Pontifícia Universidade Católica
do Rio de Janeiro

Alexandre Moreira da Silva

Two-Stage Robust Optimization Models for Power System Operation and Planning under Joint Generation and Transmission Security Criteria

DISSERTAÇÃO DE MESTRADO

Dissertation presented to the Programa de Pós-Graduação em Engenharia Elétrica of the Departamento de Engenharia Elétrica, PUC–Rio as partial fulfillment of the requirements for the degree of Mestre em Engenharia Elétrica.

Advisor: Prof. Alexandre Street de Aguiar

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Mas os que esperam no SENHOR renovarão as suas forças; subirão com asas como águias; correrão e não se cansarão; andarão e não se fatigarão. - Isaías 40:31

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Abstract

Moreira da Silva, Alexandre; Street de Aguiar, Alexandre (Advisor). **Two-Stage Robust Optimization Models for Power System Operation and Planning under Joint Generation and Transmission Security Criteria**. Rio de Janeiro, 2014. 106p. MSc Dissertation — Departamento de Engenharia Elétrica, Pontifícia Universidade Católica do Rio de Janeiro.

Recent major blackouts all over the world have been a driving force to make power system reliability, regarding multiple contingencies, a subject of worldwide research. Within this context, it is important to investigate efficient methods of protecting the system against dependent and/or independent failures. In this sense, the incorporation of tighter security criteria in power systems operation and planning became crucial.

Multiple contingencies are more common and dangerous than natural independent faults. The main reason for this lies in the complexity of the dynamic stability of power systems. In addition, the protection system, that operates in parallel to the supply system, is not free of failures. Thus, natural faults can cause subsequent contingencies (dependent on earlier contingencies) due to the malfunction of the protection mechanisms or the instability of the overall system. These facts drive the search for more stringent safety criteria, for example, $n-K$, where K can be greater than 2.

In the present work, the main objective is to incorporate the joint generation and transmission general security criteria in power systems operation and planning models. Here, in addition to generators outages, network constraints and transmission lines failures are also accounted for. Such improvement leads to new computational challenges, for which we design efficient solution methodologies based on Benders decomposition. Regarding operation, two approaches are presented. The first one proposes a trilevel optimization model to decide the optimal scheduling of energy and reserve under an $n-K$ security criterion. In such approach, the high dimensionality curse of considering network constraints as well as outages of generators and transmission assets is withstood by implicitly taking into account the set of possible contingencies. The second approach includes correlated nodal demand uncertainty in the same framework. Regarding transmission expansion planning, another trilevel optimization model is proposed to decide which transmission assets should be built within a set of candidates in order to meet an $n-K$ security criterion, and, consequently, boost the power system reliability. Therefore, the main contributions of this work are the following: 1) trilevel models to consider general $n-K$ security criteria in power systems operation and planning, 2) implicit consideration of the whole contingency set by means of an adjustable robust optimization approach, 3) co-optimization of energy and reserves for power systems operation, regarding network constraints and ensuring the deliverability of reserves in all considered post-contingency states, 4) efficient solution methodologies based on Benders decomposition that finitely converges to the global optimal solution, and 5) development of valid constraints to boost computational efficiency. Case studies highlight the effectiveness of the proposed methodologies in capturing the economic effect of nodal demand correlation on power system operation under an $n-1$ security criterion, in reducing the computational effort to consider conventional $n-1$ and $n-2$ security criteria, and in considering security criteria tighter than $n-2$, an intractable problem heretofore.

Keywords

Adjustable Robust Optimization; Benders Decomposition; Correlated Nodal Demand Uncertainty; Energy and Reserve Scheduling; Generation and Transmission $n-K$ Security Criterion; Trilevel Programming.

Resumo

Moreira da Silva, Alexandre; Street de Aguiar, Alexandre (Orientador). **Modelos Robustos de Otimização de Dois Estágios para Operação e Planejamento de Sistemas de Potência sob Critérios de Segurança de Geração e Transmissão Conjuntos**. Rio de Janeiro, 2014. 106p. Dissertação de Mestrado — Departamento de Engenharia Elétrica, Pontifícia Universidade Católica do Rio de Janeiro.

Recentes apagões em todo o mundo fazem da confiabilidade de sistemas de potência, no tocante a contingências múltiplas, um tema de pesquisa mundial. Dentro desse contexo, se faz importante investigar métodos eficientes de proteger o sistema contra falhas de alguns de seus componentes, sejam elas dependentes e/ou independentes de outras falhas. Nesse sentido, se tornou crucial a incorporação de critérios de segurança mais rigorosos na operação e planejamento de sistemas de potência.

Contingências múltiplas são mais comuns e desastrosas do que falhas naturais e independentes. A principal razão para isso reside na complexidade da estabilidade dinâmica de sistemas de potência. Além disso, o sistema de proteção que opera em paralelo ao sistema de distribuição não é livre de falhas. Portanto, interrupções naturais podem causar contingências em cascata em decorrência do mau funcionamento de mecanismos de proteção ou da instabilidade do sistema elétrico como um todo. Nesse contexto, se dá a motivação pela busca de critérios de segurança mais severos como, por exemplo, o $n - K$, onde K pode ser maior do que 2.

Nesse trabalho, o principal objetivo é incorporar o crtitério de segurança geral $n-K$ para geração e transmissão em modelos de operação e planejamento de sistemas de potência. Além de interrupções em geradores, restrições de rede, bem como falhas em linhas de transmissão também são modeladas. Esse avanço leva a novos desafios computacionais, para os quais formulamos metodologias de solução eficientes baseadas em decomposição de Benders. Considerando operação, duas abordagens são apresentadas. A primeira propõe um modelo de otimização trinível para decidir o despacho ótimo de energia e reservas sob um critério de segurança $n - K$. Nessa abordagem, a alta dimensionalidade do problema, por contemplar restrições de rede, bem como falhas de geradores e de linhas de transmissão, é contornada por meio da implícita consideração do conjunto de possíveis contingências. No mesmo contexto, a segunda abordagem leva em conta a incerteza da carga a ser suprida e a correlação entre demandas de diferentes barras. Considerando planejamento de expansão da transmissão, outro modelo de otimização trinível é apresentado no intuito de decidir quais linhas de transmissão, dentro de um conjunto de candidatas, devem ser construídas para atender a um critério de segurança $n-K$ e, consequentemente, aumentar a confiabilidade do sistema como um todo. Portanto, as principais contribuições do presente trabalho são as seguintes: 1) modelos de otimização trinível para considerar o critério de segurança $n-K$ em operação e planejamento de sistemas de potência, 2) consideração implícita de todo o conjunto de contingências por meio de uma aboradagem de otimização robusta ajustável, 3) otimização conjunta de energia e reserva para operação de sistemas de potência, considerando restrições de rede e garantindo a entregabilidade das reservas em todos os estados pós-contingência considerados, 4) metodologias de solução eficientes baseadas em decomposição de Benders que convergem em passos finitos para o ótimo global e 5) desenvolvimento de restrições válidas que alavancam a eficiência computacional. Estudos de caso ressaltam a eficácia das metodologias propostas em capturar os efeitos econômicos de demanda nodal correlacionada sob um critério de segurança $n-1$, em reduzir o esforço computacional para considerar os critérios de segurança convencionais $n-1$ e $n-2$ e em considerar critérios de segurança mais rigorosos do que o $n-2$, um problema intratável até então.

Palavras–chave

Otimização Robusta Ajustável; Decomposição de Benders; Incerteza Correlacionada das Demandas Nodais; Despacho de Energia e Reserva; Critério Geral $n-K$ de Segurança de Geração e Transmissão; Otimzação Trinível.

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Os fariseus, tendo sabido que Ele fechara a boca dos saduceus, reuniram-se; - e um deles, que era doutor da lei, propôs-lhe esta questão, para o tentar - Mestre, qual é o maior mandamento da lei? - Jesus respondeu: Amarás o Senhor teu Deus de todo o teu coração, de toda a tua alma e de todo o teu espírito; este é o maior e primeiro mandamento. E aqui tendes o segundo, semelhante a esse: Amarás o teu próximo, como a ti mesmo. - Toda a lei e os profetas se acham contidos nesses dois mandamentos.
Hearing that Jesus had silenced the Sadducees, the Pharisees got together. One of them, an expert in the law, tested him with this question: "Teacher, which is the greatest commandment in the Law?" Jesus replied: " 'Love the Lord your God with all your heart and with all your soul and with all your mind.' This is the first and greatest commandment. And the second is like it: 'Love your neighbor as yourself.' All the Law and the Prophets hang on these two commandments."

Bíblia Sagrada (Holy Bible), *S. Mateus, 22: 34-40.*

1
Introduction

In order to make an electrical system operation robust to the possibility of failures (contingencies), different models and approaches have been proposed to determine a dispatch of generators capable of ensuring the system reliability. Within this context, there are three main sources of uncertainty, namely availability of equipments, energy demand and power generation. Outages in generators and transmission lines materialize uncertainty in the availability of equipments. In the literature, this issue is addressed in a deterministic [1] and in a stochastic [2] fashion. Load variations imply demand uncertainty, for which works proposed stochastic [3] and robust approaches [4]. Finally, the complex management of reservoirs in hydrothermal systems [5,6] and the recent increasingly penetration of wind farms in power systems [7] play a major role in the power generation uncertainty, which is mainly addressed by stochastic optimization. This work focuses on uncertainty inherent to availability of equipments and energy demand.

Most power systems worldwide operate under the deterministic $n-1$ and $n-2$ security criteria [1]. Deterministic contingency-constrained models, which explicitly represent the operation under each credible contingency, are generally used to define optimal levels of reserves. Relevant applications of such models to co-optimize energy and reserves can be found in [8–11].

Stochastic models (see [2] and references therein) are also used in generation scheduling. They aim to capture probabilistic structures present in the underlying uncertainty process, e.g., correlations between nodal demands and renewable injections, by means of scenarios and their probabilities. The main goal of stochastic models is to optimize the levels of resources, e.g., energy and reserves, taking advantage of the uncertainty structure while ensuring system security in a probabilistic fashion.

Both deterministic and stochastic approaches have advantages and disadvantages. Stochastic approaches, for example, prevent the over-conservatism associated with deterministic criteria by using probabilistic information and thereby avoiding investments against unlikely events. Nevertheless, it is not trivial to obtain such probabilistic information, which is not required by deterministic approaches. To illustrate this point, consider a transmission asset that is interrupted once every ten years. In this case, too much time is required to estimate the probability of unlikely events. If the same asset suddenly fails twice in a ten-year period, then the probability will increase two-fold. Moreover, the calculation of joint probabilities of non-independent events also materializes a major problem when dealing with the stochastic framework. Therefore, it is highly difficult to accurately estimate all those probabilities and, as it is well-known, they have a deep impact on the results of stochastic approaches. On the other hand, deterministic criteria, in addition to unintentional outages, account for intentional outages, which are more dangerous and not addressed by stochastic approaches. Moreover, the large number of scenarios that may lead some stochastic problems to intractability are not needed in a deterministic framework. Despite this discussion, there is still no consensus about which approach is better. However, based on industry practice, the present work addresses the application of deterministic security criteria in power system operation and planning.

Multiple contingencies are more common and dangerous than natural independent faults. The main reason for this lies in the complexity of the dynamic stability of power systems. In addition, the protection system, that operates in parallel to the supply system, is not free of failures. Thus, natural faults can cause subsequent contingencies (dependent on earlier contingencies) due to the malfunction of the protection mechanisms or the instability of the overall system. These facts drive the search for more stringent safety criteria, for example, $n-K$, where K can be greater than 2.

In the last five years, Brazil has been victim of several blackouts, for example [12,13]. In November 2009, a load shed directly affected the life of the Brazilian population. It was a Tuesday and at least 18 states of the country became dark. According to the government, the blackout took place due to a failure in 3 transmission lines supplied by Itaipu, a bi-national hydro plant that currently supplies 17.3% of the Brazilian energy demand and 72.5% of the Paraguayan energy demand. Such failure, as confirmed by specialists in [12], was caused by adverse meteorological conditions and, as consequence, the biggest South-American country experienced 24.436 MW of load shedding. The outcome for the Brazilians could not be worse, particularly in the biggest states, such as Rio de Janeiro and São Paulo. The services of train and metro stopped working and buses became overcrowded. Moreover, violent acts were registered at some places and phone signal was interrupted. In February 2011, a significant part of the Northeast of Brazil lost energy supply for a 4-hour period, due to another failure in the electric system. The states of Bahia, Alagoas, Pernambuco, Paraíba, Ceará, Sergipe, Piauí e Rio Grande do Norte were affected. The protection mechanism of one transmission line sent a wrong signal for the system and, consequently, caused cascading outages of other transmission assets and power plants. The $n-2$ security criterion adopted in that region was not sufficient to prevent a load shed in this situation. Once again, the outcome was perverse and one of the worst consequences was the interruption of energy supply in the biggest public hospital of Pernambuco. For a wider review of major blackouts in Brazil and in other countries, the interested reader is referred to Appendix B.

The aforementioned events and other major blackouts all over the world have been a driving force to make power system reliability, regarding multiple contingencies, a subject of worldwide research [14–17]. Within this context, it is important to investigate efficient methods of protecting the system against dependent and/or independent failures. In this sense, the incorporation of security criteria in power systems operation, as well as in transmission expansion planning became crucial.

Operation and planning play complementary roles in the well-functioning of power systems. On the operative side, the major issue is to balance load and generation on an instantaneous basis by means of automatic generation control systems, preventing large frequency variations, which can cause interruptions in the energy supply. Scheduling of energy and reserves in a time frame of minutes to months depending on the system characteristics is also an important task for operators. On the planning side, one of the main points is to decide which transmission facilities should be built in order to guarantee a reliable and economic system operation in a time frame of years to decades.

The state of the art in the incorporation of security criteria in power systems operation [8, 18] lies in the contingency-constrained unit commitment (CCUC) [8, 9, 18–20]. Such model explicitly enumerates the cases of contingency in its formulation in order to determine the optimal system scheduling protected against such events.

The main drawback of explicitly considering all possible cases of contingency lies in the high-dimensionality curse. Such obstacle prevents conventional CCUC models of regarding security criteria tighter than $n-2$. Thereby, commonly, CCUC models contemplate a limited number of cases of contingency. In [10], an alternative methodology of incorporating the $n-K$ security criterion in CCUC models was presented. Basically, [10] proposed a bi-level optimization model to determine the optimal scheduling of energy and reserve under an $n-K$ security criterion, where K can be greater than 2. In such work, high-dimensionality curse was overcome through an effective approach that implicitly considers the cases of contingency by means of robust optimization techniques [21–24].

On the other hand, fewer works have addressed security in the transmission expansion planning problem. The pioneers to do so were [25] and [26]. In [25], a procedure to account single line contingencies by iteratively adding constraints in the original model was proposed. In [26], the static contingency-constrained transmission expansion planning (CC-TEP) was introduced. Latter, more effort was made concerning the well-known $n-1$ deterministic security criterion in planning problems [27]. However, due to high dimensionality and complexity, the consideration of tighter security criteria, such as $n-K$ (see [10, 11, 28]), still requires further developments.

The findings of [10] paved the way for new perspectives regarding the studies about power systems reliability. It was the first work to effectively address the $n-K(K>2)$ security criterion in the unit commitment problem. Nevertheless, [10] neglected network constraints. In the present work, the main objective is to incorporate the joint generation and transmission general security criteria in power systems operation and planning models. Here, in addition to generators outages, network constraints and transmission lines failures are also accounted for. Such improvement leads to new computational challenges. As an example, while [10] proposes a bilevel program of the robust optimization class, trilevel formulations belonging to the adjustable robust optimization class are developed in the present work. Therefore, a higher computational burden is required. To solve this problem, we design efficient solution methodologies based on Benders decomposition. Regarding operation, two approaches are presented. The first one proposes a trilevel optimization model to decide the optimal scheduling of energy and reserve under an $n-K$ security criterion. In such approach, the high dimensionality curse of considering network constraints as well as outages of generators and transmission assets is withstood by implicitly taking into account the set of possible contingencies. The second approach includes correlated nodal demand uncertainty in the same framework. Regarding transmission expansion planning, another trilevel optimization model is proposed to decide which transmission assets should be built within a set of candidates in order to meet an $n-K$ security criterion, and, consequently, boost the power system reliability.

1.1 Reliability in Power Systems Operation

The definition of the term *reliability*, according to the North American Electric Reliability Corporation (NERC), is composed of two concepts, namely *adequacy* and *operating reliability*. In [29], NERC characterizes *adequacy* as "the ability of the electric system to supply the aggregate electric power and energy requirements of the electricity consumers at all times, taking into account scheduled and reasonably expected unscheduled outages of system components" and *operating reliability* as "the ability of the electric system to withstand sudden disturbances such as electric short circuits or unanticipated loss of system components". Needless to say, reliability is a major concern when it comes to power systems operation, since, nowadays, daily life is highly dependent on electricity. Therefore, once a blackout occurs, its causes and consequences are carefully studied by specialists, in order to prevent similar events [30–32].

There are two main threats to power systems operations: unintentional outages [12, 13], often caused by weather and/or environment conditions, malfunction of protection mechanisms, and human error, and intentional outages [33], fruit of deliberate attacks against the electric system. Within this context, non-spinning and spinning reserves play a significant role in order to provide operators with necessary leeway to protect the system against both types of threats. Non-spinning reserve is defined as the off-line generation capacity that can be synchronized to the grid. On the other hand, spinning reserve is the on-line reserve capacity that is synchronized to the grid system and ready to meet electric demand. Spinning reserves can be also split into up- and down-spinning reserves, which are scheduled capacities to increase and decrease the energy production of a power plant respectively, if needed.

The aforementioned types of energy reserve are commonly used to ensure the survivability of the system against the outage of a single or two transmission or generation assets. These standards are the well-known $n-1$ and $n-2$ security criteria. Other security criteria can be also found in the literature. The $n-K^G-K^L$ contingency analysis, proposed in [15], considers the simultaneous loss of up to K^G generators and up to K^L transmission lines. The $N-1-1$ security criterion [34] regards the loss of a single generator or transmission line, accompanied by system adjustments, which are followed by another loss of a power plant or transmission asset.

Generation scheduling problems have traditionally incorporated deterministic security criteria by contingency-constrained models [8, 9, 18, 19, 35, 36].

For the sake of computational tractability, such models explicitly represent the operation of the power system under a reduced set of credible contingencies. This limitation is stressed in the current context where recent blackouts involving the loss of more than two components [12, 13] suggest that tighter security levels comprising multiple outages should be considered.

To overcome the dimensionality curse observed in conventional contingency-constrained models, Street et al. [10, 11] recently proposed robust optimization [24, 37] to schedule energy and reserves under a deterministic $n-K$ security criterion. In both works, the effect of the transmission network was neglected and only generator outages were considered. In [11], a single-period setting was used to illustrate the effectiveness of robust optimization to implicitly consider the whole contingency set. In [10], the approach was extended by analyzing a multiperiod setting and adding non-spinning reserves to the problem formulation.

The problem of specifying reserve requirements is addressed by two approaches [38]. In the first approach, the amount of reserve is predefined [39, 40]. In this case, system-wide or zonal reserve requirements are usually set to a percentage of the system load or to the capacity of the major power plant of the system. Thereby, such information is used as an input for the optimization of energy. In the second approach, energy and reserves are jointly optimized [8–10, 18, 38]. According to [18], the most appropriate process to establish reserve requirements is the second approach, since the first may lead to sub-optimal or infeasible decisions. For example, as discussed in [38], while the second approach is capable to account for the deliverability of reserves, the first approach, in some occasions, force operators to disqualify significant amounts of reserve due to transmission constraints, possibly resulting in reserve price spikes.

Within this context, Chapter 2 presents a new approach to incorporate a deterministic security criterion in the co-optimization of energy and reserves [8–11, 18, 19]. The salient feature of the proposed model over [10, 11] is the consideration of the transmission network. This modeling novelty is motivated by (i) current industry practice worldwide in the framework of the operation of electricity markets, and (ii) the need to consider line outages in power system operation problems to properly account for standard security criteria. Network constraints are needed to define locational reserves and their deliverability under a given security criterion (see [39] and references therein). Moreover, the second motivation is deemed as crucial given the recent major blackouts where failures in the transmission network played a key role [30]. It should be noted that single-bus models available in the technical literature [10, 11] are not suitable to address both aspects. Hence, new models such as the one proposed in Chapter 2 are required.

From the modeling perspective, the consideration of the transmission network gives rise to two major modifications with respect to the problem formulation presented in [10,11]: (i) down reserves are required to characterize the operation under contingency, and (ii) line outages are addressed thus allowing the consideration of transmission assets in the security criteria.

Chapter 2 also differs from [10] and [11] from the methodological perspective. The consideration of down reserves requires explicitly modeling the operation under contingency. Therefore, the robust optimization framework based on bilevel programming of [10] and [11] is not readily applicable when the effect of the transmission network is accounted for. As a distinctive feature, the proposed approach is based on adjustable robust optimization (ARO) [41,42]. Similar to robust optimization, ARO is suitable to model optimization problems where the optimal solution must remain feasible for the worst-case parameter variation in a user-defined set, denoted as uncertainty set [24, 37]. In contrast, ARO allows incorporating the flexibility of adjustable decisions, also known as recourse actions, in robust counterparts [41, 42]. In this setting, ARO involves a trilevel optimization process [41–43]. The upper level determines optimal non-adjustable decisions, i.e., decisions that must be feasible for every deviation of the uncertain parameters. The middle level identifies the worst-case parameter values leading to maximum feasibility damage of the upper-level decisions. Finally, the lower level aims at finding the best reaction, by means of adjustable variables, that minimizes the upper-level infeasibility.

In the proposed ARO-based approach for generation scheduling under a joint generation and transmission security criterion, the parameters allowed to vary represent the availability of system components under the contingency states. In addition, adjustable decisions are post-contingency operation variables such as generation levels and line flows. Similar to [44] and [4], the adjustable robust counterpart is formulated as a trilevel mixed-integer program that is solved by a Benders decomposition approach involving bilinear terms and the iterative solution of a master problem and a subproblem. It should be noted that the presence of binary variables in the middle level of the proposed trilevel program does not allow its transformation to a single-level equivalent, as done in [11] and [10]. Two methods have been proposed in the technical literature to deal with those bilinear terms: (i) a linearization scheme based on disjunctive constraints [44], which has also been widely used in the application of bilevel programming in power system planning (see [17] and references therein); and (ii) an outer approximation technique based on an iterative heuristic procedure [4]. In this work, the former method is used. Hence, the subproblem is formulated as a bilevel programming problem that is equivalently recast as a mixed-integer linear program. The master problem is a mixed-integer linear program that provides an approximation of the original trilevel problem. In order to improve the performance of the decomposition procedure, two sets of valid constraints are added to the master problem.

Regarding load variability, recent works [4, 44] propose two-stage or adjustable robust optimization (ARO) [41, 42] to deal with nodal injection uncertainty in unit commitment. In both works, a multi-level robust counterpart is formulated and Benders decomposition is applied. However, the presence of bilinear and highly nonconvex terms prevents Benders decomposition from guaranteeing the attainment of global optimality. Thus, such approaches rely on Monte Carlo sampling [44] or an iterative heuristic [4] in order to assess the quality of the suboptimal solutions achieved. Aside from their difficulties in proving optimality, the ARO-based approaches [4, 44] feature an additional shortcoming with respect to traditional stochastic approaches for unit commitment under demand uncertainty [2], namely the correlation effect between nodal demands is disregarded. Such correlation may play a crucial role in power system operation, particularly when wind power generation is accounted for in production scheduling [45].

In Chapter 3, we present a new ARO-based methodology for the co-optimization of energy and spinning reserves under both a deterministic security criterion and demand uncertainty, which, from another perspective, can be seen as generation uncertainty, mainly related to renewable energy sources. As a salient feature over previous works [44], [4], the proposed contingency-constrained model explicitly allows considering the correlation between nodal demands. Such correlation is characterized by the nodal demand covariance matrix [46], which is conveniently factorized through the Cholesky decomposition [47]. It is worth mentioning that, similar to the uncorrelated approach described in [4], a robust counterpart is formulated as a trilevel program but the incorporation of nodal demand correlation does not increase the computational complexity of the resulting optimization.

