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ANEXO 2 - O Algoritmo “Outer-Approximation”

Neste anexo apresenta-se brevemente o algoritmo “Outer-Approximation” e a seguir é apresentado sua aplicação em um exemplo numérico. Esse estudo concentrou-se basicamente no capítulo 13 do livro de Li e Sun [29].

Considere o seguinte problema geral de programação não-linear inteira mista:

(P_1)

$$\begin{aligned} &\min\ f(x,y) \\ &s.a.\ g_i(x,y) \leq 0,\ i = 1,....,q, \\ &\quad x \in X \subseteq R^n_+,\ y \in Y \subseteq Z^m, \end{aligned}$$

onde, $f: X \times Y \rightarrow R$, $g_i: X \times Y \rightarrow R$ (i = 1,...,q), $h_i: X \times Y \rightarrow R$ (i = 1,...., l), e Z^m denota o conjunto dos vetores inteiros em R^m.

Considere ainda o seguinte subproblema de programação não linear.

(*NLP(y)*)

$$\begin{aligned} &\min\ f(x,y) \\ &s.a.\ g_i(x,y) \leq 0,\ i = 1,....,q, \\ &\quad h_i(x,y) = 0,\ i = 1,....,l, \\ &\quad x \in X \subseteq R^n_+,\ y \in Y \subseteq Z^m, \end{aligned}$$

Para explicar o algoritmo, é necessário apresentar as seguintes hipóteses:

(i) $X \subseteq R^n_+$ é um conjunto compacto convexo e Y é um conjunto inteiro finito;

(ii) f e g_i (i= 1, ..., q) são funções convexas e diferenciáveis de (x, y), e h_i (i = 1, ...,l) são funções lineares de (x, y);

(iii) Para qualquer y, a solução ótima para todo subproblema viável do *NLP* é um ponto regular, isto é, os vetores dos gradientes das restrições ativas na solução ótima são linearmente independentes.

As hipóteses (i) a (iii) garantem que qualquer solução local do NLP é uma solução global, e esta solução pode ser identificada pela aplicação da condição de otimalidade KKT (Karush-Kuhn-Tucker, Bazaraa, 2006) diretamente.

Defina os conjuntos *S* e *V* com relação às desigualdades de (P_1).

$$S = \{(x,y) \in X \times Y \mid g(x,y) \leq 0\} \text{ e } V = \{y \in Y \mid \exists\, x \in X \text{ tal que } g(x,y) \leq 0\}$$

Para qualquer $y^i \in V$, seja x^i a solução ótima para ($NLP(y^i)$) e considere as hipóteses de (i) a (iii), segue que:

$$\begin{aligned}
\underset{(x,y)\in S}{\text{minimizar}}\ f(x,y) &= \underset{y^i\in V,\ x\in X}{\text{minimizar}}\{f(x,y^i) \mid g(x,y^i) \leq 0\} \\
&= \min_{y^i\in V} f(x,y^i) + \nabla^T f(x^i,y^i)\begin{pmatrix} x - x^i \\ 0 \end{pmatrix} \\
&\qquad s.a.\quad g(x^i,y^i) + \nabla g(x^i,y^i)\begin{pmatrix} x - x^i \\ 0 \end{pmatrix} \leq 0 \\
&\qquad\qquad x \in X \\
&= \min_{y^i\in V} \min \alpha \\
&\qquad s.a.\quad \alpha \geq f(x,y^i) + \nabla^T f(x^i,y^i)\begin{pmatrix} x - x^i \\ 0 \end{pmatrix} \\
&\qquad\qquad 0 \geq g(x^i,y^i) + \nabla^T g(x^i,y^i)\begin{pmatrix} x - x^i \\ 0 \end{pmatrix} \\
&\qquad\qquad x \in X,\quad \alpha \in R,
\end{aligned} \tag{40}$$

onde, a segunda equação surge da condição KKT de NLP(y^i) e sua linearização em x^i são idênticos. Seja $T = \{i \mid y^i \in V \text{ e } x^i \text{ resolve NLP}(y^i)\}$.

Considere o seguinte problema MIP Mestre:

$$\text{(MILP_M)}\quad \begin{aligned}
&\text{minimizar}\ \ \alpha \\
&s.a.\quad \alpha \geq f(x,y^i) + \nabla^T f(x^i,y^i)\begin{pmatrix} x - x^i \\ y - y^i \end{pmatrix},\ i \in T \\
&\qquad\ \ 0 \geq g(x^i,y^i) + \nabla^T g(x^i,y^i)\begin{pmatrix} x - x^i \\ y - y^i \end{pmatrix},\ i \in T \\
&\qquad\ \ x \in X,\ y \in Y,\ \alpha \in R^1.
\end{aligned}$$

Seja (x^i, y^i) a solução ótima para o (P_1), então (α^*, x^*, y^*) é uma solução ótima para a equação (38) com $\alpha^* = f(x^*, y^*)$. Da convexidade de $f(x, y)$ e $g(x, y)$, para qualquer $i \in T$, $\alpha \geq f(x^i, y^i)$ e $0 \geq g(x^i, y^i)$ implica que (α, x^i, y^i) é viável para o (MILP_M). Portanto, $\hat{\alpha} \leq \alpha^*$. Por outro lado, como existe *i* tal que (x^i, y^i) = (x^*,

y^*), isso resulta da primeira restrição do problema da equação (38), temos que $\hat{\alpha} \geq f(x^*, y^*) = \alpha$.

