W_\tau^j = \sum_{i=1}^{12} \gamma_i^j \delta_{i\tau} + \sum_{i=1}^{p} \phi_i^j W_{\tau-i}^j + \sum_{i=1}^{q} \sum_{k=1}^{4} \eta_{ik}^j I_{\tau-i}^k + \epsilon_\tau^j. \tag{4-5}$$

where the explanatory variables and its associated model coefficients are:

- : ϕ_i^j: the autoregressive coefficient for plant j in period i,
- γ_i^j: a seasonal dummy variable for each monthly period i in site j;
- $\delta_{i\tau}$: a dummy variable that assumes values 1 in the $i-th$ month of the year and 0 elsewhere, i.e., $\delta_{i\tau} = 1$ for $\tau = i + 12\ell$, $\ell = 0, 1, ...,$ and $\delta_{i\tau} = 0$ otherwise.
- I_τ^k: the inflow of market k at month τ,
- : η_{ik}^j: the regressive coefficient for inflow of market k j in period i.

Errors are assumed to be independent and normally distributed, ϵ_τ^j $N(0, \sigma_j^2)$. The model parameters may be easily estimated from historical data of the plant sites.

This model allows for negative W_τ^j, which would make no physical sense. The truncation in expressions (4-4) and (4-3), by operator $[a]_+$, ensures that the model will provide nonnegative generation profiles.

As mentioned in the Introduction, there is evidence of seasonal complementarity in generation. Wind power (WP) sources have complementary seasonality with Small Hydro (SH) sources. In Figure 4.3 we can see an example of this situation, where we display the average, 5% and 95% quantiles of the monthly generation of a wind and a small hydro generator. Generation for the WP is presented in percentage values of FEC and for the SH as the flow rate (in m^3/s). An investor could then compose a portfolio of projects mitigating the generation uncertainty, exploiting the complementarity of both plants.

Considering the aforementioned uncertainties, the stochastic process in our model is given by

$$\xi_\tau := (I_\tau, \pi_\tau, f_\tau, G_\tau), \tag{4-6}$$

where $I_\tau := (I_\tau^1, ..., I_\tau^K)$, $\pi_\tau := (\pi_\tau^1, ..., \pi_\tau^K)$ and $f_\tau := (f_\tau^1, ..., f_\tau^K)$, for the K submarkets, and $G_\tau := (G_\tau^1, ..., G_\tau^J)$, for the J projects. Correlations between windpower sites may be modelled following (110).

We use market data provided in the form of 2,000 series simulated in the NEWAVE model as described above. Generation data is obtained for the stochastic renewable sources described above. Forward contract data is obtained following the models proposed in Chapter 5. The data vector will be considered in the optimization model proposed in Chapter 6 of this

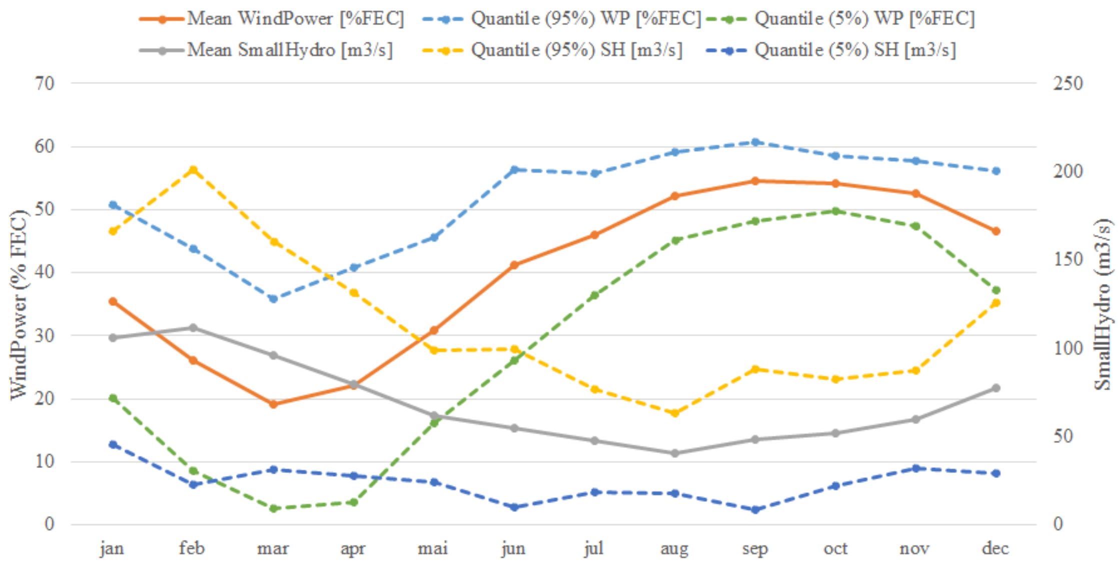

Figure 4.3: Complementary seasonality of wind power and hydro generators in the Brazilian market. Average and quantiles (5% e 95%) of monthly generation for small hydro and windpower projects.

Thesis. We will address as the *original data* for our problem the set of 2,000 simulated series of the data vector ξ_τ and quality of the policies proposed in our investment framework will ultimately be evaluated against this data.

4.2 Investment Framework

As described in the previous section, renewable uncertainty is seasonal and there is complementarity between sources. An investor or trader may exploit this behavior to mitigate generation volatility. Since his exposure to the spot price is reduced, this might allow a generator to sell forward contracts, instead of the lower priced Availability contracts. Whenever wind energy is at a low season the hydro energy may compensate, and vice versa.

In summary, we will pursue the following alternatives:

- Diversification: the investor may diversify his investment in several complementary projects.
- Partnerships: the investor may take a smaller share of the project to avoid over-concentration in few assets. Since projects are usually highly profitable, we assume it is straightforward to find a partner.
- Forward Contracts: Forward contracting may be done at market prices to reduce exposition to spot prices.
- Postponement Options: As mentioned by (27), real options, such as postponement, are available when the investor as exclusive rights or another

competitive advantage. This is usually the case in renewables investment, and we shall explore the possibility to postpone the investment, in order to wait for better market conditions.

In our strategy we will not discuss insurance. Rare, but high impact losses, such as a breached damn in the case of a hydro project, are usually avoided with insurance, and will customarily be considered as part of the project financing costs. If necessary, insurance may be accounted for in operational expenses (Opex), which are usually very low for renewables.

We will also not consider the effect of short-term derivatives trading. As discussed in Chapter 3, simply trading derivative instruments should not create value to the generator. While it can help smoothen the cash flows of the company, it has little impact on long term feasibility of the plant and its results may be alternatively achieved, for instance, with short-term debt. Intra-year liquidity will not be accounted for in this work.

Spot clearing is subject to credit risk. Counterparties in the CCEE may default their debts, which are shared by the credit receiving agents. This risk is especially high during prolonged price spikes, such as those exhibited in 2015. We will not consider credit risk in our framework.

Risk sources (such as spot credit and forward counterparty risk) not accounted for in our framework can be evaluated *ex-post*. Since policies originated in this framework are evaluated by Monte Carlo simulation, it is straightforward to consider the effect of additional modeled risk sources in the obtained policies.

In summary, the investor strategy will depend on defining the optimal investment timing and also the adequate composition of a portfolio, composed of shares in a few renewable projects where some of their FEC will be committed to forward contracts with potential customers.

The forward contract is assumed to begin delivery by the time the plants are built. Forward contract prices fluctuate along time, so there might be value in postponement of the investment decision to wait for better market prices.

Capital expenditures may also be subject to risk. There are risks associated with capex overrun, due to larger costs than originally anticipated, and delay risks, which may not only result in delayed operation but also fines from the ISO. Those risks are significant in some projects, but we will refrain from representing them in our model.

It is also worthwhile to remind that regulatory risks may present themselves in most markets. Changes in regulation after the investment decision has been made may jeopardize the future cash flows, since the problem nature relies in the current market structure. Regulatory risks will be more significant in immature or severely unbalanced markets, when there is a moral hazard associated with the market.

In our numerical results, we will consider a simplified case of an investor who considers a windpower opportunity in the Northeast market and a Small Hydro in the Southeast market. In order to reduce his volatility, he may sell forward energy in the FTE. Also, for the sake of simplicity, we will consider that the forward contract will be sold in the Southeast market, and that spot energy prices in both markets are equal. This simplification in our numerical study may not occur in practice.

Figure 4.4 illustrates the proposed strategy for an investor with a portfolio of shares (x^{WP}) of a WindPower project, shares (x^{SH}) of a Small Hydro project, and selling some fraction of his FEC in forward contracting (x^{sell}).

The geographical risks associated with trading contracts in a different market from generation is an example of a *basis risk*. Nevertheless, the proposed framework is general and we take this risk into account in Chapter 6. One may take it into consideration if this risk seems relevant in the spot price simulations.

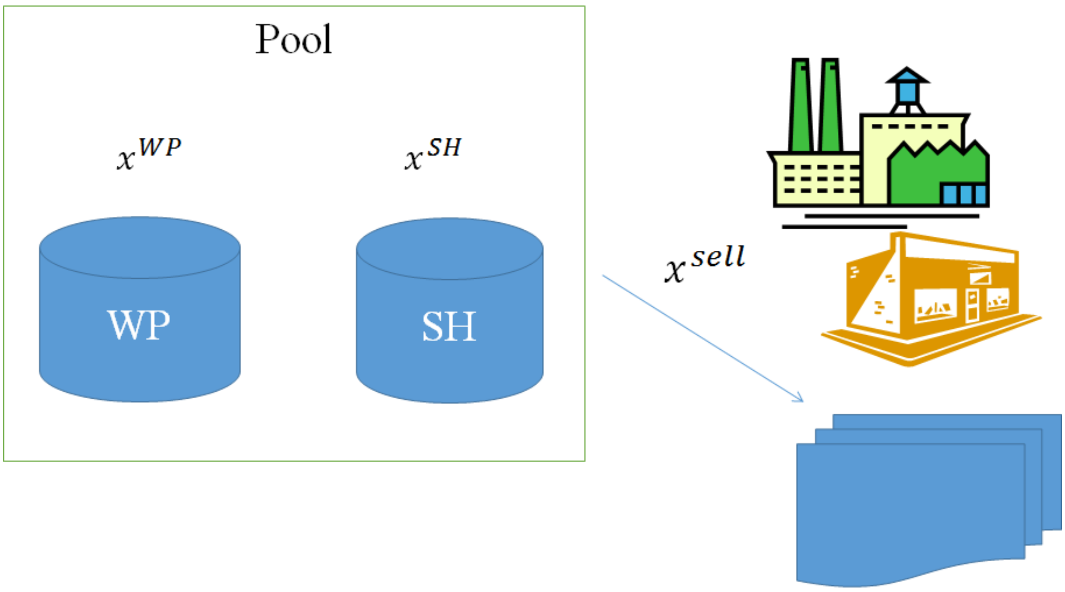

Figure 4.4: Investment strategy: generator decides for a share of each available project (x^{WP} e x^{SH}) and sells a fraction x^{sell} of his physical guarantee pool in a forward contract.

4.3 Evaluation of alternative policies

In our approach we will evaluate the value of the proposed policies by a convex combination of the expected cash flows and its AV@R. This risk functional can be parametrized with different weights, so that our model provides alternative policies.

As discussed in Chapter 3, when evaluating portfolio alternatives with different risk profiles, it is necessary to establish a criterion for portfolio comparison.

We considered in our assumptions that the investor has enough capital to undertake the investments in the portfolio. Nevertheless, usually investors face a more constraining restriction: the availability of *economic capital* (or *risk capital*). The economic capital of a company's portfolio must adhere to the company's risk appetite, otherwise the risk of not meeting its obligations may become unacceptable.

Funding for new projects is basically composed of debt and equity. While banks require enough evidence of the project's financial feasibility to lend money for capital intensive projects, investors rely on the (uncertain) free cash flows of they existing assets to fund their new ventures.

A first criterion for choosing among the available policies is straightforward: one should choose the strategy with the highest Net Present Value that adheres to both the bank's requirements and that is below the risk appetite of the firm.

When the company is faced with several investment opportunities, a more effective way to allocate capital is to do a risk budgeting process that will provide the portfolio with highest NPV that respects the available risk capital. This might be posed as a simple *Knapsack* optimization problem.

In the absence of integrality constraints, the greedy algorithm (picking the opportunities in a decreasing order of $\frac{NPV}{riskcapital}$) obtains the optimal solution.

With this idea in mind, risk adjusted measures of risk, which have been used in the banking industry for decades, have been adapted to capital projects (79) and the electricity market in special in (85). We follow the approach in (79) to reframe the RAROC definition (3-23) to a more appropriate version for investment projects, as

$$RAROC = \frac{\mathbb{E}[NPV(X)]}{PI(X) + EC(X)}, \tag{4-7}$$

where PI is the Present Value of investment cost, and we may define $EC(X) = \mathbb{E}[NPV(X)] - AV@R_\beta(X)$, for a given confidence level β.

A interesting parallel may be made with equation (4-7). In deterministic portfolio evaluation, a very popular criterion is the Profitability Index (PI - given by the Present Value of net operational cash flows divided by Present Value of investment cost), defined in equation (3-2). The above relation may be seen as a risk-adjusted version of this financial ratio.

We will use this version of RAROC, a risk adjusted return measure, to compare different investment strategies and choose the most appropriate strategy, given the investor's appetite.

5 Energy price model

We will rely on a model for the forward contract dynamics in our investment framework. We will now devise such a model to estimate the forward price curve.

We will use the Schwartz-Smith two factor model, coupled with over the counter contract data to estimate the forward curve of the Free Trading Environment in the Brazilian market. The absence of an actual futures market reduces not only the number of managerial tools available to decision makers, but from a market point of view, also the amount of available information and also information symmetry to market participants.

There are three different sources of forward contract information in Brazil. There are two trading platforms, BBCE and BRIX, and also an information pooling bulletin provided by DCIDE.

5.1 Introduction

Following deregulation in several global energy markets, competitive environment induced a sudden increase in the relevance of contracting for market agents. Forward contracting plays several major roles, such as ensuring supply adequacy (102), (76), inducing adequate economic signaling for investment and network expansion (37), risk management of cash flows (8) and marking to market one's position (59), and also reducing market power of agents (106), (9).

Modeling forward prices has been under active research in the energy literature. Forward prices may be necessary, for instance, for contracting and investment strategies (17), (20), (54), (6), (5), and evaluating risk premium (109), (8) of the market.

Financial forward contracts are usually priced by arbitrage arguments. Given a forward contract $F_{t,T}$, negotiated in period t, where contract is to be settled in maturity T, the fair price is

$$F_{t,T} = S_t(1 + r_f)^{(T-t)} \quad (5\text{-}1)$$

where S_t is the current spot price of the asset in time t and $(1+r_f)^{(T-t)}$ is the return of the risk free asset in the time interval $(T-t)$.

Forward commodities usually can not be priced as financial ones. First, storage, when possible, comes with a cost. Also, there is a competitive advantage of owning the asset over the financial obligation, since short-term shortages might happen. In this case, the relation between forward and spot prices is

$$F_{t,T} = S_t(1 + r_f + c - y)^{(T-t)}, \tag{5-2}$$

where c is the cost of carry and y represent a convenience yield for the physical asset. In practice, convenience yields are hardly static (2). In fact, single factor models (88) in general have been known to be unable to fully capture the forward term structure.

The literature of two factor models was initiated by this reason, with the models (43) and (81). The most relevant, Schwartz-Smith (89) model, displays wide acceptance in the literature due to its ability to model long term contracts using a simple Kalman Filtering (98) procedure. The model has been augmented with deterministic seasonal and jump components in (62).

Electric energy is costly or even impossible to store, so pricing by equation (5-2) may not apply. If it is possible to obtain a risk neutral measure Q, then the fair forward price is (2)

$$F_{t,T} = E_Q[S_T|\mathfrak{F}_t]. \tag{5-3}$$

In this equation the forward price is given by the expected value of the spot price in the maturity, conditional to information $\mathfrak{F}_t$available in time t.

The relation may be represented in the real measure P as

$$F_{t,T} = E_P[S_T|\mathfrak{F}_t] + RP_{t,T}, \tag{5-4}$$

where $RP_{t,T}$ is the risk premium of the contract. Expression (5-4) may also be written (83) as

$$F_{t,T} = E_P[S_T|\mathfrak{F}_t](1 + r_f - \Lambda)^{(T-t)}, \tag{5-5}$$

where Λ is the market price of risk. (84) uses a regression model to estimate the market price of risk of forward contracts in Nordpool market.

(47), (38) and (82) mention that some authors may accept zero risk premium, but empirical studies (8) suggest that for shorter maturities the risk premium is positive and long term contracts have negative premiums.

This result is coherent with market expectations, since consumers might be willing to avoid seasonal price spikes while generators might pay a premium to ensure their long term investments will be profitable, as mentioned by (30).

Alternatively to factor models, there are some attempts to reproduce the term structure of energy contracts with financial multi-factor term structure models based on the Heath, Jarrow & Morton (HJM) framework (56), (24). These models have acquired attention because in some markets, while two factor models can account for only 70% of the uncertainty, HJM models may help to explain up to 95% of the variability (2) and (56).

More details of pricing energy derivatives in energy markets may be seen in (19), (8), (24), (26), (29), (34), (48), (57).

In the Brazilian Market, contracts in the Free Trading Environment are mostly traded Over The Counter (OTC). This poses a challenge for agents who wish to develop a forward term structure model.

Our objective is to use the Schwartz-Smith two factor model, coupled with OTC contract data to estimate the forward curve of the Free Trading Environment in the Brazilian market. The assumed hypothesis is that the Schwartz-Smith model is valid for the Brazilian market, which we will justify in the following sections. We will show three different modeling alternatives and compare their relative merits.

This works contributes to the literature by creating a model for the forward curve in the OTC Brazilian market. We also show examples of how to price new (non traded) contracts with the model.

5.2 The Brazilian Contracting Market

The Brazilian Energy Market is a mixed environment, where companies are free to trade energy contracts but dispatch is centralized by a national Integrated System Operator (ISO). In this model, the spot price is given by the marginal cost of dispatch calculated by the ISO's model, but there is no future market information available. The absence of an actual futures market reduces not only the number of managerial tools available to decision makers, but from a market point of view, also the amount of available information and also information symmetry to market participants. Nonetheless, regulations require that the entire demand in this market must be backed by energy contracts, by supply adequacy security reasons. This creates a thriving bilateral OTC market with consumers, generators and traders having different strategies.

The clearing of the short-term transactions in the market is done by CCEE (99), the official clearing house. Such transactions are settled with the spot price calculated by ONS. The information from all the contracts between the companies of the market must be submitted to CCEE, but, since no information about these contracts is disclosed, the market has an Over the

Counter (OTC) dynamic. There have been attempts to change regulation as to improve information flow, by allowing CCEE to disclose overall contract statistics, but so far no such changes where possible.

There are three different sources of forward contract information in Brazil. There are two trading platforms, BBCE[1] and BRIX[2], and also an information pooling bulletin provided by DCIDE[3]. Some market participants rely on a forward curve benchmarking process done by third party company DCIDE (59). On a weekly basis, the company gathers data from over 40 participants of the market. Each participant is responsible to inform the most accurate, in their understanding, fair price for several standardized contracts. The participants then receive access to the consensus curve, which is generated by a process which encompasses a statistical step, filtering outliers and averaging the opinions, and a specialist step, where additional checks might be performed. Since participants represent generators, traders and consumers, the resulting curve approximates a market value that may later be used internally by each participant.

The BBCE trading platform is very recent, with scarce historic data, but potential for future studies. The BRIX data is open to the public and there is a large time frame available, but there is only information from short (current month) to medium term (two years) maturities. DCIDE data is available for up to four years ahead and BRIX and DCIDE data is reasonably similar when they overlap. Since we are mainly interested in long term contracts, we will rely on DCIDE data only.

5.3 Forward curve modeling with Schwartz-Smith two factor model

It has been shown (22) that at least two factors are necessary to represent the term structure of forward contracts, otherwise long term maturities are incorrectly represented.

The Schwartz-Smith (89) model is a very popular model to represent commodity prices. It is composed of two unobserved components: a long term tendency, ξ_t, which follows a Brownian Motion, to represent a moving and unknown long term equilibrium price; and a short-term deviation χ_t, to account for short-term imbalances between supply and demand, represented by a mean reverting process that reverts towards zero.