From a modeling perspective, the proposed model also extends that reported in [4] in the way a deterministic security criterion is imposed so that the system is able to withstand a set of credible contingencies. Thus, the optimal pre-contingency schedule is associated with the upper optimization where generation levels, and up- and down-spinning reserves are the decision variables. This is a relevant difference with respect to the model described in [4], where dispatching variables were determined in the lower-level problem and down-spinning reserves were disregarded. Another distinctive feature over [4] arises in the middle-level problem, which is associated with the worst-case demand. Rather than using the economic criterion of [4] relying on operation cost maximization, we argue that the worst-case demand should be defined as the demand vector yielding the largest system power imbalance.

Chapter 3 also differs methodologically from previously reported works [44], [4]. Although dual-based Benders decomposition involving bilinear terms is also used, the proposed approach is finitely convergent to global optimality and provides a measure of the distance to optimality along the iterative procedure. This is a consequence of the convexity of the resulting recourse function and the use of an effective binary expansion approach to linearize the aforementioned bilinear terms.

1.2 Transmission Expansion Planning

Decision making associated with the investment in the transmission network plays a key role in power system planning in both centralized and competitive frameworks. Such planning problem consists in determining how to expand and reinforce the transmission network in order to meet the forecast load growth over a specific time span with the available generation assets [48]. Transmission network expansion planning can be implemented via static and dynamic models [49]. In static planning, the location and number of transmission assets to be constructed are determined in a single-stage decision made at the beginning of the time span. In dynamic planning, the optimal timing for the investment in new transmission facilities is also determined within a multi-stage decision-making framework.

The primary goal of transmission network expansion planning is to supply electricity to consumers in a secure and economic fashion. Security, in a deterministic sense, is the capability of a power system to survive a specified set of credible contingencies without having to shed load [1]. However, recent major blackouts worldwide [12, 13] reveal that a number of unresolved issues remain concerning security in power system operation and planning.

Traditionally, two main approaches have been adopted in the literature to solve transmission planning problems: Mathematical Programming and Heuristic Procedures. In the former category, researchers formulate the problem by means of a mathematical optimization model and employ optimization techniques to search for an optimal expansion plan. In this setting, [50] makes use of linear programming, [51] employs dynamic programming, [52] implements nonlinear programming, [53] applies mixed-integer programming, and

[54] utilizes Benders decomposition. In order to elude the complexity of reaching the optimal solution through mathematical programming methodologies, several works designed heuristic models to find the best expansion scheme. In this sense, tabu search [55][56] and genetic algorithm [57] were proposed. A wider review on approaches to address the transmission planning problem is presented in [58], which also displays the current challenges in this field.

Despite the considerable research effort devoted to transmission network expansion planning [49, 59], most works have neglected the impact of security, being relevant exceptions the contingency-constrained models presented in [25, 26, 58, 60, 61] for unintentional outages and in [14, 16, 62] for deliberate contingencies. All those works share a major limitation: all contingency states are explicitly modeled in the problem formulation. Thus, for the sake of computational tractability, the set of credible contingencies is of reduced size. Within this context, while current system operation developments move toward tighter and more general security criteria (see [10, 11, 28] and references therein), CC-TEP models still rely on the standard $n-1$ criterion. Therefore, there is a need for new tools to expand the transmission system within the same security criterion adopted on the operative side.

Chapter 4 addresses the consideration of security in static transmission network expansion planning. Modeling security drastically increases the complexity of the resulting problem since the unavailability of system components needs to be characterized. Based on current industry practice, the security criteria considered here are deterministic [1].

Based on the findings of Chapter 2, Chapter 4 presents a new computationally efficient approach for contingency-constrained transmission expansion planning relying on two-stage robust optimization, also known as adjustable robust optimization (ARO) [41]. Within the context of transmission expansion planning, robust models were proposed in [63–65] to handle load uncertainty. However, security was disregarded in those robust models. It is worth mentioning that, differently from Chapters 2 and 3, reserves are not optimized in Chapter 4. This is justified since network expansions are generally long-term decisions and technical constraints of the generators, such as ramp-up and ramp-down limits, provide the planner with the information about how much reserve will be available to be scheduled by the operator in the future, if necessary.

As done in Chapter 2 for a different contingency-constrained problem, the adjustable robust counterpart of the original contingency-dependent expansion planing model is formulated as a trilevel mixed-integer program. In this approach, the upper level determines the least-cost non-adjustable decisions, namely the optimal investment decisions, that must be feasible for every deviation of the parameters allowed to vary. Such parameters represent the availability of system components under the contingency states. In other words, the optimal expansion plan guarantees that the system is able to withstand all contingencies associated with the security criterion adopted. For a given upper-level decision vector, the middle level identifies the worst-case parameter values leading to maximum feasibility damage. Hence, the contingency state yielding maximum system power imbalance is selected over all contingencies characterizing the security criterion. Finally, the lower level models the operator's best reaction, by means of adjustable variables, that minimizes the infeasibility for given upper- and middle-level decisions. Similar to Chapter 2, the resulting trilevel mixed-integer program is solved by a Benders decomposition approach that is finitely convergent to the optimal solution. In order to improve the performance of the decomposition, an acceleration procedure relying on an iterative column-and-constraint generation algorithm is applied.

Unlike conventional contingency-constrained models [14,16,25,26,60–62], the dimension of the resulting optimization problem does not increase with the size of the contingency set. As a result, the proposed method is a superior approach, which is corroborated by its faster performance and by its ability to solve cases for which conventional contingency-constrained models are unable to find a feasible solution.

Three recent examples of application of trilevel programming in power system planning can be found in [55,66,67]. In [55,66], a heuristic tabu search and an implicit enumeration algorithm were respectively proposed to identify the best protection scheme against deliberate attacks under the framework of a defender-attacker-defender model. In [67], the transmission network expansion planning was addressed by trilevel programming without considering security. Rather, the two lowermost optimization levels were related to equilibria associated with generation expansion and pool-based market clearing.

1.3 Contributions

The main contributions of this work are as follows:

1. Trilevel models to consider general $n - K$ security criteria in power systems operation and planning.

2. Implicit consideration of the whole contingency set by means of an adjustable robust optimization approach.

3. Co-optimization of energy and reserves for power systems operation, regarding network constraints and ensuring the deliverability of reserves in all considered post-contingency states.

4. Efficient solution methodologies based on Benders decomposition that finitely converges to the global optimal solution.

5. Development of valid constraints to boost computational efficiency.

Direct consequences of the aforementioned contributions are the reduction of the computational effort to consider conventional $n-1$ and $n-2$ security criteria and the possibility of considering security criteria tighter than $n-2$.

1.4 Outline

In this work, we present three extensions of [10]. The first one, proposed in Chapter 2, fills the main gap of [10] by considering network constraints as well as outages of transmission assets. The second one, described in Chapter 3, incorporates security to the unit commitment problem under the presence of correlated nodal demand uncertainty. The third one, developed in Chapter 4, proposes a transmission expansion planning methodology capable to meet an $n-K$ security criterion.

In addition, conclusions are drawn and future works are discussed in Chapter 5. Finally, the nomenclature used in this work is presented in Appendix A and recent major blackouts are briefly described in Appendix B.

2
Energy and Reserve Scheduling under a Joint Generation and Transmission Security Criterion: An Adjustable Robust Optimization Approach

Chapter 2 presents a new approach for energy and reserve scheduling in electricity markets subject to transmission flow limits. Security is imposed by guaranteeing power balance under each contingency state including both generation and transmission assets. The model is general enough to embody a joint generation and transmission $n-K$ security criterion and its variants. An adjustable robust optimization approach is presented to circumvent the tractability issues associated with conventional contingency-constrained methods relying on explicitly modeling the whole contingency set. The adjustable robust model is formulated as a trilevel programming problem. The upper-level problem aims at minimizing total costs of energy and reserves while ensuring that the system is able to withstand each contingency. The middle-level problem identifies, for a given pre-contingency schedule, the contingency state leading to maximum power imbalance if any. Finally, the lower-level problem models the operator's best reaction for a given contingency by minimizing the system power imbalance. The proposed trilevel problem is solved by a Benders decomposition approach. For computation purposes, a tighter formulation for the master problem is proposed. Our approach is finitely convergent to the optimal solution and provides a measure of the distance to the optimum. Simulation results show the superiority of the proposed methodology over conventional contingency-constrained models. The contents of this Chapter are directly related to a paper published in the *IEEE Transactions on Power Systems* [68].

The main contributions of this Chapter are as follows:

1. A new model is presented for the contingency-constrained energy and reserve scheduling problem under a deterministic security criterion. Unlike previously reported works, this model allows examining the effect of the transmission network while also considering line failures.

2. Adjustable robust optimization with a combinatorial uncertainty set is proposed as a suitable solution framework. The resulting problem is formulated as a trilevel programming problem.

3. A solution methodology based on Benders decomposition is presented. The performance of the proposed approach is improved by adding two sets of valid constraints that provide a tighter formulation. The superiority of the proposed method is backed by its faster performance and its ability to solve cases for which conventional contingency-constrained models are unable to find a feasible solution.

4. The proposed tool allows the system operator to assess the impact of tighter security criteria than currently used $n-1$ and $n-2$. In addition, the proposed methodology is flexible enough to comprise a wide range of security criteria such as separate criteria for generation and transmission, as well as specific criteria for subsets of system components. Finally, since the proposed model relies on the co-optimization of energy and reserves, it also constitutes a suitable methodology to define locational reserve requirements needed to implement a deterministic security criterion considering the effect of the transmission network.

The rest of this Chapter is organized as follows. Section 2.1 presents the conventional contingency-constrained formulation for the energy and reserve scheduling problem under a joint generation and transmission security criterion. In Section 2.2, the trilevel ARO counterpart is provided. Section 2.3 describes the proposed solution algorithm. Finally, in Section 2.4, two case studies are analyzed.

2.1 Conventional Contingency-Constrained Problem Formulation

The contingency-constrained generation scheduling problem determines the optimal generation schedule and reserve allocation so that the power balance is ensured under both normal and contingency states. Here we propose the explicit consideration of a joint generation and transmission security criterion. For expository purposes, we use a contingency-dependent network-constrained model based on that of [18], where a single period is analyzed and the focus is placed on synchronized reserves, specifically up-spinning and down-spinning. This model is simple to describe and analyze, yet bringing out the main features of contingency dependence. The multiperiod model would require extra indices denoting time periods and the inclusion of inter-temporal constraints such as minimum up and down times, ramping limits, and storage management. An example of multiperiod scheduling with non-synchronized reserves can be found in [10], where network constraints were neglected. The network-constrained contingency-dependent scheduling problem is formulated as:

$$\underset{\theta_b,\theta_b^k,f_l,f_l^k,p_i,p_i^k,r_i^D,r_i^U,v_i}{\text{Minimize}} \quad \sum_{i\in I}\big(C_i^P(p_i,v_i)+C_i^U r_i^U + C_i^D r_i^D\big) \tag{2-1}$$

subject to:

$$\sum_{i\in I_b} p_i + \sum_{l\in\mathcal{L}|to(l)=b} f_l - \sum_{l\in\mathcal{L}|fr(l)=b} f_l = D_b; \forall b\in N \tag{2-2}$$

$$f_l = \frac{1}{x_l}(\theta_{fr(l)}-\theta_{to(l)}); \forall l\in\mathcal{L} \tag{2-3}$$

$$-\overline{F}_l \le f_l \le \overline{F}_l; \forall l\in\mathcal{L} \tag{2-4}$$

$$\underline{P}_i v_i \le p_i \le \overline{P}_i v_i; \forall i\in I \tag{2-5}$$

$$p_i + r_i^U \le \overline{P}_i v_i; \forall i\in I \tag{2-6}$$

$$p_i - r_i^D \ge \underline{P}_i v_i; \forall i\in I \tag{2-7}$$

$$0\le r_i^U \le \overline{R}_i^U v_i; \forall i\in I \tag{2-8}$$

$$0\le r_i^D \le \overline{R}_i^D v_i; \forall i\in I \tag{2-9}$$

$$v_i\in\{0,1\}; \forall i\in I \tag{2-10}$$

$$\sum_{i\in I_b} p_i^k + \sum_{l\in\mathcal{L}|to(l)=b} f_l^k - \sum_{l\in\mathcal{L}|fr(l)=b} f_l^k = D_b; \forall b\in N, \forall k\in\mathcal{C} \tag{2-11}$$

$$f_l^k = \frac{A_l^k}{x_l}\big(\theta_{fr(l)}^k-\theta_{to(l)}^k\big); \forall l\in\mathcal{L}, \forall k\in\mathcal{C} \tag{2-12}$$

$$-\overline{F}_l \le f_l^k \le \overline{F}_l; \forall l\in\mathcal{L}, \forall k\in\mathcal{C} \tag{2-13}$$

$$A_i^k(p_i-r_i^D) \le p_i^k \le A_i^k(p_i+r_i^U); \forall i\in I, \forall k\in\mathcal{C}. \tag{2-14}$$

The objective function to be minimized (2-1) consists of the sum of the offered cost functions for generating energy plus the cost of all up- and down-spinning reserves offered by the generators.

Constraints (2-2)-(2-10), hereinafter referred to as pre-contingency scheduling constraints, impose the feasibility of the pre-contingency state schedule. Constraints (2-2) represent the nodal power balance equations. Using a dc load flow model, constraints (2-3) express the line flows in terms of the nodal phase angles, while constraints (2-4) enforce the corresponding line flow capacity limits. As is customary in generation scheduling in electricity markets $[1, 8, 9, 18, 35, 36]$, a dc load flow model is used to characterize the behavior of the network, recognizing that the use of such a simplified model leads to results that may be optimistic and that a complete study of the scheduling problem under a joint generation and transmission security criterion should also consider the effect of reactive power. This generalization would, however, render the problem essentially intractable. This modeling limitation notwithstanding, the solution of the energy and reserve scheduling problem based on the dc load flow is acceptable for the purposes of the operation of electricity markets [1, 8, 9, 18, 35, 36] and provides the system operator with a first estimate of a secure generation scheme.

Constraints (2-5) set the generation limits. Constraints (2-6) and (2-7) respectively relate the up- and down-spinning reserve contributions to the power levels produced under the pre-contingency state. Constraints (2-8)-(2-9) provide the bounds for the up- and down-spinning reserve contributions, respectively. Finally, the binary nature of scheduling variables is expressed in (2-10).

In (2-11)-(2-14), a feasible post-contingency redispatch is ensured. Analogous to (2-2)-(2-4), expressions (2-11)-(2-13) are the network constraints under contingency. Generation limits for the contingency states are set in (2-14). In (2-12) and (2-14), the statuses of system components are characterized by the generator and line availability binary parameters, A_i^k and A_l^k, respectively.

The dimension of model (2-1)-(2-14), in terms of the number of variables and constraints, and hence its computational tractability, both depend on the cardinality of $\mathcal{C}$. For the case of a joint generation and transmission security criterion, the contingency set $\mathcal{C}$ can be modeled in a compact way as:

$$\boldsymbol{f}\big(\{A_i^k\}_{i\in I}, \{A_l^k\}_{l\in L}\big) \geq \mathbf{0}; \forall k \in \mathcal{C}, \tag{2-15}$$

where $\boldsymbol{f}(\cdot)$ is a vector function. Typical joint generation and transmission security criteria can be modeled by a linear form of $\boldsymbol{f}(\cdot)$. For an $n-K$ criterion, the formulation of (2-15) would be $\sum_{i\in I} A_i^k + \sum_{l\in\mathcal{L}} A_l^k \geq n-K; \forall k \in \mathcal{C}$, where $n = |I| + |\mathcal{L}|$. Variants of such criterion such as the $n - K^G - K^L$ can also be considered in a similar fashion. Under such criteria, the size of problem (2-1)-(2-14) presents an exponential dependence with K, K^G, and K^L, which may lead to intractability even for low values of those parameters.

2.2 Adjustable Robust Optimization Approach

Problem (2-1)-(2-14) finds the least-cost schedule of power and reserves able to circumvent the contingency states included in $\mathcal{C}$. In other words, the power imbalance is explicitly set to zero for all contingencies considered in the contingency-dependent formulation. This problem can be viewed as a particular instance of ARO [41,42] wherein the parameters allowed to vary are A_i^k and A_l^k. Under this framework, the decisions modeling the reaction of the system operator against the occurrence of contingencies, i.e., decision variables with superscript k, are denoted as recourse actions or adjustable decisions [41]. Hence, the proposed ARO-based model belongs to the class of contingency-constrained generation scheduling problems, but differs from (2-1)-(2-14) in the way the operation under contingency is accounted for.

Next, the ARO-based modeling framework is described, the formulation of the proposed robust counterpart is provided, its equivalence with the original contingency-dependent model is discussed, and a simple illustrative example is analyzed.

2.2.1 ARO-Based Modeling Framework

The proposed ARO-based approach is characterized as a trilevel program [43], as shown in Fig. 2.1, which is based on the following rationale: for a given upper-level decision, the middle level problem searches in the contingency set the most damaging subset of K elements in terms of power imbalance, given the best redispatch provided by the lower level within the scheduled reserves and the remaining network and generators after contingency.

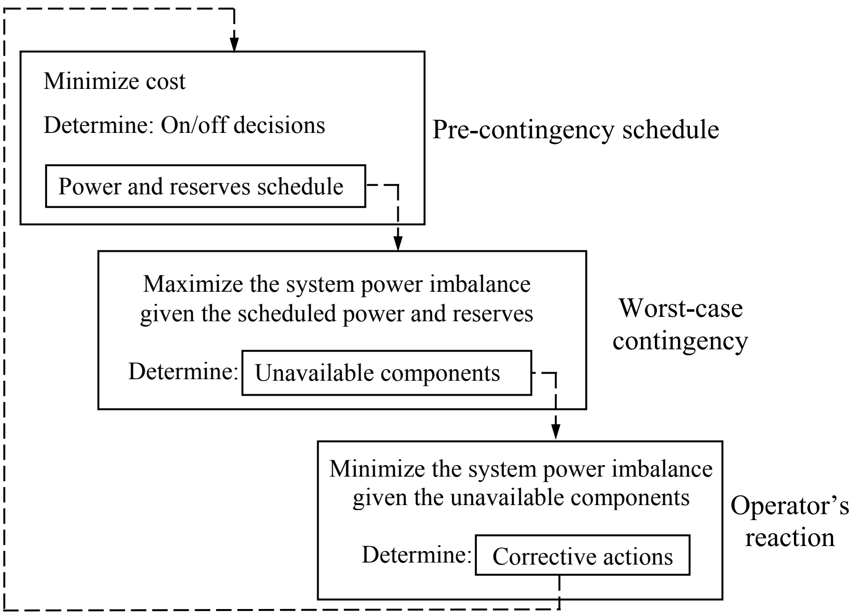


Figure 2.1: Trilevel model for the adjustable robust optimization approach

The upper level determines the cheapest pre-contingency schedule for power and reserves. In order for the pre-contingency schedule to be feasible, the system power imbalance should be equal to zero for all contingencies. Based on robust optimization [4, 41, 42, 44], pre-contingency feasibility can be modeled as a worst-case analysis requiring two additional optimization levels. For a given upper-level pre-contingency schedule, the middle level maximizes the system power imbalance over all contingencies characterizing the $n - K$

security criterion. Finally, the lower level models the operator's reaction against the contingency identified by the middle level. This reaction comprises some corrective measures, namely the adjustable decisions, to minimize the system power imbalance. Adjustable decisions include generation redispatch within the scheduled power and reserves for the available units. In each level, an objective function is optimized subject to the reaction of the subsequent level.

It should be noted that the role of the two lowermost optimizations is the identification of the contingency leading to the largest system power imbalance for each pre-contingency schedule considered in the upper level. Thus, rather than considering a single worst-case contingency associated with a base-case schedule, this framework implicitly considers all contingencies characterizing the $n-K$ security criterion for each pre-contingency schedule. It is worth mentioning that for all power and reserve schedules compliant with the security criterion, i.e., able to circumvent the loss of up to K elements, the two lowermost problems return zero system power imbalance. In other words, the worst-case system power imbalance is equal to zero since the system power imbalance is also zero for every contingency.

2.2.2 Problem Formulation

A penalized version of the contingency-dependent model (2-1)-(2-14) can be formulated as follows:

$$\underset{\theta_b,\theta_b^k,f_l,f_l^k,p_i,p_i^k,r_i^D,r_i^U,v_i}{\text{Minimize}} \sum_{i\in I}\left(C_i^P(p_i,v_i)+C_i^U r_i^U + C_i^D r_i^D\right) + C^I \max_{k\in\mathcal{C}}\left\{\sum_{b\in N}\left(\Delta P_b^k + \Delta N_b^k\right)\right\} \quad (2\text{-}16)$$

subject to:

$$\sum_{i\in I_b} p_i + \sum_{l\in\mathcal{L}|to(l)=b} f_l - \sum_{l\in\mathcal{L}|fr(l)=b} f_l = D_b; \forall b\in N \quad (2\text{-}17)$$

$$f_l = \frac{1}{x_l}(\theta_{fr(l)} - \theta_{to(l)}); \forall l \in \mathcal{L} \quad (2\text{-}18)$$

$$-\overline{F}_l \le f_l \le \overline{F}_l; \forall l\in\mathcal{L} \quad (2\text{-}19)$$

$$\underline{P}_i v_i \le p_i \le \overline{P}_i v_i; \forall i \in I \quad (2\text{-}20)$$

$$p_i + r_i^U \le \overline{P}_i v_i; \forall i\in I \quad (2\text{-}21)$$

$$p_i - r_i^D \ge \underline{P}_i v_i; \forall i \in I \quad (2\text{-}22)$$

$$0 \le r_i^U \le \overline{R}_i^U v_i; \forall i\in I \quad (2\text{-}23)$$

$$0 \le r_i^D \le \overline{R}_i^D v_i; \forall i \in I \quad (2\text{-}24)$$

$$v_i \in \{0,1\}; \forall i\in I \quad (2\text{-}25)$$

$$\sum_{i\in I_b} p_i^k + \sum_{l\in\mathcal{L}|to(l)=b} f_l^k - \sum_{l\in\mathcal{L}|fr(l)=b} f_l^k + \Delta P_b^k - \Delta N_b^k = D_b; \forall b \in N, \quad \forall k \in \mathcal{C} \tag{2-26}$$

$$f_l^k = \frac{A_l^k}{x_l}\left(\theta_{fr(l)}^k - \theta_{to(l)}^k\right); \forall l \in \mathcal{L}, \forall k \in \mathcal{C} \tag{2-27}$$

$$-\overline{F}_l \leq f_l^k \leq \overline{F_l}; \forall l \in \mathcal{L}, \forall k \in \mathcal{C} \tag{2-28}$$

$$A_i^k(p_i - r_i^D) \leq p_i^k \leq A_i^k(p_i + r_i^U); \forall i \in I, \forall k \in \mathcal{C}. \tag{2-29}$$

It is worth mentioning that, in the optimal solution of (2-16)-(2-29), $\sum_{b\in N}\left(\Delta P_b^k + \Delta N_b^k\right)$ provides the minimal system imbalance under the post-contingency state k. Therefore, for a given pre-contingency schedule, p_i, r_i^D, r_i^U, the minimal system imbalance for each state k is a phase 1 optimization problem:

$$\delta^{k*}\left(\boldsymbol{p}, \boldsymbol{r^D}, \boldsymbol{r^U}\right) = \min_{\Delta N_b^k, \Delta P_b^k, \theta_l^k, f_l^k, p_i^k} \sum_{b\in N}\left(\Delta P_b^k + \Delta N_b^k\right) \tag{2-30}$$

subject to:

$$\sum_{i\in I_b} p_i^k + \sum_{l\in\mathcal{L}|to(l)=b} f_l^k - \sum_{l\in\mathcal{L}|fr(l)=b} f_l^k + \Delta P_b^k - \Delta N_b^k = D_b; \forall b \in N, \quad \forall k \in \mathcal{C} \tag{2-31}$$

$$f_l^k = \frac{A_l^k}{x_l}\left(\theta_{fr(l)}^k - \theta_{to(l)}^k\right); \forall l \in \mathcal{L}, \forall k \in \mathcal{C} \tag{2-32}$$

$$-\overline{F}_l \leq f_l^k \leq \overline{F_l}; \forall l \in \mathcal{L}, \forall k \in \mathcal{C} \tag{2-33}$$

$$A_i^k(p_i - r_i^D) \leq p_i^k \leq A_i^k(p_i + r_i^U); \forall i \in I, \forall k \in \mathcal{C}. \tag{2-34}$$

