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Robust Regulation of a Monopolist

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Dissertation presented to the Programa de Pós-Graduação em Economia of the Departamento de Economia, PUC-Rio as partial fulfillment of the requirements for the degree of Mestre em Economia.

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Resumo

Idrogo, Carlos Antonio Burga; Carrasco, Vinicius Nascimento. **Regulação Robusta de um Monopolista**. Rio de Janeiro, 2015. 30p. Dissertação de Mestrado — Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

Este trabalho estuda o problema de um regulador que enfrenta um monopolista sem observar seus custos. Diferente de estudos anteriores, deixamos o pressuposto forte de que o regulador conhece a verdadeira distribuição de probabilidade de custos do monopolista. Em vez disso, vamos supor que o regulador têm uma distribuição prior e sua incerteza é representada pelo conjunto de distribuições mean preserving spread da sua prior. Regulador é avesso a incerteza, ou seja, ele maximiza o bem-estar social esperado sob a pior distribuição neste conjunto. Regulação ótima depende do estado da natureza e garante que o bem-estar social esperado não é afetado pela incerteza do regulador. Regulador não pode dar incentivos tão forte como os dados quando a distribuição é conhecida, o que significa que a robustez reduz o poder dos contratos.

Palavras–chave

Desenho de Mecanismo Robusto; Regulação Monopolista; Incentivos;

Abstract

Idrogo, Carlos Antonio Burga; Carrasco, Vinicius Nascimento. **Robust Regulation of a Monopolist**. Rio de Janeiro, 2015. 30p. Dissertação de Mestrado — Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

This work studies the problem of a regulator who faces a monopolist with unknown costs. Different to previous studies, we depart from the strong assumption that regulator knows the true probability distribution of monopolist costs. Instead, we assume that regulator holds a prior distribution and his uncertainty is represented by the set of mean preserving spread distributions of this prior. Regulator is uncertainty averse, i.e., he maximizes expected social welfare under the worst distribution in this set. Optimal regulation is state dependent and guarantees that expected social welfare is not affected by regulator uncertainty. Regulator can not give such strong incentive as those given when distribution is known, which means that concern for robustness reduces the power of contracts.

Keywords

Robust Mechanism Design; Monopolistic Regulation; Incentives;

Contents

1
Introduction

In a seminal paper, Baron and Myerson found the optimal regulation of a monopolist who privately observes its costs. However, this regulation requires the strong assumption that regulator knows the true probability distribution of monopolist cost. This assumption is not necessarily satisfied in many cases. A regulator facing the monopolist for the first time or a monopolist exposed to technology shocks that change cost distribution are some examples where this assumption is not satisfied and optimal regulation cannot be implemented.

In this paper we extend this problem to a context where regulator does not know the true distribution of monopolist cost. Instead, we assume that regulator holds a prior distribution such that for every point, there is a zero-mean noise that generates a spread in the distribution preserving its mean. It makes regulator to be uncertain about the true distribution, which could be any mean preserving spread of his prior. Regulator is uncertainty averse and designs a mechanism, defined by quantity, price and subsidy, in order to maximize the expected social welfare under the worst distribution within the set of mean preserving spread distributions (m.p.s.d.) of his prior. It is a problem with multiple priors and robustness is about designing a mechanism that works well in multiple cases.

Our model is built in the spirit of Baron and Myerson with two simplifications, (i) demand is constant and equal to the consumer valuation and (ii) only the variable cost is private information. The maximin problem can be understood as a zero-sum game between regulator and nature, where nature minimizes expected social welfare by choosing a distribution from the set of m.p.s.d. of regulator prior and regulator chooses a mechanism that maximizes expected social welfare. Regulator tries to reduce his uncertainty by choosing a mechanism that guarantees a linear payoff such that expected social welfare is not sensitive to nature choice, which at the same time makes nature to be indifferent between any feasible distribution. This linear payoff entails an ordinary differential equation (ODE) which describes regulator optimal mechanism.

Results show that if regulator knew the probability distribution of monopolist cost, there is a threshold which depends on this distribution, such that monopolist with lower marginal cost produces all its capacity, while monopolist with higher marginal cost is not allowed to produce. This contract is easily implemented and shows that regulator can give strong incentives for firms to report the true. On the other hand, when regulator does not know the true distribution, reducing the sensitiveness of expected social welfare to nature choice induces a state-dependent optimal quantity. It means that regulator should offer a menu of mechanisms, which implies that robustness reduces the power of contracts and makes regulator less able to give strong incentives.

The remaining of paper is organized as follows; section II places our research in the literature, section III describes the model and finds the optimal regulation under perfect information. Section IV solves the regulator problem with perfect knowledge about cost distribution, and section V derives the optimal regulation when regulator does not know the true cost distribution. Finally, section VI concludes.

2
Related Literature

Baron and Myerson (1982) explored the regulation of a monopolist with unknown costs, and several papers extended this idea to many other contexts. Baron and Besanko (1984) found the optimal regulation when regulator is able to audit monopolist costs. Lewis and Sappington (1988) explored the problem of a regulator who does not observe cost and demand of a monopolist. Amstrong and Rochet (1999) introduced a model for multi-product monopoly regulation. Biglaiser and Ma (1995) analyzed the regulation of a Stackelberg leader and Wang (2000) considered the regulation of an oligopoly with unobserved marginal cost. However these papers assume that regulator hold a correct belief about the probability distribution of private information.