Following this approach, the spot price is given by

[1] www.bbce.com.br
[2] www.brix.com.br
[3] www.dcide.com.br

$$\pi_t = e^{\xi_t + \chi_t}. \tag{5-6}$$

The unobserved state variables follow the dynamic

$$\xi_t = \mu_\xi \Delta t + \xi_{t-1} + \omega_t^\xi \qquad (\textit{long term tendency}) \tag{5-7}$$

$$\chi_t = e^{-\kappa \Delta t} \chi_{t-1} + \omega_t^\chi \qquad (\textit{short-term variations}) \tag{5-8}$$

Here, μ_ξ is the long term drift, κ is the mean reverting rate, and random errors are represented by vector

$$\begin{bmatrix} \omega_t^\xi \\ \omega_t^\chi \end{bmatrix} \sim N\left(\begin{bmatrix} \sigma_\xi & \rho \\ \rho & \sigma_\chi \end{bmatrix} \right), \tag{5-9}$$

where the short and long term errors might be correlated.

Commodity spot prices are often not observable, and all available trade information refers to forward and future prices. Due to non arbitrage, (89) show that every contract in $\{F_{t,T_i}\}_{i=1}^n$ with different maturities T_i, may be priced by the logarithm relation

$$ln\left(F_{t,T_i}\right) = e^{-\kappa(T_i - t)} \chi_t + \xi_t + \Psi(T_i - t) + \omega_t^{T_i}, \tag{5-10}$$

where $\omega_t^{T_i} \sim N(0, \sigma_{T_i}), i = 1, ..., n$ and there is no correlation in the observation errors, and $\Psi(T - t)$ is a function of the model parameters given by

$$\Psi(\tau) = \mu_\xi^* \tau - (1 - e^{-\kappa\tau}) \frac{\lambda_\chi}{\kappa} + \frac{1}{2}\left((1 - e^{-2\kappa\tau}) \frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 \tau + 2(1 - e^{-\kappa\tau}) \frac{\rho \sigma_\chi \sigma_\xi}{\kappa} \right). \tag{5-11}$$

Here, μ_ξ^* is the long term risk neutral drift and λ_χ is the short-term risk premium. This model, augmented by deterministic seasonal components, has been used by (62) to price energy contracts in the Nordic market.

The non arbitrage argument used in such models may be criticized in some energy markets, since electric energy is hard to be stored. In the Brazilian market, due to its hydroelectric basis, with large reservoirs, it is reasonable to assume that some part of the energy might be stored, which allows us to assume the non arbitrage hypothesis.

As mentioned by Schwartz & Smith, since no expected spot price curve is provided to the model, there is an impossibility to price correctly the risk premiums of the market. Errors in the estimates of the log term drift and the short-term risk premiums may cancel out in the adjustment of the forward curve, but the expected future prices will not be adjusted correctly by the model state variables.

As an example, by adding the constants $\frac{\lambda_\chi}{\kappa}$ and $\frac{-\lambda_\chi}{\kappa}$ to the state variables ξ and χ respectively, no effect occurs to the spot and forward prices, but expected future prices would change. Future work might include using expected spot data from the appropriate submarket of the NEWAVE model in the real measure price process to adjust the remaining parameters and obtain the true risk premium curve.

The two factor model is linear with normal distributed errors, so it can be estimated with the Kalman Filter. The model has seven parameters, plus the standard deviations of measurement error of the contracts $\sigma_{T_1}, ..., \sigma_{T_n}$ in the observation equation.

We will follow the notation of (98) to represent the model in the state space form. The observational and system equation are

$$y_t = \Theta_t x_t + \Gamma u_t + v_t \tag{5-12}$$

$$x_t = \phi x_{t-1} + \Upsilon u_t + w_t \tag{5-13}$$

where dimension of vector y_t is n, the number of observable contracts, x_t has dimension two, the number of state variables and u_t has dimension $n+1$. The resulting Kalman filter equations for the Schwartz-Smith model are:

$$u_t = (\Psi(T_1 - t), ..., \Psi(T_n - t), 1)' , \forall t \tag{5-14}$$

$$y_t = (log(F_{t,1}), ..., log(F_{t,n})) \tag{5-15}$$

$$\Theta_t = \begin{bmatrix} e^{-\kappa(T_1 - t)} & 1 \\ \vdots & \vdots \\ e^{-\kappa(T_n - t)} & 1 \end{bmatrix}, \forall t \tag{5-16}$$

$$\Gamma = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 \\ 0 & \ddots & \ddots & \vdots & \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \dots & 0 & 1 & 0 \end{bmatrix}, \tag{5-17}$$

$$\phi = \begin{bmatrix} e^{-\kappa dt} & 0 \\ 0 & 1 \end{bmatrix}, \tag{5-18}$$

$$\Upsilon = \begin{bmatrix} 0 & \dots & 0 & 0 \\ 0 & \dots & 0 & \mu_\xi dt \end{bmatrix}, \tag{5-19}$$

$v_t \sim N(0, R)$ and $w_t \sim N(0, Q)$, where

$$R = \begin{bmatrix} \sigma_1^2 & & 0 & \\ & \dots & & \\ 0 & & \sigma_n^2 & 0 \end{bmatrix}, \tag{5-20}$$

$$Q = \begin{bmatrix} (1 - e^{-2\kappa dt})\frac{\sigma_\chi^2}{2\kappa} & (1 - e^{-\kappa dt})\frac{\rho\sigma_\chi\sigma_\xi}{\kappa} \\ (1 - e^{-\kappa dt})\frac{\rho\sigma_\chi\sigma_\xi}{\kappa} & \sigma_\chi^2 dt \end{bmatrix}, \tag{5-21}$$

5.4
Market data

The spot data is weekly generated by ONS and is available in the CCEE website. DCIDE data is made available every Wednesday to the associated market participants, and a summarized bulletin is simultaneously made publicly available in their website.

The forward curve generated by DCIDE is composed of one spread contract, delivering in the current month, three monthly contracts for the following months, one contract for the current year and four annual contracts for the following years. Also, contracts may refer to one of the four Brazilian submarkets (Southeast, South, North and Northeast). An example of the data may be seen in Figure 5.2.

As described in (59), DCIDE monthly contracts are named $M+0, M+1, M+2$ and $M+3$ for products with monthly delivery in the current month and 1 to 3 months ahead. The contract $Y+0$ is a product with delivery staring from the fourth month ahead up to the remainder of the current year. Contracts $Y+n, n=1,...,4$ refer to products with annual delivery and maturity one to four years ahead respectively. Figure 5.1 illustrates such contracts for the week between 03/21/2016 and 03/27/2016.

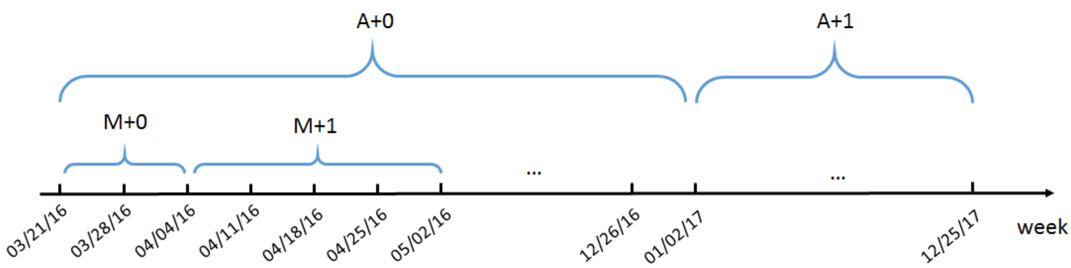

Figure 5.1: DCIDE contracts and their respective maturities on the week beginning in 03/21/2016.

Since we have weekly data, there is an additional challenge because we have moving maturities. In the last week of a month, the time to maturity of a $M+1$ contract is one week. In the following week the underlying product changes and the new $M+1$ contract now has maturity of four (or five) weeks from this date. From September to December, in order to avoid overlapping with the monthly contracts, the underlying product in the $Y+n$ contracts shifts to the next calendar year. As an example, contract $Y+1$ in October/2016 refers to deliveries from January/2018 to December/2018.

5.5 Numerical Study

Numerical results were obtained using R and RStudio, with package astsa by (103).

Data provided by DCIDE consists of 146 observations of weekly data, from January 2012 to October 2014, of conventional energy contracts. Data is composed of pooled information from the 48 most active agents in the Brazilian market, which then is cleaned by a statistical procedure for outliers and finally analyzed by a committee. Spot prices are provided by ONS (Operador Nacional do Sistema), the Brazilian ISO. All data refers to the Southeast market in Brazil.

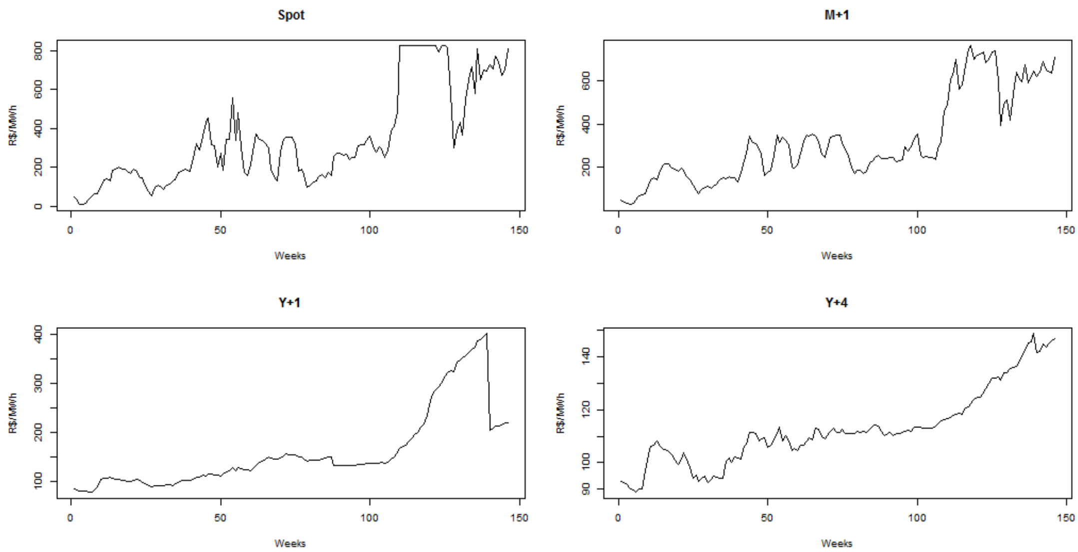

Figure 5.2: Spot and forward contract prices.

We study two monthly contracts, denoted $M+1$ and $M+2$ and also three yearly contracts, $Y+1$, $Y+3$, $Y+4$.

Visual analysis of Figure 5.2 shows that there is a noticeable change of level in yearly contracts every September, due to the change of underlying product. Figure 5.3 shows the same data but in a single plot, making evident that most of the time forward prices are lower for longer maturities, which means the market has *normal backwardation* (futures prices lower than spot, the longer the maturity).

In some European and North American markets, strong seasonal components have been observed, in what is usually associated with consumers willing to pay a premium to secure energy prices. This would imply short-term forwards with higher prices than the spot. Lack of seasonal patterns in the

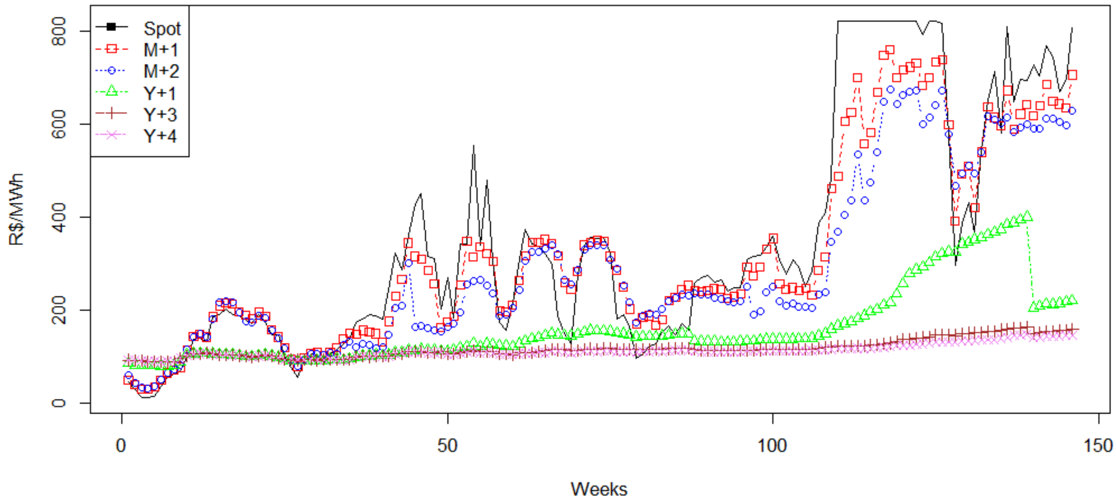

Figure 5.3: Spot and forward contracts.

Brazilian market, as identified by (59), may explain why we hardly see this behavior.

By performing Principal Component Analysis of the data, we can find out that the first component accounts for 91% of data variability and the first two components add up to 97.7% of the market data variance. This is great evidence in support of two factor models for the Brazilian market. Experience in the Nordic market by (56), (8) indicates that two factors explain no more than 75% of price variations in that market and more than ten factors were needed to account for about 98% of the variations in the empirical covariance matrix.

As mentioned before, the moving nature of the contracts' maturities, coupled with the multiple period delivery, might pose an additional modeling challenge. We provide some alternative approaches:

5.5.1 Average maturity and average delivery

In our first approach, which we call Model1, we will assume two simplifications: an average time to maturity for the contracts and that contract delivery happens in an average date in the contract lifespan. In this instance, the swap contracts are approximated by simple single delivery forward contracts with maturities given by Table 5.1. The coefficients in equation (5-16) are time-invariant in this case.

The advantage of this approach lies in its simplicity, since the regular

Schwartz-Smith model may be applied. There are several available libraries that implement this model.

We follow (89) suggestion and initialize the state vector with values zero for the short-term state variable, while the long term variable takes up the (log) average spot price, which shows to provide good results over a non informative startup. Estimation by maximum likelihood methods provides us with the results in Table 5.2. The log-likelihood function is 1692.97.

	Spot	M+1	M+2	Y+1	Y+3	Y+4
Weeks to maturity	0	3.35	7.67	69.04	173	237.3

Table 5.1: Weekly time to maturity in the average maturity and average delivery approach.

	estimate	SE	t-value
κ	0.861	0.036	23.774
σ_χ	0.973	0.075	12.922
λ_χ	1.782	0.559	3.187
μ_ξ	0.145	0.071	2.040
σ_ξ	0.119	0.094	1.265
μ_ξ^*	0.003	0.004	0.657
ρ	0.145	0.107	1.354
σ_0 (Spot)	0.245	0.069	3.570
σ_1 (M+1)	0.048	0.423	0.111
σ_2 (M+2)	0.127	0.095	1.342
σ_3 (Y+1)	0.203	0.062	3.243
σ_4 (Y+3)	0.025	0.065	0.390
σ_5 (Y+4)	0.003	1.084	0.003

Table 5.2: Model1 estimation results

This model, despite simplifications, was able to capture some features of the market. The relatively high long term drift indicates that the market expects rising prices, which agrees with the general behavior observed in Figure 5.2. The higher observation errors where found in the Spot and $Y+1$ contract. The former is related to the general volatility of the spot price. The latter is probably due to this model's inability to capture the variability of the contract maturity. Comparing the time series with the one step ahead forecasts in Figure 5.4 shows again that the model fails to capture the change in the underlying product for the yearly products, specially in the $Y+1$ contract, as we presumed by analysis of the observation error.

Analysis of residuals shows fatter tails than the normal distribution, as seen in Figure 5.5, a usual feature found when modeling prices. Nevertheless, average innovations are close to zero and volatility is approximately constant.

In order to compare the obtained fit, we modeled a GBM to each of the contracts and compared results. The Root Mean Square Error (RMSE[4]) is displayed in Table 5.3. While short-term forecasts present similar error rate, in longer forecast periods the Model1 show much better results.

	weeks ahead		
RMSE	1	5	10
Model1	37.14	38.72	42.44
GBM	39.63	90.18	110.51

Table 5.3: Comparison of RMSE of Model1 and GBM for forecasts of 1, 5 and 10 weeks ahead.

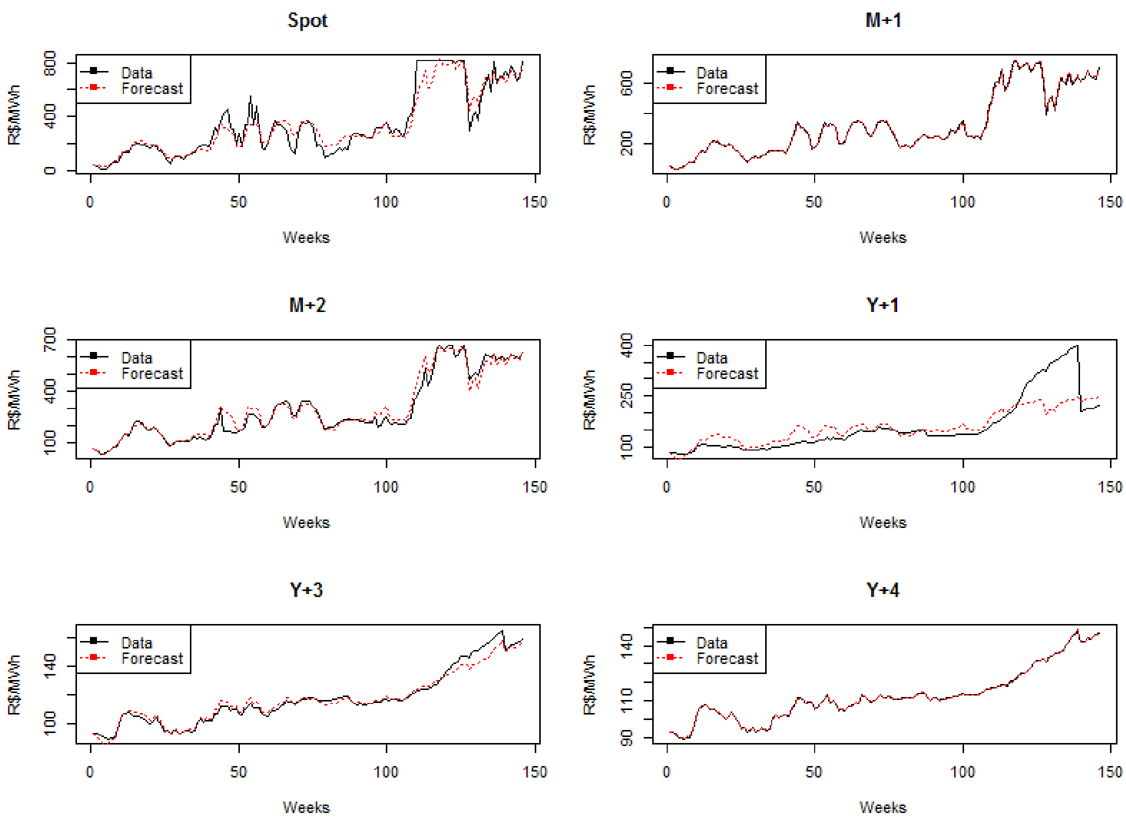

Figure 5.4: Model1 forecasted contract prices.

[4]Square root of the mean square prediction errors.

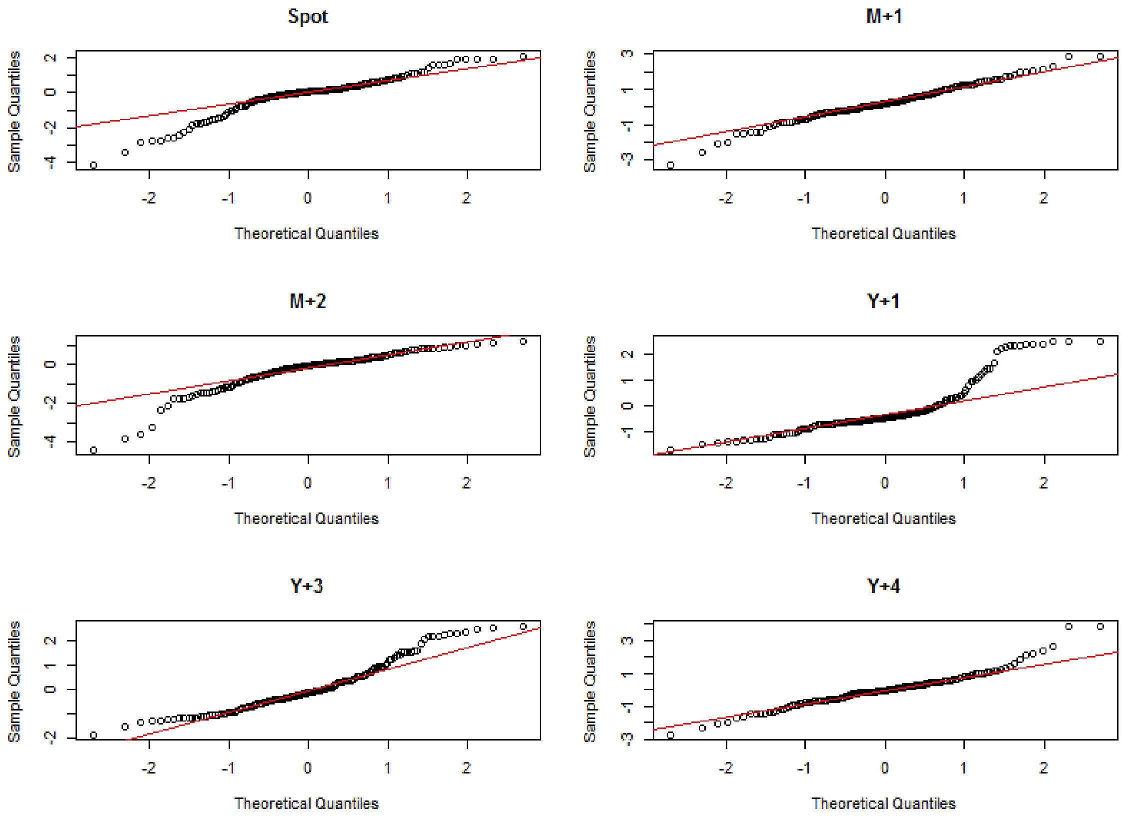

Figure 5.5: QQplot diagnostics for Model1.