A parametrization of each contingency state k can be done by means of an availability vector $\boldsymbol{a}$. For instance, if we have a toy system composed of 3 generation units and 3 transmission lines and, under contingency state k, generator 2 is out of service, the corresponding vector $\boldsymbol{a}$ will be written as: $\boldsymbol{a} = \left[-a_i^G - | - a_l^L -\right]^T = \left[1, 0, 1|1, 1, 1\right]^T$. Therefore, $\delta^{k*}\left(\boldsymbol{p}, \boldsymbol{r^D}, \boldsymbol{r^U}\right)$ can be rewritten as $\delta^*\left(\boldsymbol{p}, \boldsymbol{r^D}, \boldsymbol{r^U}, \boldsymbol{a}\right)$. Within this context, for a given pre-contingency schedule, p_i, r_i^D, r_i^U, the worst case of power system imbalance has the following definition:

$$\Delta D^{wc}\left(\boldsymbol{p}, \boldsymbol{r^D}, \boldsymbol{r^U}\right) = \max_{k\in\mathcal{C}}\left\{\delta^{k*}\left(\boldsymbol{p}, \boldsymbol{r^D}, \boldsymbol{r^U}\right)\right\} = \max_{a\in\mathcal{A}}\left\{\delta^*\left(\boldsymbol{p}, \boldsymbol{r^D}, \boldsymbol{r^U}, \boldsymbol{a}\right)\right\}, \tag{2-35}$$

where, in the case of an $n-K$ joint generation and transmission security criterion,

$$\mathcal{A} = \left\{\boldsymbol{a} \in \{0,1\}^{|\mathcal{L}|+|I|} \middle| \sum_{l\in\mathcal{L}} a_l^L + \sum_{i\in I} a_i^G \geq n - K\right\}. \tag{2-36}$$

Finally, the robust trilevel counterpart for problem (2-1)-(2-14) is formulated as follows:

$$\underset{\substack{\Delta D^{wc},\theta_b,f_l,\\ p_i,r_i^D,r_i^U,v_i}}{\text{Minimize}} \sum_{i\in I}\big(C_i^P(p_i,v_i)+C_i^U r_i^U + C_i^D r_i^D\big) + C^I \Delta D^{wc} \tag{2-37}$$

subject to:

Pre-contingency scheduling constraints (2-2)-(2-10) (2-38)

$$\Delta D^{wc}(\boldsymbol{p},\boldsymbol{r^D},\boldsymbol{r^U}) = \max_{\delta^{wc},a_i^G,a_l^L} \Bigg\{ \delta^{wc} \tag{2-39}$$

subject to:

$$\boldsymbol{f}\big(\{a_i^G\}_{i\in I},\{a_l^L\}_{l\in\mathcal{L}}\big) \geq \boldsymbol{0} \tag{2-40}$$

$$a_i^G \in \{0,1\};\forall i \in I \tag{2-41}$$

$$a_l^L \in \{0,1\};\forall l \in \mathcal{L} \tag{2-42}$$

$$\delta^{wc}(\boldsymbol{p},\boldsymbol{r^D},\boldsymbol{r^U},\boldsymbol{a}) = \min_{\Delta N_b^{wc},\Delta P_b^{wc},\theta_b^{wc},f_l^{wc},p_i^{wc}} \Bigg[\sum_{b\in N}(\Delta N_b^{wc}+\Delta P_b^{wc}) \tag{2-43}$$

subject to:

$$\sum_{i\in I_b} p_i^{wc} + \sum_{l\in\mathcal{L}|to(l)=b} f_l^{wc} - \sum_{l\in\mathcal{L}|fr(l)=b} f_l^{wc} = D_b - \Delta N_b^{wc} + \Delta P_b^{wc} : (\beta_b);\forall b\in N \tag{2-44}$$

$$f_l^{wc} = \frac{a_l^L}{x_l}\big(\theta_{fr(l)}^{wc}-\theta_{to(l)}^{wc}\big) : (\omega_l);\forall l\in\mathcal{L} \tag{2-45}$$

$$-\overline{F}_l \leq f_l^{wc} \leq \overline{F}_l : (\pi_l,\sigma_l);\forall l\in\mathcal{L} \tag{2-46}$$

$$a_i^G(p_i - r_i^D) \leq p_i^{wc} \leq a_i^G(p_i + r_i^U) : (\gamma_i,\chi_i);\forall i\in I \tag{2-47}$$

$$\Delta N_b^{wc} \geq 0, \Delta P_b^{wc} \geq 0;\forall b\in N \Bigg]\Bigg\}. \tag{2-48}$$

Problem (2-37)-(2-48) comprises three optimization levels: (i) the upper level (2-37)-(2-38), which is associated with the pre-contingency schedule; (ii) the middle level (2-39)-(2-42), characterizing the worst-case contingency for the pre-contingency schedule; and (iii) the lower level (2-43)-(2-48), corresponding to the reaction of the system operator against the worst-case contingency. Dual variables associated with the lower-level problem are in parentheses. Note that the lower level is parameterized in terms of upper-level variables (p_i, r_i^D, r_i^U) and middle-level variables (a_i^G, a_l^L).

The objective of the upper-level problem is identical to that of the contingency-dependent model (2-1) except for the last term, which penalizes the system power imbalance. A sufficiently large value for C^I ensures the feasibility of the pre-contingency schedule, which requires the largest system power imbalance, due to the worst-case contingency, to be zero. The upper-level minimization is subject to the set of pre-contingency constraints (2-2)-(2-10).

The middle-level problem (2-39)-(2-42) determines the maximum system power imbalance by the definition of new binary decision variables a_i^G and a_l^L associated with the worst-case contingency. Constraint (2-40) imposes the security criterion, whereas constraints (2-41) and (2-42) respectively set the integrality of variables a_i^G and a_l^L. The feasibility space associated with those binary variables includes all contingencies characterizing the $n - K$ security criterion. It should be noted that constraints (2-40)-(2-42) define the combinatorial (discrete) uncertainty set, which is an extension of that used in [10] and [11] by also characterizing the availability of transmission lines.

In the lower-level problem (2-43)-(2-48), the reaction of the system operator is modeled by an optimal power flow where the system power imbalance is minimized (2-43). The system power imbalance is defined as the sum over all buses of the absolute value of nodal power balance violations. The absolute value is modeled in a linear fashion by two sets of nonnegative variables ΔN_b^{wc} and ΔP_b^{wc}. Network constraints include nodal power balances (2-44), line flows (2-45), and line flow limits (2-46). Constraints (2-47) set the generation limits considering the reserves allocated by the upper level. Finally, constraints (2-48) impose the nonnegativity of nodal power-imbalance variables. It is worth mentioning that the lower-level problem (2-43)-(2-48) is always feasible and provides the upper level with a non-zero penalty when the scheduled power and reserves lead to nodal balance violations under the worst-case contingency.

In addition, once the optimal solution to the robust problem (2-37)-(2-48) is obtained, the operation under each contingency can be straightforwardly obtained by solving the lower-level problem (2-43)-(2-48) for the optimal values of the upper-level variables, p_i^*, r_i^{D*}, and r_i^{U*}, and for the values of a_i^G and a_l^L characterizing the contingency under consideration.

2.2.3
Equivalence between the Trilevel Model and the Original Contingency-Dependent Formulation

Similar to the contingency-dependent model (2-1)-(2-14), the robust trilevel counterpart (2-37)-(2-48) accounts for all contingencies characterizing the $n-K$ security criterion. In contrast, problem (2-37)-(2-48) differs from the original contingency-dependent model (2-1)-(2-14) in two aspects: (i) power-imbalance terms are included, and (ii) the operation under each contingency is not explicitly modeled. As a consequence, the feasibility search spaces of both models are different. In the original model (2-1)-(2-14), power balance under both normal and contingency states is explicitly imposed and hence pre-contingency schedules satisfying the $n-K$ security criterion are only dealt with. On the other hand, pre-contingency schedules handled by problem (2-37)-(2-48) may violate the security criterion, thereby resulting in system power imbalance. In other words, the feasibility space of the trilevel model comprises pre-contingency schedules characterized by optimal solutions to the two lowermost optimization levels (2-39)-(2-48) with system power imbalance greater than 0 MW.

In mathematical programming [69], constraint violations are customarily accounted for by including a penalty function in the objective function. Here, the penalty function is the worst-case power-imbalance cost $C^I \Delta D^{wc}$. In this framework, constraints (2-11)-(2-14) are relaxed in the robust counterpart (2-37)-(2-48) and the worst-case violation, i.e., the largest system power imbalance among all contingency states, is penalized in the objective function (2-37). Thus, the equivalence between the trilevel model and the original contingency-dependent formulation is guaranteed by the selection of a sufficiently large value for the power-imbalance cost coefficient C^I, so that a distinction is made between solutions complying with the $n-K$ security criterion and solutions violating such criterion. In other words, for a suitable value of C^I, and assuming that the system is able to withstand all contingencies without nodal balance violations, the optimal solution to (2-37)-(2-48) is identical to that of (2-1)-(2-14) in terms of system cost. Therefore, both models determine the lowest system cost incurred to meet the pre-specified security criterion defining the contingency set $\mathcal{C}$ with no power-imbalance cost. When the system is unable to meet the security criterion, the original contingency-dependent problem is infeasible, whereas the robust counterpart flags such infeasibility by attaining an optimal solution with a power-imbalance cost greater than zero.

2.2.4 Illustrative Example

The performance of the proposed trilevel methodology under an $n-1$ security criterion is illustrated with the following example with two load pockets, denoted as LPA and LPB. The contingency set comprises two line outages, namely those associated with the tie lines into LPA and LPB, respectively. For notational consistency, a^L_{LPA} and a^L_{LPB} respectively represent the binary variables associated with the availability of those lines (1 if available and 0 if unavailable).

Let us assume that three possible pre-contingency schedules can be implemented. Schedule 1 circumvents the loss of the tie line into LPA but leads to system power imbalance for the LPB contingency due to the limited transmission capacity. Schedule 2 withstands the loss of the tie line into LPB but does not cover the outage of the tie line into LPA due to network limitations. Moreover, it is assumed that the system power imbalance associated with Schedule 2 is larger than that of Schedule 1. Finally, Schedule 3 is compliant with the $n-1$ security criterion and hence guards against the loss of any single tie line. Thus, the proposed trilevel model is expected to select Schedule 3 since both Schedules 1 and 2 are infeasible for the contingency-dependent model.

In order to show that the trilevel model works as expected, the three schedules are examined on an individual basis as follows:

1. For Schedule 1, the optimal solution to the two lowermost optimization levels (2-39)-(2-48) would be $a^L_{LPA} = 1$, $a^L_{LPB} = 0$, $\Delta D^{wc} = \delta^{wc} > 0$ MW. In other words, the worst-case contingency for the schedule guarding against the loss of the tie line into LPA is precisely the loss of the tie line into LPB. Therefore, Schedule 1 would yield a system power imbalance greater than 0 MW and the value of the objective function (2-37) is denoted as C1.

2. For Schedule 2, the optimal solution to the two lowermost optimization levels (2-39)-(2-48) would be $a^L_{LPA} = 0$, $a^L_{LPB} = 1$, $\Delta D^{wc} = \delta^{wc} > 0$ MW. In other words, the worst-case contingency for the schedule guarding against the loss of the tie line into LPB is precisely the loss of the tie line into LPA. Therefore, Schedule 2 would also lead to a system power imbalance greater than 0 MW, being C2 the corresponding value of the objective function (2-37). Under the aforementioned assumption on the severity of the system power imbalance associated with Schedules 1 and 2, C2 is greater than C1.

3. For Schedule 3, all feasible combinations of binary variables a^L_{LPA} and a^L_{LPB} would be the optimum to the two lowermost optimization levels (2-39)-(2-48), namely (i) $a^L_{LPA} = 1$, $a^L_{LPB} = 1$; (ii) $a^L_{LPA} = 1$, $a^L_{LPB} = 0$, and (iii) $a^L_{LPA} = 0$, $a^L_{LPB} = 1$. Note that all combinations yield a value of the system power imbalance ΔD^{wc} equal to 0 MW since Schedule 3 meets the $n-1$ security criterion. Therefore, any of those combinations would represent the worst-case contingency for Schedule 3. Furthermore, since $\Delta D^{wc} = 0$ MW, the power-imbalance cost of Schedule 3 is \$0 and the value of the objective function (2-37) is denoted as C3.

Therefore, the three schedules constitute feasible solutions for the trilevel model, being two actually infeasible for the original contingency-dependent model since they lead to system power imbalance. The choice of a sufficiently large value for C^I would yield the following relation among the values of the objective function (2-37) for the three schedules considered: $C3 << C1 < C2$. Since the trilevel model is a minimization problem, the optimal solution would be Schedule 3, as desired. Moreover, if there were additional schedules compliant with the security criterion, the same rationale would be applied and the trilevel model would select, among those with $\Delta D^{wc} = 0$ MW, the one with the least energy and reserve cost.

2.3 Solution Methodology

Problem (2-37)-(2-48) is a mixed-integer linear trilevel program. This class of multilevel optimization is a strongly NP-hard problem [43]. As will be explained later, ΔD^{wc} is a convex function of the upper-level variables p_i, r_i^D, and r_i^U. Therefore, it can be described by an outer approximation algorithm [70]. Here, we propose a Benders decomposition approach [71], referred to as BP, that comprises the iterative solution of a master problem and a subproblem. The master problem is an approximation of the original trilevel problem where in each iteration a cutting plane or Benders cut is added to locally characterize ΔD^{wc}. The subproblem is associated with the middle- and lower-level problems for specific values of the upper-level decision variables as determined by the previous master problem. In each iteration, the solution to the subproblem provides relevant information, such as the value of ΔD^{wc} and its subgradient, to generate an additional cutting plane for the master problem.

Next, we present the mathematical formulation of the subproblem and the master problem resulting from the application of Benders decomposition to problem (2-37)-(2-48). In addition, two sets of valid constraints are provided to improve the performance of the proposed procedure.

2.3.1 Subproblem

At each iteration j, the subproblem determines the worst-case contingency for the pre-contingency schedule for power and reserves identified by the previous master problem. Mathematically, the subproblem is a mixed-integer linear max-min problem comprising the two lowermost optimization levels (2-39)-(2-48) for given values of the upper-level decision variables $p_i^{(j)}$, $r_i^{D(j)}$, and $r_i^{U(j)}$. This particular instance of bilevel programming can be recast as an equivalent single-level mixed-integer linear problem suitable for efficient off-the-shelf software based on the branch-and-cut algorithm [72].

This transformation comprises two steps:

Step 1) Based on its linearity, the lower-level problem can be replaced by its dual. Thus, the original max-min subproblem is converted into a max-max problem. Moreover, since the same objective function is optimized at both levels of the original max-min problem, the strong duality theorem can be applied. As a consequence, the max-max problem becomes a single joint maximization problem in the coupled primal and dual spaces of the middle and lower levels, respectively. Hence, this step consists in replacing (i) the lower-level problem by its dual feasibility constraints, and (ii) the middle-level objective function by the dual lower-level objective function. For further details on this transformation, the interested reader is referred to [37] and the references therein.

The single-level equivalent is formulated as:

$$\Delta D^{wc} = \max_{\substack{\beta_b,\gamma_i,\pi_l,\sigma_l,\chi_i,\omega_l,\\ a_i^G,a_l^L}} \sum_{b\in N}\beta_b D_b - \sum_{l\in\mathcal{L}}\pi_l\overline{F}_l - \sum_{l\in\mathcal{L}}\sigma_l\overline{F}_l + \sum_{i\in I}\gamma_i a_i^G\big(p_i^{(j)} - r_i^{D(j)}\big) - \sum_{i\in I}\chi_i a_i^G\big(p_i^{(j)} + r_i^{U(j)}\big) \quad (2\text{-}49)$$

subject to:

$$\boldsymbol{f}\big(\{a_i^G\}_{i\in I},\{a_l^L\}_{l\in\mathcal{L}}\big) \geq \mathbf{0} \quad (2\text{-}50)$$

$$a_i^G \in \{0,1\}; \forall i \in I \quad (2\text{-}51)$$

$$a_l^L \in \{0,1\}; \forall l \in \mathcal{L} \quad (2\text{-}52)$$

$$\beta_b + \gamma_i - \chi_i \leq 0; \forall b \in N, \forall i \in I_b \quad (2\text{-}53)$$

$$\beta_{to(l)} - \beta_{fr(l)} + \omega_l + \pi_l - \sigma_l = 0; \forall l \in \mathcal{L} \quad (2\text{-}54)$$

$$-1 \leq \beta_b \leq 1; \forall b \in N \quad (2\text{-}55)$$

$$\sum_{l\in\mathcal{L}|fr(l)=b}\frac{\omega_l a_l^L}{x_l} - \sum_{l\in\mathcal{L}|to(l)=b}\frac{\omega_l a_l^L}{x_l} = 0; \forall b \in N \quad (2\text{-}56)$$

$$\pi_l \geq 0, \sigma_l \geq 0; \forall l \in \mathcal{L} \quad (2\text{-}57)$$

$$\gamma_i \geq 0, \chi_i \geq 0; \forall i \in I. \quad (2\text{-}58)$$

In (2-49), the worst-case system power imbalance ΔD^{wc} is determined by the maximization of the dual objective function of the lower-level problem (2-43)-(2-48). This optimization is subject to constraints (2-50)-(2-52), which are respectively the same as (2-40)-(2-42); and to constraints (2-53)-(2-58), which are the dual feasibility constraints of the lower-level problem.

Step 2) The resulting single-level equivalent is a mixed-integer nonlinear programming problem. Nonlinearities arise in (2-49) and (2-56) due to the products between middle-level binary variables and lower-level dual variables. However, those bilinear terms can be recast into linear expressions using well-known algebra results [73]. The formulation of the resulting mixed-integer linear subproblem at iteration j is as follows:

$$\Delta D^{wc} = \max_{\substack{\beta_b,\gamma_i,\pi_l,\sigma_l,\chi_i,\omega_l,\\ a_i^G,a_l^L,h_i,y_l,z_i}} \sum_{b\in N}\beta_b D_b - \sum_{l\in\mathcal{L}}\pi_l\overline{F}_l - \sum_{l\in\mathcal{L}}\sigma_l\overline{F}_l + \sum_{i\in I} z_i\big(p_i^{(j)} - r_i^{D(j)}\big) - \sum_{i\in I} h_i\big(p_i^{(j)} + r_i^{U(j)}\big) \quad (2\text{-}59)$$

subject to:

$$\boldsymbol{f}\big(\{a_i^G\}_{i\in I},\{a_l^L\}_{l\in\mathcal{L}}\big) \geq \mathbf{0} \quad (2\text{-}60)$$

$$a_i^G \in \{0,1\}; \forall i \in I \quad (2\text{-}61)$$

$$a_l^L \in \{0,1\}; \forall l \in \mathcal{L} \quad (2\text{-}62)$$

$$\beta_b + \gamma_i - \chi_i \leq 0; \forall b \in N, \forall i \in I_b \quad (2\text{-}63)$$

$$\beta_{to(l)} - \beta_{fr(l)} + \omega_l + \pi_l - \sigma_l = 0; \forall l \in \mathcal{L} \quad (2\text{-}64)$$

$$-1 \leq \beta_b \leq 1; \forall b \in N \quad (2\text{-}65)$$

$$\sum_{l\in\mathcal{L}|fr(l)=b}\frac{y_l}{x_l} - \sum_{l\in\mathcal{L}|to(l)=b}\frac{y_l}{x_l} = 0; \forall b \in N \quad (2\text{-}66)$$

$$-\big(1-a_l^L\big)\overline{\omega}_l \leq \omega_l - y_l \leq \big(1-a_l^L\big)\overline{\omega}_l; \forall l \in \mathcal{L} \quad (2\text{-}67)$$

$$-a_l^L\overline{\omega}_l \leq y_l \leq a_l^L\overline{\omega}_l; \forall l \in \mathcal{L} \quad (2\text{-}68)$$

$$0 \leq \gamma_i - z_i \leq \big(1-a_i^G\big)\overline{\gamma}_i; \forall i \in I \quad (2\text{-}69)$$

$$0 \leq z_i \leq \overline{\gamma}_i a_i^G; \forall i \in I \quad (2\text{-}70)$$

$$0 \leq \chi_i - h_i \leq \big(1-a_i^G\big)\overline{\chi}_i; \forall i \in I \quad (2\text{-}71)$$

$$0 \leq h_i \leq \overline{\chi}_i a_i^G; \forall i \in I, \quad (2\text{-}72)$$

where h_i, y_l, and z_i are new variables representing the bilinear terms of (2-49) and (2-56): $h_i = \chi_i a_i^G$, $y_l = \omega_l a_l^L$, and $z_i = \gamma_i a_i^G$. Parameters $\overline{\gamma}_i$, $\overline{\chi}_i$, and $\overline{\omega}_l$ respectively represent the bounds for γ_i, χ_i, and ω_l. Since the lower level is always a feasible problem, the values of such parameters may be set based on sensitivity analysis. Note that modifying the right-hand side of (2-45) by an infinitesimal factor, the largest change in the lower-level objective function (2-43) is limited to such factor multiplied by 2. This occurs because every flow variable f_l^{wc} appears in two nodal power balance constraints respectively corresponding to the sending and receiving buses. Similarly, by perturbing (2-47), the largest change in the lower-level objective function (2-43) is limited to the magnitude of such perturbation. Therefore, the upper bounds for the dual variables associated with (2-45) and (2-47) can be set to $\overline{\omega}_l = 2$ and $\overline{\gamma}_i = \overline{\chi}_i = 1$.

Expressions (2-59)-(2-66) are respectively equivalent to (2-49)-(2-56) whereas constraints (2-67)-(2-72) represent the linearization of the bilinear products. It should be noted that, in terms of the upper-level variables, ΔD^{wc} is the maximum of affine functions within the middle-level feasibility set. Therefore, it is a convex function of the upper-level decision variables (see [74], item "3.2.3 Pointwise maximum and supremum", for a proof).

2.3.2 Master Problem

The master problem at iteration j is:

$$\underset{\substack{\alpha,\theta_b,f_l,\\ p_i,r_i^D,r_i^U,v_i}}{\text{Minimize}} \sum_{i\in I}\left(C_i^P(p_i,v_i)+C_i^U r_i^U + C_i^D r_i^D\right)+C^I\alpha \tag{2-73}$$

subject to:

$$\text{Pre-contingency feasibility constraints (2-2)-(2-10)} \tag{2-74}$$

$$\alpha \geq \Delta D^{wc(m)} + \sum_{i\in I}\Big[\big(p_i - p_i^{(m)}\big)\big(z_i^{(m)} - h_i^{(m)}\big) + \big(r_i^D - r_i^{D(m)}\big)\big(-z_i^{(m)}\big) + \big(r_i^U - r_i^{U(m)}\big)\big(-h_i^{(m)}\big)\Big]; m = 1,\ldots,j-1 \tag{2-75}$$

$$\alpha \geq 0. \tag{2-76}$$

The objective function (2-73) corresponds to (2-37), where variable α represents the approximation of ΔD^{wc}, and expressions (2-74) are identical to (2-38). At each iteration, the search space is restricted by adding a Benders cut (2-75). $\Delta D^{wc(m)}$ is obtained from the optimal solution to the subproblem (2-59)-(2-72) at iteration m for given values of the upper-level decision variables $p_i^{(m)}$, $r_i^{D(m)}$, and $r_i^{U(m)}$. In addition, coefficients $\big(z_i^{(m)} - h_i^{(m)}\big)$, $\big(-z_i^{(m)}\big)$, and $\big(-h_i^{(m)}\big)$ represent the partial subgradients of $\Delta D^{wc(m)}$ that can be derived from (2-59). Finally, constraint (2-76) sets the nonnegativity of α.