On the other hand, literature about robust design has grown in recent years. Bergemann and Schlag (2011) consider the classic problem of a monopolist who sells an indivisible good without knowing buyer valuation, extending it to a context where monopolist does not know the true distribution of private information. Optimal price is always lower and informational rents are higher for consumers with low valuation compared to the case of a monopolist who knows the true distribution. Carrasco and Moreira (2012) consider a decision maker who faces a privately informed and biased agent, both with quadratic utilities. When agent bias depends on the state of nature, optimal mechanism is stochastic, while it is constant and entails full delegation when bias is independent of the state. Carrasco et al. (2014) use the set of mean preserving spread distributions to represent the uncertainty of a monopolist who sells an indivisible good. Results suggest that monopolist should establish a linear payoff function which describes a random pricing policy. Carroll (2014) shows that linear contracts are optimal in a moral hazard problem where principal does not know what actions the agent can and can not do.

In a closer research, Garret (2014) considers a procurement problem where the principal is uncertain about the disutility of firms effort. A simple fixed-price cost-reimbursement contract minimizes the maximum expected payoff for the principal. In contrast to this result, our paper shows that concern for robustness reduces the power of contracts. While a simple contract is optimal when we assume that distribution is known, robustness under multiple priors makes it optimal to offer a menu of contracts. A simple contract is not able to give such strong incentives as those given without uncertainty about distribution.

3
The Model

We model the regulator problem by simplifying Baron and Myerson in two ways. First, we consider a constant and observed demand $p(q) = v$, which represents consumer valuation. Second, we assume that only variable cost is private information. The model consists of one consumer, one firm that behaves as a monopolist and a regulator that maximizes social welfare.

3.1
Consumer

Consumer wants to buy q units of a good, from which he obtains a constant unitary utility v. Consumer pays a constant price p for each unit of the good, and under regulation, consumer also pays taxes t to regulator. Then, consumer surplus is given by:

$$S = vq - pq - t \tag{3-1}$$

3.2
Firm

There is a monopolist who produces q units of the good, with $q \in [0, \overline{q}]$, where $\overline{q}$ is his maximum capacity. Monopolist operates with the following cost function:

$$C = a + \theta q \tag{3-2}$$

The first term is the monopolist fixed cost, which is perfectly observed, and the second term represents the variable cost. Marginal cost θ is privately observed by the monopolist and is distributed over $\left[\underline{\theta}, \overline{\theta}\right]$. Profits are given by:

$$\Pi = pq - a - \theta q + t \tag{3-3}$$

The first term is the income obtained for selling q units of the good, the second term represents fixed and variable costs, and the last term is the subsidy that regulator transfers to monopolist.

3.3
Conditions for Natural Monopoly

Lets consider a natural monopoly where fixed cost are high enough such that it is more efficient for the industry to produce with one firm only. Without regulation, firm profits are represented by equation (3-4).

$$\Pi = [p - AC]q = \left[p - \theta - \frac{a}{q}\right] q \tag{3-4}$$

Where AC is the average cost. On the other hand, consumer surplus without regulation can be defined by $S = [v - p]q$. Then, the maximum price consumer would pay is his valuation $p \leq v$. Therefore, for any produced quantity q, monopolist with marginal cost θ has profits if:

$$v \geq p \geq \theta + \frac{a}{q} \tag{3-5}$$

Finally, let C_n be the total cost of producing in an industry with n firms with the same technology and AC_n the average cost. For firms with the same θ, the average cost is:

$$AC_n = n\frac{a}{\overline{q}} + \theta \tag{3-6}$$

Then, we will consider the case where $[\underline{\theta}, \overline{\theta}] = [v - 2a/\overline{q}, v - a/\overline{q}]$, which is a sufficient condition for profitability in an industry with a single profit-maximizing firm and guarantees that industry does not have positive profits with more than one firm.

3.4 Equilibrium without Regulation

Monopolist knows the constant demand v and chooses price and quantity to solve the following optimization problem:

$$\begin{aligned} \max \quad & pq - a - \theta q \\ & Subject\ to: \\ & vq - pq \geq 0 \end{aligned} \tag{3-7}$$

Monopolist charges the highest price that makes consumer staying in the market. Then, optimal price is $p = v$, consumer gets zero surplus and firm profits are $\Pi = [v - \theta]\overline{q} - a$. However, a regulator who is worried about social welfare may want to take some monopolist profits and give it to the consumer.

3.5 Regulator

Regulator maximizes social welfare, which is defined, as usual in literature, by the sum of consumer surplus and a discounted firm profit. Let ω be the social welfare.

$$\omega = S + \alpha\Pi \tag{3-8}$$

Where $\alpha < 1$ represents the regulator bias in favor of consumer. Then, regulator wants to extract rents from monopolist and give them to consumer in order to obtain a higher social welfare.