5.5.2
Moving maturity and average delivery

The second approach, which we call Model2, improves over the previous one by allowing the model to account exactly for each week's maturity. This is accomplished by changing the coefficients of the observation matrix in equation (5-16) in each period.

We still simplify the model by assuming a single delivery on the average contract lifespan. Table 5.4 displays the approximate delivery date for each contract. As an example, for a monthly contract, we assume that delivery occurs 1.5 weeks after the month has started.

	Spot	M+1	M+2	Y+1	Y+3	Y+4
delivery week	0	1.5	1.5	26.5	26.5	26.5

Table 5.4: Average delivery week used in Model2.

In this case, maximum likelihood estimation provides us with an improved log-likelihood of 1771.26. Estimation results are displayed in Table 5.5.

	estimate	SE	t-value
κ	0.815	0.028	29.077
σ_χ	0.839	0.066	12.651
λ_χ	1.530	0.049	3.140
μ_ξ	0.150	0.069	2.160
σ_ξ	0.116	0.066	1.743
μ_ξ^*	0.005	0.004	1.439
ρ	0.073	0.101	-0.719
σ_0 (Spot)	0.279	0.081	3.437
σ_1 (M+1)	0.098	0.173	0.569
σ_2 (M+2)	0.097	0.146	0.666
σ_3 (Y+1)	0.129	0.067	1.901
σ_4 (Y+3)	0.001	8.551	0.001
σ_5 (Y+4)	0.020	0.065	0.309

Table 5.5: Model2 estimation results

Most parameters agree with the estimates of the previous model. It is noticeable that in both models the correlation between the short and long term components was hard to estimate, resulting in somewhat different estimated values, but both are close to zero. Observation error of yearly contracts $Y+1$ and $Y+3$ reduced, but there was a small increase for the $Y+4$ contract. One step ahead forecasts shown in Figure 5.6 indicate that this model was able to better capture the change of underlying contracts, which resulted in the reduction of the observation error in the aforementioned contracts.

5.5.3 Moving maturity and multiple delivery

The last model, that we will address as Model3, improves over Model2 the modeling of the multiple delivery aspect. Since the Schwartz-Smith model was designed for a single delivery, we will have to approximate its behavior.

Let's say we have a contract with multiple deliveries at a fixed single price. We will follow (19) to devise an equivalence price by non arbitrage arguments. Consider a series of contracts $F_{t,T_1}, ..., F_{t,T_n}$ with maturities $T_1, T_2, \ldots, T_n$, and a contract that establishes in time t a single fixed price K with multiple deliveries in the aforementioned maturities. This contract has a net present value (NPV) in time t of

$$NPV_t = \sum_{i=1}^{n} \frac{F_{t,T_i} - K}{(1 + r_f)^{(T_i - t)}}, \tag{5-22}$$

since it is equivalent to a series of single delivery forward contracts. In absence of arbitrage opportunities this NPV is zero, then the fair price of the contract is

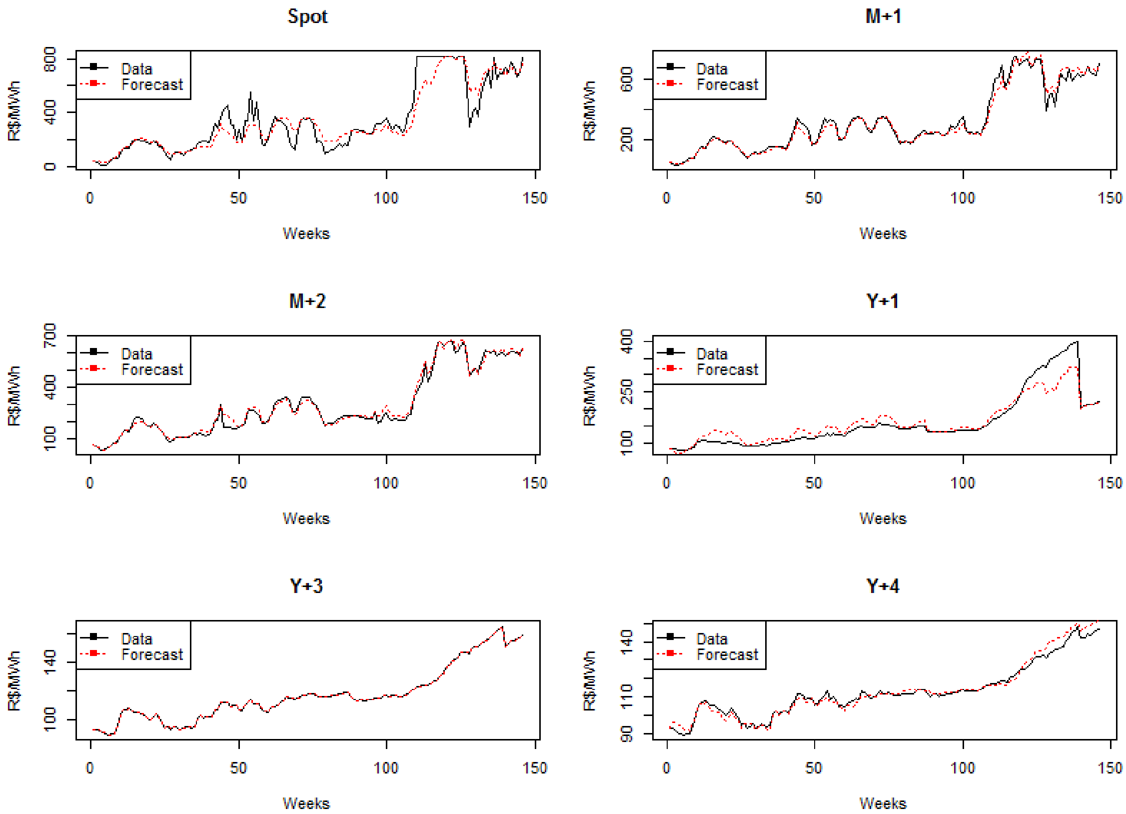

Figure 5.6: Model2 forecasted contract prices.

$$F_{t,[T_1,T_n]} = K = \frac{\sum_{i=1}^{n} \frac{F_{t,T_i}}{(1+r_f)^{(T_i-t)}}}{\sum_{i=1}^{n} \frac{1}{(1+r_f)^{(T_i-t)}}} \tag{5-23}$$

In the Schwartz-Smith model, we find the following equivalence:

$$K = \frac{\sum_{i=1}^{n} \frac{e^{-\kappa(T_i-t)}\chi_t+\xi_t+\Psi(T_i-t)}{(1+r_f)^{(T_i-t)}}}{\sum_{i=1}^{n} \frac{1}{(1+r_f)^{(T_i-t)}}} \tag{5-24}$$

$$= \frac{\sum_{i=1}^{n} \frac{e^{-\kappa(T_i-t)}}{(1+r_f)^{(T_i-t)}}}{\sum_{i=1}^{n} \frac{1}{(1+r_f)^{(T_i-t)}}}\chi_t + \xi_t + \frac{\sum_{i=1}^{n} \frac{\Psi(T_i-t)}{(1+r_f)^{(T_i-t)}}}{\sum_{i=1}^{n} \frac{1}{(1+r_f)^{(T_i-t)}}}, \tag{5-25}$$

which may also be implemented in the Kalman Filter.

Model3 requires informing an interest rate. In our numerical analysis best results (those with higher likelihood scores) where achieved with 5% per year. Maximum log-likelihood obtained was 1770.28, with estimation results displayed on Table 5.6.

Overall estimation results are similar to Model2. One distinguishing feature of this model is that it achieved reduction in the observation error of the yearly contracts with a low long term contract $(Y+4)$ error. One step ahead forecasting displayed in Figure 5.7 confirm these findings.

	estimate	SE	t-value
κ	0.822	0.027	29.822
σ_χ	0.825	0.069	11.950
λ_χ	1.573	0.478	3.287
μ_ξ	0.137	0.066	2.060
σ_ξ	0.111	0.095	1.167
μ_ξ^*	0.007	0.004	1.936
ρ	0.190	0.104	1.830
σ_0 (Spot)	0.280	0.084	3.339
σ_1 (M+1)	0.097	0.187	0.521
σ_2 (M+2)	0.097	0.157	0.616
σ_3 (Y+1)	0.138	0.069	1.986
σ_4 (Y+3)	0.019	0.075	0.252
σ_5 (Y+4)	0.003	0.924	0.004

Table 5.6: Model3 estimation results

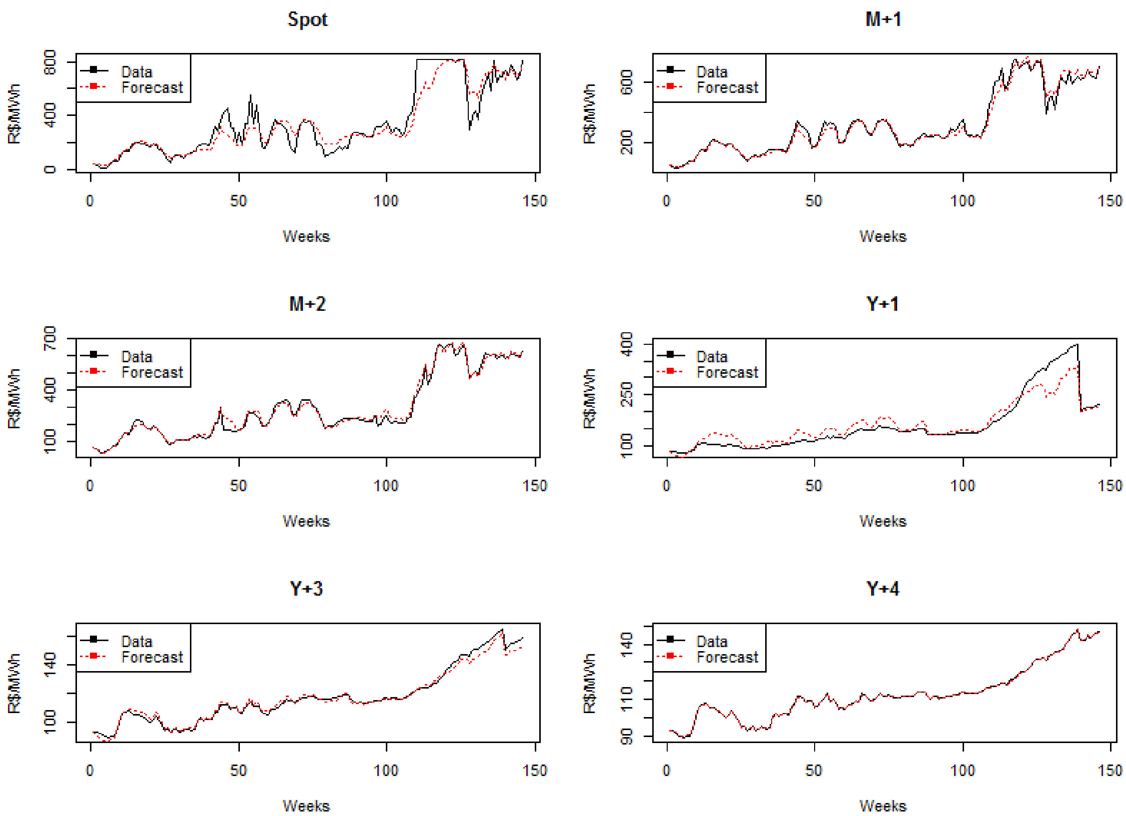

Figure 5.7: Model3 forecasted contract prices.

We show in Table 5.7 that while Model2 shows improvement over Model1 in the forecasting power for longer horizons, Model3 did not show any significant improvements over Model2 as we might expect. Since calculations of Model3 are more involved, for practical usage Model2 seems to be a more reasonable choice, unless the improved long term contract adherence is a priority.

	weeks ahead			
RMSE	1	5	10	20
Model1	37.14	38.72	42.44	51.21
Model2	39.62	40.46	41.98	48.21
Model3	39.63	40.07	42.05	48.47

Table 5.7: Comparison of forecasting error over the three different approaches.

5.5.4 Comparison with Schwartz One Factor model

The single factor model of (88) assumes that the spot prices follow a mean reverting process, which is also a very usual assumption for energy prices. Within this model, there is an equivalent pricing formula for the forward contracts, given by

$$ln\left(F_{t,T_i}\right) = e^{-\kappa(T_i-t)}\chi_t + \Psi^*(T_i - t) + \omega_t^{T_i}, \tag{5-26}$$

where κ is the mean reversion rate, $\omega_t^{T_i} \sim N(0, \sigma_{T_i}), i = 1, ..., n$ and there is no correlation in the observation errors, and $\Psi^*(T-t)$ is a function of the model parameters given by

$$\Psi^*(\tau) = (1 - e^{-\kappa\tau})\varrho^* + (1 - e^{-2\kappa\tau})\frac{\sigma^2}{4\kappa}, \tag{5-27}$$

where ϱ^* is the long term price.

This model can be estimated by Kalman filtering as the previous ones. By maximum likelihood estimation of the parameters, we are able to reproduce the one step ahead forecast as done in the previous models. Figure 5.8 shows that, as mentioned in the literature, one factor models struggle to fit long term contracts.

5.6 Example application

We provide an example usage for the proposed approach. We will price a five year forward contract with a fixed price over the whole delivery. This type of contract is also known as a *swap*. Generators may be interested in selling this contract to lock a profit margin or to use it as collateral to obtain financing for their projects. Industrial consumers might want to buy this

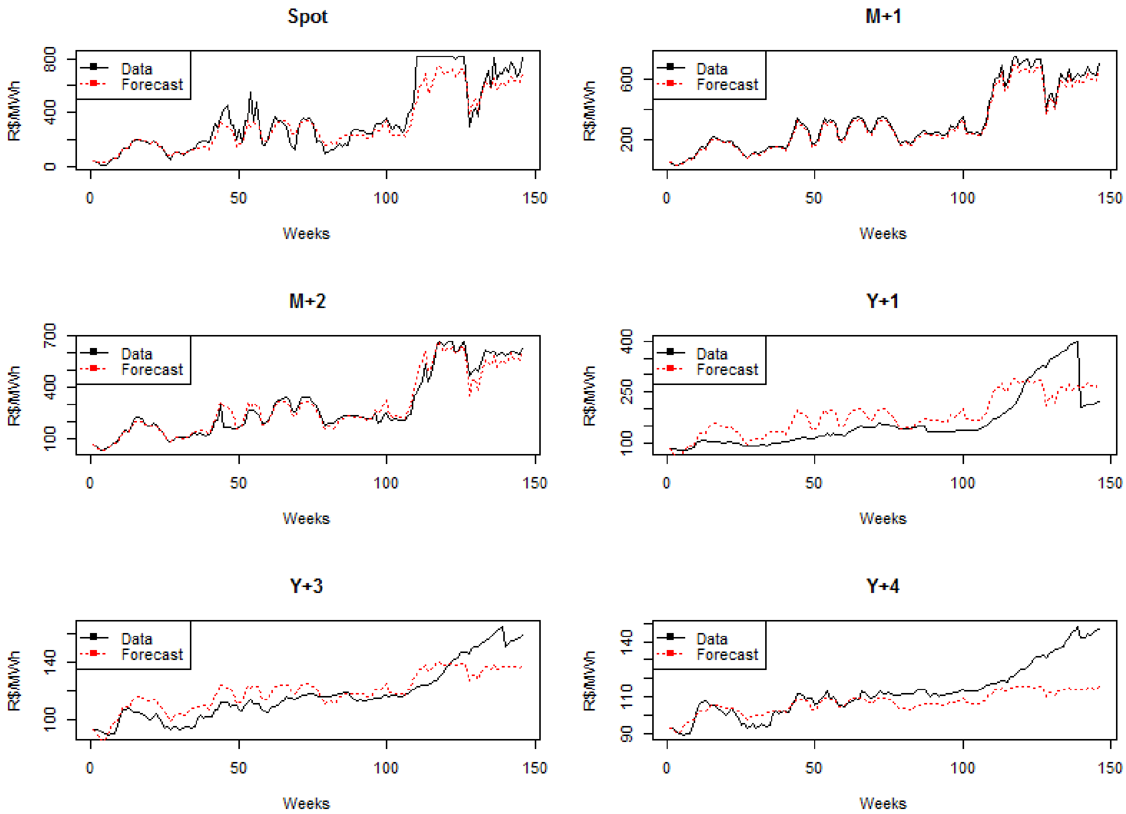

Figure 5.8: Schwartz 1 model forecasted contract prices.

contract to obtain some competitive advantage by securing predictable (and possibly lower) energy prices.

We will first price the contracts for delivery in years $Y+1, \ldots, Y+5$, then we will find the equivalent price of the proposed contract. The contract should deliver energy from January/2016 to December/2020. We assume an interest rate of 5% and a decision maker evaluating this decision with information available up to October/2014.

First, using the adjusted Model2, we obtain the contract prices in Table 5.8.

	Y+1	Y+2	Y+3	Y+4	Y+5
Forward Price (R$/MWh)	223.34	175.48	158.37	152.3	150.68

Table 5.8: Prices for the yearly contracts.

Then, using the same reasoning of equation (5-23), we can find our equivalent price as $173.70 R\$/MWh$. This price might be used as a reference for a contract negotiation.

6
Modeling the portfolio problem

We will now present the decision support model that will allow us to devise strategies using the strategic management framework proposd in the previous chapters. Several portions of this chapter are based on (17). We now introduce the mathematical formulation of the problem and then discuss the solution approach.

6.1
Problem formulation and solution approaches

Since this is a long term problem, it is convenient to deal with annual decision stages, denoted $t = 1, \ldots, T{+}1$. We will denote the monthly periods by τ, whenever necessary. We may undertake the investment decision up to period T, which we will represent by a binary variable y_t, for each period $t = 1, ..., T$. The investment decision must be undertaken simultaneously with the portfolio selection. If investment occurs in a given period t (i.e., $y_t = 1$), then the share x_t^j of each renewable plant project $j = 1, ..., J$ to be purchased and the amount of hedging by forward contracts ${x^k}_t^{sell}$, for each submarket $k = 1, ..., K$ must be decided. We will assume all plants have the same build time b (in years). We will follow the strategy devised in Chapter 4, making use of partnerships, diversification and forward contracts in our portfolio. Considering multiple periods to invest allows us to account for the postponing option in our strategy.

With a slight abuse of notation, we will denote FEC as the sum of the physical guarantees FEC^j of all the renewable projects $j = 1, ..., J$ under consideration, and h_t the sum of all the operation hours of a given year, that is:

$$FEC := \sum_{j=1}^{J} FEC^j, \tag{6-1}$$

and

$$h_t := \sum_{\tau=12(t-1)+1}^{12t} h_\tau. \tag{6-2}$$

For a given market k, the contract amount ${x^k}_t^{sell}$ is represented as a fraction of the maximum amount FEC, and all energy sold under this contract will be priced by the forward energy price f_t^k, a multi-period swap as in equation (4-2), agreed in the investment period t.

We assume that the generator will operate the plant in its full capacity, as long as there is availability of the renewable source. This will lead to absence of recourse (operational decisions) in our model.

