III

The Vehicle Routing Problem with Synchronization Constraints

In the previous chapter some of the basic concepts and main variants of the Vehicle Routing Problem were reviewed. Making use of those concepts, this chapter aims to describe the variant of the VRP that this dissertation targets, the Vehicle Routing Problem with Synchronization Constraints. Before giving any definitions, let us introduce this emerging variant of the VRP by a simple real world application.

A classic such example is a dial-a-ride system that offers transportation services to elderly or disabled patients. Some of these patients require assistance to prepare for transportation. In that case, home-care staff must be scheduled to arrive to the patient's home some time before the arrival of the transport vehicle, in order to ensure that the transfer is made safely. Depending on the type of assistance that has to be provided, the offset between the arrivals of the home-care staff and the vehicle may vary greatly. Assistance given by the home-care staff goes from checking vital signs of the patient to applying some medicine.

The fact that not all patients attended by the system will need this type of assistance, and that the arrival times of home-care staff and the transporting vehicles may differ greatly, makes it desirable to separate the schedules of both teams. However, it is clear that the schedules of the assistance teams and the vehicle fleet must be synchronized. In the classic Vehicle Routing Problem, basic synchronization is needed between vehicles, so as to determine which vehicle visits which customer. This basic synchronization is clearly not sufficient to address the problem of the given example. It is from this type of applications that the Vehicle Routing Problem with Synchronization Constraints arise.

In the above example, the required synchronization is one that imposes precedence relations between the planned tasks of the assistance teams and the operation of the vehicles. However, there are several other types of synchronization that may appear in VRPs.

From a general point of view, the Vehicle Routing Problem with Synchronization Constraints may be described as a VRP where more than one vehicle may or must be used to fulfill a task.

It is important to notice that such synchronization requirements increase the interdependency between routes. Returning to our dial-a-ride example, consider the case when a visit of an assistance team to a given customer gets delayed. Such a delay will probably cause a subsequent retardation in the schedule of the transportation vehicle assigned to that customer. This retardation may turn the route of that vehicle infeasible, forcing to recompute it. Moreover, if that vehicle had further synchronizations with other home-care teams, the schedule of those home-care teams would become infeasible as well, as they would have to be paired with another vehicle. It is clear that in a worse case scenario, a change in one route may render all other routes infeasible. This issue is known as the interdependence problem, and makes it difficult to use local search heuristic methods that are commonly used to solve large-scale instances of the VRP (ROUSSEAU *et al.*, 2003).

III.1 Types of Synchronization in VRPs

While working on this dissertation we found a number of references in the literature addressing problems that fitted the description of a Vehicle Routing Problem with Synchronization Constraints. However, we also found that the synchronization requirements of those problems were categorized by the authors with many different names, for instance some names given are, job-teaming constraints, collaborating vehicles, temporal dependencies, to cite just a few.

The lack of consensus in the literature for the classification of these type of problems makes it harder to track the advance of the research in this field. Not to mention that even among problems within the VRPs with Synchronization Constraints type, there may be substantial differences, depending on the type(s) of synchronization that they tackle.

To deal with this problem (among other reasons) a classification scheme for the different types of synchronization in VRPs was proposed by (DREXL, 2012). We will embrace this classification scheme to give definition to a specific problem that this dissertation addresses, the Vehicle Routing Problem with Exact Operation Synchronization (VRPTWEOS). A summary to the referenced classification scheme follows.

The different types of synchronization that may be present in Vehicle Routing Problems are:

1. *Task Synchronization*: it is the most basic type of synchronization and it is present in the classic VRP. As previously mentioned, this is the synchronization that occurs between vehicles as to define which vehicle visits which customer.

2. *Operation Synchronization*: this type of synchronization decides on spatial and temporal aspects of tasks. It refers to the time offset that may be required between the operation of two or more vehicles in the same or different locations. This time offset must lie within a finite interval of zero or positive length. Operation synchronization may induce *dynamic time windows*. A dynamic time window for the execution of a task depends on the execution of another task. With respect to the temporal aspect, there are three types of operation synchronization, where the overall length of the planning horizon is denoted by $T \mid T < \infty$, the offset is denoted by Δ and the interval within which it must lie by $[\alpha, \beta] \mid \alpha \leq \beta$:

    (a) Pure spatial operation synchronization: $\alpha < \beta, \beta - \alpha = T$. This is the case when the temporal aspect is irrelevant to a problem and only the spatial aspect is taken into account.

    (b) Operation synchronization with precedences: $\alpha < \beta, \beta - \alpha < T$. This is the case when two vehicles must start executing their respective operation at their respective vertex with a variable offset $\alpha \leq \Delta \leq \beta$.

    (c) Exact operation synchronization: $\alpha = \beta$. This is the case when two vehicles must start executing their task at their respective vertex either at the same time ($0 = \alpha = \Delta = \beta$) or with a fixed positive offset ($0 \neq \alpha = \Delta = \beta$).

3. *Movement Synchronization*: This is the case when two vehicles have to be synchronized in space and time, to form a single composite vehicle. Trailers and lorries are a perfect example of this requirement, as trailers must be pulled by compatible lorries. This means that for one of these vehicles to be able to move along an arc, a different but compatible vehicle must traverse the arc together. There are two types of movement synchronization:

    (a) Movement synchronization at the depot: This is the case when two vehicles may join and separate only at the depot, before the start and after the end of a route.