3.6 Optimal Regulation under Complete Information

Lets define a mechanism as the set of quantity, price and subsidy that regulator offers to monopolist $\{q(\theta),\ p(\theta),\ t(\theta)\}$. When regulator observes marginal cost, he designs a contract that guarantees non-negative profits and maximizes social welfare. Regulator problem is:

$$\begin{aligned} &\max_{\{q(.),p(.),t(.)\}} \{S(\theta) + \alpha\Pi(\theta)\} \\ &\qquad\qquad\qquad \mathit{Subject\ to}: \\ &p(\theta)q(\theta) - a - \theta q(\theta) + t(\theta) \geq 0 \end{aligned} \tag{3-9}$$

Since $\alpha < 1$, it is easy to note that $t(\theta)$ should be as little as possible.

$$t(\theta) = -p(\theta)q(\theta) + a + \theta q(\theta) \tag{3-10}$$

Under condition (3-10), monopolist produces with zero profits, while consumer gets all the surplus. Social welfare under complete information is defined by equation (3-11). Since $\theta \leq v - a/\overline{q}$, monopolist produces all its capacity.

$$\omega(\theta) = S(\theta) + \alpha\Pi(\theta) = [v - \theta]q(\theta) - a \tag{3-11}$$

Proposição 3.1 *Under complete information, regulator should establish:*

$$q(\theta) = 1 \quad \forall\theta \in [\underline{\theta}, \overline{\theta}] \tag{3-12}$$

$$t(\theta) = -p(\theta)\overline{q} + a + \theta\overline{q} \quad \forall\theta \in [\underline{\theta}, \overline{\theta}] \tag{3-13}$$

From this solution, we can observe that firms receive a total income $t(\theta)+p(\theta)\overline{q}$ which is increasing in its marginal cost θ. Then, if this information were unobservable for the regulator, monopolist may report that he is the most inefficient one, with marginal cost equal to $\overline{\theta}$, in order to obtain higher profits. Next section explores the optimal behavior of regulator when he cannot observe monopolist marginal cost, but knowing the probability distribution of this private information, he maximizes expected social welfare.

4
Asymmetric Information

Regulator does not know the true value of monopolist cost θ, but he knows that this value is drawn from a distribution $F(\theta)$. Then, he designs a truth-telling mechanism which maximizes expected social welfare. Using the Revelation Principle (Myerson (1981)), we restrict our attention on direct mechanisms, that is, regulator asks for the marginal cost, monopolist reports $\hat{\theta}$ and it is offered the mechanism $\{q(\hat{\theta}), p(\hat{\theta}), t(\hat{\theta})\}$. In this context, regulator problem is:

$$\begin{aligned} \max_{\{q(.),p(.),t(.)\}} \quad & \omega^e = E_\theta\left[\omega(\theta)\right] \\ & Subject\ to: \\ \Pi(\theta,\theta) \geq 0 \ \forall\ \theta \in [\underline{\theta}, \overline{\theta}] \quad & (IR) \\ \Pi(\theta,\theta) \geq \Pi(\theta,\hat{\theta}) \ \forall\ \theta, \hat{\theta} \in [\underline{\theta}, \overline{\theta}] \quad & (IC) \end{aligned} \tag{4-1}$$

Where $\Pi(\theta, \hat{\theta})$ represents the monopolist profits when its marginal cost is θ but he reports $\hat{\theta}$. IR describes the individual rationality constraint, which requires that for any mechanism provided by the regulator, monopolist obtains non-negative profits reporting its true marginal cost. IC is the incentive compatibility constraint, which means that regulator should offer mechanisms such that it is always better for the monopolist to report its true marginal cost. Finally, $F(\theta)$ satisfies the follow monotonicity condition:

$$\frac{F(\theta_0)}{f(\theta_0)} \leq \frac{F(\theta_1)}{f(\theta_1)} \ \forall\, \theta_0 < \theta_1 \tag{4-2}$$

Proposição 4.1 *From Mirrless (1971), IC can be replaced by the following conditions:*

$$\Pi(\theta) = \Pi(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} q(\tau)d\tau \tag{4-3}$$

$$q(\theta) \textit{is non-increasing.} \tag{4-4}$$

On the other hand, IR can be replaced by $\Pi(\overline{\theta}, \overline{\theta}) \geq 0$. Finally, replacing these results on social welfare, regulator problem is:

$$\max_{\{q(.)\}} E\left[\left(v - \theta - \frac{(1-\alpha)F(\theta)}{f(\theta)}\right) q(\theta) - a\right] \tag{4-5}$$

Finally, optimal regulation is given by Proposition 4.2.

Proposição 4.2 *When regulator does not observe monopolist cost but knows the probability distribution, and this distribution satisfies (4-2), optimal regulation is:*

$$q(\theta) = \begin{cases} \overline{q}, & \text{if } \theta \leq v - \dfrac{a}{\overline{q}} - \dfrac{(1-\alpha)F(\theta)}{f(\theta)} \\ 0, & \text{if } \theta > v - \dfrac{a}{\overline{q}} - \dfrac{(1-\alpha)F(\theta)}{f(\theta)} \end{cases} \tag{4-6}$$

5
Robust Design

In previous section we derived the optimal regulation when regulator knows the probability distribution of monopolist cost. However, when regulator faces the monopolist for the first time or for few periods, it seems to be very strong to assume that regulator has correct beliefs about cost distribution. In industries with large technology shocks, monopolist cost distribution may change over time and regulator may not know how these changes are, even if he faced the monopolist many times. Literature has attempted to find optimal contracts in contexts where principal does not know the distribution of private information. In this section we derive the optimal mechanism given the regulator uncertainty about cost distribution.

The timing of the model is as follows. In $t = 0$, regulator holds a prior belief about the ex-ante monopolist cost. Then, monopolist receives information about his cost, and regulator is uncertain about this information acquisition. Prior belief about cost distribution is represented by $\hat{F}(\theta)$ and for every point θ_0, regulator is exposed to a zero mean noise that perturbs the distribution generating a spread that preserves the mean. Therefore, regulator is uncertain about the true distribution of monopolist cost, but he knows that it is related to his prior in such a way that we can define regulator uncertainty by the set of mean preserving spread distributions (m.p.s.d.) of his prior. Regulator want to design a mechanism robust to any information acquisition technology, that is, regulator is uncertainty averse and maximizes the expected social welfare under the worst possible distribution in the m.p.s.d. set of his prior.