Alternatively, instead of penalizing ΔD^{wc} in (2-37), a constraint imposing $\Delta D^{wc} \leq 0$ could be added to the set of constraints (2-38). In this case, the expression (2-73) would not account for the term $C^I\alpha$. In addition, the left-hand side of (2-75) would be replaced by 0 and such expression would represent feasibility cuts instead of optimality cuts. Computationally, such modification would not be appealing, since the consideration of $C^I\alpha$ in the objective function of the master problem provides the algorithm (further described in subsection 2.3.4) with an interesting gradient to find the optimal solution within the search space.

Another important feature here is the possibility of regarding different values for the security parameter K in the same master problem. In addition, it is also possible to associate an acceptable amount of system power imbalance to each considered value of K. Let W be the set of indexes of security parameters. Then, in order to ensure that the worst case of system imbalance will not be larger than an maximum level of system power imbalance $\overline{\Delta}_w$, under a security criterion $n - K_w$, for all $w \in W$, the master problem should be formulated as:

$$\underset{\substack{\alpha_w,\theta_b,f_l,\\ p_i,r_i^D,r_i^U,v_i}}{\text{Minimize}} \sum_{i \in I}\left(C_i^P(p_i,v_i) + C_i^U r_i^U + C_i^D r_i^D\right) + \sum_{w \in W} C_w^I \alpha_w \quad (2\text{-}77)$$

subject to:

$$\text{Pre-contingency feasibility constraints (2-2)-(2-10)} \quad (2\text{-}78)$$

$$\alpha_w \geq \Delta D_w^{wc(m)} + \sum_{i \in I}\Big[\left(p_i - p_i^{(m)}\right)\left(z_{w,i}^{(m)} - h_{w,i}^{(m)}\right) + \left(r_i^D - r_i^{D(m)}\right)\left(-z_{w,i}^{(m)}\right) + \left(r_i^U - r_i^{U(m)}\right)\left(-h_{w,i}^{(m)}\right)\Big]; w \in W, m = 1,\ldots,j-1 \quad (2\text{-}79)$$

$$0 \leq \alpha_w \leq \overline{\Delta}_w; w \in W. \quad (2\text{-}80)$$

Despite of these considerations, expressions (2-73)–(2-76) are considered in this Chapter the core of the master problem, further enhanced by means of valid constraints in the next subsection.

2.3.3 Valid Constraints

In the proposed Benders decomposition approach, the master problem implements a cutting-plane approximation of function ΔD^{wc}, that is iteratively improved. In addition, it should be noted that reserves are penalized at the objective function of the master problem through their respective cost rates. As a consequence, the first iterations of BP are prone to yield solutions with no scheduled reserves, i.e., infeasible solutions that would lead to nodal power imbalances and thereby violate the security criterion.

Based on the findings of [11] and [4], two sets of valid constraints can be added to the master problem. These constraints provide a tighter formulation that avoids dealing with infeasible solutions, i.e., the search space is narrowed without removing the optimal solution. Thus, the performance of the proposed BP is improved.

Generation outage constraints:

In [11], the $n - K$ contingency-constrained problem was addressed by considering only generator outages in a single-bus system. Based on robust optimization theory, a set of linear inequalities (expressions (9.7)-(9.10) of [11]) equivalently represent the effect of the two lowermost optimization levels (2-39)-(2-48) considered here for the case of a single-bus setting, regarding only generator outages. The objective here is twofold: (i) to derive the a set of valid constraints as a relaxation of the objective function ΔD^{wc} of the two lowermost levels and (ii) to show that the consideration of such constraints in the Benders master problem produces a tighter formulation. In the following three steps, (i) is derived. Afterwards, (ii) is addressed.

Step 1) Starting by the second-level, the set of constraints that define the joint GT criteria can be found as a particularization of (2-40)-(2-42). For the joint GT $n-K$ criterion we have:

$$\sum_{i \in I} a_i^G + \sum_{l \in \mathcal{L}} a_l^L \geq |I| + |\mathcal{L}| - K \tag{2-81}$$

$$a_i^G \in \{0,1\}; \forall i \in I \tag{2-82}$$

$$a_l^L \in \{0,1\}; \forall l \in \mathcal{L}, \tag{2-83}$$

and, for the $n - K^G - K^L$,

$$\sum_{i \in I} a_i^G \geq |I| - K^G \tag{2-84}$$

$$\sum_{l \in \mathcal{L}} a_l^L \geq |\mathcal{L}| - K^L \tag{2-85}$$

$$a_i^G \in \{0,1\}; \forall i \in I \tag{2-86}$$

$$a_l^L \in \{0,1\}; \forall l \in \mathcal{L}. \tag{2-87}$$

In both cases, the constrained version can be found by fixing $a_l^L = 1, \forall l \in \mathcal{L}$. In both criteria, it implies that only generators contingencies are considered for the same K and K^G. Therefore, the constrained second-level produces a lower bound for the load shed. Under such constraints, expression (2-81) simplifies to (2-84), by letting $K^G = K$, and expression (2-85) can be dropped. In both criteria, lines-availability variables can also be dropped. Under this setting, the remaining expressions result in a criterion in which only generators contingencies are considered:

$$\sum_{i \in I} a_i^G \geq |I| - K^G \tag{2-88}$$

$$a_i^G \in \{0,1\}; \forall i \in I \tag{2-89}$$

These constraints are the same used to characterized the $n-K$ generation criterion in [11].

Step 2) Moving on to the third-level problem (2-43)-(2-48), a relaxed version of such network-constrained problem can be found by dropping network constraints (2-45), (2-46), dropping the left-hand-side bound of (2-47), and replacing (2-44) by its summation. This leads to the following single-node dispatch model:

$$\delta^{wc.1bus} = \min_{\Delta D^{wc}, p_i^{wc}} \quad \Delta D^{wc} \tag{2-90}$$

subject to:

$$\sum_{i \in I} p_i^{wc} = \sum_{b \in N} D_b - \Delta D^{wc} \tag{2-91}$$

$$p_i^{wc} \leq a_i^G \big(p_i + r_i^U\big); \forall i \in I \tag{2-92}$$

$$\Delta D^{wc} \geq 0. \tag{2-93}$$

In (2-90)-(2-93), ΔD^{wc} represents the total system load shed variable. This model aims to minimize the total system load shed within the available pre-contingency schedule of energy and up-spinning reserves. Since (2-90)-(2-93) is a single-bus relaxed version of (2-43)-(2-48), it is easy to see that $\delta^{wc.1bus} \leq \delta^{wc}$.

According to expression (2-91), the objective function (2-90) can be replaced by $(\sum_{b \in N} D_b - \sum_{i \in I} p_i^{wc})^+$, where $(\cdot)^+ = max\{0, \cdot\}$. This let us to drop ΔD^{wc} and rewrite the model as follows:

$$\delta^{wc.1bus} = \min_{p_i^{wc}} \left(\sum_{b \in N} D_b - \sum_{i \in I} p_i^{wc} \right)^+ \tag{2-94}$$

subject to:

$$p_i^{wc} \leq a_i^G (p_i + r_i^U); \forall i \in I. \tag{2-95}$$

According to (2-94) and (2-95), $\delta^{wc.1bus}$ will be zero whenever a sufficient capacity of energy and up reserves is available to meet the total system load. Otherwise, it will assume the value of the system capacity deficit. This rationale conduces to the following closed form:

$$\delta^{wc.1bus} = \left(\sum_{b \in N} D_b - \sum_{i \in I} a_i^G (p_i + r_i^U) \right)^+. \tag{2-96}$$

Step 3) In order to achieve the valid constraints formulas, the single-node system load shed $\delta^{wc.1bus}$ must be coupled into the objective function of the constrained version of the second-level problem. This results in the following model:

$$\underline{\Delta D}^{wc} = \max_{a_i^G} \left(\sum_{b \in N} D_b - \sum_{i \in I} a_i^G (p_i + r_i^U) \right)^+ \tag{2-97}$$

subject to:

$$\sum_{i \in I} a_i^G \geq |I| - K^G \tag{2-98}$$

$$a_i^G \in \{0, 1\}; \forall i \in I. \tag{2-99}$$

The optimal value $\underline{\Delta D}^{wc}$ constitutes a lower bound for the original worst-case load shed function (2-39) since it is built as a maximum of a lower objective function ($\delta^{wc.1bus} \leq \delta^{wc}$) within a constrained version of the original set. Hence, the set of pre-contingency schedules, $\{p_i, r_i^U, r_i^D\}_{i\in I}$, for which $\underline{\Delta D}^{wc} \leq 0$ contains the set of schedules that satisfies the security criterion, i.e., that satisfies $\Delta D^{wc} \leq 0$.

According to (2-97)-(2-99), $\underline{\Delta D}^{wc}$ meets zero whenever the minimum system available capacity, given by the second term of the objective function (2-97), exceeds the total system load. Therefore, the set of schedules that satisfies (2-100)-(2-104) is the same as the set that satisfies $\underline{\Delta D}^{wc} \leq 0$. The following set of constraints is the second-level problem of the bilevel program proposed in [11] to model the generation $n - K$ security criterion.

$$D^* \geq \sum_{b\in N} D_b \tag{2-100}$$

$$D^* = \min_{a_i^G} \sum_{i\in I} a_i^G(p_i + r_i^U) \tag{2-101}$$

subject to:

$$\sum_{i\in I} a_i^G \geq |I| - K^G : (\lambda) \tag{2-102}$$

$$a_i^G \leq 1 : (\xi_i); \forall i \in I \tag{2-103}$$

$$a_i^G \in \{0, 1\}. \tag{2-104}$$

Due to the unimodular matrix structure of (2-102)-(2-103), (2-101)-(2-104) can be replaced by its linear relaxation. The recast of (2-100)-(2-104) to the valid constraints (2-105)-(2-108) is based on week-duality that holds for the linear relaxation of (2-101)-(2-104). Replacing D^* in (2-100) by the dual objective function of the linear relaxation of (2-101)-(2-104) and replacing (2-101)-(2-104) by the dual feasibility constraints of its linear relaxation, (2-105)-(2-108) are found (for an interested reader we refer to [11] and references therein). As a consequence, the set of pre-contingency schedules that satisfies (2-105)-(2-108), hereinafter referred to as generator outage constraints, contains the set of schedules that satisfies $\Delta D^{wc} \leq 0$.

$$\big(|I| - K^G\big)\lambda - \sum_{i\in I} \xi_i \geq \sum_{b\in N} D_b \tag{2-105}$$

$$\lambda - \xi_i \leq p_i + r_i^U; \forall i \in I \tag{2-106}$$

$$\xi_i \geq 0; \forall i \in I \tag{2-107}$$

$$\lambda \geq 0, \tag{2-108}$$

where λ and ξ_i are dual variables of the lower-level problem defining the worst-case generation outage in [11].

Finally, it should be noted that the initial step of the BP consists in the solution of the master problem with no cuts to approximate ΔD^{wc}. This first master problem minimizes the cost of energy and reserves, but has no information on contingencies and their costs, which are approximated by Benders cuts in subsequent iterations. Therefore, the optimal solution to this first master problem typically yields $\alpha^* = 0$ and does not schedule reserves. However, this solution is unable to meet the security criterion and in general will be far from the optimal solution, which is characterized by non-zero reserve contributions. The incorporation of the valid constraints in the master problem cuts lots of pre-contingency schedules that do not commit sufficient up reserves to meet the $n - K^G$ single-node security criterion. This can be easily understood by means of (2-100)-(2-104): since a feasible schedule accomplishes $\sum_{i \in I} p_i = \sum_{b \in N} D_b$, if no up reserves are scheduled, the loss of any online generator with $p_i > 0$ makes (2-100) infeasible. As a conclusion, the incorporation of the proposed valid constraints (2-105)-(2-108) in the Benders master problem (2-73)-(2-76) provides a tighter formulation and considerably improves convergence, as backed by numerical simulations.

Redispatch constraints:

According to [4], the convergence of BP can be accelerated by cutting off the infeasible schedules identified by the subproblem along the iterative process. Thus, at each iteration j, the following redispatch constraints are added to the master problem:

$$\sum_{i \in I_b} p_i^m + \sum_{l \in \mathcal{L} | to(l) = b} f_l^m - \sum_{l \in \mathcal{L} | fr(l) = b} f_l^m = D_b; \forall b \in N, m = 1, \dots, j-1 \quad (2\text{-}109)$$

$$f_l^m = \frac{a_l^{L(m)}}{x_l} \big(\theta_{fr(l)}^m - \theta_{to(l)}^m\big); \forall l \in \mathcal{L}, m = 1, \dots, j-1 \quad (2\text{-}110)$$

$$-\overline{F}_l \leq f_l^m \leq \overline{F}_l; \forall l \in \mathcal{L}, m = 1, \dots, j-1 \quad (2\text{-}111)$$

$$a_i^{G(m)} \big(p_i - r_i^D\big) \leq p_i^m \leq a_i^{G(m)} \big(p_i + r_i^U\big); \forall i \in I, m = 1, \dots, j-1, \quad (2\text{-}112)$$

where f_l^m, p_i^m, and θ_b^m constitute decision variables of the tight master problem. These variables model the operation under contingency m as identified by the subproblem at that iteration through $a_i^{G(m)}$ and $a_l^{L(m)}$. Constraints (2-109)-(2-112) respectively correspond to post-contingency redispatch constraints (2-11)-(2-14), where A_i^k and A_l^k are replaced by $a_i^{G(m)}$ and $a_l^{L(m)}$, respectively.

2.3.4 Algorithm

The proposed methodology works as follows:

1. *Initialization.*

   – Initialize the iteration counter: $j \leftarrow 1$;
   – Solve the master problem without cuts. This step provides $p_i^{(1)}$, $r_i^{D(1)}$, $r_i^{U(1)}$, $\alpha^{(1)}$, and a lower bound for the optimal cost $LB = \sum_{i \in I} \big(C_i^P \big(p_i^{(1)}, v_i^{(1)} \big) + C_i^U r_i^{U(1)} + C_i^D r_i^{D(1)} \big)$.

2. *Subproblem solution.* Solve the subproblem for the given $p_i^{(j)}$, $r_i^{D(j)}$, and $r_i^{U(j)}$. This step provides $z_i^{(j)}$, $h_i^{(j)}$, $\Delta D^{wc(j)}$, and an upper bound for the optimal cost $UB = \sum_{i \in I} \big(C_i^P \big(p_i^{(j)}, v_i^{(j)} \big) + C_i^U r_i^{U(j)} + C_i^D r_i^{D(j)} \big) + C^I \Delta D^{wc(j)}$.

3. *Iteration counter updating.* Increase the iteration counter: $j \leftarrow j + 1$.

4. *Master problem solution.* Solve the full master problem. This step provides $p_i^{(j)}$, $r_i^{D(j)}$, $r_i^{U(j)}$, $\alpha^{(j)}$, and a lower bound for the optimal cost $LB = \sum_{i \in I} \big(C_i^P \big(p_i^{(j)}, v_i^{(j)} \big) + C_i^U r_i^{U(j)} + C_i^D r_i^{D(j)} \big) + C^I \alpha^{(j)}$.

5. *Convergence checking.* If a solution with a level of accuracy ϵ has been found, i.e., $\frac{(UB - LB)}{LB} \leq \epsilon$, then stop; otherwise go to step 2.

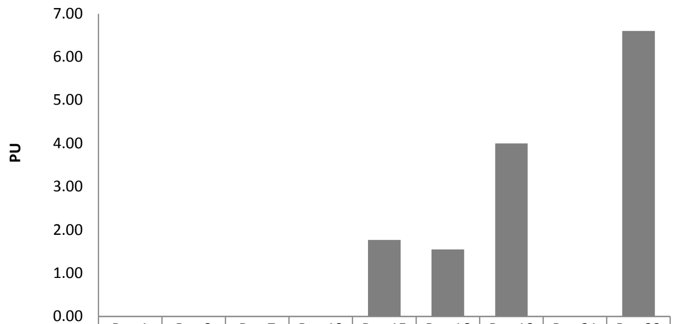
Since ΔD^{wc} is a convex function of the upper-level variables p_i, r_i^D, and r_i^U, and the master problem is a mixed-integer linear program, BP finitely converges to optimality. In addition, the upper and lower bounds provide a measure of the distance to the optimum.

2.4 Case Studies

This section presents results from two test cases based on the 24-bus IEEE Reliability Test System (RTS) [75] and the IEEE 118-bus system [76], respectively. For the sake of simplicity, generators offer linear cost functions of the form $C_i^P(p_i, v_i) = C_i^f v_i + C_i^v p_i$. For all simulations, C^I was set equal to $\$10^6$/MWh. The model has been implemented on an Amazon virtual machine [77] with 32 Intel Xeon Cloud Computing, 2.63-GHz processors with 60.5 GB of RAM, using Xpress-MP 7.2 under MOSEL [72].

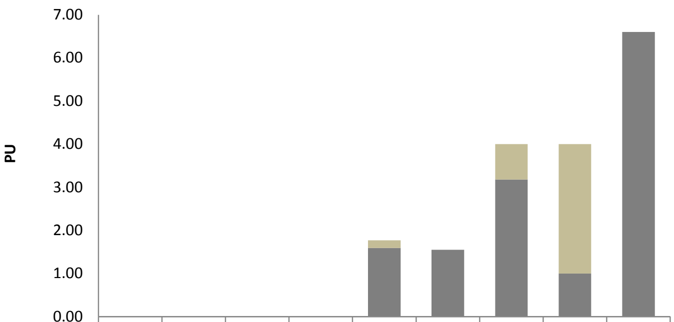
2.4.1 RTS-Based Case

This case study illustrates the performance of BP under an $n - K$ security criterion. The 24-bus IEEE Reliability Test System [75] comprises 26 generators and 38 transmission assets. The data for the generators can be found in [19]. Coefficients C_i^f and C_i^v respectively correspond to the intercept and the linear coefficient of the cost function provided in [19]. The load profile corresponds to Monday of week 48 at 3:00 a.m. The resilience of the system against multiple contingencies is increased by adding three circuits in line 7-8, and one circuit in lines 1-2, 1-3, 1-5, 2-4, 2-6, 3-9, 3-24, 4-9, 5-10, 6-10, 8-9, 8-10, 11-14, 12-23, 13-23, 14-16, 15-16, 15-24, 16-17, and 16-19. As a consequence, the system is able to be operated under the $n - 3$ security criterion.



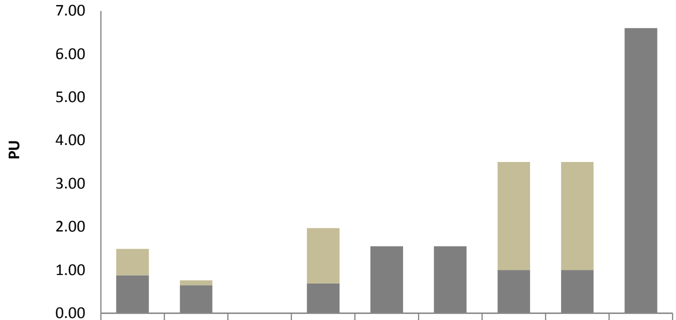
	Bus 1	Bus 2	Bus 7	Bus 13	Bus 15	Bus 16	Bus 18	Bus 21	Bus 23
Down-spinning reserves	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Up-spinning reserves	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Power	0.00	0.00	0.00	0.00	1.77	1.55	4.00	0.00	6.60

Figure 2.2: Energy and reserves scheduled for each bus with generation units, considering $K = 0$.



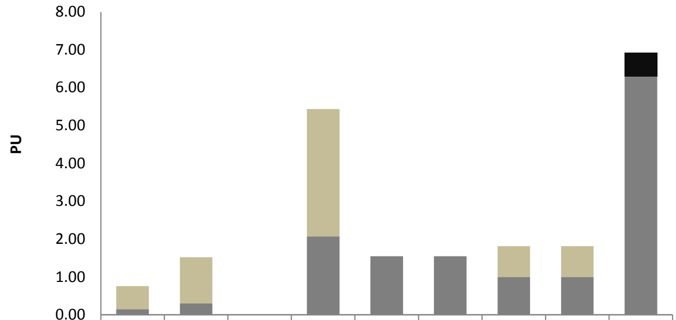
	Bus 1	Bus 2	Bus 7	Bus 13	Bus 15	Bus 16	Bus 18	Bus 21	Bus 23
Down-spinning reserves	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Up-spinning reserves	0.00	0.00	0.00	0.00	0.18	0.00	0.82	3.00	0.00
Power	0.00	0.00	0.00	0.00	1.59	1.55	3.18	1.00	6.60

Figure 2.3: Energy and reserves scheduled for each bus with generation units, considering $K = 1$.



	Bus 1	Bus 2	Bus 7	Bus 13	Bus 15	Bus 16	Bus 18	Bus 21	Bus 23
Down-spinning reserves	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Up-spinning reserves	0.61	0.11	0.00	1.28	0.00	0.00	2.50	2.50	0.00
Power	0.88	0.65	0.00	0.69	1.55	1.55	1.00	1.00	6.60

Figure 2.4: Energy and reserves scheduled for each bus with generation units, considering $K = 2$.



	Bus 1	Bus 2	Bus 7	Bus 13	Bus 15	Bus 16	Bus 18	Bus 21	Bus 23
Down-spinning reserves	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.63
Up-spinning reserves	0.61	1.22	0.00	3.37	0.00	0.00	0.81	0.81	0.00
Power	0.15	0.30	0.00	2.07	1.55	1.55	1.00	1.00	6.30

Figure 2.5: Energy and reserves scheduled for each bus with generation units, considering $K = 3$.

This case study has been solved by five approaches: (i) the mixed-integer linear contingency-dependent model (2-1)-(2-14), referred to as CD; (ii) the original adjustable robust optimization approach without valid constraints, denoted as O-BP; (iii) the tight robust method with generation outage constraints only, labeled as T(G)-BP; (iv) the tight robust approach with redispatch constraints only, denoted as T(R)-BP; and (v) the tight robust technique with both sets of valid constraints, referred to as T-BP. Tables 2.1 and 2.2 summarize the results obtained for different values of the security parameter K ranging between 0 and 5. For all simulations, the level of accuracy ϵ was set at 10^{-3}. Given the huge number of constraints that have to be explicitly considered in CD, a time limit of 4 h (14400 s) was set for the execution of Xpress.

Figures 2.2, 2.3, 2.4, and 2.5 display the allocation of scheduled energy and reserves among the buses with generation units. Such allocation, as can be seen, becomes more spread over the mentioned buses as the security parameter K is increased. This pattern is coherent, since one of the objectives of our methodology is to ensure deliverability of energy and reserves considering network constraints as well as outages of transmission assets.

Table 2.1: RTS-Based Case: System Costs ($)

K	CD	O-BP	T(G)-BP	T(R)-BP	T-BP
0	2068020	2068020	2068020	2068020	2068020
1	2688760	2688760	2688760	2688760	2688760
2	3751680	3751680	3751680	3751680	3751680
3	Time exceeded	5232740	5232740	5232740	5232740
4	Out of memory	Infeasible	Infeasible	Infeasible	Infeasible
5	Out of memory	Infeasible	Infeasible	Infeasible	Infeasible

Table 2.2: RTS-Based Case: Computing Times (s)

K	CD	O-BP	T(G)-BP	T(R)-BP	T-BP
0	0.30	0.33	0.33	0.58	0.34
1	3.60	1.36	0.72	1.75	0.75
2	334.78	4.96	1.70	5.76	1.79
3	14400.00	48.33	27.14	28.88	16.77
4	Out of memory	371.75	384.37	4.85	1.31
5	Out of memory	1816.71	10.95	4.59	2.45

Table 2.1 provides information on the quality of the solutions attained by the proposed adjustable robust approaches in terms of system cost. As can be seen, all methods achieved the same optimal solution identified by CD for values of K up to 2. For an $n-3$ security criterion, CD was unable to find a feasible solution within the pre-specified 4-h time limit. In contrast, the adjustable robust models attained a feasible solution meeting the $n-3$ security criterion. As expected, tighter security criteria yield higher system costs. For this case study, imposing an $n-3$ security criterion incurs a 39.5% cost increase over the operation under an $n-2$ security criterion. Tighter security criteria than $n-3$ led to intractable contingency-dependent models that ran out of memory. On the other hand, the adjustable robust approaches converged to infeasible solutions resulting in power imbalances. In other words, the adjustable robust models were able to identify that the system is unable to be operated under such tight security criteria. These results provide the system operator with valuable information on the ability of the power system to withstand multiple contingencies.