Given the vector of portfolio decisions $x_t := (x_t^1, \dots, x_t^J, {x^1}_t^{sell}, \dots, {x^K}_t^{sell})$, we define the contract limit constraint

$$X_t := \left\{ x_t \in [0,1]^{J+K} : \sum_{j=1}^{J} FEC^j x_t^j \geq FEC \sum_{k=1}^{K} {x^k}_t^{sell} \right\}, \forall t, \tag{6-3}$$

ensuring that investors have physical guarantees for their contracts. As previously mentioned, one may not sell energy contracts in the Brazilian market if they are not backed by physical guarantees from existing plants.

Generalizing equation (3-4), profit of the investor in period t may be represented as a function $R_t(x_t, \xi_t)$, given by

$$R_t(x_t, \xi_t) = FECh_t \sum_{k=1}^{K} f_t^k {x^k}_t^{sell} + \sum_{\tau=12(t-1)+1}^{12t} \sum_{j=1}^{J} \pi_\tau^{m_j} G_\tau^j x_t^j - \sum_{\tau=12(t-1)+1}^{12t} FEC \sum_{k=1}^{K} \pi_\tau^k h_\tau {x^k}_t^{sell}, \tag{6-4}$$

where $m_j \in \{1, ..., K\}$ is the submarket where plant j belongs and $\pi_\tau^{m_j}$ the spot price in that market during period τ. The differences between the generated energy and the contract are settled in the spot market at the spot price π_τ^k. The settlement revenue is a function of uncertain spot prices and generation. *Basis risk*[1] of the different submarkets $k = 1, ..., K$ is accounted for by considering the (possibly) different prices in the markets. To simplify notation, we disregard operational expenses. This is a reasonable assumption, since, in general, renewable sources have very low operational costs.

We present the optimal investment problem as

[1]In our case, geographical risks associated with trading contracts in a different market from generation.

$$\underset{x_t,y_t}{\text{Max}} \quad \mathbb{E}\left[\sum_{t=1}^{T} g_t(x_t,\xi_t)y_t(1+r)^{1-t}\right] \tag{6-5}$$

$$\text{s.t.} \qquad x_t \in X_t, \tag{6-6}$$

$$\sum_{t=1}^{T} y_t \leq 1, \tag{6-7}$$

$$y_t \in \{0,1\}, \tag{6-8}$$

where

$$g_t(x_t,\xi_t) := \sum_{t'=t+b}^{t+b+l-1} R_{t'}(x_t,\xi_t)\big(1+r\big)^{t-t'} - \sum_{j=1}^{J} v^j x_t^j. \tag{6-9}$$

where b is the build time, r is the proper discount rate and v^j is the present value of investment in the renewable source j.

The projects revenues begin b years after the investment decision in time t, extending up to the end of the project lifetime, up to period $t+b+l-1$.

The decision variables x_t (portfolio) and y_t (decision to invest) are adapted to the respective filtration $\mathfrak{F}_t$ (the nonanticipativity constraint), and the expectation in (6-5) is taken with respect to the random data process defined in Chapters 4 and 5. We will assume ξ_t (the yearly data process) to be *Markovian* henceforth and that, at time $t=1$, ξ_1 is known (deterministic).

Here, we maximize the expected net present value over the horizon, considering a project lifetime l. In the period when the investment decision occurs, the investor must decide his portfolio mix, without knowledge of future price and generation. In the following operations years, after a build time b, cash flows due to the profit and losses from the spot market clearing are incurred during the project lifetime.

The above formulation assumes a risk neutral investor. We will use a coherent risk measure to represent risk aversion in our problem, as described in Chapters 3 and 4. Additional constraints could be imposed on variables x_t and y_t to account for other business rules. In both cases, the problem at hand is a nonlinear multistage stochastic problem. This class of problems normally can't be solved to optimality and in practice heuristics are usually employed.

The presented problem is an instance of the *optimal stopping problem* (31) and can be represented as a mixed integer multistage stochastic optimization model.

Multistage optimization problems with integer variables may be solved by the dual decomposition method of (21). There are two major difficulties related to the generation of scenarios and the resolution method. Scenario generation is usually accomplished by Sample Average Approximation (55), which entails approximating continuous distributions of the problems by Monte

Carlo discretization. In the multistage case it was demonstrated by (94), (95) that, in order to properly approximate the original continuous distributions by Monte Carlo discretization, it is necessary to generate more scenarios in the sampling process, and that this number of scenarios grows exponentially with the number of stages. This may lead to a extremely large number of scenarios with even a few number of stages, thus rendering it unsolvable with reasonable computational efforts.

The problem at hand has the block-separable recourse property, as seen in Chapter 3. Exploiting block separability would lead to a mixed integer first stage and a linear second stage. Still, this requires solving a difficult decomposition problem. We will rather rely on dynamic programming methods, such as used in American options problems, which enable efficient problem resolution.

In the next section we will propose a solution method to the investment problem.

6.2 Stochastic Dual Dynamic Programming heuristic

The Stochastic Dual Dynamic Programming (SDDP) method can be applied to multistage linear optimization problems with stagewise independence. The method was originated in (78) and (77) and its convergence properties were analyzed in (80) and (91). The main advantage of this method is the ability to solve stochastic optimization problems with a large number of stages and scenarios, thus allowing the use of the SAA method for a multistage problem. We will now show the steps to formulate our problem in order to apply a SDDP approach.

Since the problem (6-5)-(6-8) is nonlinear, we will introduce an approximate formulation. The main simplification is that we will allow the project lifetime to extend up until period $T + l$, regardless of which period the investment decision was made. This allows us to avoid extra state variables to account for the end of project lifetime. In practice, this will increase the plant operation period for investments undertaken before stage T, which could bias the model to favor early investment. To prevent this bias, we suggest adjusting capital expenditures. This should be done as to roughly maintain the internal rate of return or the profitability index unchanged.

In order to present our reformulation, we will make an additional assumption: we will consider build time $b = 1$. This simplification may be avoided with a more involved modeling process.

To help rewriting the problem, we will introduce a binary variable z_t, which indicates that the investment has been decided in the current or a previous period. Constraint (6-7) may be replaced by balance (or inventory) equations $z_t = y_t + z_{t-1}$, where $z_0 = 0$.

In order to perform our reformulation, equation (6-9) must be broken into two parts: fixed cash flows ($g_t^F(x_t, \xi_t)$) and operational clearing ($g_t^{OPER}(x_t, \xi_t)$). As we will see, $g_t^F(x_t, \xi_t)$ will be multiplied by y_t and ($g_t^{OPER}(x_t, \xi_t)$) will be multiplied by z_t, allowing the linearization of the objective function.

The forward contract is a multi-delivery contract with continuous delivery from the beginning of the plants' operation up to the end of the lifetime. The revenue from the contract in year $\tilde{t}$ is $FECh_{\tilde{t}} f_t^k {x^k}_t^{sell}$. Assuming all years have the same number of operating hours and considering an annuity factor α with duration equivalent to the contract, revenue present value from the contract is $\alpha FECh_t f_t^k {x^k}_t^{sell}$, as seen in Section 3.1.

Once the investment decision is made in a given period t, the associated net present value of the cash flow related to the capital expenditures (capex) and the net present value at time t under the forward contracts is

$$g_t^F(x_t, \xi_t) = (1+r)^{-b} \alpha FECh_t \sum_{k=1}^{K} f_t^k {x^k}_t^{sell} - \sum_{j=1}^{J} v^j x_t^j, \tag{6-10}$$

where α is the aforementioned annuity factor and K is the set of submarkets. Note that (6-10) is a linear expression composed of fixed income and expenses that in practice will occur in several years, but can be valued in period t by proper discounting.

During operation phase, differences between energy generated by the project portfolio and sold in the contracts are settled in the spot market. The cash flows from such settlements during an operation year $\tilde{t}$ is

$$g_{\tilde{t}}^{OPER}(x_t, \xi_t) = \sum_{\tau=12(\tilde{t}-1)+1}^{12\tilde{t}} \sum_{j=1}^{J} \pi_\tau^{m_j} G_\tau^j x_t^j - \sum_{\tau=12(\tilde{t}-1)+1}^{12\tilde{t}} FEC \sum_{k=1}^{K} \pi_\tau^k h_\tau {x^k}_t^{sell}, \tag{6-11}$$

where $m_j \in 1, ..., K$ is the submarket to which the plant belongs.

This results in equation (6-9) been rewritten as

$$g_t(x_t, \xi_t) := g_t^F(x_t, \xi_t) + \sum_{t'=t+1}^{t+l} g_{t'}^{OPER}(x_t, \xi_t)\big(1+r\big)^{t-t'}. \tag{6-12}$$

The new variable z_t also simplifies the objective function formulation. Since the portfolio may not be changed after the investment decision, the cash flow in the investment period (when $y_t = 1$) is $g_t^F(x_t, \xi_t)$ and in the following periods (during operations, after build time, when $z_{t-1} = 1$) is $g_t^{OPER}(x_t, \xi_t)$. Then we obtain the equivalent formulation:

$$\underset{x_t,y_t,z_t}{\text{Max}}\,\mathbb{E}\left[\sum_{t=1}^{T}\left(g_t^F(x_t,\xi_t)y_t + g_t^{OPER}(x_t,\xi_t)z_{t-1}\right)\left(1+r\right)^{1-t} + \sum_{t=T+1}^{T+l} g_t^{OPER}(x_T,\xi_t)\left(1+r\right)^{1-t}\right] \tag{6-13}$$

$$\text{s.t.}\quad x_t \in X_t, t = 1, ..., T+1 \tag{6-14}$$

$$z_t = y_t + z_{t-1}, t = 1, ..., T \tag{6-15}$$

$$x_t \leq z_t, t = 1, ..., T \tag{6-16}$$

$$x_t \geq x_{t-1}, t = 1, ..., T+1 \tag{6-17}$$

$$x_t \leq x_{t-1} + 1 - z_{t-1}, t = 1, ..., T+1 \tag{6-18}$$

$$y_t, z_t \in \{0,1\}, t = 1, ..., T, \tag{6-19}$$

with initial conditions $z_0 = 0$, $x_0 = 0$.

The first summation in the objective function is the present value associated with investment or operation in that period. In the last stage $(T+1)$ there is no investment decision, just settlement cash flows according to the pre-defined portfolio, up to the last operation period $T+l$, represented by the second summation. The inequalities (6-16), (6-17) and (6-18) ensure that x_t will be defined in the investment period (when $y_t = 1$) and will stay fixed thereafter. After investment decision is made, we have $x_t = x_{t-1}$, i.e. the decision of portfolio of projects and contracts can not be changed.

	$(x_t - x_{t-1})$	$g_t^F(x_t,\xi_t)y_t$	$g_t^F(x_t - x_{t-1},\xi_t)$
$y_t = 0$	0	0	0
$y_t = 1$	x_t	$g_t^F(x_t,\xi_t)$	$g_t^F(x_t,\xi_t)$

Table 6.1: Expression $g_t^F(x_t,\xi_t)y_t$ has linear counterpart $g_t^F(x_t - x_{t-1},\xi_t)$.

The objective function (6-13) can also be linearized: as mentioned above, equations (6-16), (6-17) and (6-18) will not allow x_t to differ from x_{t-1} unless it is the investment period ($y_t = 1$). This allows us to replace expression $g_t^F(x_t,\xi_t)y_t$ with $g_t^F(x_t - x_{t-1},\xi_t)y_t$, since, as illustrated in Table 6.1, whenever y_t equals zero both expressions equal zero and also, when $y_t = 1$, both expressions evaluate to $g_t^F(x_t,\xi_t)$. Likewise, by constraint (6-16) we have

$$z_{t-1} = 0 \rightarrow x_{t-1} = 0, \tag{6-20}$$

while equations (6-17) and (6-18) allows us to derive

$$z_{t-1} = 1 \rightarrow x_t = x_{t-1}. \tag{6-21}$$

	x_{t-1}	x_t	$g_t^{OPER}(x_t,\xi_t)z_{t-1}$	$g_t^{OPER}(x_{t-1},\xi_t)$
$z_{t-1}=0$	0	x_t	0	0
$z_{t-1}=1$	x_{t-1}	x_{t-1}	$g_t^{OPER}(x_{t-1},\xi_t)$	$g_t^{OPER}(x_{t-1},\xi_t)$

Table 6.2: Expression $g_t^{OPER}(x_t,\xi_t)z_{t-1}$ has linear counterpart $g_t^{OPER}(x_{t-1},\xi_t)$.

Following the previous linearization, we provide Table 6.2 with a summary of the equivalences. This way, expression $g_t^{OPER}(x_t,\xi_t)z_{t-1}$ may be replaced by the linear counterpart $g_t^{OPER}(x_{t-1},\xi_t)$.

We may now present the following equivalent formulation:

$$\begin{array}{ll}
\underset{x_t,y_t,z_t}{\text{Max}} & \mathbb{E}\left[\sum_{t=1}^{T}\frac{g_t^F(x_t-x_{t-1},\xi_t)+g_t^{OPER}(x_{t-1},\xi_t)}{(1+r)^{t-1}}+\sum_{t=T+1}^{T+l}\frac{g_t^{OPER}(x_T,\xi_t)}{(1+r)^{t-1}}\right]\\
\text{s.t.} & x_t\in X_t, t=1,...,T\\
& z_t=y_t+z_{t-1}, t=1,...,T\\
& x_t\le z_t, t=1,...,T\\
& x_t\ge x_{t-1}, t=1,...,T\\
& x_t\le x_{t-1}+1-z_{t-1}, t=1,...,T\\
& y_t,z_t\in\{0,1\}, t=1,...,T.
\end{array} \tag{6-22}$$

The model (6-22) is linear, if we disregard the integrality constraints. These simplifications will allow a dynamic programming reformulation of the problem. In fact, the last stage is $T+1$, when there is no recourse decision.

In this case, considering the state variables (x_t,z_T) and a realization of the random vector ξ_{T+1}, there is no portfolio decision, only settlement of cashflows in the spot market, to which there is no associated decision. The trivial optimal value is $\sum_{t=T+1}^{T+l}g_t^{OPER}(x_T,\xi_t)(1+r)^{1-t}$, the net present value of the spot market clearance. We will denote this subproblem $\mathcal{Q}_{T+1}(x_T,z_T,\xi_{T+1})$.

The remaining periods $t=2,...,T$ are represented by $Q_t(x_{t-1},z_{t-1},\xi_t)$, given by

$$\begin{array}{ll}
\underset{x_t,y_t,z_t}{\text{Max}} & g_t^F(x_t-x_{t-1},\xi_t)+g_t^{OPER}(x_{t-1},\xi_t)+\mathcal{Q}_{t+1}(x_t,z_t,\xi_t)(1+r)^{-1}\\
\text{s.t.} & x_t\in X_t,\\
& z_t=y_t+z_{t-1},\\
& x_t\le z_t,\\
& x_t\ge x_{t-1},\\
& x_t\le x_{t-1}+1-z_{t-1},\\
& y_t,z_t\in\{0,1\},
\end{array} \tag{6-23}$$

where

$$\mathcal{Q}_{t+1}(x_t, z_t, \xi_t) := \mathbb{E}\left[Q_{t+1}(x_t, z_t, \xi_{t+1})\middle|\xi_t\right], \tag{6-24}$$

and $\mathbb{E}[\cdot|\xi_t]$ denotes the corresponding conditional expectation. At the first stage we solve the problem (recall that $z_0 = 0, x_0 = 0$)

$$\begin{aligned} \underset{x_1, y_1, z_1}{\text{Max}} \quad & g_1^F(x_1, \xi_1) + \mathbb{E}\left[Q_2(x_1, z_1, \xi_2)\right]\left(1+r\right)^{-1} \\ \text{s.t.} \quad & x_1 \in X_1, \\ & z_1 = y_1, \\ & x_1 \leq z_1, \\ & y_1 \in \{0, 1\}. \end{aligned} \tag{6-25}$$

In order to represent a general build time b, we would need to include additional state variables in our model. While this might be efficiently accomplished with the aid of McCormick inequalities (70), we will leave this generalization to future work.

Notice that the problem presents *relatively complete recourse*. It is easy to see that there is no constraint enforcing investment and, after an investment decision is made, the defined portfolio remains feasible in the remaining stages. This allows us to avoid the inclusion of feasibility cuts into the decomposition method.

It is worthwhile to stress the importance of keeping the number of state variables as low as possible. The overall computational complexity of the SDDP method is determined by the number of the state variables, and typically grows linearly with respect to the number of stages (93), (96), (17).

The SDDP method requires stagewise independent problems. If this hypothesis would hold true, the conditional expectation in the previous cost-to-go functions might be replaced by the (unconditional) expectation, so that $\mathbb{E}[Q_{t+1}(x_t, z_t, \xi_{t+1})]$ may be approximated by cutting planes. Also, building a scenario tree from the 2,000 samples of our original data is simple under this hypothesis: independence allow us to sample paths from the vector ξ_t independently for each (yearly) stage t. We may create a SAA problem as in (91) based on a random sample $\xi_t^n, n = 1, ..., N_t$, where $N_t \leq 2,000$. With $N = N_t, \forall t$, this synthetic tree has N^T branches, but in practice the SDDP algorithm only samples random paths from the tree and converges in quite few iterations. As we will see in the next section, the independence hypothesis rarely holds true for most data processes, which usually require the usage of some SDDP method variant to adequately model dependency.

The SDDP method converges within a confidence interval to its optimal policies by improving the approximation of the cost-to-go functions through the inclusion of new cutting planes at each iteration. Computational cost is low, since each backward pass of the algorithm demands solving $N \times T$ linear subproblems per iteration.

6.2.1 Modeling dependency

As will be seen in the numerical results, the necessary independence hypothesis does not apply to our case. Even if we consider that yearly renewable generation is relatively independent (from the previous year), the same cannot be said from the market prices.

If dependency on a linear stochastic problem is on the right hand side (RHS) of the problem's constraints, then it is easy to augment the state vector to represent autoregressive processes and apply the SDDP method, without losing convexity. Unfortunately, in our case, dependency occurs in the cost vector, and convexity is lost.

In this case, we may follow previous work of (13) and (44), who apply a Markov Chain variant of the SDDP method, creating a cost-to-go function for each discrete state. More details about the Markov Chain SDDP may be obtained in (93), (92). The Markovian property of the data process is held by approximating the process by a discrete Markov Chain.

The Markov Chain introduces two difficulties: First, we have to establish transition probabilities for the states. Several approaches are proposed in (45) for financial option problems. In general, numerical solution requires estimating transition probabilities empirically, by Monte Carlo simulation. The second difficulty is that computational cost increases, since we have to build a cutting plane approximation to the cost-to-go function of each state, demanding the evaluation of $N^2 \times T$ linear subproblems per iteration.

We will avoid calculating the transition probabilities by making two assumptions: first, we will consider (yearly) stagewise independence for the inflows and renewable sources. We also obtain each state $\xi_t^k, k = 1, ..., N_t$ of the Markov Chain by Monte Carlo sampling over the original data. Then, due to Monte Carlo sampling, each discrete state has the same probability $\frac{1}{N}$, and the sample $\{\xi_t^k\}_{n=1}^{N_t}$ approximates the original distribution, as exemplified in Figure 6.1. Since stages are independent, transition probabilities are equiprobable, i.e., the conditional probability of arriving at state ξ_{t+1}^n from ξ_t^k is $\frac{1}{N}, \forall n, k$.