Definição 5.1 *1. The distribution G is a mean preserving spread of the distribution $\hat{F}$ if there is a family of distributions $\{H(.|k)\}_{k\in R}$, such that $\int x dH(x|k) = k$ and for every Borelian $B \subset R$*

$$G(B) = \int H(B|k) d\hat{F}(k) \tag{5-1}$$

Regulator problem is to chose the quantity that maximizes the expected social welfare induced by the worst distribution inside the set of mean preserving spread of his prior.

$$\max_{q(\cdot)} \inf_{G(.)\in S(\hat{F})} \int_{\underline{\theta}}^{\overline{\theta}} \left[vq(\theta) - \theta q(\theta) - a - (1-\alpha) \int_{\theta}^{\overline{\theta}} q(\tau) d\tau \right] dG(\theta) \tag{5-2}$$

Let $W(\theta)$ be the argument of this integral, which is $\omega(\theta)$ under both participation and inventive constraints, and $S(\hat{F})$ be the set of m.p.s.d. of regulator prior. This problem can be understood as a zero-sum game between regulator and nature, where nature chooses the distribution that minimizes the expected social welfare within the set of m.p.s.d. of regulator prior and regulator chooses the quantity that maximizes expected social welfare. Regulator will design a mechanism that reduces the sensitiveness of expected social welfare on nature decision, that is, on his uncertainty.

Definição 5.2 *2. The convex hull of a function $W : [\underline{\theta}, \overline{\theta}] \to R$ is the greatest convex function that is below W. Let Φ denote the convex hull of W. Following Rockafeller:*

$$\Phi(\theta) = \inf\{\sum_{i=1}^{n} \lambda_i W(x^i) : \lambda_i \geq 0, x^i \in [\underline{\theta}, \overline{\theta}], \sum_{i=1}^{n} \lambda_i = 1, \sum_{i=1}^{n} \lambda_i x_i = \theta, n \in N\} \tag{5-3}$$

Proposição 5.3 *Suppose $W : [\underline{\theta}, \overline{\theta}] \to R$ is non-increasing and define $W^+(\theta) = W(\theta+)$ if $\theta < \overline{\theta}$ and $W^+(\overline{\theta}) = W(\overline{\theta})$. Then*

$$\inf\left\{\int W(\theta)dG(\theta) : G \in M(k)\right\} = \min\left\{\int W^+(\theta)dG(\theta) : G \in M(k)\right\} \tag{5-4}$$

Where $M(k) = \{G : G$ is a distribution and $\int \theta dG(\theta) = k\}$.

Proposition 5.3 says that we can replace the infimum with a minimum, just if the objective function were right-continuous. With the following definitions, we can simplify the regulator problem.

Definição 5.4 *3. Let $\boldsymbol{W} = (R_+)^{[\underline{\theta}, \overline{\theta}]}$ be the set of functions $W : [\underline{\theta}, \overline{\theta}] \to R_+$ with consider the sets:*

$\mathcal{W} = \{W \in \boldsymbol{W} :$ *W is non-decreasing, right-continuous and* $W(\overline{\theta}) = 0\}$;

$\boldsymbol{H} = \{(H(.|k))_{k \in [\underline{\theta}, \overline{\theta}]} : H(.|k) \in M(k)\}$ *y*

$\mathcal{H} = \{H \in \boldsymbol{H} : k \to H(.|k)$ *is mesurable*$\}$.

Proposição 5.5 *It is true that:*

$$\inf_{G \in S(\hat{F})} \int W(\theta)dG(\theta) = \inf_{H \in \mathcal{H}} \int \int W(\theta)dH(\theta|k)d\hat{F}(k) = \int \Phi(k)d\hat{F}(k) \tag{5-5}$$

Where Φ is the convex hull of W.

Proposition 5.5 says that regulator problem is equivalent to maximize the expected convex hull of social welfare under the prior distribution. Integrating by parts, this objective function can be replaced as follows:

$$\int \Phi(\theta)d\hat{F}(\theta) = \int (1-\hat{F}(\theta))\Phi'(\theta)d\theta \tag{5-6}$$

On the other hand, social welfare is a function of quantity and therefore its convex hull is too. Differentiating $W(\theta)$ gives the following result:

$$\frac{W'(\theta)}{(v-\theta)^{1-\alpha}} = \frac{d\left[(v-\theta)^{\alpha}\, q(\theta)\right]}{d\theta} \tag{5-7}$$

From the definition of convex hull we have that for every non-negative function $\eta(\theta)$, $\int \eta(\theta)W'(\theta)d\theta \geq \int \eta(\theta)\Phi'(\theta)d\theta$. Therefore, convex hull should satisfy the following constraint:

$$-(v-\underline{\theta})^{\alpha}\overline{q} \geq \int \frac{\Phi'(\theta)d\theta}{(v-\theta)^{1-\alpha}} \tag{5-8}$$

Therefore, we have the following regulator problem.

$$\begin{aligned} &\max_{\Phi(.)} \int (1-\hat{F}(\theta))\Phi'(\theta)d\theta \\ &\qquad\qquad \textit{Subject to}: \\ &-(v-\underline{\theta})^{\alpha}\overline{q} \geq \int \frac{\Phi'(\theta)d\theta}{(v-\theta)^{1-\alpha}} \end{aligned} \tag{5-9}$$