Podemos verificar a viabilidade de *NPLF*(y^i) através de

$$(NLP(y^i)) \quad \text{minimizar } \beta$$
$$\text{s.a.} \quad \beta \geq g(x^i, y^i), \quad i = 1, ..., q$$
$$x \in X$$

Pode-se mostrar que *NLP*(y^i) é inviável se e somente se *NLPF*(y^i) tem um valor ótimo positivo $\beta^* > 0$. Seja ainda, x^i a solução ótima para a checagem de viabilidade do problema *NLPF*(y^i) modificado. *Então* y^i é inviável para o seguinte sistema de equações:

$$0 \geq g_j(x^i, y^i) + \nabla^T g_j(x^i, y^i) \begin{pmatrix} x - x^i \\ y - y^i \end{pmatrix}, \; j = 1, ..., \text{q}, \text{ para todo x} \in \text{X}. \quad (41)$$

Seja F o conjunto indexado de todos $y^i \in Y$ tal que *NLP*(y^i) seja inviável, então deve-se excluir todos $y^i \in F$. Portanto, incorporando a equação (41) ao problema MILP_M e substituindo *V por Y* e considerando o número de iterações geradas por *k*, teremos:

$$(MILP\_M_k) \quad \text{minimizar } \alpha$$
$$\text{s.a.} \quad \alpha \geq f(x, y^i) + \nabla^T f(x^i, y^i) \begin{pmatrix} x - x^i \\ y - y^i \end{pmatrix}, \; i \in T^k$$
$$0 \geq g(x^i, y^i) + \nabla g(x^i, y^i) \begin{pmatrix} x - x^i \\ y - y^i \end{pmatrix}, i \in T^k$$
$$0 \geq g(x^i, y^i) + \nabla g(x^i, y^i) \begin{pmatrix} x - x^i \\ y - y^i \end{pmatrix}, i \in F^k$$
$$x \in X, \; y \in Y, \; \alpha \in R^1.$$

onde

$$T^k = \{ i \mid y^i \in V \text{ e } x^i \text{ resolve } NLP(y^i), \; i = 1, ..., k \},$$
$$F^k = \{ i \mid NLP(y^i) \text{ é inviável e } x^i \text{ resolve } NLPF(y^i), \; i = 1, ..., k \}.$$

Resumindo, é necessário seguir os seguintes passos para a aplicação do algoritmo "Outer-Approximation".

Passo 1: Escolha $y^1 \in Y$. Defina LB = $-\infty$, UB = $+\infty$, $T^0 = F^0 = \emptyset$, k = 1.

Passo 2: Resolva o problema NLP(y^k).

(i) Se NLP(y^k) é viável, obtenha uma solução ótima x^k do problema. Faça $UB^k = f(x^k, y^k)$ e $T^k = T^{k-1} \cup \{k\}$. Faça UB = min {UB, UB^k}. Se UB = UB^k, então $(x^*, y^*) = (x^k, y^k)$.

(ii) Se NLP(y^k) for inviável, resolva o NLPF(y^k) e obtenha uma solução ótima x^k, redefina $F^k = F^{k-1} \cup \{k\}$.

Passo 3: Resolva o problema (MIP Mestre) e obtenha uma solução ótima $(\alpha^k, \bar{x}^{k+1}, y^{k+1})$ .Faça $LB^k = \alpha^k$. Se $LB^k \geq UB$, pare e (x^*, y^*) é a solução ótima para o $MINLP_1$. Caso contrário, k := k + 1 e vá para o passo 2.

Exemplo de Aplicação do algoritmo "Outer-Approximation".

$$(E_z) \quad \text{minimizar } f(x,y) = 5y + \ln(x+2)$$
$$s.a. \quad g_1(x,y) = e^{x/3} - \sqrt{y} - 1 \leq 0$$
$$g_2(x,y) = -2\ln(x+2) - \left(1/4\right)y + 2.5 \leq 0$$
$$g_3(x,y) = x + 2y - 9 \leq 0$$
$$x \in [0, 2.5], \quad y \in [1,4] \text{ inteiro}$$

A representação gráfica da região viável de (E_z) é apresentada no gráfico da Figura 19.