Second, in our *approximate dynamic programming* approach, we will represent energy spot prices as a function of inflows. We approximate (log) spot price π_τ^s of period τ and submarket s by a linear model dependent on the inflows I_m^k of every market k in current and past periods m, following

$$\log \pi_\tau^s = \sum_{m=\tau}^{\tau-12} \sum_{k=1}^{4} \psi_{\tau m}^k I_m^k + \varepsilon_\tau^s \tag{6-26}$$

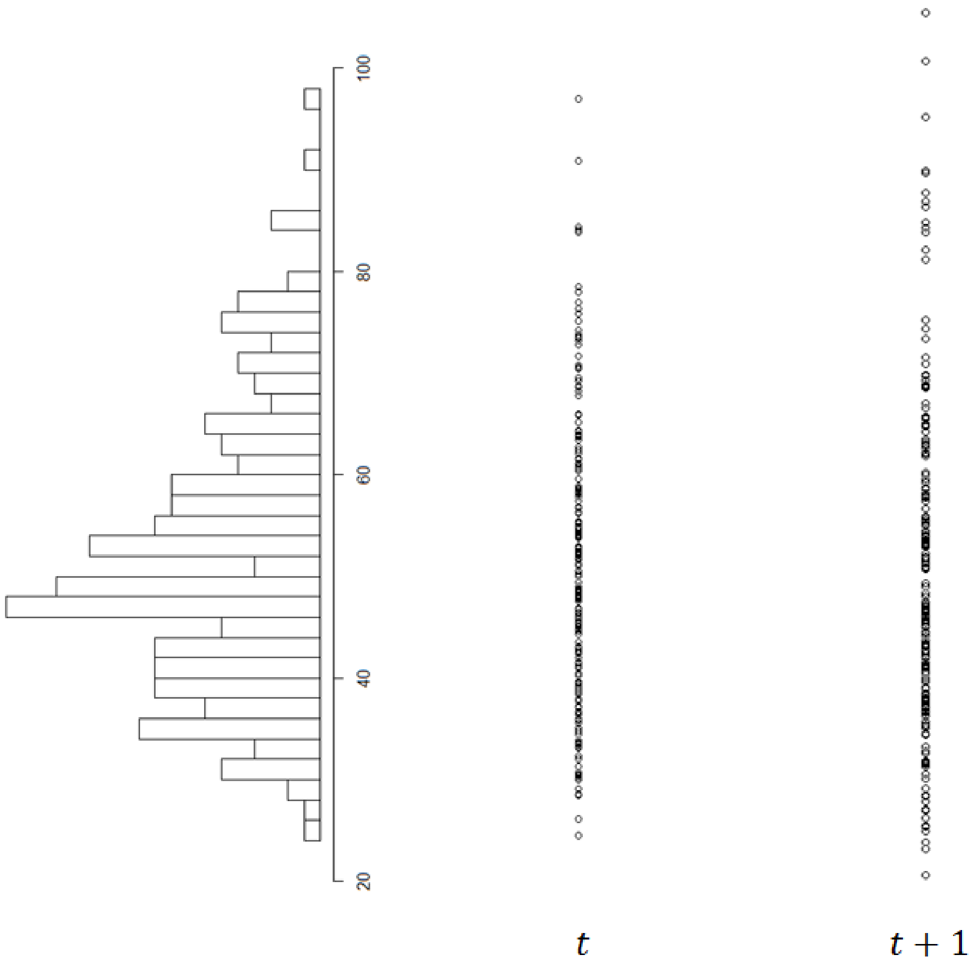

Figure 6.1: Monte Carlo sampling of the inflows approximates the original distribution. On the left, a histogram of the sample inflows of period t is displayed. On the right, sampled inflows of period t and $t+1$ are displayed in the same scale. All data refers to the Southeast market in January.

where $\psi^k_{\tau m}$ is the regression coefficient of the inflows of market k and I^k_m is the inflow of the market k, for the current and previous monthly periods $m = \{\tau, ..., \tau - 12\}$. This choice of regressors allows exploiting the relationship between prices and inflows but respects the *Markovian* assumption on ξ_t. The estimation of the coefficients is accomplished by Ordinary Least Squares method, using data from the available 2,000 NEWAVE simulations. The forward prices f^k_t are calculated as a function of the spot price as in equations (5-4) and (5-23).

This couple of assumptions (independence hypothesis over (I_t, G_t) and approximate spot prices by the inflows) allows us to introduce dependence between the stages. We will now illustrate the proposed approach considering a simplified problem where the uncertainty vector is represented only by inflows I_t and the dependency occurs in the cost vector c_t (following notation in Section

3.2), i.e., spot price is a function of inflows.

If spot price depends only of inflows of the current stage such that $c_t(I_t)$, then we might represent the subproblems as

$$\begin{array}{c} V_t(x_{t-1}, I_t) := \max_{x_t} c_t(I_t)x_t + \mathcal{V}_{t+1}(x_t) \\ x_t \in X_t(x_{t-1}) \end{array}, \tag{6-27}$$

where $\mathcal{V}_{t+1}(x_t) = \mathbb{E}\left[V_{t+1}(x_t, I_{t+1})\right]$. The cost vector is a (deterministic) function of inflows in the current stage, then the problem may be solved with the regular SDDP method.

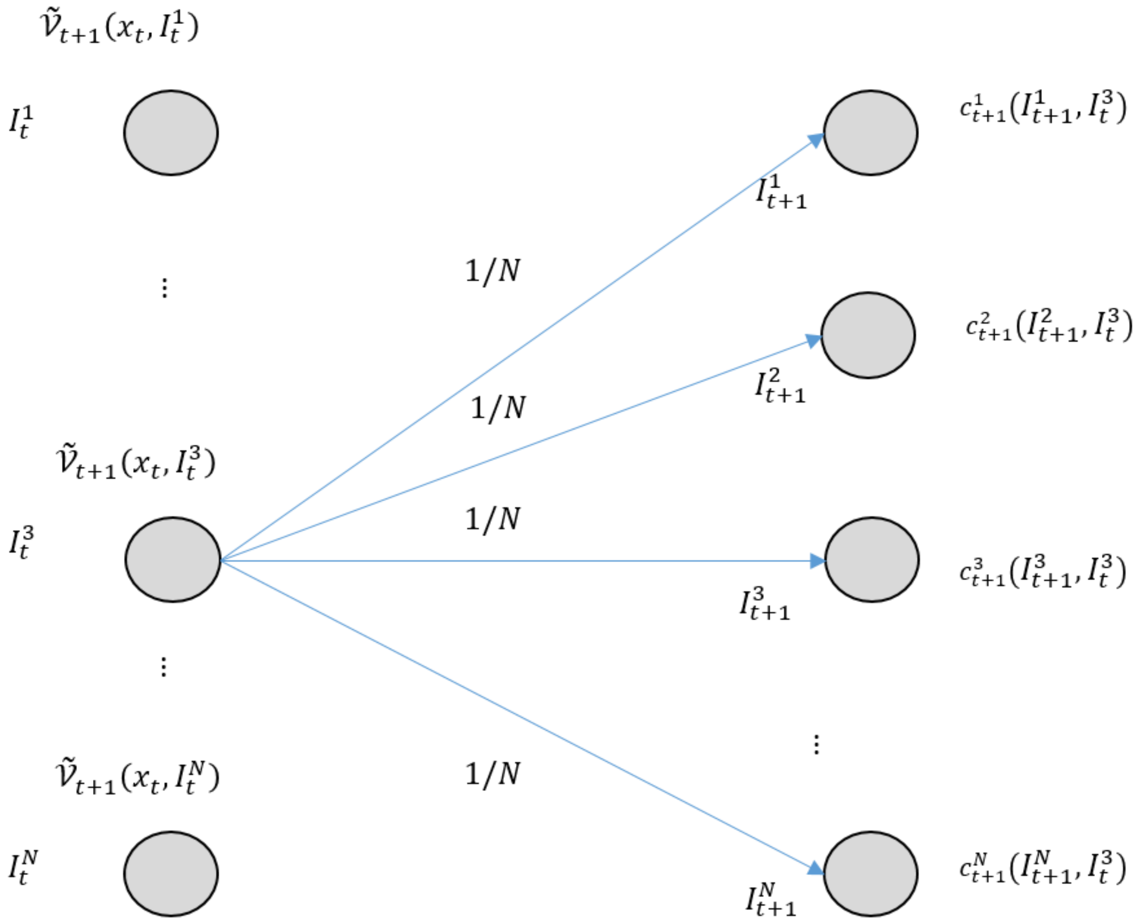

Figure 6.2: Example of an equiprobable Markov chain. In this case, the cost vector is a Markovian function of inflows and each state has its own future cost function.

Consider now that spot prices follow a Markovian function of the inflows of both the current and last periods. In this case, $c_t(I_t, I_{t-1})$ and we will assume stagewise independence for the inflows. The subproblem may be formulated as

$$\begin{array}{c} V_t(x_{t-1}, I_t, I_{t-1}) := \max_{x_t} c_t(I_t, I_{t-1})x_t + \mathcal{V}_{t+1}(x_t, I_t) \\ x_t \in X_t(x_{t-1}) \end{array}, \tag{6-28}$$

where $\mathcal{V}_{t+1}(x_t, I_t) = \mathbb{E}\left[V_{t+1}(x_t, I_{t+1}, I_t)\right]$.

Here, there is no convex reformulation to the problem, but we may apply the Markov Chain variant of the SDDP method to problem (6-28) in order to (approximately) solve it.

In this case, in the forward step we sample $I_t^i, i \in N_t$ for each stage, assuming transition probabilities between states I_{t-1}^n and I_t^i equal to $\frac{1}{N_t}$, because of our independence hypothesis. For each stage, we solve problem $\tilde{\mathcal{V}}_t(x_{t-1}, I_t^i, I_{t-1}^n)$, given by

$$\begin{array}{c} \tilde{\mathcal{V}}_t(x_{t-1}, I_t^i, I_{t-1}^n) := \max_{x_t} c_t(I_t^i, I_{t-1}^n)x_t + \tilde{\mathcal{V}}_{t+1}(x_t, I_t^i) \\ x_t \in X_t(x_{t-1}) \end{array}, \tag{6-29}$$

where $\tilde{\mathcal{V}}_{t+1}(x_t, I_t^i)$ is a piecewise linear approximation of the cost-to-go function for the discrete state I_t^i.

For a sample N_t of discrete inflow states on each stage t, we need to store $\#N_t$ cost-to-go functions for each stage. Figure 6.2 illustrates the problem.

Consider now the backward step of such procedure. For each stage $t \in T-1, \ldots, 1$ and a given candidate solution x_t provided by the forward step, we would like to improve the approximation $\tilde{\mathcal{V}}_{t+1}(x_t, I_t^i)$ for each state $I_t^i, i \in N_t$. This requires solving, for each $I_t^n, n \in N_{t+1}$, problem

$$\begin{array}{c} \tilde{\mathcal{V}}_{t+1}(x_t, I_{t+1}^n, I_t^i) := \max_{x_{t+1}} c_{t+1}(I_{t+1}^n, I_t^i)x_{t+1} + \tilde{\mathcal{V}}_{t+2}(x_{t+1}, I_{t+1}^n) \\ x_{t+1} \in X_{t+1}(x_t) \end{array}. \tag{6-30}$$

This allows including a new cut to the piecewise approximation $\tilde{\mathcal{V}}_{t+1}(x_t, I_t^i)$, remembering that each state I_{t+1}^n is equally probable in the constructed cut.

There is a final limitation to our approach, namely, the integrality constraints. We will relax integrality in the backward step and enforce it in the forward pass. Numerical results provide evidence that this is a successful approach.

Policies obtained by a SAA approach are typically evaluated to assess their performance for the true problem. Here, we will consider our original data 2,000 samples as the data set for this evaluation. The true value of the policies (disregarding any approximations assumed in our heuristic) may be obtained in this fashion.

The presented formulation assumes risk neutrality and we use the proper discount rate r given the risk profile of the market. When considering risk aversion, we will use a weighted average of the expectation and AV@R at the objective function. This approach, performed as in (96), won't directly increase computational cost, but might require an increased amount of samples N in the backward pass, increasing the computational effort. Also, we must apply the risk free rate r_f to discount the cash flows, otherwise risk would be accounted twice, as mentioned in (27) and (50).

7
Numerical Results

We now present a numerical assessment of the proposed framework. We will introduce a small portfolio problem and analyze it in two steps. First, we will show the risk neutral formulation and how the numerical results provide policies that are inconsistent with risk management strategies. Then, we will introduce a convex combination of the expectation and AV@R, illustrating how different policies may be obtained by changing the relative weights of the measures. We will provide some arguments to justify why the heuristic approach is considered efficient, obtaining good investment policies, and then will use the RAROC criteria to help evaluating different investment strategies.

Our numerical study was implemented in AIMMS 3.13 and optimized in GUROBI 5.6, on a personal computer with Intel Core i-7 4500U @ 1.8 Ghz, 8GB RAM memory, under Windows 8 64-bit. We did not use parallel processing, but SDDP based methods offer plenty of opportunities for enhancement. There is also literature on the subject of speeding up the SDDP method (28).

In our example, we consider a portfolio with one Wind Power project (WP) and one Small Hydro plant (SH). For simplicity, we assume that both are in the southeast market, where we will also sell forward contracts, if necessary. In practical cases there are often two or more markets, and one should account for basis risk, as mentioned in Chapter 4. In Brazil, energy prices will differ in the submarkets whenever transmission network congestion occurs, and our general framework is able to handle those risks.

The projects share common characteristics of typical renewable plants. We will denote the currency, Brazilian Reais, by the "$" symbol. The deterministic data of the numerical example is:

- T: 4 years
- r: 10%
- r_f: 4%
- C^{SH}=30 MW
- C^{WP}=30 MW

- FEC^{SH}=17,6 avg-MW
- FEC^{WP}=12,0 avg-MW
- v^{SH}: $ 134,9 million
- v^{WP}: $ 91,6 million
- l: 20 years
- b: 1 year
- $\Lambda_t(\tau) = 0, \forall t, \forall \tau$[1]

Note that, in this numerical study, equation (4-1) applies with zero risk premium and forward price calculations are simplified. Expectations of spot prices were taken in accordance with the evaluated method. We will also present in Section 7.4 the case were forward prices are obtained fro the Schwartz-Smith model studied in Chapter 5.

In our tests, we first evaluate convergence of the algorithm and then its out-of-sample performance. In order to assess the quality of our solutions, we must point out that there are three approximation levels in our method. First, we realize that our "true" model is only an approximation of reality. Furthermore, while we solve the discretized SAA problem, we are really interested in the true problem with continuous distributions. The finite sample size and the stagewise independence assumption lead to an approximation of the true problem. Next, the SDDP method approximates the cost-to-go functions by piecewise linear functions and its convergence is only attained after a large number of iterations.

We will evaluate the policies of the SAA problem on the original 2,000 series, in order to estimate the out-of-sample optimal value of the true problem. An additional approximation is due to the integrality constraint. Our approach requires the relaxation of integrality of the investment decision variable in the backward step. We present numerical evidence that this heuristic is adequate for the problem at hand.

We will compare the results of our heuristic considering stagewise independence and also using the Markov Chain approximation. We will refer to the latter as the *dependent* method, as opposed to the former, stagewise independent, SDDP, which we will denote as the *independent* method. Next, we present the numerical results for the independent and the dependent methods in the risk neutral setting.

[1] $\Lambda_t(\tau)$ is the risk premium, introduced in equation (4-1)

7.1 Risk neutral results

In all numerical tests, we perform only one sample per forward step iteration. This strategy proved to be computationally efficient and is consistent with the results of the literature (80), where problems requiring few iterations to achieve convergence were accelerated by applying one sample per iteration. The disadvantage here is that confidence intervals for the forward step can no longer be calculated. We use as stop criterion a maximum of one hundred iterations, which proved enough for all tests. In Section 7.3 we present the numerical experiments that motivated our parameter choices. More details on acceleration of the SDDP method available in (28).

After convergence, it is possible to evaluate in-sample the forward step to a large number of samples, thus obtaining a confidence interval for the lower limit of the method.

As mentioned in Section 6.2, scenarios will be generated by yearly independent samples of the 2,000 Monte Carlo series of process ξ_t, in a SAA fashion.

For comparison purposes, we show in Figure 7.1 the histograms of the NPV obtained if one invests immediately in any of the candidate projects without signing any forward contracts. Despite positive expected value (\$8.9 million for WP and \$19.1 million for SH), skewness of the returns imply in very risky ventures. There is 43.4% risk of negative results in the WP investment and 38.2% for SH. If one acquires 100% of both projects, some diversification allows for a 40.3% combined probability of negative free cash flows. On the other side, large profits are attained if spot prices rise for long periods of time. This characteristic supports the empirical evidence that contracts are fundamental to allow funding renewable projects.

7.1.1 Independent model for risk neutral setting

We generate M = 10 instances of the SAA problem considering $N_t = 30$ samples per stage at the backward step (more details in Section 7.3). The N_t parameter also equals the number of states in the Markov chain for the dependent case.

Table 7.1 summarizes the independent method main results for each of the instances. The Upper Bound refers to the last upper bound obtained by the first stage problem. The Forward Average and Standard Deviations for the in-sample evaluation of the forward step are presented next. The last two columns

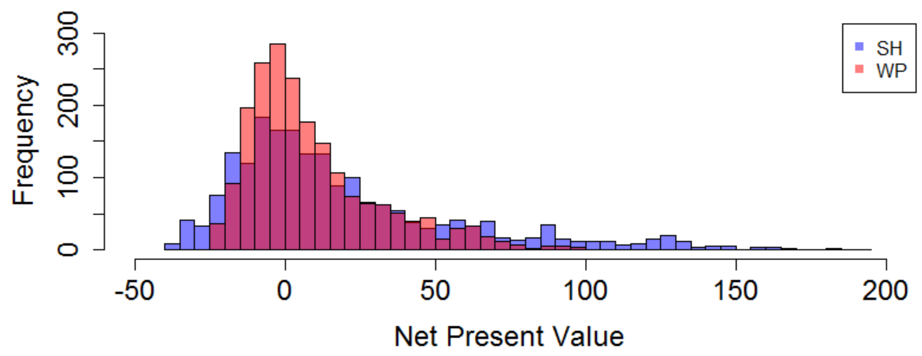

Figure 7.1: Net Present Value Histogram of investment in both sources immediately, with no forward contracting.

refer to out-of-sample evaluation, as will be explained next. All monetary units are displayed in millions of \$.

We first confirm that the SDDP algorithm terminated adequately in our problem for most of the instances. Using data from the three first columns of Table 7.1, we obtain the corresponding 95% confidence intervals displayed in Table 7.2. Note that the standard deviations displayed in the following tables are given for the values rather than their averages. For the respective averages these standard deviations should be adjusted by dividing by the square root of the employed sample size, i.e., by dividing these standard deviations by $\sqrt{2000} = 44.7$. Convergence is verified in all instances.

Next, we address the relaxation of the integrality constraint in the backward step of our method.

We obtain an upper bound to the independent problem by analyzing its convex relaxation. Evaluating the relaxation policies in the same fashion as done in the integer problem, we built a comparison in Table 7.3. One might believe that the integer policies would be far inferior, but, as our numerical results show, practically there is no gap. This is due to the bang-bang characteristic of the solutions to our problem. This feature is common in other cases in the Real Options literature (see, for instance, (31)).

Recall that the obtained bounds refer to the approximated problem, built by assuming stagewise independence of randomly sampled scenarios. We are actually interested in the performance of the policies over the true problem. It is worth to remind that in the SDDP method all the information regarding the policy is available in the cost-to-go functions. Fortunately, we may evaluate the policies of each SAA problem over the original 2,000 series of the 'true' problem. We then can choose the best out-of-sample policy, as usually done in

Table 7.1: Main results for independent model considering $M = 10$ different SAA instances. First three columns refer to the bounds obtained in the last iteration of the algorithm. Last two columns refer to policy evaluation over the original 2,000 series.

		In-sample		Out-of-sample	
M	Upper Bound	Forward Average	Forward Standard Deviation	Forward Average	Forward Standard Deviation
1	40.24	40.02	52.85	45.72	49.01
2	53.2	51.19	56.98	45.53	54.85
3	61.59	61.87	66.99	45.53	54.85
4	46.44	46.14	52.65	45.53	54.85
5	62.85	62.41	54.95	45.72	49.01
6	36.86	36.45	39.53	45.72	49.01
7	39.05	37.99	42.62	45.72	49.01
8	37.93	38.82	56.11	45.72	49.01
9	40.43	39.92	45.96	45.72	49.01
10	39.11	38.96	49.55	45.72	49.01

Table 7.2: Analysis of convergence of independent problem instances. Confidence interval for the lower bound is built considering the average and standard deviation of the forward step.

		In-sample		Confidence Interval	
M	Upper Bound	Forward Average	Forward Standard Deviation	Lower	Upper
1	40.24	40.02	52.85	37.7	42.33
2	53.2	51.19	56.98	48.7	53.69
3	61.59	61.87	66.99	58.94	64.81
4	46.44	46.14	52.65	43.84	48.45
5	62.85	62.41	54.95	60	64.81
6	36.86	36.45	39.53	34.72	38.19
7	39.05	37.99	42.62	36.12	39.85
8	37.93	38.82	56.11	36.36	41.28
9	40.43	39.92	45.96	37.91	41.94
10	39.11	38.96	49.55	36.79	41.13

a SAA approach (93). The last couple columns of Table 7.1 show that most policies have expected result inferior to $46 million. The best obtained policy consists of the deferral of investments, acquiring 100% in both projects in the third period, selling no forward contract. The second best available strategy (in instances 2, 3 and 4) delays investment only one year.

The point estimations of the out-of-sample policy values should be used carefully. As mentioned before, standard deviations of the averages may be retrieved by dividing the standard deviations of the last columns of Table 7.1 by $\sqrt{2000} = 44.7$. One might notice that in the independent case the obtained confidence intervals would overlap, thus it is unclear what would be the best policy.

Table 7.3: Comparison of integer versus convex relaxation for independent problem instances. Forward Average difference between integer and convex relaxation is measured by their Gap.