Note that we have a convex constraint, which allows us to use the Lagrange theorem. Regulator chooses the convex hull that maximizes the following Lagrangian:

$$L(\Phi,\lambda) = \int_0^{\tilde{v}} \left(1-\hat{F}(\theta) - \frac{\lambda}{(v-\theta)^{1-\alpha}}\right)\Phi'(\theta)d\theta \tag{5-10}$$

Since $\lambda > 0$, there exists $\hat{\theta}$ such that for every $\theta \in (\hat{\theta}, \overline{\theta})$, the optimal convex hull satisfies $\Phi' = 0$. Let $H(x) = \int_{\underline{\theta}}^{x} \hat{F}(u)du - (x-\underline{\theta}) - \lambda\left[\frac{(v-x)^{\alpha}-(v-\underline{\theta})^{\alpha}}{\alpha}\right]$. Regulator problem is equivalent to:

$$\max_{\Phi(.)} \int_{\underline{\theta}}^{\hat{\theta}} H(\theta)\Phi''(\theta)d\theta \tag{5-11}$$

Finally, we can conjecture a multiplier that makes $H(\theta) \leq 0$, such that it is optimal for regulator to choose a piece-wise linear social welfare, linear in $(\underline{\theta}, \hat{\theta})$, and constant for $(\hat{\theta}, \overline{\theta})$. This linear social welfare entails an ODE, which defines the optimal regulation policy established in Theorem 5.6.

Teorema 5.6 *When an uncertainty averse regulator holds a prior belief* $\hat{F}(.)$ *about the true probability distribution of monopolist cost, optimal regulation is* *given by equations (5-12) and (5-13).*

$$q(\theta) = \left[\frac{(v-\underline{\theta})^{\alpha}}{(v-\underline{\theta})^{\alpha}-(v-\hat{\theta})^{\alpha}}\right]\left[\frac{(v-\theta)^{\alpha}-(v-\hat{\theta})^{\alpha}}{(v-\theta)^{\alpha}}\right]\overline{q} \tag{5-12}$$

Where:

$$\hat{\theta} = \arg\max_{\tau} \frac{\tau - \int_{\underline{\theta}}^{\overline{\theta}} k d\hat{F}(k)}{(v-\underline{\theta})^{\alpha}-(v-\tau)^{\alpha}} \tag{5-13}$$

Optimal quantity is state dependent and guarantees a linear social welfare which is insensitive to nature choice, because the mean is known by regulator. In other words, this mechanism makes that the objective function -expected social welfare- does not depend on regulator uncertainty, which is in line with the fact that regulator designs a mechanism that works well in many cases. Concern for robustness induces a mechanism that is not as easy to implement as the other one derived in previous section and implemented by one simple contract. Now, implementation requires a menu of contracts given the incapacity of regulator to give such strong incentives as before. Thus, robustness reduces power of contracts.

5.1 Degenerate Prior

Lets consider the case of a simple degenerate prior $\hat{F} = \delta_z$. The set of m.p.s.d. is defined by all distributions with mean z. We can analyze the nature problem, which is to minimize the expected social welfare.

$$\begin{aligned} \min_{G(.)} \int_{\underline{\theta}}^{\overline{\theta}} &\left[vq(\theta) - \theta q(\theta) - a - (1-\alpha)\int_{\theta}^{\tilde{v}} q(\tau)d\tau\right] dG(\theta) \\ &\qquad\qquad Subject\, to: \\ &\qquad\qquad \int_{\underline{\theta}}^{\overline{\theta}} \theta dG(\theta) = z \\ &\qquad\qquad \int_{\underline{\theta}}^{\overline{\theta}} dG(\theta) = 1 \end{aligned} \tag{5-14}$$

For a degenerate prior, these constraints define the set of mean preserving spread distributions. Since both constraints are convex, we can use the sufficient condition of Lagrange theorem to argue that the solution of previous problem minimizes the following expression:

$$L(G,\mu,\epsilon) = \int_{\underline{\theta}}^{\overline{\theta}} \left[vq(\theta) - \theta q(\theta) - a - (1-\alpha)\int_{\theta}^{\overline{\theta}} q(\tau)d\tau + \mu\theta - \epsilon\right] dG(\theta) \tag{5-15}$$

Let $m(\theta, \mu, \epsilon)$ be the argument of the integral in equation (5-15). For $m(\theta, \mu, \epsilon) < 0$, nature will assign infinite measure to the minimum value. On the other hand, if $m(\theta, \mu, \epsilon) > 0$, nature solves the minimization problem assigning zero-measure to these points. Therefore, for almost every point, μ and ϵ satisfy $m(\theta, \mu, \epsilon) = 0$. It means that regulator should choose a linear social welfare with parameters μ and ϵ, which makes nature becoming indifferent between any choice. Regulator behavior is very intuitive because expected social welfare depends on nature choice, that is, on regulator uncertainty, and the only way that regulator can reduce his exposure to nature choice is setting a linear social welfare.