Table 2.2 presents the computational results for all methods. As above mentioned, the computational burden of CD is prohibitive for more than 2 simultaneous out-of-service components, whereas the adjustable robust approaches converge in moderate computing times. Even for the conventional $n-1$ and $n-2$ security criteria, the robust methods also outperform CD. These results clearly back the superiority of the adjustable robust approaches over the contingency-dependent formulation from a computational viewpoint. Moreover, the results shown in Table 2.2 highlight the computational advantage of jointly considering both sets of valid constraints included in T-BP. While these constraints do not affect the quality of the solution in terms of system cost (Table 2.1), they yield large reductions in computing time with respect to O-BP. As can be observed, computing time reductions are particularly significant for tighter security criteria. For these particular cases, redispatch constraints are more effective than generation outage constraints when considered separately.

2.4.2
IEEE 118-Bus System

This case study shows the behavior of BP under an $n-K^G-K^L$ security criterion. The IEEE 118-bus system consists of 54 thermal generators and 186 transmission lines [76]. Coefficients C_i^f and C_i^v respectively correspond to the intercept and the linear coefficient of the cost function provided in [76]. Nodal peak load data were obtained from [78] and were modulated with the same factors presented in [75]. The load profile corresponds to Monday of week 48 at 10:00 p.m. reduced by 50%. Similar to the RTS-based case, an additional circuit was considered in lines 9-10, 12-117, 68-116, 71-73, 85-86, 86-87, 110-111, and 110-112. Moreover, generator 5 was also duplicated.

Table 2.3 compares the performance of T-BP and CD for different values of K^G and K^L and a level of accuracy ϵ equal to 10^{-2}. As can be seen, T-BP attained either the optimum or an ϵ-optimal solution in reasonable times for all cases but one. The $n-5-1$ criterion resulted in the most challenging case from a computational perspective given the vast feasible search space to be explored. For this case, the optimality gap could only be reduced down to 2.4% after 7897.73 s. Note also that T-BP converged to infeasible solutions leading to power imbalances for all cases with $K^L=2$. In other words, T-BP allowed identifying that the system is unable to withstand the loss of more than one transmission line.

In contrast, CD attained the optimal solution in only 4 out of 18 cases. It is worth mentioning that CD could not be loaded into the computer memory

Table 2.3: Results for the IEEE 118-Bus System

		T-BP		CD	
K^G	K^L	System Cost ($)	Time (s)	System Cost ($)	Time (s)
0	0	12826.6	0.17	12826.6	0.81
0	1	12826.6	2.31	12826.6	115.06
0	2	Infeasible	6.55	-	14400.00
1	0	15450.7	0.47	15450.7	15.77
1	1	15643.4	50.91	-	14400.00
1	2	Infeasible	14.13	-	Out of memory
2	0	17507.4	0.61	17507.4	8070.47
2	1	17641.6	276.66	-	Out of memory
2	2	Infeasible	8.42	-	Out of memory
3	0	18503.7	1.05	-	14400.00
3	1	18503.7	38.60	-	Out of memory
3	2	Infeasible	15.37	-	Out of memory
4	0	19356.5	1.22	-	Out of memory
4	1	19881.2	453.62	-	Out of memory
4	2	Infeasible	43.04	-	Out of memory
5	0	20343.5	2.17	-	Out of memory
5	1	21520.5#	7897.73	-	Out of memory
5	2	Infeasible	96.47	-	Out of memory

Optimality gap = 2.4%

when considering security criteria with $K^G + K^L > 2$. Moreover, the computing times required by CD to attain optimality were significantly larger than those of T-BP. These results also substantiate the superior performance of T-BP over the contingency-dependent formulation.

3
Energy and Reserve Scheduling under Correlated Nodal Demand Uncertainty: An Adjustable Robust Optimization Approach

Chapter 3 presents a nonparametric approach based on adjustable robust optimization to consider correlated nodal demand uncertainty in a joint energy and reserve scheduling model with security constraints. In this model, up- and down-spinning reserves provided by generators are endogenously defined as a result of the optimization problem. Adjustable robust optimization is used to characterize the worst-case load variation under a given user-defined uncertainty set. This approach differs from recent previous work in two respects: (i) nonparametric correlations between nodal demands are accounted for in the uncertainty set, and (ii) based on the binary expansion linearization approach, a mixed-integer linear model is provided for the optimization related to the worst-case demand. The resulting problem is formulated as a trilevel program and solved by means of Benders decomposition. Empirical results suggest that the incorporation of nodal correlations can be effectively captured by the robust scheduling model. The methodology developed in this Chapter can be also used to account for correlated generation uncertainty, mainly associated with renewable energy sources. The contents of this Chapter are partially related to a paper presented at the *18th Power Systems Computation Conference, PSCC'14* [79].

The remainder of this Chapter is organized as follows. Section 3.1 presents the problem formulation. Section 3.2 describes the proposed solution methodology. Finally, in Section 3.3, two case studies are analyzed.

3.1
Problem Formulation

The joint energy and reserve scheduling problem addressed in this Chapter determines the optimal generation schedule and reserve allocation so that the uncertain power demand is supplied under both normal and contingency states. Unlike [44] and [4], spatial correlation between nodal demands is explicitly modeled through the nodal demand covariance matrix [46]. For expository purposes, a single period is considered. The extension to a multiperiod model can be achieved based on the findings of [4]. Within an ARO-based framework, demand uncertainty can be modeled through a bilevel program embedded in the original optimization problem, thereby yielding a trilevel robust counterpart as follows:

$$\underset{\substack{\Delta D^{wc},\theta_b,f_l,p_i,\\ r_i^D,r_i^U,v_i}}{\text{Minimize}} \sum_{i\in I}\left(C_i^P(p_i,v_i)+C_i^U r_i^U + C_i^D r_i^D\right) + C^I \Delta D^{wc} \tag{3-1}$$

subject to:

$$\sum_{i\in I_b} p_i + \sum_{l\in\mathcal{L}|to(l)=b} f_l - \sum_{l\in\mathcal{L}|fr(l)=b} f_l = \hat{D}_b; \forall b\in N \tag{3-2}$$

$$f_l = \frac{1}{x_l}\left(\theta_{fr(l)} - \theta_{to(l)}\right); \forall l\in\mathcal{L} \tag{3-3}$$

$$-\overline{F}_l \le f_l \le \overline{F}_l; \forall l\in\mathcal{L} \tag{3-4}$$

$$\underline{P}_i v_i \le p_i \le \overline{P}_i v_i; \forall i\in I \tag{3-5}$$

$$p_i + r_i^U \le \overline{P}_i v_i; \forall i\in I \tag{3-6}$$

$$p_i - r_i^D \ge \underline{P}_i v_i; \forall i\in I \tag{3-7}$$

$$0 \le r_i^U \le \overline{R}_i^U v_i; \forall i\in I \tag{3-8}$$

$$0 \le r_i^D \le \overline{R}_i^D v_i; \forall i\in I \tag{3-9}$$

$$v_i \in \{0,1\}; \forall i\in I \tag{3-10}$$

$$\Delta D^{wc}\left(\boldsymbol{p},\boldsymbol{r^D},\boldsymbol{r^U}\right) = \max_{\delta^{wc},a_i^G,a_l^L,D_b,e_b^{(+)},e_b^{(-)}} \Big\{ \delta^{wc} \tag{3-11}$$

subject to:

$$D_b = \hat{D}_b + Z \sum_{b'\in N|b'\le b} L_{b,b'}\left(e_{b'}^{(+)} - e_{b'}^{(-)}\right); \forall b\in N \tag{3-12}$$

$$\boldsymbol{f}\left(\{a_i^G\}_{i\in I}, \{a_l^L\}_{l\in\mathcal{L}}\right) \ge \boldsymbol{0} \tag{3-13}$$

$$\sum_{b\in N} W_{b,q} D_b \le h_q; \forall q\in Q \tag{3-14}$$

$$\sum_{b\in N}\left(e_b^{(+)} + e_b^{(-)}\right) \le \Gamma \tag{3-15}$$

$$0 \le e_b^{(+)} \le 1; \forall b\in N \tag{3-16}$$

$$0 \le e_b^{(-)} \le 1; \forall b\in N \tag{3-17}$$

$$a_i^G \in \{0,1\}; \forall i\in I \tag{3-18}$$

$$a_l^L \in \{0,1\}; \forall l\in\mathcal{L} \tag{3-19}$$

$$\delta^{wc}\left(\boldsymbol{p},\boldsymbol{r^D},\boldsymbol{r^U},\boldsymbol{a},\boldsymbol{D}\right) = \min_{\substack{\Delta N_b^{wc},\Delta P_b^{wc},\\ \theta_b^{wc},f_l^{wc},p_i^{wc}}} \Bigg[\sum_{b\in N}\left(\Delta N_b^{wc} + \Delta P_b^{wc}\right) \tag{3-20}$$

subject to:

$$\sum_{i \in I_b} p_i^{wc} + \sum_{l \in \mathcal{L}|to(l)=b} f_l^{wc} - \sum_{l \in \mathcal{L}|fr(l)=b} f_l^{wc} - \Delta P_b^{wc} + \Delta N_b^{wc}$$

$$= D_b : \big(\beta_b\big); \forall b \in N \quad (3\text{-}21)$$

$$f_l^{wc} = \frac{a_l^L}{x_l}\big(\theta_{fr(l)}^{wc} - \theta_{to(l)}^{wc}\big) : \big(\omega_l\big); \forall l \in \mathcal{L} \quad (3\text{-}22)$$

$$-\overline{F}_l \leq f_l^{wc} \leq \overline{F}_l : \big(\pi_l, \sigma_l\big); \forall l \in \mathcal{L} \quad (3\text{-}23)$$

$$a_i^G\big(p_i - r_i^D\big) \leq p_i^{wc} \leq a_i^G\big(p_i + r_i^U\big) : \big(\gamma_i, \chi_i\big); \forall i \in I \quad (3\text{-}24)$$

$$\Delta N_b^{wc} \geq 0; \forall b \in N \quad (3\text{-}25)$$

$$\left.\left.\Delta P_b^{wc} \geq 0; \forall b \in N\right]\right\}. \quad (3\text{-}26)$$

The goal of the upper-level problem (3-1)–(3-10) is to minimize the total cost including the production cost, up- and down-spinning reserve costs, and the system imbalance cost, for which a sufficiently large imbalance penalty cost is used (3-1). Based on the formulation presented in [8], expressions (3-2)–(3-10) model energy and reserve scheduling for the pre-contingency state. Expressions (3-2)–(3-4) define a dc-power flow model, (3-5)–(3-7) ensure that the levels of energy and spinning reserves lie in the feasible generation region of each scheduled unit, and (3-8) and (3-9) set the spinning reserve limits. Finally, the binary nature of the scheduling on/off variables is imposed in (3-10). Note that the set of upper-level decision variables related to generation includes not only scheduling variables v_i, as done in [4], but also production levels and reserve contributions. This modeling aspect is crucial in order to appropriately model the pre-contingency system state.

The middle-level problem (3-11)–(3-19) determines the worst-case demand vector for the pre-contingency schedule identified by the upper level. The middle-level optimization is driven by the maximization of the system power imbalance (3-11). As done in [4], nodal demands are middle-level decision variables lying in a given user-defined polyhedral region. Notwithstanding, it is worth emphasizing that, unlike [4], correlation between nodal demands is explicitly incorporated in the uncertainty set by applying a linear transformation to the nodal demand error vectors (3-12). Such transformation is implemented through a lower triangular matrix $\boldsymbol{L}$, which is obtained by means of the Cholesky decomposition of the estimated covariance matrix $\boldsymbol{\Sigma}$ [47]. A scaling factor Z is added to allow the enlargement of the error variability if needed. This is justified in cases where observed data exhibit well-known correlated patterns. Expression (3-13) imposes the security criterion, which can be $n - K$. Expressions (3-14) are general polyhedral constraints used to characterize the feasibility space for nodal demands. Expressions (3-15)–(3-17) limit the number of demand deviations across buses to a given user-defined uncertainty budget Γ, also known as the conservativeness parameter [24]. Note that if $\mathbf{\Sigma}$ is diagonal and $Z = 1$, the correlation of nodal demands is neglected and each nodal demand may vary, at most, one standard deviation around its nominal value. It is worth mentioning that, in addition to correlated energy demand, correlated renewable generation uncertainty can be also regarded if we consider such generation as negative load. Constraints (3-18) and (3-19) respectively set the integrality of variables a_i^G and a_l^L.

The lower-level problem (3-20)–(3-26) identifies the optimal post-contingency dispatch for the pre-contingency schedule determined in the upper level and for the worst case of contingency as well as demand vector determined in the middle level. The goal of the lower level is to minimize the system power imbalance (3-20). The system power imbalance is defined as the sum over all buses of the absolute value of the nodal power imbalances. Network constraints are modeled in (3-21)–(3-23) whereas constraints (3-24) set generation limits given the reserves scheduled in the upper-level and under the contingency state decided in the middle-level. Finally, (3-25)–(3-26) impose the nonnegativity of nodal power-imbalance variables.

This model is general and nonparametric, i.e., it can be used without requiring the association of $\boldsymbol{L}$ with a particular probabilistic or statistical model. Notwithstanding, it can still be used assuming a parametric multivariate Gaussian noise, in which case Z can be interpreted as the quantile function for a given confidence level.

3.2 Solution Methodology

Problem (3-1)–(3-26) is a mixed-integer linear trilevel program. As will be explained in Subsection 3.2.1, ΔD^{wc} is a convex function of the upper-level variables, p_i, r_i^D, and r_i^U. Therefore, it can be described by an outer approximation algorithm. Here, we propose a Benders decomposition approach [71] comprising the iterative solution of a master problem and a subproblem. The master problem is an approximation of the original trilevel problem where in each iteration a cutting plane or Benders cut is added to locally characterize the recourse function ΔD^{wc}. The subproblem is associated with the middle- and lower-level problems for specific values of the upper-level decision variables as determined by the previous master problem. At each iteration, the solution to the subproblem provides relevant information, such as the value of ΔD^{wc} and its subgradient, to generate an additional cutting plane for the master problem. The convexity of the subproblem guarantees finite convergence to global optimality. Moreover, at each iteration, a measure of the distance to optimality is set by comparing the upper and lower bounds for the optimal cost respectively provided by the master problem and the subproblem. Hence, a stopping criterion relying on a user-defined tolerance level can be implemented.

Next, we present the mathematical formulation of the subproblem and the master problem resulting from the application of Benders decomposition to problem (3-1)–(3-26).

3.2.1 Subproblem

At each iteration t, the subproblem determines the worst-case contingency as well as the worst-case demand for the pre-contingency schedule for power and reserves identified by the previous master problem. Mathematically, the subproblem is a mixed-integer linear max-min problem comprising the two lowermost levels (3-11)–(3-26) for given values of p_i, r_i^D, and r_i^U. Such bilevel instance can be reformulated as an equivalent single-level mixed-integer linear program. This transformation consists of the following steps:

Step 1) Based on [37], the middle-level objective function δ^{wc} is replaced in (3-11) by the dual lower-level objective function, and the lower-level problem (3-20)–(3-26) is replaced by its dual feasibility constraints. Thus, the two lowermost levels are recast as:

$$\Delta D^{wc} = \max_{\substack{\beta_b,\gamma_i,\pi_l,\\ \sigma_l,\chi_i,\omega_l,\\ a_i^G, a_l^L, D_b,\\ e_b^{(+)}, e_b^{(-)}}} \left\{ \sum_{b \in N} \beta_b D_b - \sum_{l \in \mathcal{L}} \pi_l \overline{F}_l - \sum_{l \in \mathcal{L}} \sigma_l \overline{F}_l + \sum_{i \in I} \gamma_i a_i^G \left(p_i - r_i^D\right) \right.$$

$$- \sum_{i \in I} \chi_i a_i^G \left(p_i + r_i^U\right) \quad (3\text{-}27)$$

subject to:

$$\text{Constraints (3-12)–(3-19)} \quad (3\text{-}28)$$

$$\beta_b + \gamma_i - \chi_i \leq 0; \forall b \in N, \forall i \in I_b \quad (3\text{-}29)$$

$$\beta_{to(l)} - \beta_{fr(l)} + \omega_l + \pi_l - \sigma_l = 0; \forall l \in \mathcal{L} \quad (3\text{-}30)$$

$$-1 \leq \beta_b \leq 1; \forall b \in N \quad (3\text{-}31)$$

$$\sum_{l \in \mathcal{L} | to(l) = b} \frac{a_l^L}{x_l} \omega_l - \sum_{l \in \mathcal{L} | fr(l) = b} \frac{a_l^L}{x_l} \omega_l = 0; \forall b \in N \quad (3\text{-}32)$$

$$\pi_l \geq 0, \sigma_l \geq 0; \forall l \in \mathcal{L} \quad (3\text{-}33)$$

$$\left. \gamma_i \geq 0, \chi_i \geq 0; \forall i \in I \right\}. \quad (3\text{-}34)$$

Step 2) Bilinear terms $\beta_b D_b$ in (3-27) are linearized through the binary expansion approach described in [80]. First, one set of the variables is discretized using equally-sized levels. Such discretization is subsequently represented as a sum of binary variables, which can reproduce all of the discretization levels.

In contrast to lower-level variables β_b, D_b are not dual variables, thereby being the appropriate choice for discretization. As a result, dual sub-optimality is avoided while keeping the model with the minimum number of binary variables. Hence, assuming that, for each bus b, the nodal demand D_b is discretized into H_b equally-sized levels (with step size s_b), the binary representation of D_b requires at least $J_b = \lceil log_2 H_b \rceil$ binary variables. Thus, the discretization of D_b can be represented as follows:

$$D_b = \underline{D}_b + s_b \sum_{j=1}^{J_b} 2^{j-1} u_{jb}. \tag{3-35}$$

Using (3-35) in (3-27) yields products between continuous variables β_b and binary variables u_{jb}, which are subsequently linearized through well-known disjunctive expressions [80]. Under such linearization scheme, the resulting mixed-integer linear subproblem at iteration t is then formulated as:

$$\Delta D^{wc(t)} = \max_{\substack{\beta_b,\gamma_i,\mu_b,\\ \pi_l,\sigma_l,\chi_i,\\ \psi_{jb},\omega_l,a_i^G,\\ a_l^L,D_b,e_b^{(+)},\\ e_b^{(-)},h_i,u_{jb},\\ y_l,z_i}} \Bigg\{ \sum_{b\in N} \mu_b - \sum_{l\in\mathcal{L}} \pi_l \overline{F}_l - \sum_{l\in\mathcal{L}} \sigma_l \overline{F}_l + \sum_{i\in I} z_i \big(p_i^{(t)} - r_i^{D(t)}\big) - \sum_{i\in I} h_i \big(p_i^{(t)} + r_i^{U(t)}\big) \tag{3-36}$$

subject to:

$$\text{Constraints (3-28)–(3-31) and (3-33)–(3-34)} \tag{3-37}$$

$$\sum_{l\in\mathcal{L}|fr(l)=b} \frac{y_l}{x_l} - \sum_{l\in\mathcal{L}|to(l)=b} \frac{y_l}{x_l} = 0; \forall b \in N \tag{3-38}$$

$$-\big(1-a_l^L\big)\overline{\omega}_l \le \omega_l - y_l \le \big(1-a_l^L\big)\overline{\omega}_l; \forall l \in \mathcal{L} \tag{3-39}$$

$$-a_l^L \overline{\omega}_l \le y_l \le a_l^L \overline{\omega}_l; \forall l \in \mathcal{L} \tag{3-40}$$

$$0 \le \gamma_i - z_i \le \big(1-a_i^G\big)\overline{\gamma}_i; \forall i \in I \tag{3-41}$$

$$0 \le z_i \le \overline{\gamma}_i a_i^G; \forall i \in I \tag{3-42}$$

$$0 \le \chi_i - h_i \le \big(1-a_i^G\big)\overline{\chi}_i; \forall i \in I \tag{3-43}$$

$$0 \le h_i \le \overline{\chi}_i a_i^G; \forall i \in I, \tag{3-44}$$

$$\mu_b = \beta_b \underline{D}_b + s_b \sum_{j=1}^{J_b} 2^{j-1} \psi_{jb}; \forall b \in N \tag{3-45}$$

$$D_b = \underline{D}_b + s_b \sum_{j=1}^{J_b} 2^{j-1} u_{jb}; \forall b \in N \tag{3-46}$$

$$-Mu_{jb} \leq \psi_{jb} \leq Mu_{jb}; \forall b \in N, \forall j = 1, \ldots, J_b \tag{3-47}$$

$$-M\big(1-u_{jb}\big) \leq \psi_{jb} - \beta_b \leq M\big(1-u_{jb}\big); \forall b \in N, \forall j = 1, \ldots, J_b \tag{3-48}$$

$$\left. u_{jb} \in \{0,1\}; \forall b \in N, \forall j = 1, \ldots, J_b \right\}, \tag{3-49}$$

where μ_b, h_i, y_l, and z_i are newly added variables representing the products $\beta_b D_b$, $\chi_i a_i^G$, $\omega_l a_l^L$, and $\gamma_i a_i^G$ respectively, and ψ_{jb} is a new variable equal to $\beta_b u_{jb}$. Expressions (3-36)–(3-37) respectively correspond to (3-27)–(3-34), except constraint (3-32). Expression (3-38) is the linearized version of (3-32). Expressions (3-39)–(3-44) represent the linearization of bilinear terms, whereas expressions (3-45)–(3-49) characterize the binary expansion linearization (see [80] for further details on this procedure).

It should be noted that the worst-case imbalance variable ΔD^{wc} can be viewed as a function of the upper-level variables. Moreover, from (3-27), ΔD^{wc} is the maximum of affine functions within the middle-level feasibility set. Therefore, it is a convex function of the upper-level decision variables (see [74], item 3.2.3).