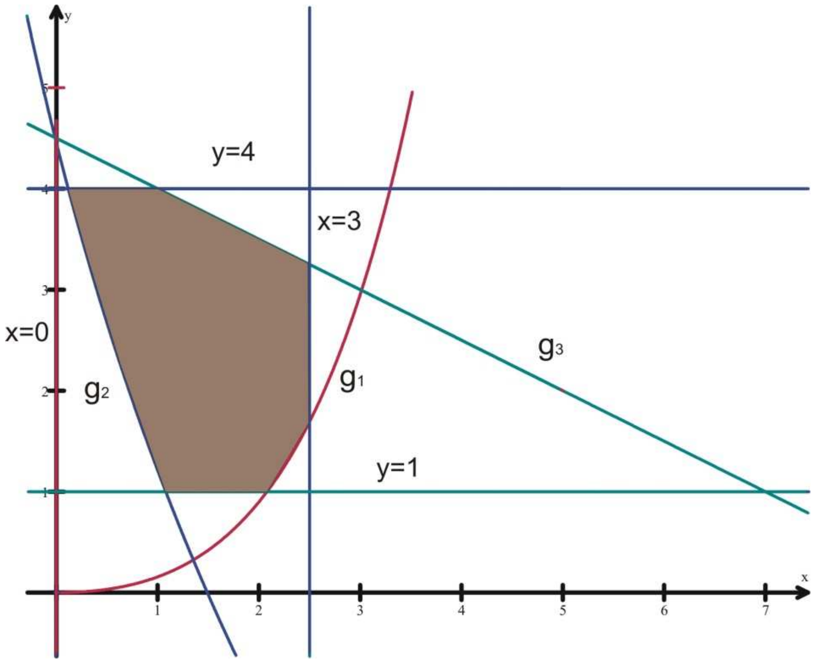

Figura 19 – Região viável do exemplo numérico E_z

Para resolver (E_z), é aplicado o algoritmo Outer-approximation.

Iteração 1

Passo 1: Escolha y^1 = 4. Seja LB = -∞, UB = +∞, $T^0 = F^0$ = Ø, k = 1. Temos o seguinte problema NLP(y^1):

$$\begin{aligned}&\text{minimizar } f(x,y) = \ln(x+2) + 20\\ &s.a.\quad g_1(x,y) = e^{x/3} - 3 \le 0\\ &\qquad\quad g_2(x,y) = -2\ln(x+2) + 1.5 \le 0\\ &\qquad\quad g_3(x,y) = x - 1 \le 0\\ &\qquad\quad x \in [0,2.5]\end{aligned}$$

Passo 2: Resolvendo o $NLP(y^1)$ obtém-se x^1 = 0,117 e UB^1 = 20.75 para T^1 = {1}.

Passo3: O problema Mestre (MILP_M$_1$) é

$$\begin{aligned}&\min \alpha\\ &s.a.\quad \alpha \ge \left(20 + \ln(2.117)\right) + \left(1/2.117\right)\cdot(x - 0.117) + 5\cdot(y-4)\\ &\qquad\quad 0 \ge \left(e^{\frac{0.117}{3}} - 3\right) + \left(e^{\frac{0.117}{3}}\Big/3\right)\cdot(x-0.117) - \left(1/4\right)\cdot(y-4)\\ &\qquad\quad 0 \ge \left(-2\ln(2.117) + 1.5\right) - \frac{2}{2.117}\cdot(x-0.117) - \left(1/4\right)\cdot(y-4)\\ &\qquad\quad 0 \ge -0.883 + (x - 0.117) + 2\cdot(y-4)\\ &\qquad\quad x \in [0,2.5],\quad y \in [1,4] \text{ inteiro.}\end{aligned}$$

A solução ótima para o (MILP_M$_1$) é $(\alpha^1, \bar{x}^2, y^2) = (6.125, 0.9109, 1)$. Sejam $LB^1 = \alpha^1 = 6.125$, k=2.

Iteração 2

Passo 1: Utilizando y^2 = 1. Seja LB = *6.125*, UB = 20.75, k = 2. Temos o seguinte NLP(y^2):

$$\begin{aligned}&\min\ f(x,y) = \ln(x+2) + 5\\ &s.a.\quad g_1(x,y) = e^{x/3} - 2 \le 0\\ &\qquad\quad g_2(x,y) = -2\ln(x+2) + 9/4 \le 0\\ &\qquad\quad g_3(x,y) = x - 7 \le 0\\ &\qquad\quad x \in [0,2]\end{aligned}$$

Passo 2: Resolvendo o $NLP(y^2)$ que é viável, obtém-se x^3=1.0802 com UB^2 = 6.125 e para T^2= {1,3}.