This first conclusion entails the following ODE for every $\theta \in (\underline{\theta}, \hat{\theta})$.

$$(v - \theta)q'(\theta) - \alpha q(\theta) = -\mu \tag{5-16}$$

Where $\hat{\theta}$ is the threshold until which regulator does not close the industry. Given the previous theorem, we assume that $\hat{\theta}$ exists and regulator chooses it in an optimal way. Solving this ODE, the optimal mechanism when regulator holds a degenerate prior in z is defined by the following equation.

$$(v - \hat{\theta})^{\alpha} q(\hat{\theta}) - (v - \theta)^{\alpha} q(\theta) = \frac{\mu}{\alpha}\left[(v - \hat{\theta})^{\alpha} - (v - \theta)^{\alpha}\right] \tag{5-17}$$

However, since $W(\hat{\theta}) = 0$, we obtain $\epsilon = \mu\hat{\theta}$. From Proposition 5.5 and the linear shape of social welfare, regulator problem is to choose μ and $\hat{\theta}$ to maximize $-\mu(z - \hat{\theta})$. From equation (5-17) we obtain μ as a function of $\hat{\theta}$.

$$\mu = \frac{\alpha(v - \underline{\theta})^{\alpha}\overline{q}}{(v - \underline{\theta})^{\alpha} - (v - \hat{\theta})^{\alpha}} \tag{5-18}$$

Finally, regulator problem is equivalent to choose $\hat{\theta}$. The optimal mechanism is given by the following theorem.

Teorema 5.7 *When an uncertainty averse regulator holds a degenerate prior δ_z about the true probability distribution of monopolist cost, optimal regulation is given by equations (5-19) and (5-20).*

$$q(\theta) = \left[\frac{(v - \underline{\theta})^{\alpha}}{(v - \underline{\theta})^{\alpha} - (v - \hat{\theta})^{\alpha}}\right]\left[\frac{(v - \theta)^{\alpha} - (v - \hat{\theta})^{\alpha}}{(v - \theta)^{\alpha}}\right]\overline{q} \tag{5-19}$$

Where:

$$\hat{\theta} = \arg\max_{\tau}\left[\frac{\tau - z}{(v - \underline{\theta})^{\alpha} - (v - \tau)^{\alpha}}\right] \tag{5-20}$$

In line with the general distribution problem, regulator chooses a mechanism that reduces his exposure to the uncertainty represented by the choice of nature. This exposure disappears when social welfare is linear because it makes that regulator expected payoff depends just on the mean of theta, which is perfectly known by regulator because it is not affected by the unobserved technology of information acquisition that monopolist receives.

6
Conclusion

This paper explores the problem of a regulator who faces a monopolist with private information about costs, without knowing the true probability distribution of this private information. To do so, we have built a model in the spirit of Baron and Myerson, simplifying it by assuming that demand is constant and the only private information is marginal cost.

When regulator knows the probability distribution of monopolist cost, optimal quantity is a two-point function. Regulator induces efficient monopolists to produce all its capacity and close the industry for inefficient ones. However, as we argued before, it is not usually reasonable to assume that regulator knows the true cost distribution. Then, leaving this assumption, the optimal quantity is a continuous decreasing function contingent to the state of nature, guaranteeing a linear social welfare which is insensitive to regulator uncertainty.

The first mechanism is easily implemented by a single contract, while the state dependent regulation requires a menu of contracts. In contrast with Garret (2014), who concludes that robust contracts in a procurement problem can be implemented with a simple contract compared to that provided by Lafont and Tirole, our paper shows that actually, concern for robustness reduces the power of incentives that principal is able to give, making it more difficult to implement contracts.

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8
Appendix

Proof of Proposition 2.

From monopolist profits we have:

$$\Pi(\theta) = p(\theta)q(\theta) - a - \theta q(\theta) + t(\theta) \tag{8-1}$$

From Envelope Theorem:

$$\Pi'(\theta) = -q(\theta) \tag{8-2}$$

Integrating from θ to $\overline{\theta}$:

$$\Pi(\overline{\theta}) - \Pi(\theta) = \int_{\theta}^{\overline{\theta}} [-q(\tau)]d\tau \tag{8-3}$$

$$\Pi(\theta) = \Pi(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} q(\tau)d\tau \tag{8-4}$$

Proof of Proposition 3.

Replacing firm profits in equation (8-4) we have:

$$p(\theta)q(\theta) - a - \theta q(\theta) + t(\theta) = \Pi(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} q(\tau)d\tau \tag{8-5}$$

$$t(\theta) = \Pi(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} q(\tau)d\tau - p(\theta)q(\theta) + a + \theta q(\theta) \tag{8-6}$$

$$W^e(\theta) = E\left[vq(\theta) - p(\theta)q(\theta) - t(\theta) + \alpha\left\{\Pi(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} q(\tau)d\tau\right\}\right] \tag{8-7}$$

$$W^e(\theta) = E\left[(v-\theta)q(\theta) - (1-\alpha)\Pi(\overline{\theta}) - (1-\alpha)\int_{\theta}^{\overline{\theta}} q(\tau)d\tau - a\right] \tag{8-8}$$

Regulator chooses $\Pi(\overline{\theta})$ with the only constraint that it can not be negative. Therefore, it is zero.

$$W^e(\theta) = E\left[(v-\theta)q(\theta) - (1-\alpha)\int_{\theta}^{\overline{\theta}} q(\tau)d\tau - a\right] \tag{8-9}$$

$$W^e(\theta) = E\left[\left(v - \theta - \frac{(1-\alpha)F(\theta)}{f(\theta)}\right)q(\theta) - a\right] \tag{8-10}$$

$$q(\theta) = \begin{cases} 1 & \text{if } \theta \leq v - \dfrac{a}{\overline{q}} - \dfrac{(1-\alpha)F(\theta)}{f(\theta)} \\ 0 & \text{if } \theta > v - \dfrac{a}{\overline{q}} - \dfrac{(1-\alpha)F(\theta)}{f(\theta)} \end{cases} \tag{8-11}$$

$$t(\theta) = \begin{cases} \theta\overline{q} + a - p(\theta)\overline{q} + \int_{\theta}^{\overline{\theta}} q(\tau)d\tau & \text{if } \theta \leq v - \dfrac{a}{\overline{q}} - \dfrac{(1-\alpha)F(\theta)}{f(\theta)} \\ 0 & \text{if } \theta > v - \dfrac{a}{\overline{q}} - \dfrac{(1-\alpha)F(\theta)}{f(\theta)} \end{cases} \tag{8-12}$$

Proof of Proposition 4.