3.2.2 Master Problem

The master problem at iteration t is:

$$\underset{\substack{\Delta D_b^{+m}, \Delta D_b^{-m}, \alpha, \theta_b, \theta_b^m, \\ \lambda, \xi_i, f_l, f_l^m, p_i, p_i^m, \\ r_i^D, r_i^U, v_i}}{\text{Minimize}} \sum_{i \in I} \big(C_i^P(p_i, v_i) + C_i^U r_i^U + C_i^D r_i^D \big) + C^I \alpha \tag{3-50}$$

subject to:

$$\text{Pre-contingency constraints (3-2)–(3-10)} \tag{3-51}$$

$$\alpha \geq \sum_{b \in N} \mu_b^{(m)} - \sum_{l \in \mathcal{L}} \pi_l^{(m)} \overline{F}_l - \sum_{l \in \mathcal{L}} \sigma_l^{(m)} \overline{F}_l + \sum_{i \in I} z_i^{(m)} \big(p_i - r_i^D\big) - \sum_{i \in I} h_i^{(m)} \big(p_i + r_i^U\big); \forall m = 1, \ldots, t-1 \tag{3-52}$$

$$\big(|I| - K^G\big)\lambda - \sum_{i \in I} \xi_i \geq \sum_{b \in N} \hat{D}_b \tag{3-53}$$

$$\lambda - \xi_i \leq p_i + r_i^U; \forall i \in I \tag{3-54}$$

$$\xi_i \geq 0; \forall i \in I \tag{3-55}$$

$$\lambda \geq 0 \tag{3-56}$$

$$\sum_{i \in I_b} p_i^m + \sum_{l \in \mathcal{L}|to(l)=b} f_l^m - \sum_{l \in \mathcal{L}|fr(l)=b} f_l^m = D_b^{(m)} + \Delta D_b^{+m} - \Delta D_b^{-m};$$
$$\forall b \in N, m = 1, \ldots, t-1 \tag{3-57}$$

$$f_l^m = \frac{a_l^{L(m)}}{x_l}\left(\theta_{fr(l)}^m - \theta_{to(l)}^m\right); \forall l \in \mathcal{L}, m = 1, \ldots, t-1 \tag{3-58}$$

$$-\overline{F}_l \leq f_l^m \leq \overline{F}_l; \forall l \in \mathcal{L}, m = 1, \ldots, t-1 \tag{3-59}$$

$$a_i^{G(m)}\left(p_i - r_i^D\right) \leq p_i^m \leq a_i^{G(m)}\left(p_i + r_i^U\right); \forall i \in I, m = 1, \ldots, t-1 \tag{3-60}$$

$$\alpha \geq \sum_{b \in N}\left(\Delta D_b^{+m} + \Delta D_b^{-m}\right); m = 1, \ldots, t-1 \tag{3-61}$$

$$\Delta D_b^{+m} \geq 0, \Delta D_b^{-m} \geq 0; \forall b \in N, m = 1, \ldots, t-1. \tag{3-62}$$

The objective function (3-50) corresponds to (3-1), where variable α represents the approximation of ΔD^{wc}. Expressions (3-51) are the same pre-contingency constraints (3-2)–(3-10) described in Section 3.1. At each iteration, the search space is restricted by adding a Benders cut (3-52). $\Delta D^{wc(m)}$ is obtained from the optimal solution to the subproblem (3-36)–(3-49) at iteration m for given values of the upper-level decision variables $p_i^{(m)}$, $r_i^{D(m)}$, and $r_i^{U(m)}$. In addition, the values of coefficients $\mu_b^{(m)}$, $\pi_l^{(m)}$, $\sigma_l^{(m)}$, $z_i^{(m)}$, $h_i^{(m)}$, $D_b^{(m)}$, $a_i^{G(m)}$, and $a_l^{L(m)}$ are also provided by the solution of the subproblem. Expressions (3-53)–(3-56) represent the generation outage valid constraints developed in Chapter 2. Expressions (3-57)–(3-60) are a relaxed version of the redispacth valid constraints presented in Chapter 2. Here, instead of imposing the power imbalance to be zero, it is minimized. The sum over all buses of added variables ΔD_b^{+m} and ΔD_b^{-m} provides the power imbalance under contingency m. Finally, constraints (3-62) set the nonnegativity of ΔD_b^{+m}, ΔD_b^{-m}, and consequently α.

3.3 Case Studies

In this section, we examine the performance of the proposed model and solution methodology with two test systems. The first case is an illustrative three-bus example, whereas the second case study is based on the 24-bus IEEE Reliability Test System (RTS) [75]. We assume that generators offer linear cost functions of the form $c_i^P(p_i, v_i) = C_i^f v_i + C_i^v p_i$, and C^I is set equal to \$5 $\times 10^4$/MWh. At each bus b, the maximum demand is expressed as $\overline{D}_b = \hat{D}_b + Z\sqrt{\Sigma_{b,b}}$, whereas the minimum demand is modeled as $\underline{D}_b = \hat{D}_b - Z\sqrt{\Sigma_{b,b}}$.

Such limits are imposed in the problem formulation by means of $W_{b,q}$ and h_q. Moreover, the discretization step for nodal demands is $s_b = 10$ MW. For reproducibility purposes, input data for all case studies can be downloaded from [81].

Different correlation levels have been accounted for. In addition, the impact of security on generation scheduling has been analyzed by comparing the results obtained when no security criterion is considered with those attained when an $n-1$ security criterion is imposed. The proposed approach was implemented on an Intel® Core$^{\text{TM}}$ i7-3960C with a CPU of 3.3 GHz and 64 GB of RAM, using Xpress-MP 7.5 under Mosel [72]. Simulations were stopped when the relative difference between the corresponding upper and lower bounds for the optimal cost was within 0.02%.

3.3.1 Three-Bus Case

As shown in Fig. 3.1, this system comprises three buses, three lines, three generators, and two loads. Rated capacities and minimum power outputs for all generators are respectively equal to 200 MW and 10 MW whereas up- and down-spinning reserve contributions are all limited to 60 MW. Generation cost data are provided in Table 3.1. Line reactances are all 0.63 p.u. on a base of 100 MVA and 138 kV, whereas the flow of all lines is limited to 100 MW. Nodal demands are both equal to 100 MW.

This case study is useful to illustrate how the proposed model is capable

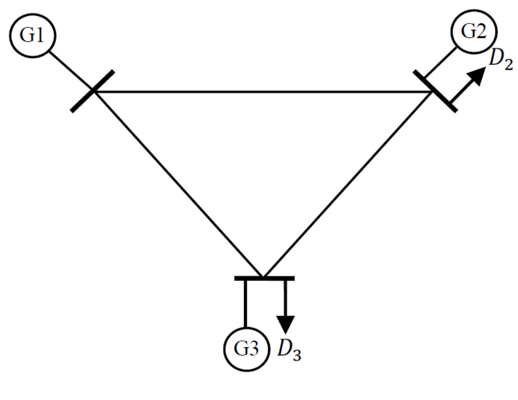


Figure 3.1: Three-bus system.

Table 3.1: Cost Data of Generators

Generator	C_i^f (\$)	C_i^v (\$/MWh)	C_i^D (\$/MW)	C_i^U (\$/MW)
1	10	40	4	4
2	10	50	5	5
3	10	150	15	15

of capturing the economic effect of considering the correlation between uncertain nodal demands. We assume that nodal demands may deviate from the nominal value in the range between plus and minus one standard deviation, which is set equal to 31 MW. Thus, $Z = 1$, the entries of the main diagonal of $\mathbf{\Sigma}$ are set to 31^2, and the other entries are $\Sigma_{1,2} = \Sigma_{2,1} = \rho \times 31^2$, where ρ represents the correlation level. Since there are only two sources of uncertainty, namely D_2 and D_3, the conservativeness parameter Γ is set to 1 for all simulations.

Table 3.2: Three-Bus System–Scheduling Costs with No Correlation

With No Security Criterion		With an $n-1$ Security Criterion	
Energy Cost ($)	Reserve Cost ($)	Energy Cost ($)	Reserve Cost ($)
8120.0	384.0	11340.0	1564.0

Table 3.3: Three-Bus System–Computing Times and System Power Imbalance with No Correlation

With No Security Criterion	With an $n-1$ Security Criterion	
Computing Time (s)	Computing Time (s)	System Imbalance (MW)
0.49	1.22	0.00

Table 3.4: Three-Bus System–Scheduling Costs with Correlation

ρ	With No Security Criterion		With an $n-1$ Security Criterion	
	Energy Cost (%)	Reserve Cost (%)	Energy Cost (%)	Reserve Cost (%)
-1.0	100	46	87	85
-0.5	100	82	91	99
0.5	100	114	128	104
1.0	103	143	141	106

Table 3.5: Three-Bus System–Computing Times and System Power Imbalance with Correlation

ρ	With No Security Criterion	With an $n-1$ Security Criterion	
	Computing Time (s)	Computing Time (s)	System Imbalance (MW)
-1.0	0.56	1.33	0.00
-0.5	0.95	1.75	0.00
0.5	0.55	1.33	0.00
1.0	0.36	0.83	7.67

Tables 3.2 and 3.3 show the optimal results for the uncorrelated case, i.e., with $\rho = 0$. The influence of correlation on energy and reserve costs is summarized in Tables 3.4 and 3.5. In Table 3.4, cost information is reported

in relation to that of the uncorrelated case. As can be seen, the correlation level significantly impacts on energy and reserve costs thereby corroborating the capability of the proposed approach of recognizing the effect of correlated nodal demands on generation scheduling. For the cases where no security criterion is enforced, reserves are the resources that mostly compensate for load variability. In contrast, under an $n-1$ security criterion, reserves are required to cope with credible contingencies and, as a consequence, the energy schedule presents a higher dependence upon the demand correlation. As shown in Table 3.5, for the security-constrained case with $\rho = 1$, the system is unable to simultaneously withstand the set of credible contingencies while addressing such nodal demand correlation. As a consequence, the optimal solution yields a level of system power imbalance equal to 7.67 MW, which represents 3.83% of the system load. Finally, from Table 3.5 it can also be inferred that accounting for correlation does not substantially modify computing times.

3.3.2 RTS-Based Case

The purpose of this case study is to assess the performance of the proposed approach with a standard and well-known benchmark such as the RTS [75]. Uncertainty takes place in the demands of buses 1, 2, 4, 5, 10, and 14. The standard deviations of such nodal demands are set equal to 6 MW, 5 MW, 4 MW, 4 MW, 10 MW, and 10 MW, respectively, while Z is set equal to 1. In this context, we set $\Sigma_{1,1} = 6^2$, $\Sigma_{2,2} = 5^2$, $\Sigma_{4,4} = 4^2$, $\Sigma_{5,5} = 4^2$, $\Sigma_{10,10} = 10^2$, and $\Sigma_{14,14} = 10^2$ in the main diagonal of $\mathbf{\Sigma}$. In addition, we consider the presence of correlation between demands of buses 1 and 2, 4 and 5, and 10 and 14, which is mathematically characterized as $\Sigma_{1,2} = \Sigma_{2,1} = \rho \times 6 \times 5$, $\Sigma_{4,5} = \Sigma_{5,4} = \rho \times 4^2$, and $\Sigma_{10,14} = \Sigma_{14,10} = \rho \times 10^2$. All the other entries of $\mathbf{\Sigma}$ are set to 0. In this case, since the presence of uncertainty is considered in a larger number of buses, the conservativeness parameter Γ is increased and set to 2.

Table 3.6: RTS-Based Case–Scheduling Costs with No Correlation

With No Security Criterion		With an $n-1$ Security Criterion	
Energy Cost (10^6 \$)	Reserve Cost (\$)	Energy Cost (10^6 \$)	Reserve Cost (\$)
2.10	176.8	2.72	2153.2

Tables 3.6–3.9 present the optimal results attained by the proposed approach. As can be seen, nodal demand correlation only impacts on reserve costs. This result is consistent since reserves are significantly cheaper than energy for this particular test system. Moreover, for all correlation levels

Table 3.7: RTS-Based Case–Computing Times and System Power Imbalance with No Correlation

With No Security Criterion	With an $n-1$ Security Criterion	
Computing Time (s)	Computing Time (s)	System Imbalance (MW)
8.71	4598.69	0.00

Table 3.8: RTS-Based Case–Scheduling Costs with Correlation

ρ	With No Security Criterion		With an $n-1$ Security Criterion	
	Energy Cost (%)	Reserve Cost (%)	Energy Cost (%)	Reserve Cost (%)
-1.0	100	15	100	89
-0.5	100	65	100	95
0.5	100	115	100	102
1.0	100	145	100	106

Table 3.9: RTS-Based Case–Computing Times and System Power Imbalance with Correlation

ρ	With No Security Criterion	With an $n-1$ Security Criterion	
	Computing Time (s)	Computing Time (s)	System Imbalance (MW)
-1.0	1.78	12.81	0.00
-0.5	5.91	2221.40	0.00
0.5	3.01	2685.85	0.00
1.0	1.24	7.71	0.00

considered, the optimal solutions were compliant with the $n-1$ security criterion. Finally, the computational burden associated with the proposed approach is mainly affected by the incorporation of the security criterion rather than by the consideration of correlation.

4
An Adjustable Robust Optimization Approach for $n-K$-Constrained Transmission Expansion Planning

Chapter 4 presents a novel approach for the transmission network expansion planning under generalized joint generation and transmission $n-K$ security criteria. The proposed methodology identifies the optimal expansion plan while guaranteeing power balance under both normal and contingency states. An adjustable robust optimization approach is presented to circumvent the tractability issues associated with conventional contingency-constrained methods relying on explicitly modeling the whole contingency set. The adjustable robust model is formulated as a trilevel programming problem. The upper-level problem aims at minimizing the investment and operation cost while ensuring that the system is able to withstand all contingencies. The middle-level problem identifies, for a given expansion plan, the contingency state leading to maximum power imbalance if any. Finally, the lower-level problem models the operator's best reaction for a given contingency and investment plan by minimizing the system power imbalance. The resulting trilevel program is solved by a primal-dual algorithm based on Benders decomposition combined with a column-and-constraint generation procedure. The proposed approach is finitely convergent to the optimal solution and provides a measure of the distance to the optimum. Simulation results show the superiority of the proposed methodology over conventional contingency-constrained models. The contents of this Chapter are directly related to a paper published in the *IEEE Transactions on Power Systems* [82].

The main contributions of this Chapter are as follows:

1. The application scope of adjustable robust optimization, which was previously used within the framework of power system operation in Chapters 2 and 3, is broadened to power system planning.

2. Adjustable robust optimization with a combinatorial uncertainty set is proposed as a suitable solution framework for the contingency-constrained transmission network expansion planning problem under a joint generation and transmission security criterion. Unlike previous works, the security criterion includes both existing and candidate lines,

and power imbalance within a pre-specified limit is allowed. The resulting problem is formulated as a trilevel mixed-integer program.

3. An effective solution methodology based on Benders decomposition is presented. The proposed approach is finitely convergent to optimality and provides a measure of the distance to the optimal solution thereby allowing the network planner to control the tradeoff between solution quality and computational effort.

4. An acceleration scheme relying on an iterative column-and-constraint generation procedure is provided to improve the computational performance. The enhanced solution methodology can be viewed as a primal-dual algorithm.

The rest of this Chapter is organized as follows. Section 4.1 presents the conventional contingency-constrained formulation for transmission network expansion planning. In Section 4.2, the trilevel ARO counterpart is provided. Section 4.3 describes the proposed solution methodology. Finally, in Section 4.4, two case studies are analyzed.

4.1 Conventional Contingency-Constrained Transmission Expansion Problem

Based on the models presented in [14, 26, 61, 62], a deterministic security criterion can be incorporated in transmission expansion planning through the following contingency-constrained formulation relying on mixed-integer programming:

$$\underset{\substack{\Delta D^{wc}, \Delta D_b^{+k}, \Delta D_b^{-k}, \\ \theta_b, \theta_b^k, f_l, f_l^k, p_i, p_i^k, v_l}}{\text{Minimize}} \sum_{i \in I} C_i^P(p_i) + \sum_{l \in \mathcal{L}^C} C_l v_l + C^I \Delta D^{wc} \tag{4-1}$$

subject to:

$$\sum_{i \in I_b} p_i + \sum_{l \in (\mathcal{L} \cup \mathcal{L}^C) | to(l) = b} f_l - \sum_{l \in (\mathcal{L} \cup \mathcal{L}^C) | fr(l) = b} f_l = D_b; \forall b \in N \tag{4-2}$$

$$f_l = \frac{1}{x_l}\big(\theta_{fr(l)} - \theta_{to(l)}\big); \forall l \in \mathcal{L} \tag{4-3}$$

$$- M_l\big(1 - v_l\big) \le f_l - \frac{1}{x_l}\big(\theta_{fr(l)} - \theta_{to(l)}\big) \le M_l\big(1 - v_l\big); \forall l \in \mathcal{L}^C \tag{4-4}$$

$$- \overline{F}_l \le f_l \le \overline{F}_l; \forall l \in \mathcal{L} \tag{4-5}$$

$$- v_l \overline{F}_l \le f_l \le v_l \overline{F}_l; \forall l \in \mathcal{L}^C \tag{4-6}$$

$$0 \le p_i \le \overline{P}_i; \forall i \in I \tag{4-7}$$

$$v_l \in \{0, 1\}; \forall l \in \mathcal{L}^C \tag{4-8}$$

$$\Delta D^{wc} \le \overline{\Delta} \tag{4-9}$$

$$\sum_{i\in I_b} p_i^k + \sum_{l\in(\mathcal{L}\cup\mathcal{L}^C)|to(l)=b} f_l^k - \sum_{l\in(\mathcal{L}\cup\mathcal{L}^C)|fr(l)=b} f_l^k - \Delta D_b^{+k} + \Delta D_b^{-k} = D_b;$$
$$\forall b\in N, \forall k\in\mathcal{C} \quad (4\text{-}10)$$
$$f_l^k = \frac{A_l^k}{x_l}\big(\theta_{fr(l)}^k - \theta_{to(l)}^k\big); \forall l\in\mathcal{L}, \forall k\in\mathcal{C} \quad (4\text{-}11)$$
$$-M_l\big(1-v_lA_l^k\big) \le f_l^k - \frac{1}{x_l}\big(\theta_{fr(l)}^k - \theta_{to(l)}^k\big) \le M_l\big(1-v_lA_l^k\big); \forall l\in\mathcal{L}^C,$$
$$\forall k\in\mathcal{C} \quad (4\text{-}12)$$
$$-\overline{F}_l \le f_l^k \le \overline{F}_l; \forall l\in\mathcal{L}, \forall k\in\mathcal{C} \quad (4\text{-}13)$$
$$-v_lA_l^k\overline{F}_l \le f_l^k \le v_lA_l^k\overline{F}_l; \forall l\in\mathcal{L}^C, \forall k\in\mathcal{C} \quad (4\text{-}14)$$
$$A_i^k\big(p_i - R_i^D\big) \le p_i^k \le A_i^k\big(p_i + R_i^U\big); \forall i\in I, \forall k\in\mathcal{C} \quad (4\text{-}15)$$
$$0 \le p_i^k \le \overline{P}_i; \forall i\in I, \forall k\in\mathcal{C} \quad (4\text{-}16)$$
$$\Delta D_b^{+k} \ge 0, \Delta D_b^{-k} \ge 0; \forall b\in N, \forall k\in\mathcal{C} \quad (4\text{-}17)$$
$$\Delta D^{wc} \ge \sum_{b\in N}\Big(\Delta D_b^{+k} + \Delta D_b^{-k}\Big); \forall k\in\mathcal{C}. \quad (4\text{-}18)$$

Problem (4-1)–(4-18) determines the subset of lines, within a set of candidates, to be built so that the total cost is minimized while modeling the system operation under both normal and contingency states. As done in [14,26,61,62], a static planning model is considered where generation sites are known, the duration of contingencies is disregarded, and a single load scenario is modeled, typically corresponding to the highest load demand forecast for the considered planning horizon.

The objective function to be minimized (4-1) comprises three terms, namely production costs, investment costs, and system power imbalance costs. This latter term penalizes the worst-case system power imbalance associated with the security criterion adopted by the network planner. Since the normal state does not belong to the contingency set characterizing the security criterion, no power imbalance is allowed under such state. Constraints (4-2)–(4-8), hereinafter referred to as pre-contingency constraints, model the operation under the normal state. Constraints (4-2) represent the nodal power balance equations. Using the dc load flow model of [54], constraints (4-3) and (4-4) express line flows in terms of nodal phase angles for existing and candidate lines, respectively. Constraints (4-5) and (4-6) respectively enforce power flow capacity limits for existing and candidate lines. Constraints (4-7) set the generation limits. Finally, the binary nature of investment variables is modeled in (4-8).

Constraints (4-9) set the maximum level of system power imbalance associated with the security criterion. Constraints (4-10)–(4-18) model the operation under all contingency states characterizing the security criterion. Analogous to (4-2)–(4-6), expressions (4-10)–(4-14) are the network constraints under contingency. Note however that, unlike in the pre-contingency state, nodal power imbalance is allowed through variables ΔD_b^{+k} and ΔD_b^{-k}. For each contingency k, the system power imbalance is defined as the summation over all buses of the nodal power surplus and deficit, ΔD_b^{+k} and ΔD_b^{-k}, considered in the nodal power balances (4-10). Generation limits for the contingency states are set in (4-15) and (4-16) whereas the nonnegativity of nodal power imbalance variables is imposed in (4-17). Constraints (4-18) in conjunction with the minimization of the total cost (4-1) characterize the worst-case system power imbalance, which is defined as the maximum system power imbalance over all contingencies.

This formulation allows accommodating a wide range of security criteria by means of the contingency set $\mathcal{C}$, where each contingency k is characterized by binary parameters A_i^k and A_l^k. Such parameters respectively represent the availability of generating units and transmission lines, including both existing and candidate transmission assets. The adopted security criterion, i.e., the definition of the contingency set $\mathcal{C}$, can be modeled in a compact way as:

$$\boldsymbol{f}\left(\{A_i^k\}_{i\in I}, \{A_l^k\}_{l\in(\mathcal{L}\cup\mathcal{L}^C)}\right) \geq \mathbf{0}. \tag{4-19}$$

Thus, the contingencies k included in $\mathcal{C}$ are those for which the corresponding binary availability parameters satisfy (4-19). It is worth emphasizing that (4-19) can handle extended security criteria including multiple outages such as the $n-K$ and the $n-K^G-K^L$ criteria. However, under such criteria, the size of problem (4-1)–(4-18) is exponentially dependent on K, K^G, and K^L, which may lead to intractability even for low values of those parameters. Therefore, there is a need for new tools to efficiently solve the contingency-constrained transmission expansion planning problem under joint generation and transmission security criteria.

4.2 Adjustable Robust Optimization Approach

Problem (4-1)–(4-18) is a contingency-constrained model relying on the explicit formulation of all contingency states included in $\mathcal{C}$. As described in Chapter 2, ARO [41] is suitable for contingency-dependent models such as (4-1)–(4-18). In this setting, the uncertainty set associated with ARO is characterized by $\{A_i^k\}_{i\in I}$, $\{A_l^k\}_{l\in(\mathcal{L}\cup\mathcal{L}^C)}$, and (4-19), while the operation under all contingency states is implicitly modeled through a trilevel robust

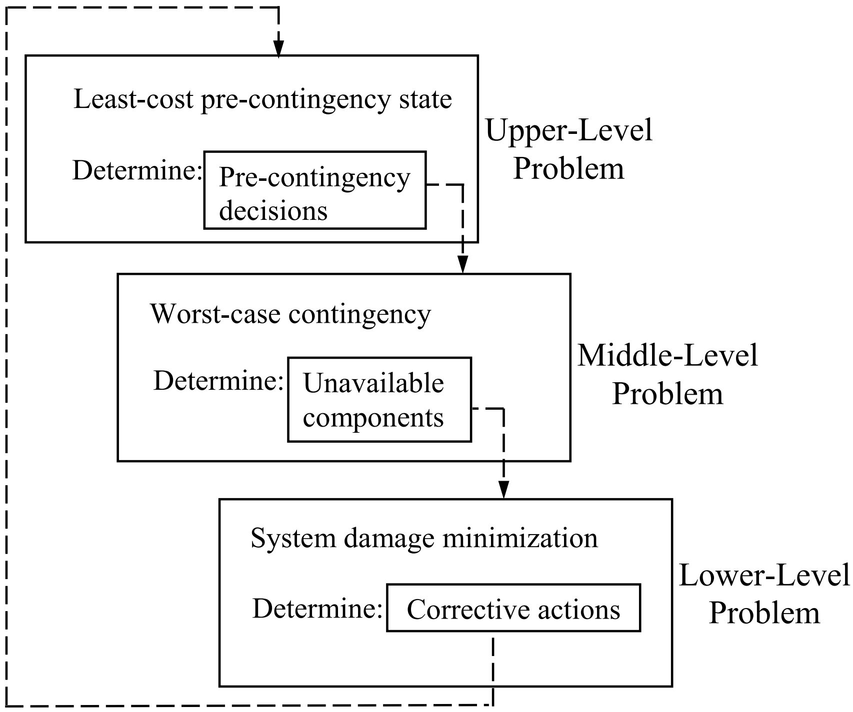


Figure 4.1: Adjustable robust optimization framework for contingency-dependent models.

counterpart, which is outlined in Fig. 4.1. Such implicit representation of contingency states is implemented by modeling binary parameters by a new set of binary decision variables that are constrained by the same functions $\boldsymbol{f}(\cdot)$ defining the contingency set in (4-19). Thus, for each feasible pre-contingency state, the robust counterpart determines the worst-case feasible realization of the binary variables. In addition, for each feasible combination of those new decision variables, the operation of the system is characterized by an optimal power flow, where adjustable decisions are determined in order to minimize the system damage, which is measured in terms of the system power imbalance.