	In-sample integer		In-sample relaxation		Gap
M	Forward Average	Forward Standard Deviation	Forward Average	Forward Standard Deviation	
1	45.72	49.01	45.72	49.01	0%
2	45.53	54.85	45.53	54.85	0%
3	45.53	54.85	45.53	54.85	0%
4	45.53	54.85	45.53	54.85	0%
5	45.72	49.01	45.72	49.01	0%
6	45.72	49.01	45.72	49.01	0%
7	45.72	49.01	45.72	49.01	0%
8	45.72	49.01	45.72	49.01	0%
9	45.72	49.01	45.72	49.01	0%
10	45.72	49.01	45.72	49.01	0%

7.1.2 Dependent model for risk neutral setting

We will now evaluate the dependent method. Similar to the previous case, Table 7.4 contains a summary of the main results of the dependent method for each of the instances. The difference in this case is that there is a cost-to-go function for each state of the Markov chain. In order to evaluate the out-of-sample solutions of the original set of 2,000 scenarios, it is not clear which cost-to-go function to use in each stage, since the scenario will not habitually correspond to a state. We opt to project the out-of-sample scenarios onto the discrete states using an Euclidean distance criterion, i.e. we apply the cost to go function of the state with the smallest distance to the scenario at hand.

In order to represent spot price dependency, we proceed as described in Section 6.2. The regression equation (6-26) is adjusted to the (log) spot price of each monthly period considering the linear model with respect to inflows. The model has a good fit, with high $adjusted - R^2$. For instance, the January 2016 model has $adjusted - R^2 = 0.8436$.

The termination criterion is again adequate for all cases evaluated in Table 7.5. The analysis of the convex relaxation in Table 7.6 also shows good results.

Regarding the out-of-sample quality of the policies, it is possible to see now in Tables 7.4 and 7.7 that most policies agree with the aforementioned *deferral* strategy, but with an overall improvement the longer we wait. Instances 3, 6 and 9 present out-of-sample results over $50 million, all of them delaying investment but eventually investing 100% in both projects in any scenario. For Instance 9, the policy value has a 95% confidence interval of $[51.84, 54.22]$. Since the lower end of this confidence interval is not greater than the higher end of the confidence interval of the other two aforementioned policies, we should further evaluate them, possibly sampling more scenarios, in order to make our decision. Figure 7.3 summarizes the portfolio decisions of all instances.

In Instance 9, valued at $ 53.26 million, we see that investment was postponed to the last period in all scenarios. In this last period both projects are integrally acquired for all scenarios and on average 32% of the available contract capacity is acquired. Careful analysis shows that the model is actually searching for arbitrage opportunities, since it opts for no or maximum (29.53 avg-MW) forwarding contracting depending on high or low observed forward prices. This behavior is expected in a risk-neutral approach. Figure 7.2 shows the NPV histogram comparison of the dependent and independent approaches. Despite the improvement over the previous approach, there is still risk of high losses.

7.2 Risk averse results

We will consider the following risk measure

$$\mathcal{R}[Z] := (1-\lambda)\mathbb{E}[Z] + \lambda AV@R_\beta[Z], \tag{7-1}$$

where $AV@R_\beta$, $\beta \in (0,1)$, is the Average Value-at-Risk risk measure, defined in Section 3.2.1.

The dynamic programming equations associated with risk measure (7-1) are obtained from equations (6-23) – (6-25) by replacing equation (6-24) with

Table 7.4: Main results for dependent model considering $M = 10$ different SAA instances. First three columns refer to the bounds obtained in the last iteration of the algorithm. Last two columns refer to policy evaluation over the original 2,000 series.

		In-sample		Out-of-sample	
M	Upper Bound	Forward Average	Forward Standard Deviation	Forward Average	Forward Standard Deviation
1	52.41	52.54	30.41	48.33	43.43
2	46.65	46.32	32.82	45.42	40.77
3	58.98	58.47	30.08	50.14	43.38
4	46.37	46.97	28.39	44.51	38.67
5	54.41	54.89	34.63	49.84	44.53
6	53.3	53.36	29.47	50.96	43.63
7	32.33	32.72	20.06	39.43	41.18
8	44.22	44.18	27.1	46.18	44.31
9	64.86	65.85	32.6	53.26	43.69
10	48.89	48.97	27.96	46.88	43.84

Table 7.5: Analysis of convergence of dependent problem instances. Confidence interval for the lower bound is built considering the average and standard deviation of the forward step.

		In-sample		Confidence Interval	
M	Upper Bound	Forward Average	Forward Standard Deviation	Lower	Upper
1	52.41	52.54	30.41	51.21	53.88
2	46.65	46.32	32.82	44.88	47.75
3	58.98	58.47	30.08	57.15	59.79
4	46.37	46.97	28.39	45.73	48.22
5	54.41	54.89	34.63	53.37	56.41
6	53.3	53.36	29.47	52.07	54.65
7	32.33	32.72	20.06	31.84	33.6
8	44.22	44.18	27.1	42.99	45.37
9	64.86	65.85	32.6	64.42	67.28
10	48.89	48.97	27.96	47.75	50.2

Table 7.6: Comparison of integer versus convex relaxation for independent problem instances. Forward Average difference between integer and convex relaxation is measured by their Gap.

	In-sample integer		In-sample relaxation		Gap
M	Forward Average	Forward Standard Deviation	Forward Average	Forward Standard Deviation	
1	52.54	30.41	52.54	30.41	0.00%
2	46.32	32.82	46.32	32.82	0.00%
3	58.47	30.08	58.47	30.08	0.00%
4	46.97	28.39	46.97	28.39	0.00%
5	54.89	34.63	54.89	34.63	0.00%
6	53.36	29.47	53.36	29.47	0.00%
7	32.72	20.06	32.72	20.06	0.00%
8	44.18	27.1	44.18	27.1	0.00%
9	65.85	32.6	65.85	32.6	0.00%
10	48.97	27.96	48.97	27.96	0.00%

Table 7.7: Average investment (x^{WP} and x^{SH}) and contracts (x^{sell}) for $M = 10$ instances of the risk neutral dependent model. There is zero probability of investment in the first period in all policies. Investment in the second period only occurs in few scenarios of policies 2 and 7. Most policies present some chance of investment in the third period, but policy 9, with best out-of-sample performance, postpones investment up to the fourth period.

t	1	1	1	2	2	2	3	3	3	4	4	4
	x^{WP}	x^{SH}	x^{sell}	x^{WP}	x^{SH}	x^{sell}	x^{WP}	x^{SH}	x^{sell}	x^{WP}	x^{SH}	x^{sell}
M												
1							0.02	0.02	0.02	1	1	0.27
2				0.01	0.01	0.01	0.01	0.01	0.01	1	1	0.32
3							0.04	0.04	0.04	1	1	0.29
4							0.01	0.01	0.01	1	1	0.43
5										1	1	0.26
6							0.09	0.09	0.09	1	1	0.33
7				0.03	0.03	0.03	0.15	0.15	0.09	1	1	0.38
8							0.03	0.03	0.03	1	1	0.24
9							0	0	0	1	1	0.32
10							0.05	0.05	0.03	1	1	0.27

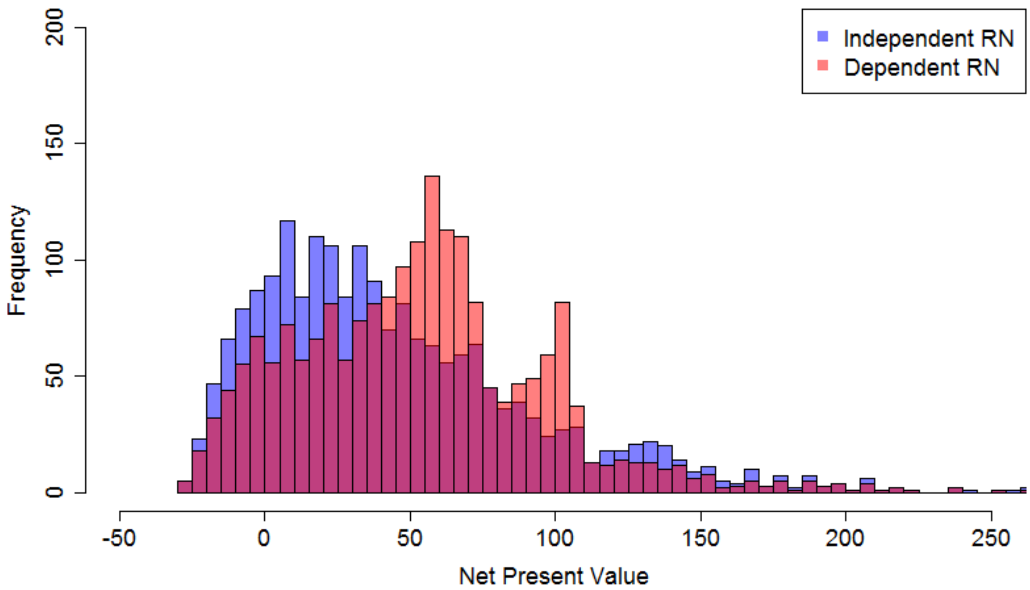

Figure 7.2: Net Present Value Histogram of risk neutral policies obtained by the independent and dependent approaches.

$$\mathcal{Q}_{t+1}(y_t, z_t, \xi_t) := \mathcal{R}_{|\xi_t}\left[Q_{t+1}(y_t, z_t, \xi_{t+1})\right], \tag{7-2}$$

where

$$\mathcal{R}_{|\xi_t}[\,\cdot\,] := (1-\lambda)\mathbb{E}[\,\cdot\,|\xi_t] + \lambda AV@R_\beta[\,\cdot\,|\xi_t] \tag{7-3}$$

is the conditional analogue of the risk measure $\mathcal{R}$.

While in the risk neutral approach we discounted the cash flows in the real measure, using the appropriate discounting by r, here a small adjustment is necessary. Since we are using a coherent measure of risk, $\mathcal{R}[Z]$ corresponds to a certainty equivalent, thus we must use the risk free rate r_f, as would be done in a real option pricing framework to avoid double counting the risk (see (27), (50) for more details).

7.2.1 Independent model for risk averse setting

As in the risk neutral case, we present Table 7.8 to summarize our findings. Here we consider $N_t = 50$ samples per stage, for all t, $\beta = 0.9$ and $\lambda = 0.4$. We can no longer use the forward pass in-sample simulation to obtain a confidence interval to the lower bound of the problem. Here, we would expect that an increase in the risk aversion (given by λ) would correspond to a reduction in the uncertainty of the policy. In Figure 7.4 we plot the out-of-sample expected return versus expected return minus AV@R of the different

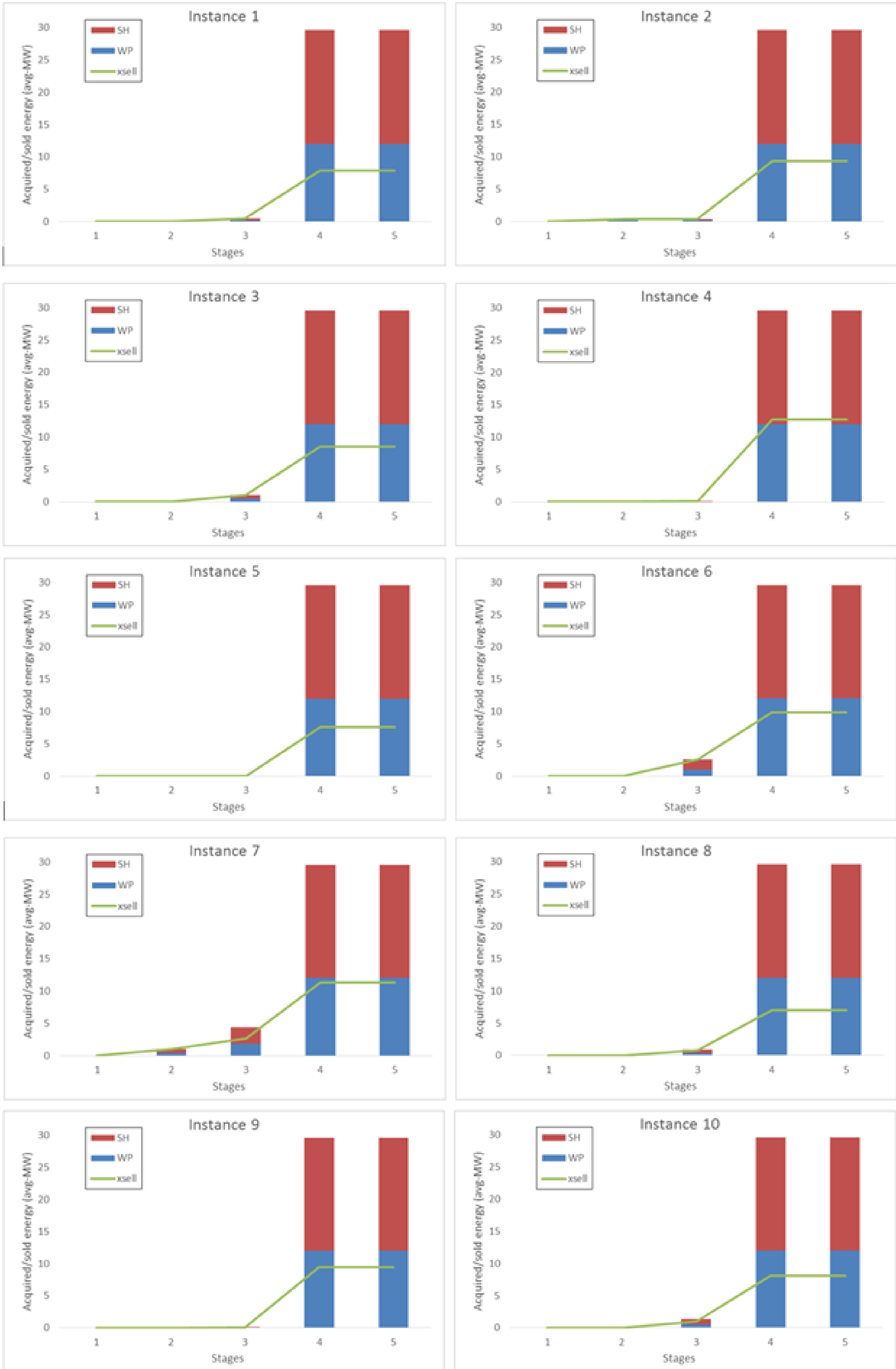

Figure 7.3: Average investment in WindPower(WP) and Small Hydro (SH) and sold contracts ($xsell$) for $M = 10$ instances of the risk neutral dependent model.

samples for several different risk aversion levels. It is possible to notice that the anticipated relation between risk aversion and policy uncertainty did not occur.

Figure 7.4: Risk and return of instances of the independent problem for different λ values

7.2.2
Dependent model for risk averse setting

We repeat the tests considering the dependent approach. As before, in-sample and out-of-sample results are displayed in Table 7.9, for $\lambda = 0.4$. Again, we cannot evaluate the convergence using the forward simulation since it is not the lower bound to the method in the risk averse case.

We repeat the previous experiment and sample some problems with different values for λ. In Figure 7.5 it is possible to see that an risk-reward frontier was built by sampling different problems. One would select a policy that best suits the risk management requirements from this set of Pareto optimal solutions.

Analysis of the solutions show that risk averse policies in general agree with postponement of the investment and acquisition of close to 100% shares on both projects, as in the risk-neutral case. Here, the main difference is that, the higher the risk aversion level, the bigger the amount of forward contracts is signed. This means, the proposed policies reduce the cash flow uncertainty by acquiring some amount of forward contracts.

As an example, one obtained policy [2] with out-of-sample expected payoff of \$ 36.65 million is displayed in Figure 7.6, compared to the risk neutral

[2] 3^{rd} instance of risk averse setting with $\lambda = 0.8$

Table 7.8: Main results for independent risk averse model considering $\lambda = 0.4$ and 10 different SAA instances. First three columns refer to the bounds obtained in the last iteration of the algorithm. Last two columns refer to policy evaluation over the original 2,000 series.

	In-sample			Out-of-sample	
M	Upper Bound	Forward Average	Forward Standard Deviation	Forward Average	Forward Standard Deviation
1	23.09	24.51	9.26	23.5	11.22
2	22.33	46.43	53.87	45.72	49.01
3	29.24	59.3	64.41	45.72	49.01
4	21.95	45.95	51.65	45.72	49.01
5	35.83	62.41	54.95	45.72	49.01
6	21.09	36.45	39.53	45.72	49.01
7	21.22	22	10.44	24.89	12.79
8	20.97	22.4	12.53	24.32	12.07
9	22.55	23.99	10.06	24.66	12.49
10	22.35	38.96	49.55	45.72	49.01

alternative. There is a noticeable reduction in risk of this policy. There is only 0.05% probability of negative net cash flows, with positive AV@R of $ 17.48 million. This policy consists in delaying investment for two years in most scenarios. The SH project was integrally acquired in every scenario, while the WP project share stayed between 95% and 100% in all scenarios. Forward contracting was done in every scenario, ranging from 60% to 75%, with an average of 68% of maximum FEC (20.1 avg-MW).

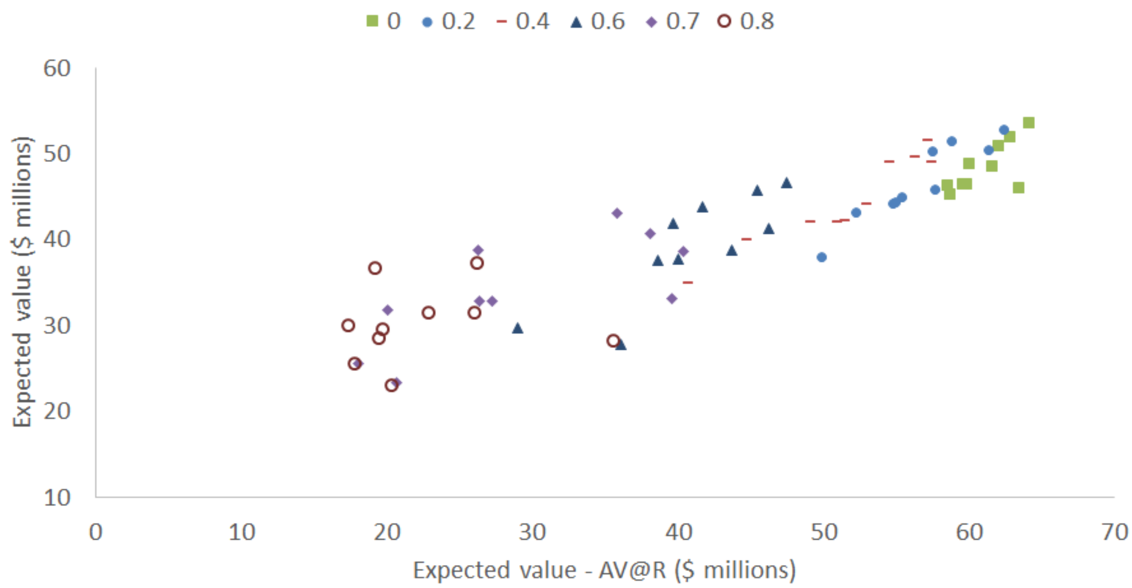

Figure 7.5: Risk-reward frontier for the dependent problem

Table 7.9: Main results for dependent risk averse model considering $\lambda = 0.4$ and 10 different SAA instances. First three columns refer to the bounds obtained in the last iteration of the algorithm. Last two columns refer to policy evaluation over the original 2,000 series.