Let Phi be the convex hull of W. From continuity of convex hull and non-increasing welfare function we have:

$$\Phi \leq W^+ \leq W \tag{8-13}$$

Using Jensen inequality:

$$\Phi\left(\int \theta dG(\theta)\right) \leq \int \Phi(\theta)dG(\theta) \leq \int W^+(\theta)dG(\theta) \leq \int W(\theta)dG(\theta) \tag{8-14}$$

$$\Phi(k) \leq \min\left\{\int W^+(\theta)dG(\theta) : G \in M(k)\right\} \leq \inf\left\{\int W(\theta)dG(\theta) : G \in M(k)\right\} \tag{8-15}$$

From Rockafeller, convex hull of $W(\theta)$ is defined by the following expression:

$$\Phi(k) = \inf\left\{\sum_{i=1}^{n} \lambda_i W(x_i) : \lambda_i \geq 0,\ x_i \in [\underline{\theta}, \overline{\theta}],\ \sum_{i=1}^{n} \lambda_i = 1,\ \sum_{i=1}^{n} \lambda_i x_i = k,\ n \in N\right\} \tag{8-16}$$

We can understand λ_i as a discrete probability with mean k. Therefore:

$$\Phi(k) \geq \inf\left\{\int W(\theta)dG(\theta) : G \in M(k)\right\} \tag{8-17}$$

$$\Phi(k) = \min\left\{\int W^+(\theta)dG(\theta) : G \in M(k)\right\} = \inf\left\{\int W(\theta)dG(\theta) : G \in M(k)\right\} \tag{8-18}$$

Proof of Proposition 5.

The first equality is given by definition 2. The second one is associated to the previous proposition because for every k:

$$\Phi(k) = \inf\left\{\int W(\theta)dG(\theta) :\ G \in M(k)\right\} \tag{8-19}$$

Which is equivalent, for every $H \in \mathcal{H}$, to:

$$\Phi(k) \leq \int W(\theta)dH(\theta|k) \tag{8-20}$$

Finally, for every $H \in \mathcal{H}$:

$$\int \Phi(k)dF(k) \leq \int \int W(\theta)dH(\theta|k)dF(k) \tag{8-21}$$

On the other hand, we have that for every k:

$$\begin{aligned} \Phi(k) &\geq \inf\left\{\int W(\theta)dG(\theta) : G \in M(k)\right\} \\ &\geq \inf\left\{\int W(\theta)dH(\theta|k) : H(\theta|k) \in \mathcal{H}\right\} \end{aligned} \tag{8-22}$$

Therefore:

$$\int \Phi(k)dF(k) \geq \inf_{H\in\mathcal{H}} \int \int W(\theta)dH(\theta|k)dF(k) \tag{8-23}$$

Proof of Theorem 1.

Lagrangian of regulator problem is equivalent to:

$$\max_{\Phi(.)} \int_{\underline{\theta}}^{\hat{\theta}} -H'(\theta)\Phi'(\theta)d(\theta) \tag{8-24}$$

Where:

$$H(x) = \int_{\underline{\theta}}^{x} \hat{F}(u)du - (x-\underline{\theta}) - \lambda\left[\frac{(v-x)^\alpha - (v-\underline{\theta})^\alpha}{\alpha}\right] \tag{8-25}$$

Integrating by parts, we have:

$$\begin{aligned} -\int_{\underline{\theta}}^{\hat{\theta}} H'(\theta)\Phi'(\theta)d\theta &= -H(x)\Phi'(x)|_{\underline{\theta}}^{\hat{\theta}} + \int_{\underline{\theta}}^{\hat{\theta}} H(\theta)d\Phi'(\theta) = \\ = \int_{\underline{\theta}}^{\overline{\theta}}\left[\int_{\underline{\theta}}^{\theta} F(u)du - (\theta-\underline{\theta}) - \lambda\left(\frac{(v-\theta)^\alpha - (v-\underline{\theta})^\alpha}{\alpha}\right)\right]&\Phi''(\theta)d\theta \end{aligned} \tag{8-26}$$

For $\lambda = sup_\tau\left[\dfrac{\int_{\underline{\theta}}^{\tau} F(u)du - (\tau-\underline{\theta})}{(v-\tau)^\alpha - (v-\underline{\theta})^\alpha}\right]$, we have $\Phi''(\tau) = 0$.