Thus, according to Chapter 2, the trilevel robust counterpart of (4-1)–(4-18) is formulated as follows:

$$\underset{\Delta D^{wc},\theta_b,f_l,p_i,v_l}{\text{Minimize}} \sum_{i \in I} C_i^P(p_i) + \sum_{l \in \mathcal{L}^C} C_l v_l + C^I \Delta D^{wc} \tag{4-20}$$

subject to:

$$\text{Constraints (4-2)–(4-8)} \tag{4-21}$$

$$\Delta D^{wc} \leq \overline{\Delta} \tag{4-22}$$

$$\Delta D^{wc} = \max_{\delta, a_i^G, a_l^L} \left\{ \delta \right. \tag{4-23}$$

subject to:

$$\boldsymbol{f}\big(\{a_i^G\}_{i\in I},\{a_l^L\}_{l\in(\mathcal{L}\cup\mathcal{L}^C)}\big)\geq \mathbf{0} \quad (4\text{-}24)$$

$$a_i^G\in\{0,1\};\forall i\in I \quad (4\text{-}25)$$

$$a_l^L\in\{0,1\};\forall l\in\big(\mathcal{L}\cup\mathcal{L}^C\big) \quad (4\text{-}26)$$

$$\delta=\min_{\substack{\Delta D_b^{+wc},\Delta D_b^{-wc},\\ \theta_b^{wc},f_l^{wc},p_i^{wc}}}\Bigg[\sum_{b\in N}\Big(\Delta D_b^{+wc}+\Delta D_b^{-wc}\Big) \quad (4\text{-}27)$$

subject to:

$$\sum_{i\in I_b}p_i^{wc}+\sum_{l\in(\mathcal{L}\cup\mathcal{L}^C)|to(l)=b}f_l^{wc}-\sum_{l\in(\mathcal{L}\cup\mathcal{L}^C)|fr(l)=b}f_l^{wc}-\Delta D_b^{+wc}$$
$$+\Delta D_b^{-wc}=D_b:\big(\beta_b\big);\forall b\in N \quad (4\text{-}28)$$

$$f_l^{wc}=\frac{a_l^L}{x_l}\big(\theta_{fr(l)}^{wc}-\theta_{to(l)}^{wc}\big):\big(\omega_l\big);\forall l\in\mathcal{L} \quad (4\text{-}29)$$

$$-M_l\big(1-v_la_l^L\big)\leq f_l^{wc}-\frac{1}{x_l}\big(\theta_{fr(l)}^{wc}-\theta_{to(l)}^{wc}\big)\leq M_l\big(1-v_la_l^L\big):$$
$$\big(\pi_l,\sigma_l\big);\forall l\in\mathcal{L}^C \quad (4\text{-}30)$$

$$-\overline{F}_l\leq f_l^{wc}\leq\overline{F}_l:\big(\xi_l,\phi_l\big);\forall l\in\mathcal{L} \quad (4\text{-}31)$$

$$-v_la_l^L\overline{F}_l\leq f_l^{wc}\leq v_la_l^L\overline{F}_l:\big(\gamma_l,\chi_l\big);\forall l\in\mathcal{L}^C \quad (4\text{-}32)$$

$$a_i^G\big(p_i-R_i^D\big)\leq p_i^{wc}\leq a_i^G\big(p_i+R_i^U\big):\big(\zeta_i,\lambda_i\big);\forall i\in I \quad (4\text{-}33)$$

$$0\leq p_i^{wc}\leq\overline{P}_i:\big(\mu_i\big);\forall i\in I \quad (4\text{-}34)$$

$$\Delta D_b^{+wc},\Delta D_b^{-wc}\geq 0;\forall b\in N\Bigg]\Bigg\}. \quad (4\text{-}35)$$

Problem (4-20)–(4-35) comprises three optimization levels: (i) the upper level (4-20)–(4-22), which is associated with the pre-contingency state including expansion decisions and generation dispatch; (ii) the middle level (4-23)–(4-26), characterizing the worst-case contingency for the pre-contingency state; and (iii) the lower level (4-27)–(4-35), related to the system's reaction against the contingency identified by the middle level. Dual variables associated with the lower-level problem are in parentheses. Note that the lower level is parameterized in terms of upper-level variables p_i, v_l, and middle-level variables a_i^G, a_l^L.

The objective of the upper-level problem (4-20) is identical to that of the contingency-dependent model (4-1). The upper-level minimization is subject to the set of pre-contingency constraints (4-2)–(4-8), as imposed in (4-21), and to the upper bound on the worst-case system power imbalance (4-22).

The middle-level problem (4-23)–(4-26) determines the worst-case contingency for the solution identified by the upper level. To that end, out of all combinations of availability binary variables for generators and transmission lines, $\{a_i^G\}_{i\in I}$ and $\{a_l^L\}_{l\in(\mathcal{L}\cup\mathcal{L}^C)}$, complying with (4-24), the worst-case contingency corresponds to the combination yielding the largest system power imbalance (4-23). It is worth mentioning that functions $\boldsymbol{f}(\cdot)$ in (4-24) are identical to those in (4-19). Hence, the feasibility set of the middle-level problem precisely represents the contingency set $\mathcal{C}$. The integrality of a_i^G and a_l^L is respectively modeled in (4-25) and (4-26).

In the lower-level problem (4-27)–(4-35), the system power imbalance associated with upper-level variables p_i, v_l, and middle-level variables a_i^G, a_l^L is minimized in (4-27). The system power imbalance is defined as the summation over all buses of the nodal power surplus and deficit, ΔD_b^{+wc} and ΔD_b^{-wc}, considered in the nodal power balances (4-28). Constraints (4-28)–(4-35) respectively correspond to (4-10)–(4-17).

In essence, the trilevel problem (4-20)–(4-35) results from replacing constraints (4-10)–(4-18) in the original contingency-dependent model (4-1)–(4-18) by the two lowermost optimization levels (4-23)–(4-35). As a consequence, variables with superscript k are dropped and hence contingency dependence is avoided.

4.3 Solution Methodology

Based on the successful application of Benders decomposition to a structurally similar problem in Chapter 2, we propose using such master-subproblem framework for the mixed-integer trilevel program (4-20)–(4-35), where ΔD^{wc} constitutes the recourse function. In this setting, the master problem is formulated as a mixed-integer program. Moreover, ΔD^{wc} is a convex function of the upper-level variables p_i and v_l since it is the pointwise maximum of affine functions within the feasibility set of the subproblem [74]. Hence, the proposed algorithm finitely converges to a globally optimal solution. Additionally, the upper and lower bounds provided along the iterative process allow measuring the distance to the optimum.

4.3.1 Subproblem

At each iteration j, the subproblem determines the optimal value of the recourse function ΔD^{wc} for a given upper-level decision provided by the previous master problem. Mathematically, the subproblem is a mixed-integer linear max-min problem comprising the two lowermost optimization levels (4-23)–(4-35) parameterized in terms of the upper-level decision variables $p_i^{(j)}$

and $v_l^{(j)}$. Such bilevel problem can be reformulated as an equivalent single-level mixed-integer linear program through the following two-step procedure:

Step 1) A nonlinear single-level equivalent is obtained by replacing (i) the middle-level objective function by the dual lower-level objective function, and (ii) the lower-level problem by its dual feasibility constraints. The resulting equivalent is formulated as:

$$\Delta D^{wc} = \underset{\substack{\beta_b,\gamma_l,\zeta_i,\lambda_i,\mu_i,\xi_l,\pi_l,\\ \sigma_l,\phi_l,\chi_l,\omega_l,a_i^G,a_l^L}}{\text{Maximize}} \sum_{b\in N} D_b\beta_b - \sum_{l\in\mathcal{L}^C} v_l^{(j)}\overline{F}_l a_l^L\gamma_l - \sum_{l\in\mathcal{L}^C} v_l^{(j)}\overline{F}_l a_l^L\chi_l$$
$$- \sum_{l\in\mathcal{L}^C} M_l\big(1-v_l^{(j)}a_l^L\big)\sigma_l - \sum_{i\in I}\overline{P}_i\mu_i + \sum_{i\in I}\big(p_i^{(j)}-R_i^D\big)a_i^G\zeta_i$$
$$- \sum_{i\in I}\big(p_i^{(j)}+R_i^U\big)a_i^G\lambda_i - \sum_{l\in\mathcal{L}^C} M_l\big(1-v_l^{(j)}a_l^L\big)\pi_l - \sum_{l\in\mathcal{L}}\overline{F}_l\xi_l - \sum_{l\in\mathcal{L}}\overline{F}_l\phi_l \quad (4\text{-}36)$$

subject to:

$$\boldsymbol{f}\big(\{a_i^G\}_{i\in I},\{a_l^L\}_{l\in(\mathcal{L}\cup\mathcal{L}^C)}\big) \geq \mathbf{0} \quad (4\text{-}37)$$
$$a_i^G \in \{0,1\}; \forall i\in I \quad (4\text{-}38)$$
$$a_l^L \in \{0,1\}; \forall l\in \big(\mathcal{L}\cup\mathcal{L}^C\big) \quad (4\text{-}39)$$
$$\beta_b+\zeta_i-\lambda_i-\mu_i \leq 0; \forall b\in N, \forall i\in I_b \quad (4\text{-}40)$$
$$\beta_{to(l)}-\beta_{fr(l)}+\pi_l-\sigma_l+\gamma_l-\chi_l = 0; \forall l\in\mathcal{L}^C \quad (4\text{-}41)$$
$$\beta_{to(l)}-\beta_{fr(l)}+\omega_l+\xi_l-\phi_l = 0; \forall l\in\mathcal{L} \quad (4\text{-}42)$$
$$-1\leq\beta_b\leq 1; \forall b\in N \quad (4\text{-}43)$$
$$\sum_{l\in\mathcal{L}^C|to(l)=b}\left(\frac{\pi_l}{x_l}-\frac{\sigma_l}{x_l}\right) - \sum_{l\in\mathcal{L}^C|fr(l)=b}\left(\frac{\pi_l}{x_l}-\frac{\sigma_l}{x_l}\right) + \sum_{l\in\mathcal{L}|to(l)=b}\frac{a_l^L}{x_l}\omega_l$$
$$- \sum_{l\in\mathcal{L}|fr(l)=b}\frac{a_l^L}{x_l}\omega_l = 0; \forall b\in N \quad (4\text{-}44)$$
$$\zeta_i,\lambda_i,\mu_i \geq 0; \forall i\in I \quad (4\text{-}45)$$
$$\xi_l,\phi_l \geq 0; \forall l\in\mathcal{L} \quad (4\text{-}46)$$
$$\pi_l,\sigma_l,\gamma_l,\chi_l \geq 0; \forall l\in\mathcal{L}^C. \quad (4\text{-}47)$$

In (4-36), the worst-case system power imbalance ΔD^{wc} is determined by the maximization of the objective function of the dual lower level. Constraints (4-37)–(4-39) are respectively identical to middle-level constraints (4-24)–(4-26), whereas (4-40)–(4-47) are the dual feasibility constraints of the lower-level problem (4-27)–(4-35).

Step 2) Well-known algebra results [73] are applied to recast products between middle-level binary variables and lower-level dual (continuous) variables in (4-36) and (4-44) as linear expressions. The equivalent mixed-integer linear subproblem is formulated as:

$$\Delta D^{wc} = \underset{\substack{\beta_b,\gamma_l,\zeta_i,\lambda_i,\mu_i,\xi_l,\pi_l,\\ \sigma_l,\phi_l,\chi_l,\omega_l,a_i^G,a_l^L,d_l,\\ e_i,h_l,q_l,r_l,y_i,z_l}}{\text{Maximize}} \sum_{b\in N} D_b\beta_b - \sum_{l\in\mathcal{L}^C} v_l^{(j)}\overline{F}_l d_l - \sum_{l\in\mathcal{L}^C} v_l^{(j)}\overline{F}_l r_l$$
$$-\sum_{l\in\mathcal{L}^C} M_l\big(\sigma_l - v_l^{(j)}h_l\big) - \sum_{i\in I}\overline{P}_i\mu_i + \sum_{i\in I}\big(p_i^{(j)} - R_i^D\big)y_i$$
$$-\sum_{i\in I}\big(p_i^{(j)} + R_i^U\big)e_i - \sum_{l\in\mathcal{L}^C} M_l\big(\pi_l - v_l^{(j)}z_l\big) - \sum_{l\in\mathcal{L}}\overline{F}_l\xi_l - \sum_{l\in\mathcal{L}}\overline{F}_l\phi_l \tag{4-48}$$

subject to:

$$\text{Constraints (4-37)–(4-43), (4-45)–(4-47)} \tag{4-49}$$

$$\sum_{l\in\mathcal{L}^C|to(l)=b}\left(\frac{\pi_l}{x_l}-\frac{\sigma_l}{x_l}\right) - \sum_{l\in\mathcal{L}^C|fr(l)=b}\left(\frac{\pi_l}{x_l}-\frac{\sigma_l}{x_l}\right) + \sum_{l\in\mathcal{L}|to(l)=b}\frac{q_l}{x_l} - \sum_{l\in\mathcal{L}|fr(l)=b}\frac{q_l}{x_l} = 0; \forall b\in N \tag{4-50}$$

$$-\overline{\omega}_l\big(1-a_l^L\big) \le q_l - \omega_l \le \overline{\omega}_l\big(1-a_l^L\big); \forall l\in\mathcal{L} \tag{4-51}$$
$$-\overline{\omega}_l a_l^L \le q_l \le \overline{\omega}_l a_l^L; \forall l\in\mathcal{L} \tag{4-52}$$
$$-\overline{\pi}_l\big(1-a_l^L\big) \le z_l - \pi_l \le \overline{\pi}_l\big(1-a_l^L\big); \forall l\in\mathcal{L}^C \tag{4-53}$$
$$0 \le z_l \le \overline{\pi}_l a_l^L; \forall l\in\mathcal{L}^C \tag{4-54}$$
$$-\overline{\sigma}_l\big(1-a_l^L\big) \le h_l - \sigma_l \le \overline{\sigma}_l\big(1-a_l^L\big); \forall l\in\mathcal{L}^C \tag{4-55}$$
$$0 \le h_l \le \overline{\sigma}_l a_l^L; \forall l\in\mathcal{L}^C \tag{4-56}$$
$$-\overline{\zeta}_i\big(1-a_i^G\big) \le y_i - \zeta_i \le \overline{\zeta}_i\big(1-a_i^G\big); \forall i\in I \tag{4-57}$$
$$0 \le y_i \le \overline{\zeta}_i a_i^G; \forall i\in I \tag{4-58}$$
$$-\overline{\lambda}_i\big(1-a_i^G\big) \le e_i - \lambda_i \le \overline{\lambda}_i\big(1-a_i^G\big); \forall i\in I \tag{4-59}$$
$$0 \le e_i \le \overline{\lambda}_i a_i^G; \forall i\in I \tag{4-60}$$
$$-\overline{\gamma}_l\big(1-a_l^L\big) \le d_l - \gamma_l \le \overline{\gamma}_l\big(1-a_l^L\big); \forall l\in\mathcal{L}^C \tag{4-61}$$
$$0 \le d_l \le \overline{\gamma}_l a_l^L; \forall l\in\mathcal{L}^C \tag{4-62}$$
$$-\overline{\chi}_l\big(1-a_l^L\big) \le r_l - \chi_l \le \overline{\chi}_l\big(1-a_l^L\big); \forall l\in\mathcal{L}^C \tag{4-63}$$
$$0 \le r_l \le \overline{\chi}_l a_l^L; \forall l\in\mathcal{L}^C. \tag{4-64}$$

In (4-48)–(4-64), d_l, e_i, h_l, q_l, r_l, y_i, and z_l are new variables representing the nonlinear terms in (4-36) and (4-44): $d_l = \gamma_l a_l^L$, $e_i = \lambda_i a_i^G$, $h_l = \sigma_l a_l^L$, $q_l = \omega_l a_l^L$, $r_l = \chi_l a_l^L$, $y_i = \zeta_i a_i^G$, and $z_l = \pi_l a_l^L$.

It should be noted that an upper bound for the optimal cost is obtained by using $p_i^{(j)}$, $v_l^{(j)}$, and the optimal value of the objective function of the subproblem, ΔD^{wc}, in the objective function of the original trilevel program (4-20).

4.3.2 Master Problem

The master problem constitutes a relaxation for problem (4-20)–(4-35) where the recourse function ΔD^{wc} is approximated by a set of cutting planes referred to as Benders cuts, which are set up with information from the subproblem. For a given iteration j, the master problem can be formulated as the following mixed-integer program:

$$\underset{\alpha,\theta_b,f_l,p_i,v_l}{\text{Minimize}} \sum_{i\in I} C_i^P(p_i) + \sum_{l\in\mathcal{L}^C} C_l v_l + C^I\alpha \tag{4-65}$$

subject to:

$$\text{Constraints (4-2)–(4-8)} \tag{4-66}$$

$$\alpha \leq \overline{\Delta} \tag{4-67}$$

$$\begin{aligned} \alpha \geq \Delta D^{wc(m)} &+ \sum_{i\in I}\left(p_i - p_i^{(m)}\right)\left(y_i^{(m)} - e_i^{(m)}\right) \\ &+ \sum_{l\in\mathcal{L}^C}\left(v_l - v_l^{(m)}\right)\Big[\left(z_l^{(m)} + h_l^{(m)}\right)M_l \\ &- \left(d_l^{(m)} + r_l^{(m)}\right)\overline{F}_l\Big]; m = 1,\ldots,j-1 \end{aligned} \tag{4-68}$$

$$\alpha \geq 0. \tag{4-69}$$

In the objective function (4-65), α corresponds, at the optimal solution, to the pointwise maximum within all linear approximations of ΔD^{wc}. Moreover, the optimal value of the objective function is a lower bound for the optimal cost. Constraints (4-66) are identical to (4-21) whereas expression (4-67) corresponds to (4-22). Expressions (4-68) represent the Benders cuts, i.e., the local linear approximations of ΔD^{wc}, available until iteration j. Parameters $p_i^{(m)}$ and $v_l^{(m)}$ are the optimal values of the upper-level variables p_i and v_l obtained at a previous iteration m. Analogously, $\Delta D^{wc(m)}$, $d_l^{(m)}$, $e_i^{(m)}$, $h_l^{(m)}$, $r_l^{(m)}$, $y_i^{(m)}$, and $z_l^{(m)}$ result from the optimal solution to the subproblem at iteration m. Finally, the nonnegativity of α is imposed in (4-69).

As done in Chapter 2, the proposed algorithm can also be accelerated by means of valid constraints in the master problem. The key idea is to iteratively cut off expansion plans leading to a level of system power imbalance greater than $\overline{\Delta}$ under contingency states already identified by previous subproblems. This procedure can be viewed as a primal-type approximation for the original problem, where new columns (comprising generation levels, nodal power imbalances, nodal phase angles, and line flows) and new constraints (associated with the operation under a given contingency state) are added to the master problem. Hence, at each iteration j, the master problem includes the following set of linear constraints, hereinafter referred to as redispatch constraints:

$$\sum_{i \in I_b} p_i^m + \sum_{l \in (\mathcal{L} \cup \mathcal{L}^C) | to(l)=b} f_l^m - \sum_{l \in (\mathcal{L} \cup \mathcal{L}^C) | fr(l)=b} f_l^m - \Delta D_b^{+m} + \Delta D_b^{-m} = D_b;$$
$$\forall b \in N, m = 1, \ldots, j-1 \quad (4\text{-}70)$$

$$f_l^m = \frac{a_l^{L(m)}}{x_l} \big(\theta_{fr(l)}^m - \theta_{to(l)}^m\big); \forall l \in \mathcal{L}, m = 1, \ldots, j-1 \quad (4\text{-}71)$$

$$- M_l\big(1 - v_l a_l^{L(m)}\big) \le f_l^m - \frac{1}{x_l}\big(\theta_{fr(l)}^m - \theta_{to(l)}^m\big) \le M_l\big(1 - v_l a_l^{L(m)}\big);$$
$$\forall l \in \mathcal{L}^C, m = 1, \ldots, j-1 \quad (4\text{-}72)$$

$$- \overline{F}_l \le f_l^m \le \overline{F}_l; \forall l \in \mathcal{L}, m = 1, \ldots, j-1 \quad (4\text{-}73)$$

$$- v_l a_l^{L(m)} \overline{F}_l \le f_l^m \le v_l a_l^{L(m)} \overline{F}_l; \forall l \in \mathcal{L}^C, m = 1, \ldots, j-1 \quad (4\text{-}74)$$

$$a_i^{G(m)}\big(p_i - R_i^D\big) \le p_i^m \le \big(p_i + R_i^U\big) a_i^{G(m)}; \forall i \in I, m = 1, \ldots, j-1 \quad (4\text{-}75)$$

$$0 \le p_i^m \le \overline{P}_i; \forall i \in I, m = 1, \ldots, j-1 \quad (4\text{-}76)$$

$$\Delta D_b^{+m}, \Delta D_b^{-m} \ge 0; \forall b \in N, m = 1, \ldots, j-1 \quad (4\text{-}77)$$

$$\alpha \ge \sum_{b \in N} \Big(\Delta D_b^{+m} + \Delta D_b^{-m}\Big); m = 1, \ldots, j-1, \quad (4\text{-}78)$$

where the additional decision variables of the master problem, ΔD_b^{+m}, ΔD_b^{-m}, θ_b^m, f_l^m, and p_i^m, are associated with the contingency state identified by the subproblem at iteration m through $a_i^{G(m)}$ and $a_l^{L(m)}$. Constraints (4-70)–(4-77) respectively correspond to lower-level constraints (4-28)–(4-35) with $a_i^{G(m)}$ and $a_l^{L(m)}$ equal to the optimal values obtained by the subproblem at iteration m. Finally, the system power imbalance corresponding to the contingency identified at iteration m represents a lower bound for α, as modeled in (4-78).

4.3.3 Algorithm

The proposed algorithm works as follows:

1. *Initialization.*
   - Initialize the iteration counter: $j \leftarrow 1$;
   - Solve the master problem without cuts. This step provides $\alpha^{(1)}$, $p_i^{(1)}$, $v_l^{(1)}$, and a lower bound for the optimal cost $LB = \sum_{i \in I} C_i^P\big(p_i^{(1)}\big) + \sum_{l \in \mathcal{L}^C} C_l v_l^{(1)}$.
2. *Subproblem solution.* Solve the subproblem for the given $p_i^{(j)}$ and $v_l^{(j)}$. This step provides $\Delta D^{wc(j)}$, $d_l^{(j)}$, $e_i^{(j)}$, $h_i^{(j)}$, $r_l^{(j)}$, $y_i^{(j)}$, $z_l^{(j)}$, and an upper bound for the optimal cost $UB = \sum_{i \in I} C_i^P\big(p_i^{(j)}\big) + \sum_{l \in \mathcal{L}^C} C_l v_l^{(j)} + C^I \Delta D^{wc(j)}$.

3. *Iteration counter updating*. Increase the iteration counter: $j \leftarrow j+1$.

4. *Master problem solution*. Solve the full master problem. This step provides $\alpha^{(j)}$, $p_i^{(j)}$, $v_l^{(j)}$, and a lower bound for the optimal cost $LB = \sum_{i \in I} C_i^P\big(p_i^{(j)}\big) + \sum_{l \in \mathcal{L}^C} C_l v_l^{(j)} + C^I \alpha^{(j)}$.

5. *Convergence checking*. If a solution with a level of accuracy ϵ has been found, i.e., $\frac{\left(UB-LB\right)}{LB} \leq \epsilon$, then stop; otherwise go to step 2.

The master problem is a mixed-integer linear program. Moreover, ΔD^{wc} is a convex function of the upper-level variables p_i and v_l since it is the point-wise maximum of affine functions within the feasibility set of the subproblem [74]. Hence, the proposed algorithm finitely converges to a globally optimal solution. Additionally, the upper and lower bounds provide a measure of the distance to the optimum.

