6 Conclusions and further work

The contribution of this work is fourfold. First, using previous ideas from de Almeida & Haeusler (1999), we propose a novel encoding of Petri Nets as programs of a Dynamic Logic. This is a modular and compositional encoding: we have three basic types of Petri Nets, and using a composition operator we can build more complex Nets. Second, we introduce a Propositional Dynamic Logic which the programs are Marked Petri Nets. Unlike previous approaches Hull (2005); Hull & Su (2005); Tuominen (1990), which translate Petri Nets into Dynamic Logic, in this work we have Petri Nets encoded as programs of PDL yielding a new Dynamic Logic tailored to reasoning about Petri Nets in a more natural way. We present an axiomatization to Petri-PDL and prove its soundness and completeness. Then, we establish the decidability and finite model property of the logic. A polynomial reduction of the two-person tiling corridor game to Petri-PDL SAT problem is presented, showing its EXPTime-hardness. A Natural Deduction system with labels and a Resolution based system is presented.

The first extension of Petri-PDL in here presented is a Dynamic Logic tailored to reasoning about Marked Stochastic Petri Nets is the $\mathcal{DS}_3$ logic, not only increasing its expressiveness but also presenting a modular and compositional approach to probabilistic modal logic. This system aims to be an alternative to model performance evaluation. We present an axiomatization and prove its soundness and completeness regarding our semantics. After that we establish the decidability and finite model property and EXPTime-hardness of its SAT problem. A Natural Deduction system with labels for $\mathcal{DS}_3$ is presented. The Natural Deduction calculus is proved to be sound and complete; the Resolution based calculus is proved to be sound.

To simplify the probabilistic computations we present a third extension of Petri-PDL yielding a variant for $\mathcal{DS}_3$, the $\mathcal{DS}_3^\star$. This last system conceives in a Dynamic Logic to deal with Marked Stochastic Petri Nets with a transitive closure operator. Its usage leads to a simpler and more natural tracking of the probability computations. An axiomatization is presented and its soundness and completeness are presented regarding our semantics. The finite property model, decidability and EXPTime-hardness of its SAT problem are also presented.

By using $\mathcal{DS}_3$ or $\mathcal{DS}_3^\star$ it is possible to take advantage of systems that generate SPN automatically from UML diagrams López-Grao et al. (2004), used in software specification, to verify properties. The behaviour of the system can also be translated to a CTMC. Some usage examples of Petri-PDL and $\mathcal{DS}_3$ are presented.

It is also possible to define a conservative extension of Petri-PDL (extensible to the two variants) with a fixed-pointer operator on the trace of the Petri Net program. This extension would include an operator in the form $\mu.s\langle s, \pi\rangle\varphi$ where this formula will be true in a world w from a model $\mathcal{M}$ if exists worlds $v_1, v_2, \ldots, v_n$ in the same model such that $wRv_1 \circ v_1Rv_2 \circ \cdots \circ v_{n-1}Rv_n$ such that $\mathcal{M}$,$v_{n+1} \Vdash \langle s, \pi\rangle\varphi$ and $w \neq v_n$. The usage of this operator can provide shorten proofs.

Further work include prospecting meaningful case studies to apply in concrete situations, the extension of the Resolution based system for the other systems beyond Petri-PDL and a Natural Deduction with labels system for $\mathcal{DS}_3^\star$. We intend to investigate the usage of Petri-PDL in the formal verification of software, game modelling (da Costa et al., 2013) and multi-agent systems verification (Lopes et al., 2013). Future work also includes the extension of our approach to other Petri Nets, like Timed and Stochastic Petri Nets. Finally we would like to study issues concerning Model Checking and implement an Automatic Theorem Prover to the logics.