	In-sample			Out-of-sample	
M	Upper Bound	Forward Average	Forward Standard Deviation	Forward Average	Forward Standard Deviation
1	35.38	50.34	26.05	41.98	31.79
2	30.4	44.79	28.38	42.21	35.67
3	39.56	56.14	27.02	49.03	37.36
4	27.51	43.85	23.91	40.03	29.64
5	34.98	52.91	30.01	49.1	38.99
6	35.48	52.41	26.49	49.73	38.11
7	18.07	30.12	15.72	35.03	29.93
8	0.17	42.58	23.74	44.26	38.84
9	45.08	64.08	29.18	51.61	36.75
10	31.83	46.72	23.24	42.22	33.88

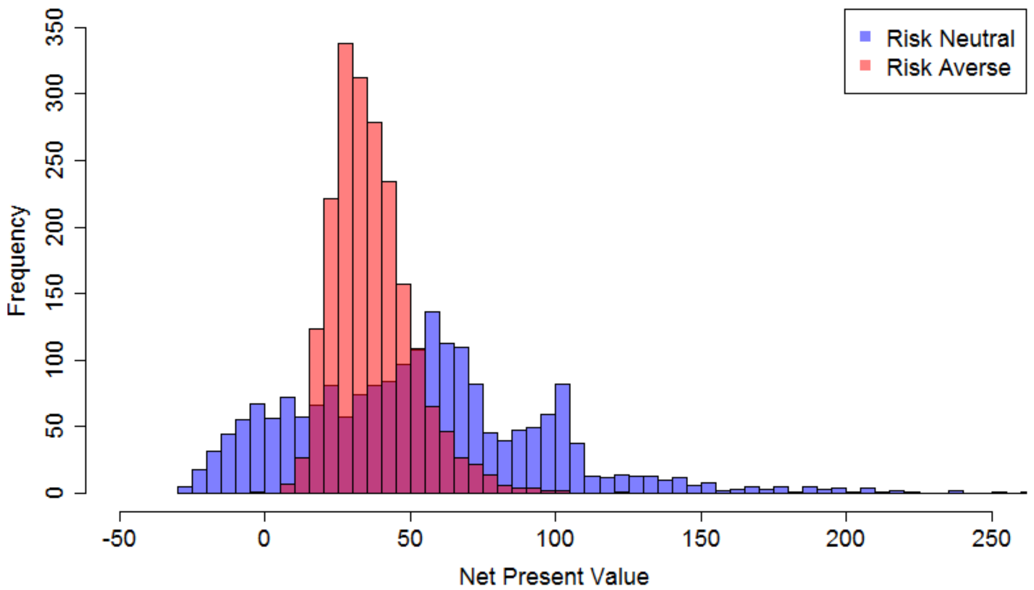

Figure 7.6: Comparison of Net Present Value Histogram of risk neutral and risk averse policy considering $\lambda = 0.8$

7.3 Sensitivity Analysis

We perform now some additional tests to access the effect of the model's chosen parameters and the quality of the obtained solutions.

7.3.1
Bound Analysis

Since the devised policies are obtained in a heuristic fashion, it is worthwhile to try to estimate the quality of our solution by analysis of bounds to the optimal solution of the problem. We will use a rolling horizon approach to provides us with a lower bound to the optimal solution and wait-and-see analysis to obtain upper bounds.

In the wait-and-see approach (11), we evaluate the optimal solution for each scenario independently (disregarding nonanticipativity constraints). The value of the wait-and-see solution is obtained by averaging the optimal value of such problems. Despite reliance on clairvoyance, such solutions, due to relaxation of nonanticipativity, present themselves as an upper bound to our problem.

The computed wait-and-see solution for the risk neutral case is valued at \$ 104.23 million. This solution, similarly to the policy evaluated in Section 7.1.2, relies heavily in forward contract arbitrages. In practice risk premiums are non-positive and one should not expect to profit from such arbitrages. It is thus valuable to evaluate the wait-and-see solution when forward contracting is not possible. In this case, the value of the wait-and-see solution is \$ 59.67 million. This poses as a much closer value to the policy values obtained in Section 7.1.2.

A rolling horizon approach allows us to compute suboptimal policies for the problem at hand. Here, we may use a two stage problem formulation to evaluate such decisions. For each stage t, we solve a two stage problem, where nonanticipativity in the portfolio is enforced throughout the 2,000 scenarios. The policy with higher optimal value among the stages is selected as the candidate solution.

The best policy provided by the rolling horizon approach to the risk neutral case is to invest in the last stage in 100% of both projects, without acquiring any contracts. This policy yields a value of \$ 45.72 million. Comparison of this value with the policies obtained in the risk neutral independent case show us that the policies obtained then are no better than this bound. Policies generated in Section 7.1.2, which incorporate dependency, present significant improvements over this bound.

7.3.2
Number of scenarios in backward step

As mentioned in the previous chapter, the computational effort is proportional to the number of scenarios N_t used in the backward step of the algorithm. While it may be advantageous to use as few as possible scenarios, a minimum number is deemed necessary so that sampling of the cutting planes can be accurate, thus creating valuable policies.

A larger number of scenarios is necessary in the risk averse case, since the AV@R functional requires evaluating the new cuts using the $(1-\beta)\%$ worst valued scenarios. Even for reasonable valued, say $\beta = 0.1$, and for instance $N_t = 100$, it means computing the cuts using information from only ten scenarios.

In Figure 7.7, we present policies obtained in the dependent model for the risk averse setting with different values for N_t. We make $N_t = N, \forall t$, as in the remaining numerical experiments. All instances were generated with risk aversion parameter $\lambda = 0.8$. Comparison with the risk-reward frontier in Figure 7.5 shows that higher values of N leads to policies that consistently present a risk averse profile. In the other hand, as we reduce the number of backward step scenarios, some policies begin to resemble the risk-reward profile of instances with a smaller risk averse coefficient λ. With $N \leq 30$, it is possible to notice that, despite the high risk aversion coefficient $\lambda = 0.8$, some policies may even present risk-reward profile similar to the risk neutral instances.

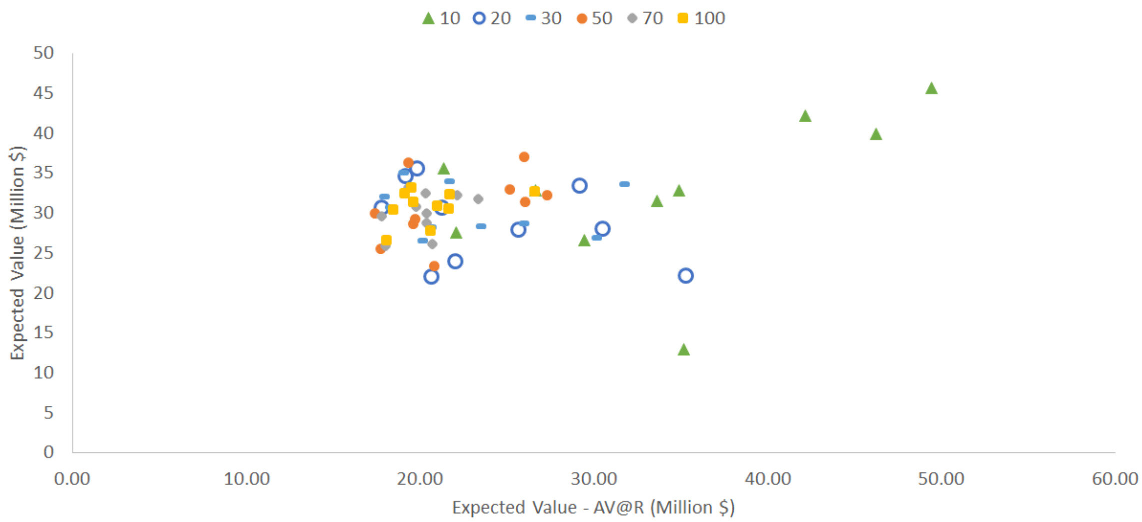

Figure 7.7: Sensitivity to the number of samples N in backward step.

This sensitivity analysis may help us evaluate the minimum number of backward step scenarios to consider in the algorithm. This allowed us determine that, while $N_t = 30, \forall t$ is sufficient in the risk neutral case, more samples are needed in the risk averse setting.

7.3.3 Stopping criterion

As mentioned in Section 7.1, we only evaluate a single sample for each forward step of the SDDP algorithm. While this greatly reduces the computational effort, calculation of bounds for the traditional stopping criteria of the SDDP method is not possible. Alternatively, we monitor the improvement of the policy values, as displayed in Figure 7.8. Lowerbound is obtained from a sample of the last 30 forward iterations (hence no values before iteration 30). As consistently verified in our numerical tests, in the risk neutral setting upperbounds converge in very few iterations, and after 100 or less iterations policy values stop improving. In the example depicted in Figure 7.8, while a traditional convergence criteria might lead to stopping in iteration 186, policy values have stopped improving significantly since iteration 65. If we stop updating the cost to go functions after iteration 65, a traditional stopping criteria would still lead us to stop before 200 iterations. Complementary analysis done in the previous section confirms policy convergence in all instances. Likewise, in the risk averse setting 40 iterations where deemed sufficient for our termination criteria.

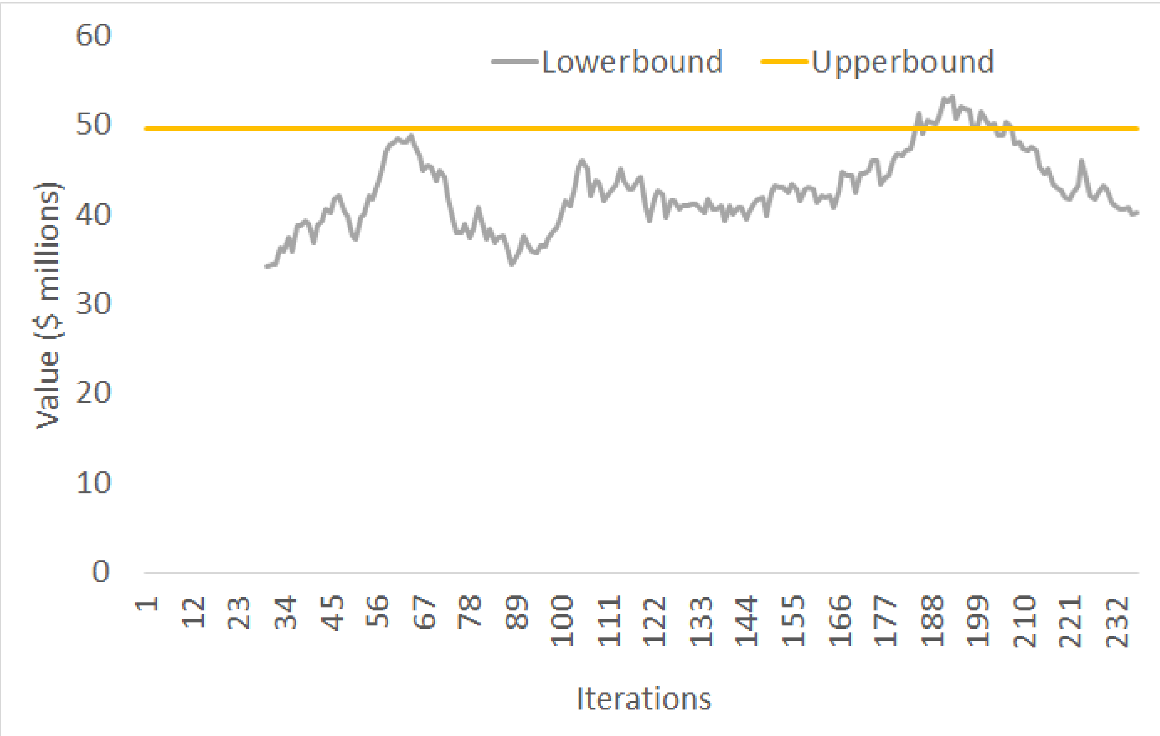

Figure 7.8: Bounds of the SDDP method for the dependent model for each iteration of the algorithm.

7.4
Forward prices by Schwartz-Smith model

We now price forward contracts by the Schwartz-Smith model, using the approach introduced in Section 5.6. Differently from the previous results, the market risk premium will be positive or negative, according to price dynamics. In Figure 7.9 we present average spot and forward prices for the 2,000 sample series. As can be seen, on average forward prices are lower than spot prices. This negative premium for long term contracts may be understood as evidence that generators may be willing to accept lower income in order to make their investments possible.

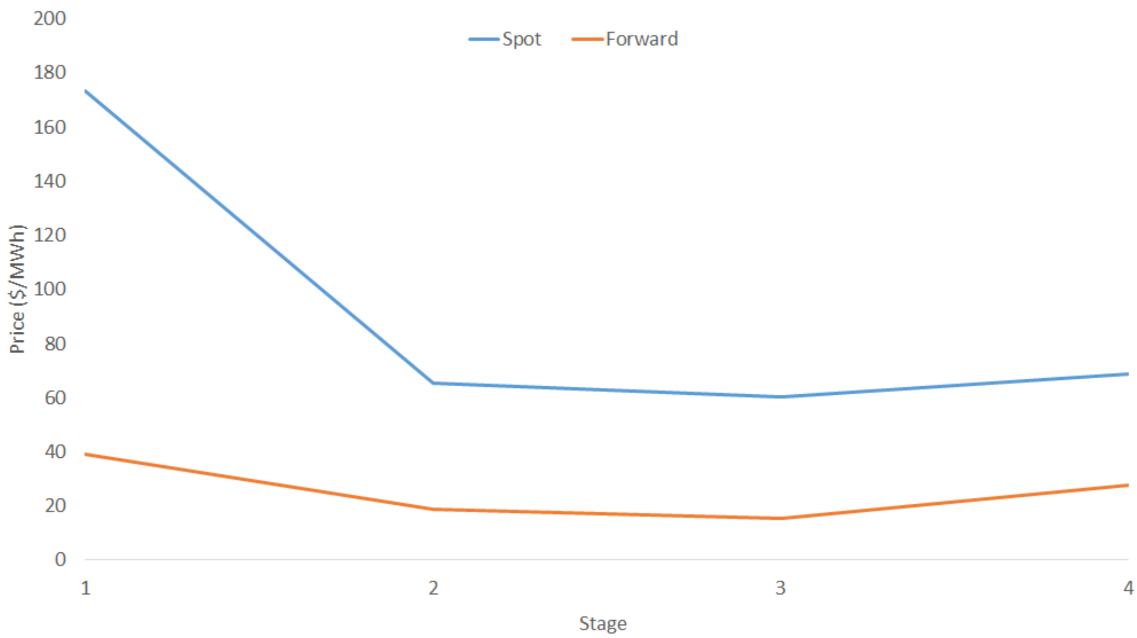

Figure 7.9: Comparison of average spot and forward prices using the Schwartz-Smith model.

For each of the available 2,000 data scenarios, forward prices f_τ are obtained by Kalman smoothing techniques using the relations of equation 5-23 and using observable spot prices π_t. In the backward step of the algorithm forward prices are determined using relation 6-26.

Reproducing the exercise of the previous section, we obtain in Figure 7.10 the risk-reward frontier when considering forward prices by the Schwartz-Smith model. In the figure there is a noticeable difference between contracts clustered in two ranges: the first group has the highest expected return, close to \$ 50 million, and also higher risk[3]. The second group has expected return

[3]Remember that we are denoting as risk the difference between the expected value and the AV@R of the policies)

always lower than \$ 40 million, with risk ranging from as low as \$ 20 million up to \$ 65 million.

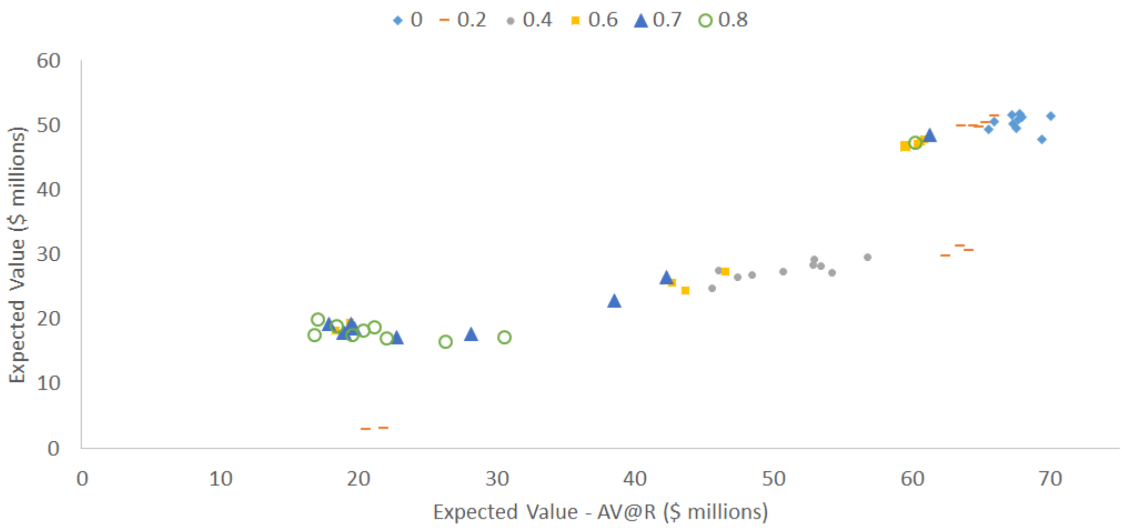

Figure 7.10: Risk-reward frontier when considering forward prices by Schwartz-Smith model

A closer look into the policies reveals that the first group is composed of policies where investment occurs from the second stage onwards, never investing in the first period. Those policies are associated with low amounts of forward contracting (at most 20% of available FEC). On the other hand, all the policies in the second group refer to investment strategies with no postponement, i.e., investment always occurs in the first period. In this policy group, forward contracting increases as the level of risk aversion (defined by the weight λ) increases. A sample risk averse policy, with $\lambda = 0.8$, may yield average return of \$ 20 million and consists of investment in the first period, acquiring 100% of the WP project and 95% of the SH, while selling 70% of maximum FEC (20.7 avg-MW) in forward contracts.

The lower risk-reward ratio may be attributed to the negative risk premium on the forward contracts, as well as difficulties in devising good policies by the model. Future work may include studying different strategies to represent the forward price model.

As seen from this exercise, the policies obtained under the zero market risk premium model may be far too optimistic if real market risk premiums are negative. For instance, consider the risk averse policy studied in Section 7.2.2 and displayed in Figure 7.6. Under the forward prices obtained by the Schwartz-Smith model, the AV@R is \$ -2.93 million, as opposed to the \$ 17.48 million in the zero premium instance. The probability of negative cash flows is 5.90%, while in the previous section we found it to be only 0.05%. If the risk premiums are not correctly accounted for, the investor may incur in *model risk*, leading to inadequate investment decisions.

7.5 Choosing a strategy

As mentioned in Chapter 4, after obtaining the risk-return frontier, we must define which policy to follow. In order to add maximum value, the chosen strategy should be the one with the highest NPV that meets both the regular capital expenditures and risk capital budgets of the investor. The risk capital budget is defined by his risk appetite, as defined in Chapter 3.

If the investor has opportunities to allocate his capital to other markets, he might use the RAROC criterion (4-7) to choose the portfolio with best risk/return profile. Any remaining capital can then be allocated to other ventures. This would hold true for a company that has investments in non renewable sources, transmission, distribution, or other industries.

In this case, an optimal strategy would be to choose high RAROC policies and allocate his remaining capital to those other ventures, trying to obtain the overall portfolio with highest NPV inside his risk budget.

	λ					
M	0	0.2	0.4	0.6	0.7	0.8
1	0.202	0.196	0.189	0.174	0.157	0.137
2	0.211	0.197	0.189	0.177	0.149	0.129
3	0.218	0.217	0.214	**0.200**	**0.184**	**0.177**
4	0.202	0.193	0.184	0.126	0.125	0.124
5	0.209	0.220	**0.217**	0.192	0.154	0.146
6	**0.227**	0.223	0.216	**0.205**	0.185	0.150
7	0.188	0.171	0.163	0.143	0.112	0.111
8	0.201	0.200	0.196	0.187	0.178	0.150
9	0.221	**0.226**	**0.223**	**0.206**	0.197	0.174
10	0.197	0.197	0.188	0.173	0.156	0.142

Table 7.10: RAROC of the different policies, according to risk aversion parameter λ. Values in bold lie in the risk-return frontier.

Table 7.10 summarizes our findings for all the evaluated policies in the dependent approach. We highlight in bold type those policies that lie in the risk-return frontier. Risk neutral solutions are included here in the set of policies with $\lambda = 0$. RAROC of the best risk neutral solution ($\lambda = 0$) may be high, but in practice arbitrage of forward contracts is not possible. For comparison purposes, a implementable policy would be to invest without any contracts, as we saw in Section 7.1.1. Despite a potential for upside during price spikes, there is still the unaccounted credit risk of defaulting counterparties in the CCEE.

This distinction of unfeasible portfolios allows us to raise an important issue. What might sound as value destruction (by applying risk management efforts) in fact should be reframed: if risk mitigation allows undertaking an otherwise unfeasible portfolio, then this strategy creates value proportional to the additional net present value. As seen in the beginning of this chapter, it would be unfeasible for most investors to execute the immediate investment strategy. This would result in project abandonment, with zero value. Policies devised here reduce the probability and intensity of losses, making the investment a feasible venture.

In our numerical results, policies for $\lambda = 0.4$ and $\lambda = 0.6$ still provide a relatively high RAROC ratio. One should avoid as much as possible resorting to policies with higher λ, since there are diminishing benefits from the risk management efforts. The reduction of expected profit occurs at a higher rate than the reduction in risk capital for the instance at hand. Risk averse policy depicted in Figure 7.6 may have very low risk, but we also loose valuable upside potential of profiting from some sales in the spot market in high price periods.

8
Conclusions and Future Work

In this Thesis, we presented an integrated framework to devise multistage optimal investment strategies in renewable energy portfolios. Such framework allows employing commonly used risk management tools. We were able to apply the most important managerial flexibilities, namely diversification, partnerships, postponement options and use of forward contracts. The approach is suited for the FTE or similar markets.