Passo3: O problema Mestre (MILP_M_2) é

$$
\begin{aligned}
&\min \alpha \\
&s.a.\quad \alpha \geq \left(20+\ln(2.117)\right)+\left(1/2.117\right)\cdot(x-0.117)+5\cdot(y-4) \\
&\qquad \alpha \geq (5+\ln(3.0802))+\left(1/3.0802\right)\cdot(x-1.0802)+5\cdot(y-1) \\
&\qquad 0 \geq \left(e^{\frac{0.117}{3}}-3\right)+\left(e^{\frac{0.117}{3}}/3\right)\cdot(x-0.117)-\left(1/4\right)\cdot(y-4) \\
&\qquad 0 \geq \left(e^{\frac{1.0802}{3}}-2\right)+\left(e^{\frac{1.0802}{3}}/3\right)\cdot(x-1.0802)-\left(1/2\right)\cdot(y-1) \\
&\qquad 0 \geq \left(-2\ln(2.117)+1.5\right)-\frac{2}{2.117}\cdot(x-0.117)-\left(1/4\right)\cdot(y-4) \\
&\qquad 0 \geq \left(-2\ln(3.0802)+2.25\right)-\frac{2}{3.0802}\cdot(x-1.0802)-\left(1/4\right)\cdot(y-1) \\
&\qquad 0 \geq -0.883+(x-0.117)+2\cdot(y-4) \\
&\qquad 0 \geq -5.9198+(x-1.0802)+2\cdot(y-1) \\
&\qquad x\in[0{,}2.5],\quad y\in[1{,}4]\ \text{inteiro.}
\end{aligned}
$$

A solução ótima do (MILP_M_2) é $(\alpha^2, \bar{x}^3, y^3) = (6.205, 1.0802, 1)$. Faça LB^2=α^2=6.205, k = 3. Pelo critério de parada indicado no terceiro passo do algoritmo, temos $LB^2 \geq UB^2$ (6.205 > 6.125). Logo o algoritmo termina na solução ótima de (x^*, y^*)=(1.0802, 1). O ponto em questão pode ser visualizado no gráfico da Figura 20.

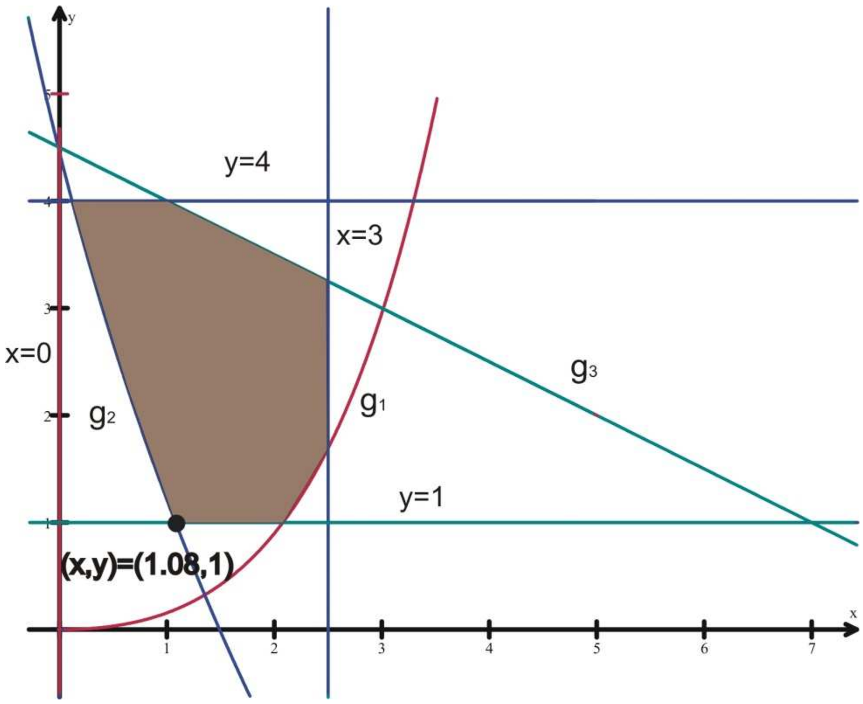

Figura 20 – Região viável e solução ótima do algoritmo AO para o exemplo numérico E_z

A Tabela 18 apresenta o resumo dos resultados das duas iterações do exemplo apresentado quando aplicamos o algoritmo AO.

Tabela 18 – Solução do exemplo de aplicação do algoritmo OA.

Iteração	x^k	y^k	LB^k	UB^k
1	0.1170	4	6.125	20.75
2	1.0802	1	6.205	6.125

Para este exemplo, usou-se a ferramenta de modelagem AIMMS 3.8 para a modelagem matemática do problema proposto, e o solver CPLEX para resolver o problema de programação linear inteira mista ($MILP\_M_j$) através do algoritmo branch-and-bound e o solver MINOS para resolver o problema de programação não linear ($NLP(y^i)$ e $NLPF(y^i)$).

.