4.4 Case Studies

The performance of the proposed model and solution methodology is illustrated with three cases respectively based on the 24-bus IEEE Reliability Test System (RTS) [75], on the IEEE 118-bus system [76], and on the IEEE 300-bus system [83]. A joint generation and transmission $n-K$ security criterion is considered. Thus, the formulation of (4-24) is $\sum_{i \in I} a_i^G + \sum_{l \in (\mathcal{L} \cup \mathcal{L}^C)} a_l^L \geq n-K$, where $n = |I| + |\mathcal{L}| + |\mathcal{L}^C|$. It is assumed that generators offer linear cost functions of the form $C_i^P\big(p_i\big) = C_i^v p_i$ whereas C^I is set equal to $\$4 \times 10^9$/MWh. For reproducibility purposes, input data for all case studies can be downloaded from [84].

The three case studies have been solved by the contingency-dependent model (4-1)–(4-18), denoted as CD, and by the proposed primal-dual Benders decomposition, hereinafter referred to as PDBD, with a stopping criterion based on an optimality gap equal to 0.001%. Numerical testing has been conducted with different values of the security parameter K ranging between 0 and 5. For the unconstrained case and conventional $n-1$ and $n-2$ security criteria, no system power imbalance is allowed, i.e., $\overline{\Delta} = 0$ MW. For K equal to 3, 4, and 5, $\overline{\Delta}$ has been set to 5%, 10%, and 15% of the system load, respectively. Simulations have been implemented on a Dell Precision T7600 workstation with two Intel® Xeon® E5-2687W processors at 3.1 GHz and 128 GB of RAM, using Xpress-MP 7.5 under MOSEL [72].

4.4.1 RTS-Based Case

For this case study comprising 24 buses, 26 generators, and 49 existing transmission assets, a set of 12 candidate lines is available for expansion decisions [84].

Table 4.1: RTS-Based Case–System Costs and Computing Times

K	PDBD		CD	
	System Cost ($)	Time (s)	System Cost ($)	Time (s)
0	1.60E+08	0.06	1.60E+08	0.06
1	1.63E+08	1.36	1.63E+08	2.49
2	1.66E+08	2.36	1.66E+08	388.31
3	1.92E+08	87.47	1.92E+08	29985.30
4	6.51E+11	29.14	Out of Memory	Out of Memory
5	1.00E+12	88.01	Out of Memory	Out of Memory

Table 4.1 lists the system costs and computing times attained by PDBD and CD. As can be seen, PDBD solved all instances in less than 89 s. In contrast, CD was successful only for values of K up to 3 since tighter security criteria led to intractable contingency-dependent models. Moreover, PDBD was considerably faster than CD for K equal to 2 and 3. These results clearly substantiate the superiority of PDBD over CD from a computational perspective.

Table 4.1 also shows that, for values of K up to 3, the system costs are of the same order of magnitude and grow with the security level, as expected. Such cost increases over the case with no security range between 1.88% for $K = 1$ and 20.00% for $K = 3$. For $K > 3$ the system costs sharply rise due to the presence of system power imbalance.

Table 4.2: RTS-Based Case–Expansion Plans and Levels of System Power Imbalance for PDBD

K	$\overline{\Delta}$ (%)	Expasion Plan	System Power Imbalance (%)
0	0	7-8	0.00
1	0	6-10, 7-8	0.00
2	0	6-10(2), 7-8	0.00
3	5	1-5, 2-4, 2-6, 6-10(2), 7-8(2), 11-14	0.00
4	10	2-6, 6-10(2), 7-8	9.54
5	15	6-10, 7-8	14.67

Table 4.2 presents the expansion plans attained by PDBD and the corresponding levels of system power imbalance in percent of the system load. The figures in brackets in the third column represent the number of parallel

lines built in the corresponding corridor. This table shows that, for the available set of candidate lines, the system is able to be expanded with no system power imbalance even under an $n-3$ security criterion. Moreover, the size of the expansion plans grows from one line for $K=0$ to eight lines for $K=3$. However, tighter security criteria lead to solutions incurring system power imbalance within the pre-specified limit $\overline{\Delta}$. It is worth pointing out that for $K=4$ fewer lines are built than for $K=3$. This seemingly counterintuitive result stems from the lack of expansion plans under an $n-4$ security criterion without requiring system power imbalance. Thus, while eight lines are needed to meet power balance under an $n-3$ criterion, building more than four lines would not reduce the level of system power imbalance and hence the total cost attained for $K=4$. A similar explanation holds for $K=5$.

4.4.2 IEEE 118-Bus System

The second case study is based on the IEEE 118-bus system and comprises 55 generating units and 186 existing transmission assets. For this system, 70 new circuits are considered as candidate lines as described in [84].

Table 4.3: IEEE 118-Bus System–System Costs and Computing Times

K	PDBD		CD	
	System Cost ($)	Time (s)	System Cost ($)	Time (s)
0	1.08E+08	0.22	1.08E+08	0.52
1	1.10E+08	8.56	1.10E+08	291.54
2	4.18E+08	2261.99	4.66E+08*	269087.00*
3	1.60E+11	26.76	Out of Memory	Out of Memory
4	2.40E+11	46.00	Out of Memory	Out of Memory
5	2.40E+11	716.97	Out of Memory	Out of Memory

*Unfinished (Optimality gap = 32.34%).

The superior performance of the proposed PDBD over CD is illustrated in Table 4.3, where the system costs and computing times for both methods are presented. Unlike CD, PDBD attained either the optimum or an ϵ-optimal solution in reasonable times for all security criteria. On the other hand, due to the dimensionality issue characterizing contingency-dependent models, CD was only capable of solving the expansion planning problem for $K \leq 1$. For an $n-2$ security criterion, CD was unable to reach an acceptable solution within 74.75 hours, whereas for tighter security criteria the model could not even be loaded in the computer memory.

Table 4.4: IEEE 118-Bus System–Levels of System Power Imbalance and Sizes of Expansion Plans for PDBD

K	$\overline{\Delta}$ (%)	System Power Imbalance (%)	Number of Circuits Built
0	0	0.00	2
1	0	0.00	3
2	0	0.00	33
3	5	3.10	3
4	10	4.65	3
5	15	4.65	8

Table 4.3 also shows that higher values of the security parameter yield higher system costs, as expected. Moreover, from this table it can be inferred that $K = 2$ is the maximum level of security for which no system power imbalance is required. This result is corroborated in Table 4.4 where the levels of system power imbalance and the numbers of new lines provided by PDBD are reported. It should be noted that for $K \leq 2$ the size of the expansion plans grows as K increases. More specifically, two new lines are built for $K = 0$ whereas the $n-2$ security criterion leads to the construction of 33 lines. A discontinuity in this tendency arises for $K = 3$, which is the lowest value of the security level requiring system power imbalance.

4.4.3 IEEE 300-Bus System

The scalability of the proposed approach is illustrated with a case study based on the IEEE 300-bus system that consists of 107 generators, 411 existing lines, and 109 candidate transmission assets [84].

Table 4.5: IEEE 300-Bus System–System Costs and Computing Times

K	PDBD		CD	
	System Cost ($)	Time (s)	System Cost ($)	Time (s)
0	2.06E+09	0.78	2.06E+09	1.01
1	2.08E+09	108.50	2.08E+09	30145.80
2	3.26E+09	7222.27	Out of Memory	Out of Memory
3	6.18E+11	1125.91	Out of Memory	Out of Memory
4	1.64E+12	188.49	Out of Memory	Out of Memory
5	2.28E+12	66.61	Out of Memory	Out of Memory

As can be seen in Table 4.5, while CD could only handle the cases with $K = 0$ and $K = 1$, PDBD solved all instances with moderate computational effort. Similar to the previous case studies, the computing time required by PDBD increases with K when the corresponding optimal or ϵ-optimal solution does not require system power imbalance. In contrast, the computational effort drops as K increases when the level of system power imbalance is greater than 0 MW. This result reveals that the computational performance of PDBD is case dependent.

Table 4.6: IEEE 300-Bus System–Levels of System Power Imbalance and Sizes of Expansion Plans for PDBD

K	$\overline{\Delta}$ (%)	System Power Imbalance (%)	Number of Circuits Built
0	0	0.00	2
1	0	0.00	10
2	0	0.00	46
3	5	1.26	13
4	10	3.35	6
5	15	4.65	4

Finally, Table 4.6 shows that while the number of newly added lines grows from two for $K = 0$ to 46 for $K = 2$, for $K \geq 3$ the size of the expansion plans drops as K increases due to the presence of system power imbalance.

5
Conclusions and Future Works

The research conducted in this work focused on developing two stage robust models to address security in power systems operation and planning. Chapter 2 proposes a novel formulation and solution methodology to solve the contingency-constrained scheduling of energy and reserves considering a joint generation and transmission security criterion and ensuring deliverability of reserves. The distinctive modeling features are (i) the consideration of the effect of the transmission network, which requires not only up-spinning reserves but also down-spinning reserves, and (ii) the inclusion of transmission lines outages in the security criterion. The proposed approach is based on adjustable robust optimization by which the original contingency-constrained model is formulated as a trilevel programming problem. In order to solve the resulting mixed-integer linear trilevel program, a Benders decomposition technique is applied. The proposed methodology comprises the iterative resolution of a master problem and a subproblem. Both problems are formulated as mixed-integer linear programs suitable for efficient off-the-shelf branch-and-cut software. Two sets of valid constraints are also proposed to improve the computational performance of the presented approach. Numerical results show that the adjustable robust approach is able to attain optimal or high-quality near-optimal solutions with reasonable computational effort. Moreover, the superiority of the proposed methodology over the conventional contingency-constrained formulation is shown.

The consideration of some relevant information about the uncertainty modeling in generation scheduling via robust optimization models is not explored yet in the literature. In Chapter 3, the correlation between nodal demands is explicitly considered to provide a least-cost schedule of energy and reserves based on adjustable robust optimization. The resulting model is formulated as a trilevel program that is effectively solved by the combined use of Benders decomposition, a binary expansion approach, and a linearization scheme based on disjunctive constraints. Numerical results highlight the ability of the proposed approach in capturing the economic effect of nodal demand correlation on power system operation. It is worth mentioning that correlated generation uncertainty, mainly associated with renewable energy sources, can be straightforwardly addressed by the methodology presented in Chapter 3.

Finally, Chapter 4 has presented a novel approach to incorporate a general joint generation and transmission security criterion in transmission expansion planning problems. Based on adjustable robust optimization, the conventional contingency-dependent model is formulated as a trilevel programming problem where contingencies are implicitly embedded. The resulting trilevel program is solved by a primal-dual Benders decomposition that finitely converges to a globally-optimal solution. Numerical results reveal the computational superiority of the proposed approach over the conventional contingency-dependent formulation as well as the effectiveness of adjustable robust optimization to solve the problem with reasonable computational effort. While the proposed approach has been illustrated with $n-K$ security criteria extending traditional $n-1$ and $n-2$ standards, it is also applicable to other forms of security criteria involving the loss of subsets of generators or transmission assets. Moreover, the proposed tool relies on a worst-case framework that may be of interest to address expansion planning under deliberate outages.

Since all formulations proposed in this work were designed in a single-period setting, a natural continuation of our research is the consideration of the multiperiod case. It should be noted that the main steps used in the proposed solution approaches are readily applicable to the multiperiod instance with time-coupling constraints and some additional notation to properly index variables and parameters over the time periods. In addition, due to the increasingly wind power penetration in worldwide power systems, the modeling of wind uncertainty under the presented framework is also an interesting avenue of research. Finally, current research is underway to investigate alternatives of linearization procedures to the equivalent second level of the model presented in Chapter 3.

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A
Nomenclature

For the sake of clarity, we present here the notation related to this work. Note that few symbols may have different meanings depending on the Chapter. In this case, it be will explicitly highlighted in their respective descriptions.

Functions

$C_i^P(\cdot)$: Energy cost function offered by generator i.

$\boldsymbol{f}(\cdot)$: Vector of functions defining the security criterion.

Constants

Γ : Conservativeness parameter.

$\overline{\Delta}$: Maximum level of system power imbalance.

$\overline{\gamma}_i$: Bounding parameter for dual variable γ_i.

$\overline{\gamma}_l$: Bounding parameter for dual variable γ_l in Chapter 4.

$\overline{\zeta}_i$: Bounding parameter for dual variable ζ_i.

$\overline{\lambda}_i$: Bounding parameter for dual variable λ_i in Chapter 4.

$\overline{\pi}_l$: Bounding parameter for dual variable π_l in Chapter 4.

ρ : Correlation parameter.

$\boldsymbol{\Sigma}$: Estimated nodal demand covariance matrix.

$\Sigma_{b,b'}$: Element $(b,\, b')$ of $\boldsymbol{\Sigma}$.

$\overline{\sigma}_l$: Bounding parameter for dual variable σ_l in Chapter 4.

$\overline{\chi}_i$: Bounding parameter for dual variable χ_i.

$\overline{\chi}_l$: Bounding parameter for dual variable χ_l in Chapter 4.

$\overline{\omega}_l$: Bounding parameter for dual variable ω_l.

A_i^k : Availability parameter that is equal to 0 if generator i is unavailable under contingency state k, being 1 otherwise.

A_l^k : Availability parameter that is equal to 0 if line l is unavailable under contingency state k, being 1 otherwise.

C_i^D : Cost rate offered by generator i to provide down-spinning reserve.

C_i^f, C_i^v : Coefficients of the energy cost function offered by generator i.

C^I : Power-imbalance cost coefficient.

C_l : Construction cost of candidate line l.

C_i^U : Cost rate offered by generator i to provide up-spinning reserve.

D_b : Demand at bus b, only in Chapters 2 and 4.

$\hat{D}_b$: Nominal demand at bus b.

$\overline{D}_b$: Maximum demand level at bus b.

$\underline{D}_b$: Minimum demand level at bus b.

$\overline{F}_l$: Power flow capacity of line l.

$fr(l)$: Sending or origin bus of line l.

h_q : Bound of the q-th general polyhedral constraint.

H_b : Number of discretization levels of D_b.

J_b : Number of binary variables used in the discretization of D_b.

K, K^G, K^L : Number of unavailable system components, generators, and transmission lines, respectively.

$\boldsymbol{L}$: Lower triangular matrix that satisfies the equality $\boldsymbol{\Sigma} = \boldsymbol{L}\boldsymbol{L}^T$.

$L_{b,b'}$: Element $(b,\, b')$ of $\boldsymbol{L}$.

M : Big number used in the disjunctive constraints.

M_l : Sufficiently large constant in Chapter 4.

n : Number of system components.

$\overline{P}_i$: Capacity of generator i.

$\underline{P}_i$: Minimum power output of generator i.

$\overline{R}_i^D$: Upper bound for the down-spinning reserve contribution of generator i in Chapters 2 and 3.

R_i^D : Ramp-down limit of generator i in Chapter 4.

$\overline{R}_i^U$: Upper bound for the up-spinning reserve contribution of generator i in Chapters 2 and 3.

R_i^U : Ramp-up limit of generator i in Chapter 4.

s_b : Discretization step for D_b.

$to(l)$: Receiving or destination bus of line l.

$W_{b,q}$: Element (b, q) of the matrix representing a general polyhedral constraint bounding the demand.

x_l : Reactance of line l.

Z : Scaling factor.

Decision Variables

α : Approximation of the system power imbalance in the master problem.

ΔD^{wc} : System power imbalance under the worst-case contingency.

ΔD_b^+ : Power surplus at bus b.

ΔD_b^{+k} Power surplus at bus b under contingency k.

ΔD_b^{+wc} Power surplus at bus b under the worst-case contingency.

ΔD_b^- : Power deficit at bus b.

ΔD_b^{-k} Power deficit at bus b under contingency k.

ΔD_b^{-wc} Power deficit at bus b under the worst-case contingency.

ΔN_b^{wc} : Auxiliary variable used in the linearization of the absolute value of the power imbalance at bus b under the worst-case contingency.

ΔP_b^{wc} : Auxiliary variable used in the linearization of the absolute value of the power imbalance at bus b under the worst-case contingency.

δ : System power imbalance, given a_i^G, a_l^L, p_i, and v_l in Chapter 4.

δ^{wc} : Auxiliary variable representing the worst-case system power imbalance.

θ_b : Phase angle at bus b in the pre-contingency state.

θ_b^k : Phase angle at bus b under contingency k.

θ_b^{wc} : Phase angle at bus b under the worst-case contingency.

μ_b : Variable equal to the product $\beta_b D_b$.

ψ_{jb} : Variable equal to the product $\beta_b u_{jb}$.

a_i^G : Binary variable that is equal to 0 if generator i is unavailable under the worst-case contingency, being 1 otherwise.

a_l^L : Binary variable that is equal to 0 if line l is unavailable under the worst-case contingency, being 1 otherwise.

D_b : Demand at bus b, only in Chapter 3.

d_l : Variable equal to the product $\gamma_l a_l^L$.

e_i : Variable equal to the product $\lambda_i a_i^G$ in Chapter 4.

$e_b^{(+)}$: Positive error on the demand at bus b.

$e_b^{(-)}$: Negative error on the demand at bus b.

f_l : Power flow of line l in the pre-contingency state.

f_l^k : Power flow of line l under contingency k.

f_l^{wc} : Power flow of line l under the worst-case contingency.

h_i : Variable equal to the product $\chi_i a_i^G$.

h_l : Variable equal to the product $\sigma_l a_l^L$ in Chapter 4.

p_i : Power output of generator i in the pre-contingency state.

p_i^k : Power output of generator i under contingency k.

p_i^{wc} : Power output of generator i under the worst-case contingency.

q_l : Variable equal to the product $\omega_l a_l^L$.

r_l : Variable equal to the product $\chi_l a_l^L$ in Chapter 4.

r_i^D : Down-spinning reserve provided by generator i.

r_i^U : Up-spinning reserve provided by generator i.

u_{jb} : Binary variable used in the discretization of D_b.

v_i : Binary variable that is equal to 1 if generator i is scheduled in the pre-contingency state, being 0 otherwise, in Chapters 2 and 3.

v_l : Binary variable that is equal to 1 if candidate line l is built, being 0 otherwise, in Chapter 4.

y_l : Variable equal to the product $\omega_l a_l^L$.

y_i : Variable equal to the product $\zeta_i a_i^G$ in Chapter 4.

z_i : Variable equal to the product $\gamma_i a_i^G$.

z_l : Variable equal to the product $\pi_l a_l^L$ in Chapter 4.

Dual Variables

β_b : Dual variable associated with the power balance equation at bus b under the worst-case contingency.

γ_i : Dual variable associated with the lower bound for p_i^{wc}.

γ_l : Dual variable associated with the lower bound for f_l^{wc} for candidate line l in Chapter 4.

ζ_i : Dual variable associated with the lower bound for p_i^{wc}.

λ : Dual variable associated with the $n - K^G$ security constraint in the robust approach for energy and reserve scheduling under a generation security criterion.

λ_i : Dual variable associated with the upper bound for p_i^{wc} in Chapter 4.

μ_i : Dual variable associated with the capacity constraint for p_i^{wc} in Chapter 4.

ξ_i : Dual variable associated with the upper bound for generator i availability in the robust approach for energy and reserve scheduling under a generation security criterion.

ξ_l : Dual variable associated with the lower bound for f_l^{wc} for existing line l in Chapter 4.

π_l : Dual variable associated with the lower bound for f_l^{wc}.

π_l : Dual variable associated with the lower bound constraint relating power flow and phase angles for candidate line l under the worst-case contingency in Chapter 4.

σ_l : Dual variable associated with the upper bound for f_l^{wc}.

σ_l : Dual variable associated with the upper bound constraint relating power flow and phase angles for candidate line l under the worst-case contingency in Chapter 4.

ϕ_l : Dual variable associated with the upper bound for f_l^{wc} for existing line l in Chapter 4.

χ_i : Dual variable associated with the upper bound for p_i^{wc}.

χ_l : Dual variable associated with the upper bound for f_l^{wc} for candidate line l in Chapter 4.

ω_l : Dual variable associated with the equation relating power flow and phase angles for line l under the worst-case contingency.

ω_l : Dual variable associated with the equation relating power flow and phase angles for existing line l under the worst-case contingency in Chapter 4.

Sets

$\mathcal{C}$: Set of contingency indexes.

I : Set of generator indexes.

I_b : Set of indexes of generators connected to bus b.

$\mathcal{L}$: Set of transmission line indexes.

$\mathcal{L}$: Set of indexes of existing transmission lines in Chapter 4.

$\mathcal{L}^C$: Set of indexes of candidate transmission lines in Chapter 4.

N : Set of bus indexes.

Q : Set of polyhedral constraints bounding the demand.

B
Recent Blackouts

Brazil has faced various blackouts since 2011 until the present date. According to the largest Brazilian media vehicles, in 2012, 62 blackouts took place in different locations of the country and, in 2013, at least 100 MW of load was shed 45 times. Among all these events, the following major occurrences caught special attention due to the high number of affected people.

- ***In October* 2012** [85], an impressive blackout turned off the lights of the whole Northeast Region of Brazil, which has a population size similar to Italy, and 77% of the North, the largest Brazilian region in territorial terms. A short circuit occurred in one transmission line (500 kV Imperatriz-Colinas) of the system. However, the main and the alternative protection mechanisms of such asset did not work properly. In consequence, cascading outages in other transmission lines took place. As a result, $11,789$MW of load was shed.
- ***In August* 2013** [86], the Brazilian Northeast was strongly hit once again. A fire damaged two transmission lines of 500 kV connecting substations Ribeiro Gonçalves and São João do Piauí. The adopted $n-1$ security criterion was not sufficient to prevent a blackout. Consequently, $10,900$ MW of load was shed.
- ***In February* 2014** [87], another major blackout exposed the vulnerability of the Brazilian power system. Due to short-circuits, two transmission circuits of 500 kV connecting substations Miracema and Colinas were interrupted. At that moment, such asset was transporting energy from the North to South and Southeast Brazilian Regions. In order to alleviate the possible consequences, the Brazilian ISO turned off other circuits. As a result, a blackout affected 4.9 million homes.

Other countries have been also facing blackouts during the last years. Some of them are briefly described below and discussed in detail in [88], which also presents a review on major blackouts all over the world since the 1970s.

- ***In August* 2003**, the Northeast of the United States and Canada experienced a huge blackout. Some adverse conditions preceded the occurrence. Firstly, key generators were out of service due to planned generation outages, which did not shed load. Secondly, the MISO (Midwest Independent System Operator) was not communicated about outages in two transmission lines (Bloomington-Denois Creek 230-kV and Stuart-Atlanta 345-kV), which forced MlSO's state estimator to operate incorrectly. Thirdly, the Eastlake 5 generating unit was also interrupted, resulting in a harder voltage control in northern Ohio. Even under such adverse conditions, the system was able to meet demand. Nevertheless, outages of lines Chamberlin-Harding 345 kV, Stuart-Atlanta 345 kV, and Hanna-Juniper 345-kV started cascading outages and a heavy blackout, in which 62,000 MW of load was shed, took place.

– ***In May* 2005**, a quarter of the electricity demand of Moscow was cut off by a blackout. In this case, short circuits interrupted five 220kV transmission lines, provoking cascading outages. Consequently, a 2,500MW load shedding happened in the capital of Russia.

– ***In November* 2006**, blackout cut off the lights of many European countries. Two months before the event, a shipyard asked the local transmission system operator (TSO) to disconnect a double circuit 380 kV line Conneforde-Diele in North Germany on November 5, in order to permit the transfer of a ship on the river Ems to the North Sea. Promptly, the TSO took all the necessary precautions to do so. Nevertheless, on November 3, the same shipyard asked the TSO to make the disconnection one day before, on November 4. Consequently, there was no feasible time to perform all the indispensable studies and protection actions. As a result, European countries such as Austria, Spain, Portugal, France, Italy, Belgium, Luxemburg, The Netherlands, Switzerland, Germany, Slovenia, and Croatia were significantly affected by a system power imbalance.