

**Andrei Alhadeff Monteiro**

**Mapping Cohesive Fracture and Fragmentation  
Simulations to GPUs**

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Thesis presented to the Programa de Pós-Graduação em  
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Advisor: Prof. Waldemar Celes Filho

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## Abstract

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A GPU-based computational framework is presented to deal with dynamic failure events simulated by means of cohesive zone elements. We employ a novel and simplified topological data structure relative to CPU implementation and specialized for meshes with triangles or tetrahedra, designed to run efficiently and minimize memory requirements on the GPU. We present a parallel, adaptive and distributed explicit dynamics code that implements an extrinsic cohesive zone formulation where the elements are inserted “on-the-fly”, when needed and where needed. The main challenge for implementing a GPU-based computational framework using an extrinsic cohesive zone formulation resides on being able to dynamically adapt the mesh, in a consistent way, by inserting cohesive elements on fractured facets and inserting or removing bulk elements and nodes in the adaptive mesh modification case. We present a strategy to refine and coarsen the mesh to handle dynamic mesh modification simulations on the GPU. We use a reduced scale version of the experimental specimen in the adaptive fracture simulations to demonstrate the impact of variation in floating point operations on the final fracture pattern. A novel strategy to duplicate *ghost* nodes when distributing the simulation in different compute nodes containing one GPU each is also presented. Results from parallel simulations show an increase in performance when adopting strategies such as distributing different jobs amongst *threads* for the same element and launching many *threads* per element. To avoid concurrency on accessing shared entities, we employ graph coloring for non-adaptive meshes and node traversal for the adaptive case. Experiments show that GPU efficiency increases with the number of nodes and bulk elements.

## Keywords

Fragmentation simulation; GPUs; Finite Element Method; Cohesive elements; CUDA;

## Resumo

Monteiro, Andrei Alhadeff; Filho, Waldemar Celes. **Mapeamento de Simulação de Fratura e Fragmentação Coesiva para GPUs**. Rio de Janeiro, 2015. 155p. Tese de Doutorado — Departamento de Informática, Pontifícia Universidade Católica do Rio de Janeiro.

Apresentamos um método computacional na GPU que lida com eventos de fragmentação dinâmica, simulados por meio de zona coesiva. Implementamos uma estrutura de dados topológica simples e especializada para malhas com triângulos ou tetraedros, projetada para rodar eficientemente e minimizar ocupação de memória na GPU. Apresentamos um código dinâmico paralelo, adaptativo e distribuído que implementa a formulação de modelo zona coesiva extríntrica (CZM), onde elementos são inseridos adaptativamente, onde e quando necessários. O principal objetivo na implementação deste *framework* computacional reside na habilidade de adaptar a malha de forma dinâmica e consistente, inserindo elementos coesivos nas facetas fraturadas e inserindo e removendo elementos e nós no caso da malha adaptativa. Apresentamos estratégias para refinar e simplificar a malha para lidar com simulações dinâmicas de malhas adaptativas na GPU. Utilizamos uma versão de escala reduzida do nosso modelo para demonstrar o impacto da variação de operações de ponto flutuante no padrão final de fratura. Uma nova estratégia de duplicar nós conhecidos como *ghosts* também é apresentado quando distribuindo a simulação em diversas partições de um cluster. Deste modo, resultados das simulações paralelas apresentam um ganho de desempenho ao adotar estratégias como distribuir trabalhos entre *threads* para o mesmo elemento e lançar vários *threads* por elemento. Para evitar concorrência ao acessar entidades compartilhadas, aplicamos a coloração de grafo para malhas não-adaptativas e percorrimento nodal no caso adaptativo. Experimentos demonstram que a eficiência da GPU aumenta com o número de nós e elementos da malha.

## Palavras-chave

Simulação de fragmentação; GPUs; Método dos Elementos Finitos; Elementos Coesivos; CUDA;

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*Insanity: doing the same thing over and over  
again and expecting different results.*

**Albert Einstein**