VII
Conclusions

This dissertation addressed the Vehicle Routing Problem with Synchronization Constraints, an emergent VRP variant for which attention has increased in the past years. This problem was described in the context of a classification scheme that was proposed by (DREXL, 2012).

More specifically, we focused on a special case of VRP with synchronization, the Vehicle Routing Problem with Exact Operation Synchronization (VRPTWEOS). This is a problem that arose from a real world application in the mining industry. A solution to that problem was built on top of an assignment model, and presented in chapter IV.

We introduced the VRPTWEOS to the reader by using classic examples, and then a formal definition of the problem was given. This dissertation suggested a time indexed flow formulation to model such a problem. However, we claim that using that formulation can become impractical in some cases, due to its pseudo-polynomial number of variables an restrictions.

To deal with that problem, the use of two column generation techniques were proposed. Firstly, the standard column generation approach based in the Dantzig-Wolfe decomposition of the flow model was reviewed. Secondly, the column generation for extended formulations approach, which works on a restricted version of the original variable formulation, was described. As those column generations techniques are intended to solve the linear relaxation of the problem, another complementary technique has to be used in order to solve the integer program. We described a well known alternative, the branch and bound algorithm, which receives the name of branch-and-price when used with column generation.

Computational experiments were conducted using some of the above techniques. For that purpose a set of instances, derived from the well known Solomon benchmark for the VRPTW, was developed. We suggested a method to derive such type of instances starting from a known solution. The purpose of these method is to add synchronization requirements to the VRPTW instances. For the sake of simplicity, this method was described and used

to generate instances based on solution with three vehicles. However, this derivation technique can be extended to generate more complex instances.

The results of the computational experiments allowed us to make some observations, regarding the increased level of difficulty brought to VRPs by synchronization constraints. Our results comply with the general perception in the literature that this increased difficulty relies on a problem known as interdependency. To support this, we gave a practical example in which the same problem instance was tested with and without synchronization, arriving at the conclusion that the first case was harder to solve.

Moreover, using the developed set of instances, we tested the performance of some of the algorithmic approaches here suggested against the known generic MIP solver Gurobi. As a result of that comparison it was observed that the column generation approach for extended formulations is a promising alternative to the solely use of Gurobi, as optimal or near optimal bounds were found for all instances with synchtonization. However, the algorithms presented in this dissertation shall be further developed, in order to be able to solve harder instances.

VII.1 Future Work

The topics addressed in this dissertation provided a range of future works. Some of these are:

- An extended set of benchmark instances for VRPs with synchronization can be developed to have a proper testbed for this type of problem.
- The branch and price algorithm described in Section V.5(a) can be enhanced with a better branching technique as well as other techniques such as dual stabilization or variable fixation by reduced cost.
- The column generation approaches can further be extended with the use of state-of-the-art pricing techniques, such as ng-route pricing (PECIN *et al.*, 2013).