Our data process includes not only the main risk factors, renewable generation and spot and forward energy prices, but also the market inflows, which proved invaluable to assess the investment policies. We introduced a forward contract model using the Schwartz-Smith two factor approach, coupled with spot and OTC data.

The available data is provided in the form of Monte Carlo simulated time-series. Since there is no filtration available, an approximate dynamic programming solution, such as the Lest Squares Monte Carlo, is more suited than a scenario tree approach.

We followed a SAA approach, generating sample paths by drawing scenarios randomly from simulated time-series. Our approximate dynamic programming approach to generate investment policies is based on Stochastic Dual Dynamic Programming. Non linearity was overcome with the use of a Markov Chain, to represent the future cost function for each state, and relaxation of integrality constraints in the backward step, taking advantage of the fact that relaxation solutions tend to be very similar to their integer counterparts. In order to define the portfolio, we evaluate all the candidate policies over the original 2,000 simulated time series and select one that best adheres to our risk-return profile.

Incorporation of risk averse by means of an AV@R measure allowed us to build a risk-return frontier, from which to choose candidate solutions from.

Representing interstage dependency of energy prices proved to be fundamental in obtaining good policies in our numerical examples. We obtained good results with our price regression, but there is no general case approximation.

Valuation of such portfolios under high uncertainty is hard to accomplish and is under active research investigation. In the energy industry, proper valuation plays a major role due to the high volatilities associated with the sources of uncertainty of this business.

It is common for positive NPV renewable projects to suffer from financing issues due to high revenue uncertainty. Since it is possible to obtain a set of Pareto efficient strategies, one might choose the one that most adheres to the investor financing needs. This allows the investor to better shape his capital budgeting and financing decisions.

This framework may be valuable for an investor deciding his optimal strategy to join the market or to support an energy trader in his pricing strategy.

The proposed approach, despite its heuristic nature, in practical instances may present near optimal solutions, due to the bang-bang feature of the problem. This may encourage its application to other classes of problems.

The policies provided by the model are evaluated by Monte Carlo simulation, and can be compared to any strategy otherwise devised by a decision maker by introducing some additional constraints to the presented solution approach. It is thus very easy to gain insight in solution quality.

The main disadvantage of the method is that it may become computationally infeasible for a large number of state variables, i.e., for a portfolio with a large number of plants and contracts under evaluation.

In future work, the Markov Chain approach could be reassessed. By devising a strategy to calculate transition probabilities, for instance, using information from the NEWAVE model, improvements to the policies could be found.

A Least Square Monte Carlo approach such as (20) might benefit from two outcomes of this Thesis: first, the forward contract model proposed here might be used to price energy contacts. Second, experience from our price regression might provide valuable information in building the *basis*[1] of such LSMC procedure.

From the stochastic modeling standpoint, we might pursue improvements in the stochastic model of the generated power. Renewable energy models for monthly generation using the LASSO method such as (102) show improvements over the VAR model that is generally used and applied in this work.

The proposed contract model allows modeling the term structure of forward contracts, which may be used in our valuation model, but also for several other applications.

[1]Explanatory variables of the regression are called the basis of the LSMC method.

One limitation of our approach is that we can not calculate the risk premium over the expected spot prices. Future work includes using the spot price forecasts of the NEWAVE model to obtain an expected spot price curve. By augmenting the Schwartz-Smith model with the real measure price process of the expected price curve, we could be able to estimate the risk premiums in the Brazilian market.

Extension of the Schwartz-Smith model with seasonal components was done in (62). We suggest another extension, tailored to the Brazilian market, by adding inflows as explanatory variables to the forward curve model. This might further improve the model fit, since inflows are able to explain price behavior.

For applications of midterm contract pricing, BRIX data might aid improving results, since there are several contract maturities available on this time-frame. In future work, one might use BBCE transaction information to improve the forward curve modeling, as more data becomes available in this platform.

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A
Probability background

Let's define a discrete set of scenarios $\omega \in \Omega$. We will also define a collection $\mathfrak{F}$ of Ω subsets. This collection is denoted a σ-algebra if it follows:

- $\Omega \in \mathfrak{F}$,

- If $A \in \mathfrak{F}$, then its complement $A^c \in \mathfrak{F}$,

- If $A_1, A_2, A_3, \ldots \in \mathfrak{F}$, then $\bigcup_{i=1}^{\infty} A_i \in \mathfrak{F}$.

The σ-algebra created by open subsets of $\mathbb{R}^n$ is named a Borel σ-algebra and is represented by $\mathcal{B}$.

The set Ω equipped with $\mathfrak{F}$ is called a measurable space and denoted by $(\Omega, \mathfrak{F})$.

A probability measure $P : \mathfrak{F} \to [0, 1]$ is a function that associates a measure to an element of a σ-algebra, having the following properties:

- $P(\Omega) = 1$,

- If $A_1, A_2, A_3, \ldots \in \mathfrak{F}$ are disjoint, then $P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$.

The measurable space $(\Omega, \mathfrak{F})$ equipped with P is denoted $(\Omega, \mathfrak{F}, P)$ and is called a probability space. Notice that we can only measure the probability of elements in the σ-algebra. Following (97), we can say that the σ-algebra represents a record of the available information.

A function $g : \Omega \to \mathbb{R}^n$ is measurable if its inverse image is in σ-algebra $\mathfrak{F}$, i.e., $g^{-1}(A) := \{\omega \in \Omega | g(\omega) \in A\} \in \mathfrak{F}, \forall A \in \mathcal{B}$.

B
The Capital Asset Pricing Model

The Mean-Variance framework was soon extended by several contributions. One of main improvements one might consider is the inclusion of a risk free asset in the portfolio. Capital allocated to the new risk free asset is rewarded by a known return r_f. The modified model may be represented by

$$\underset{w}{\text{Max}}\, w^T \Sigma w \tag{B-1}$$

$$\mu^{\mathsf{T}} w + r_f(1 - e^{\mathsf{T}} w) \geq \hat{r}. \tag{B-2}$$

The new efficient solutions of this problem lie on the Capital Market Line (CML), with performance superior to the previous efficient frontier. The linear equation of the CML has intercept r_f and there is only one portfolio in common with the previous efficient frontier, the *tangency portfolio*. Figure B.1 illustrates the CML and the solutions to Markowitz's problem. The expected return of an efficient portfolio P is given by the relation

$$\mathbb{E}[r_p] = r_f + \frac{\mathbb{E}[r_t] - r_f}{\sigma_t} \sigma_p, \tag{B-3}$$

where r_t and σ_t represent return and variance of the tangency portfolio and σ_p is the variance of the portfolio P. Notice that this only applies to efficient portfolios. All efficient portfolios are composed of a weighted combination of the risk free asset and the tangency portfolio.

The Capital Asset Pricing Model (CAPM), independently proposed by Lintner and Sharpe, is a theorem that provides a relation to the expected returns of any asset or portfolio.

Considering that the market is on equilibrium, where every investor has access to the same information and all necessary transactions were undertaken, then all investors should agree on the same tangency portfolio. In this case, this portfolio is known as the *market portfolio*. In this way, it is possible to determine the market portfolio as the value-weighted average of all market traded assets, given their current prices and number of available shares. The results of the CAPM are valid for a two period investment problem and assuming relatively fair hypothesis, such as normality of returns or quadratic

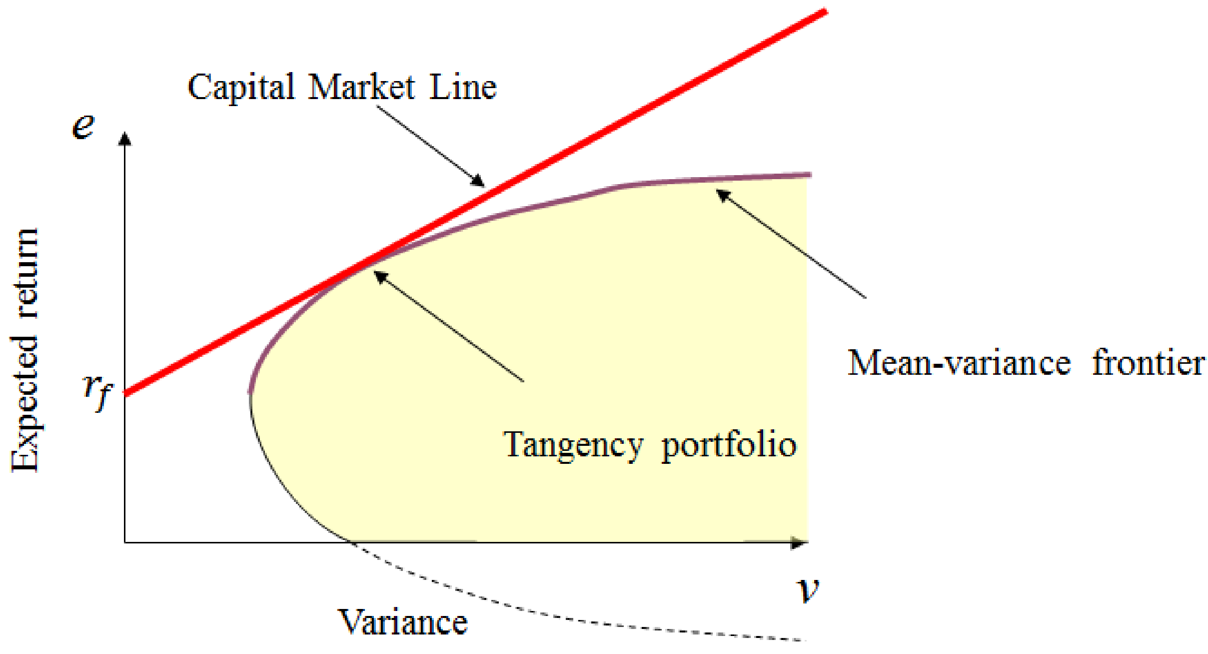

Figure B.1: The Capital Market Line touches the previous efficient frontier in the tangency portfolio

utility functions (there are several versions of the CAPM, leading to similar results). The main result of the CAPM is an equation to price (value) an asset, known as the Security Market Line (SML):

$$\mathbb{E}[r_A] = r_f + (\mathbb{E}[r_M] - r_f)\beta_A \tag{B-4}$$

The SML equation provides a relation of the expected returns $\mathbb{E}[r_A]$ of any portfolio or asset A to the risk measure beta. Here $\mathbb{E}[r_M]$ is the market expected return. The coefficient β_A of an asset A is given by $\beta_A = \frac{\sigma_{M,A}}{\sigma_M^2}$. It represents the amount of nondiversifiable (systematic) risk that the portfolio (or asset) holds.

The slope of the SML is given by $(\mathbb{E}[r_M] - r_f)$ and represents a market risk premium. The beta represents the market risk exposure.

C
Modigliani-Miller and the Value of Risk Management

This section follows (15) and (52).

In 1958, Modigliani and Miller (MM) wrote a proposition over the capital structure of companies that remains one of the most important results in financial management. MM's Proposition 1 states that, in a perfect market, in the absence of taxes and other inefficiencies, the level of leverage of a company does not affect its value. The basic idea is that cash flows are additive, then, splitting the cash flows between equity and debt holders does not generate additional value.

MM theorem states that under these conditions, there is no difference in the capital structure for adding value, so the risk of highly leveraged firms should not matter.

This result implies that there is no difference between a company solely funded by the investor's equity or highly leveraged with debt. Despite obvious increase in the expected return on equity, due to additional firm's risk, value of a leveraged firm stays the same, since it may not be able to fulfill its obligations to their debt holders.

In other words, in perfect markets, value of a company is define by the left hand side of the balance sheet - it's Assets. The Liabilities, that lie on the right hand side, have no place in adding value to the company. Funding decision such as deciding to lease or invest in new equipment, or how to issue debt, should make no difference to the company.

What might sound as a controversial result, should be really interpreted backwards: Since markets are not perfect, one should search how to exploit those imperfections to add value. The right hand side of the balance sheet can not directly add value to the firm, but proper management of the liabilities might add indirect value by enabling the company's strategy, allowing it to add value to the assets.

The first and easiest to understand imperfection is taxes. Since debt payments are tax deductible in most countries, there is an incentive to acquire debt, known as *tax shield*. Tax shield exists and adds value because the government's take in the company's cash flows is reduced. This alone would create an incentive to arbitrarily large leverage ratios, but this is not noticed in practice.

What counterbalances the tax shield effect is the cost of financial distress. As the company increases the amount of liabilities it is subject to, the risk of not being able to fulfill it increases equivalently. The costs associated with those risks are generally called *cost of financial distress*.

The first source of those costs is the bankruptcy costs. Once a company goes bankrupt, there are several costs associated with the process, such as court and legal fees. The *expected* cost of bankruptcy then is accounted for by the debt holders, that required increasing interest rates when they notice higher probability of default in a company.

There are also indirect costs associated with bankruptcy, such as reputation and managerial costs. For instance, it is harder to a bankrupt company to be found trustworthy in commercial agreements and to maintain its market share. Those indirect costs are also priced by debt holders.

Financial distress may present to a company much sooner than in a bankrupt state. When a company is in bad financial shape, third parties might engage in some value destructing behavior, due to the fear for unavailability of inventories and fear of not being paid for goods supplied. Also, debt or derivative positions may have covenants (or trigger clauses) that allow an early liquidation of the debt or require additional collateral.

The resulting balance may be seen in Figure C.1. The tax shield and financial distress costs create a break even point that might be seen as an optimal debt ratio. Firms that base their financing decisions using this rationale are said to follow the *Trade-off Theory*[1].

We can extend the rationale behind MM framework to understand the value of risk management. As stated in MM proposition, financing decisions should not affect the value of the firms, since the added cash flows are not changed. Risk management may be seen equivalently, since in a hedging activity the net result of both sides of a contract (such as a derivative) is zero.

Also, as mentioned before in the CAPM section, risk management might be performed by investors, by diversification or hedging of their risks away, to the exact degree they fell right. Those questions pose as an argument in favor of the idea that companies should not practice any risk management policies.

[1]In practice, there is better empirical support for the Pecking Order Theory, that in essence states that companies, due to information asymmetry, follow a specific order of financing sources when they need funding. Also, since reputation is easier to lose than to build, most firms (like Disney (27)) choose to keep their leverage below the optimal ratio.

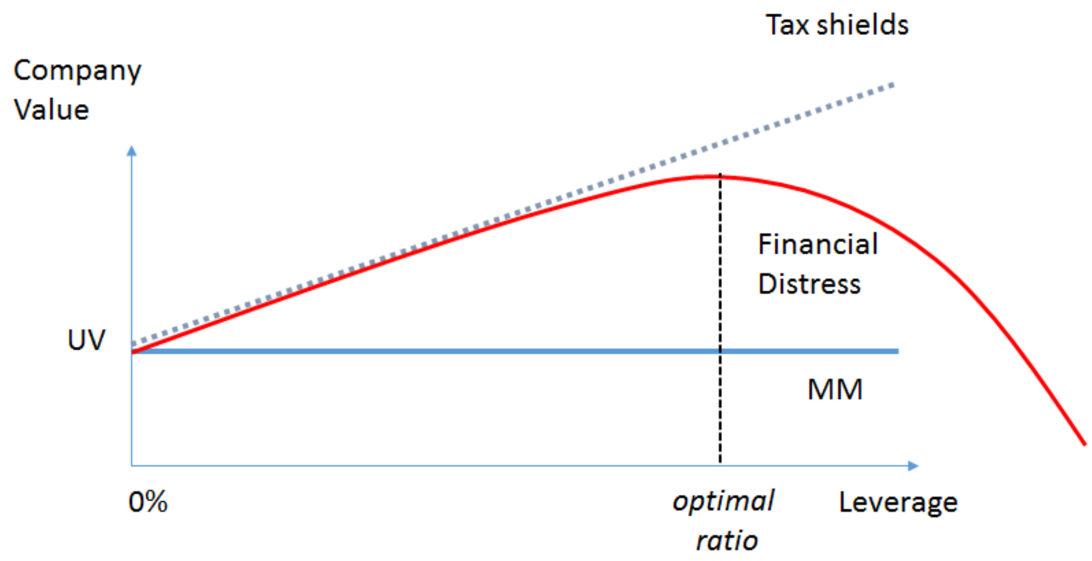

Figure C.1: The optimal debt ratio by the Trade-off theory.

Again, there are some indirect effects that bring value to risk management. The first one, as seem in the trade-off theory case, is to reduce the cost of financial distress. By creating some hedging structures that reduce the chance of financial distress a company might shift its financial distress costs and allow it to obtain an higher debt optimal ratio, thus safely increasing leverage.

It is worth remembering that value of the risk management effect of reducing financial distress is indirect. A reduction of the volatility of the cash flows might save a company from abandoning a profitable operation due to lack of financial resources, or increased leverage might allow the company to expedite growth plans. In either case, value is added because of increase in the asset value enabled by risk management. Figure C.2 illustrates the effects of risk management to the optimal debt ratio.

This allows us to point out an imperative objective in risk management or hedging: we are not interested in obtaining low cost of capital sources to fund the assets or to profit from derivatives trading. A company should not expect to make money with trading unless it its its core business, after all we expect derivatives to be fairly priced. It would be incorrect for a company to try to maximize its profits with a portfolio of investment and contracts. Any anticipated gain from trading in reality is probably due to lack of proper information or to risk mispricing. The company should only engage in hedge contracts to help shape the company cash flows to its needs. Ideally, uncertainty of the liabilities should be designed to (at least partially) match the uncertainty of the assets. This notion motivates the discipline of

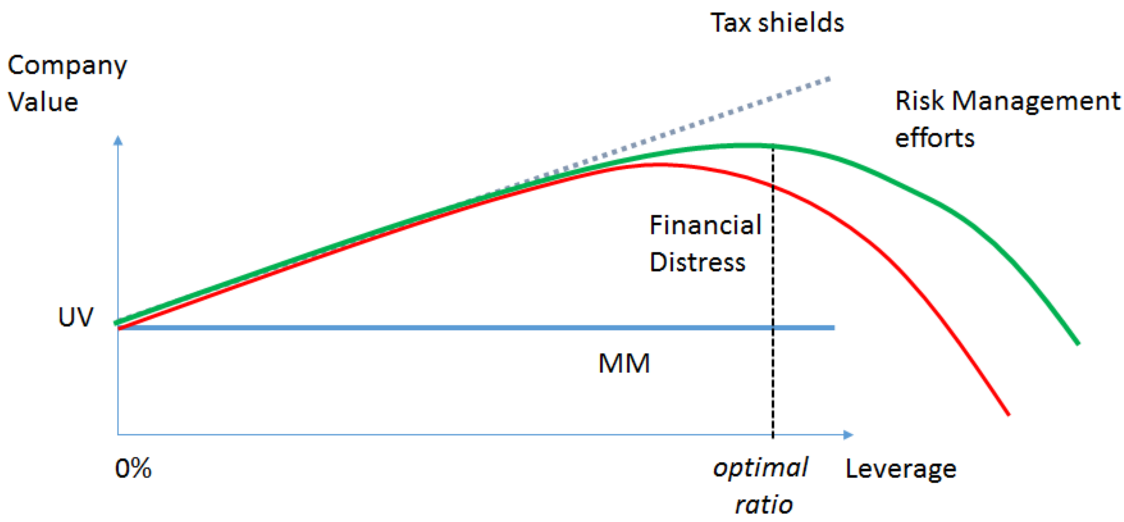

Figure C.2: Effect of risk management efforts to the debt structure.

Asset-Liability Management. If the balance sheet uncertainty profile displays an imbalance larger than some risk tolerance defined by the company, then management has to review its strategy.

Proper risk management might allow shaping the cash flows of the company so that it does not have to access external capital in bad years. Allowing it to use internal funding to finance it project portfolio might avoid a rise in the cost of capital. A higher cost of capital might reduce the Net present value (NPV) of projects, or even make them negative. This is known as the *underinvestment problem*. Again hedging might avoid the value loss of the firm.

Another positive side effect of hedging is the reduction of so-called *agency costs*. Agency costs are those associated with money spent to monitor management and costs incurred by mismanagement, mainly because managers may try to maximize *their* value, instead of the company's. By using derivative contracts to lock some prices, it is easier to known which business units are running profitable operations or are simply gambling with shareholders money.

Reduction of cash flow volatility also may have a positive effect on tax payments. Most countries constrain the amount of tax reduction that can be achieved by declaring losses in previous years. Reducing the volatility of the cash flows might lead to a reduction in tax payments over a longer period.

Finally, there are investors that are heavily invested in a company, such as the owner of a startup or the government with a state owned company. In this case, risk aversion makes sense, since the owner may be unable to diversify and may have big plans to the company's future cash flows.

Ultimately, strategic risk management is a key enabler of the company's strategy.