That means $\Phi'(\theta)$ is constant, and from the constraint we have:

$$\Phi'(\theta) = \frac{\alpha(v-\underline{\theta})^\alpha}{(v-\hat{\theta})^\alpha - (v-\underline{\theta})^\alpha}\overline{q} \tag{8-27}$$

$$\Phi(\hat{\theta}) - \Phi(\theta) = \frac{\alpha(v-\underline{\theta})^\alpha(\hat{\theta}-\theta)}{(v-\hat{\theta})^\alpha - (v-\underline{\theta})^\alpha}\overline{q} \tag{8-28}$$

$$\Phi(\theta) = \frac{\alpha(v-\underline{\theta})^\alpha(\hat{\theta}-\theta)}{(v-\underline{\theta})^\alpha - (v-\hat{\theta})^\alpha}\overline{q} \tag{8-29}$$

Finally, evaluating the linear social welfare from θ to $\hat{\theta}$, optimal mechanism is defined by the following equations.

$$q(\theta) = \left[\frac{(v-\underline{\theta})^\alpha}{(v-\underline{\theta})^\alpha - (v-\hat{\theta})^\alpha}\right]\left[\frac{(v-\theta)^\alpha - (v-\hat{\theta})^\alpha}{(v-\theta)^\alpha}\right]\overline{q} \tag{8-30}$$

$$\hat{\theta} = \arg\max_{\tau}\left[\frac{\tau - \int_{\underline{\theta}}^{\overline{\theta}} k dF(k)}{(v-\tau)^\alpha - (v-\underline{\theta})^\alpha}\right] \tag{8-31}$$

Proof of Theorem 2.

Lagrangian is given by equation (5-15) and its argument should satisfy the following equation.

$$vq(\theta) - \theta q(\theta) - (1-\alpha)\int_{\theta}^{\overline{\theta}} q(\tau)d\tau - a = -\mu\theta + \epsilon \tag{8-32}$$

Lets define $\hat{\theta}$ such that for every $\theta \in (\underline{\theta}, \hat{\theta})$, the following ODE, which comes from the derivate of previous expression, defines $q(.)$:

$$(v-\theta)q'(\theta) - \alpha q(\theta) = -\mu \tag{8-33}$$

The ODE solution is given by equation 8-40.

$$q'(\theta) - \frac{\alpha}{v-\theta}q(\theta) = \frac{-\mu}{v-\theta} \tag{8-34}$$

$$e^{\int \frac{-\alpha}{v-\tau}d\tau}q'(\theta) + e^{\int \frac{-\alpha}{v-\tau}d\tau}\frac{-\alpha}{v-\theta}q(\theta) = e^{\int \frac{-\alpha}{v-\tau}d\tau}\frac{-\mu}{v-\theta} \tag{8-35}$$

$$\frac{\partial\left[e^{\int \frac{-\alpha}{v-\tau}d\tau}q(\theta)\right]}{\partial\theta} = e^{\int \frac{-\alpha}{v-\tau}d\tau}\frac{-\mu}{v-\theta} \tag{8-36}$$

$$e^{\int \frac{-\alpha}{v-\tau}d\tau} = e^{\alpha\ln(v-\theta)} = (v-\theta)^\alpha \tag{8-37}$$

$$d\left[(v-\theta)^\alpha q(\theta)\right] = \frac{-\mu}{(v-\theta)^{1-\alpha}}\partial\theta \tag{8-38}$$

$$\int d\left[(v-\theta)^\alpha q(\theta)\right] = \int \frac{-\mu}{(v-\theta)^{1-\alpha}}d\theta \tag{8-39}$$

$$(v-\hat{\theta})^\alpha q(\hat{\theta}) - (v-\theta)^\alpha q(\theta) = \frac{\mu}{\alpha}\left[(v-\hat{\theta})^\alpha - (v-\theta)^\alpha\right] \tag{8-40}$$

Evaluating solution at $\overline{\theta}$ we have a value for μ as a function of $\hat{\theta}$.

$$-(v-\underline{\theta})^\alpha\overline{q} = \mu\left[\frac{(v-\hat{\theta})^\alpha}{\alpha} - \frac{(v-\underline{\theta})^\alpha}{\alpha}\right] \tag{8-41}$$

$$-(v-\underline{\theta})^\alpha\overline{q} = -\mu\left[\frac{(v-\underline{\theta})^\alpha - (v-\hat{\theta})^\alpha}{\alpha}\right] \tag{8-42}$$

$$\frac{-\alpha(v-\underline{\theta})^{\alpha}}{(v-\underline{\theta})^{\alpha}-(v-\hat{\theta})^{\alpha}}\overline{q}=-\mu \tag{8-43}$$

Regulator maximizes $-\mu z+\epsilon$, since $W(\hat{\theta})=0=-\mu\hat{\theta}+\epsilon$, regulator maximizes $-\mu(z-\hat{\theta})$. Then, $\hat{\theta}$ satisfies:

$$\hat{\theta}=\arg\max_{\tau}\left[\frac{-\alpha(v-\underline{\theta})^{\alpha}(z-\tau)\overline{q}}{(v-\underline{\theta})^{\alpha}-(v-\tau)^{\alpha}}\right] \tag{8-44}$$

$$\hat{\theta}=\arg\max_{\tau}\left[\frac{\tau-z}{(v-\underline{\theta})^{\alpha}-(v-\tau)^{\alpha}}\right] \tag{8-45}$$

Finally, evaluating (8-40) at θ, optimal mechanism is given by equation (8-46).

$$q(\theta)=\left[\frac{(v-\underline{\theta})^{\alpha}}{(v-\underline{\theta})^{\alpha}-(v-\hat{\theta})^{\alpha}}\right]\left[\frac{(v-\theta)^{\alpha}-(v-\hat{\theta})^{\alpha}}{(v-\theta)^{\alpha}}\right]\overline{q} \tag{8-46}$$