   (b) Movement synchronization en route: This is the case when two vehicles may join and separate at different location that they may visit during their route.

4. *Load Synchronization*: This type of synchronization refers to problems where load transhipment occur between vehicles. In that case, the load that the *active* vehicle unload is exactly equal to the load that *passive* vehicle receives; no load gets lost. Therefore, at each vertex the difference between the total amount of load unloaded by all active vehicles and the total amount of load received by all passive vehicles must be equal to the specified demand of the vertex. There are three types of load synchronization:

   (a) Fixed load synchronization: This is the case when the amount of load to be delivered, collected or transferred is fixed in advance, e.g. when the application requires that the active vehicle always unload completely.

   (b) Discretized load synchronization: In this case, the possible amounts of load that can be delivered, collected or transferred are finitely discretized.

   (c) Continuous load synchronization: This is the case when the amount of load that can be delivered, collected or transferred may be any real number between 0 and the respective upper bound.

5. *Resource Synchronization*: This type of synchronization is needed when different vehicles compete for common, limited resources. In that case, at any point of time the total utilization by all vehicles of a specified resource must be less or equal to the specified limit for that resource.

III.2 The Vehicle Routing Problem with Exact Operation Synchronization

From now on, we will focus in a specific type of VRPs with Synchronization Constraints. This case arose from a real world application at a major mining company in Brazil. Sticking to the classification scheme described in the previous section, we classified this problem as the Vehicle Routing Problem with Time Windows and Exact Operation Synchronization (VRPTWEOS). Next we introduce the VRPTWEOS by describing our real world application.

Figure III.1: Synchronized operation of mine vehicles.
Vehicles from left to right: *Rubber Tire Dozer, Bulldozer, Truck, Shovel*

Mining operations require the use of heavy duty vehicles to perform tasks such as digging, carrying, hauling and dumping materials. Some of these tasks require the collaboration of two or more types of vehicles. To illustrate this behavior, let us show a common operation.

Figure III.1 shows a typical shovel-truck mining operation. In this operation four types of vehicles collaborate. The *shovel* will load the *truck* with material, while the *bulldozer* and the *rubber tire dozer* maintain the loading area clean. All vehicles have to be in place in order to be able to begin the operation. After the completion of the task, vehicles may part ways to perform other tasks alone or with other vehicles.

In our example application, tasks like these originate from 27 different locations involving 50 different types of vehicles. Moreover, each task has an associated time window within which it can be served. Vehicle capacities are not considered as a limiting factor, as customer generated demands will already take care of this by requiring a specific type of vehicle that they already know will suit its capacity demands. In this specific example, as there are a limited number of vehicles, the objective is to maximize the number of tasks performed.

Generally speaking, the VRPTWEOS is a generalization of the VRP where customers may require two or more vehicles of different types to perform a task collaboratively. Similarly to the VRPTW, each customer has an associated time window within which it can be served. In fact, when all customers require only one type of vehicle and vehicles are of sufficient capacity, this problem can be viewed as an straightforward generalization of the VRPTW. In the VRPTWEOS, the required vehicles are allowed to arrive at the customer location at different times but they must begin to operate at the same time and remain together during the task processing time. These type of synchronization fits the definition of exact operation synchronization given in 2c.

(a) Problem Definition

The Vehicle Routing Problem with Time Windows and Exact Operation Synchronization is defined over a multi-graph $G = (V, A)$ where V is the set of vertices and A is the set of arcs. Vertices $i = \{1, \ldots, n\}$ correspond to customers while vertices 0 and $n + 1$ correspond to the depot. Additionally a function $f : A \rightarrow V \times V = \{(i, j) \mid i, j \in V\}$ defines that arcs a_1 and a_2 are *multiple arcs* if $f(a_1) = f(a_2)$.

Each vertex i has an associated service time s_i and an associated time window $[\alpha_i, \beta_i]$ where α_i and β_i correspond to the earliest and latest times, respectively, in which the customer i can start to be serviced.

Let E be the set of types of vehicles available. There is an arc $(i, j) \in A$ for each pair of vertices $i, j \in V$ and each vehicle type $e \in E$. There is also a non-negative cost function $c_{i,j} : A \rightarrow \mathbb{Z}^+$ and a non-negative travel time t_{ij} that is associated with each arc of G.

Moreover, for each vertex $i \in V \setminus \{0, n + 1\}$ and each vehicle type $e \in E$ there is an associated demand function:

$$d(i, e) = \begin{cases} 1 & \text{if customer } i \text{ requires a vehicle of type } e \\ 0 & \text{otherwise} \end{cases} \tag{III.1}$$

Notice that for each pair of vertices $i, j \in V \setminus \{0, n+1\}$ the arc $(i, j) \in A$ associated to vehicle type $e \in E$ can be eliminated if $d(i, e) = 0$ or $d(j, e) = 0$. An arc $(i, j) \in A$ can also be eliminated if $\alpha_i + s_i + t_{ij} > \beta_j$

The objective is to find a set of minimum cost routes such that:

1. All routes start at end at the depot.
2. All customers are visited exactly once by one vehicle of each required vehicle types.
3. For each customer i the service starts within time interval $[\alpha_i, \beta_i]$ and all servicing vehicles remain s_i time instants.
4. For each customer i all servicing vehicles begin operating at the same time.

Finally it is important to note that the Vehicle Routing Problem with Time Windows and Exact Operation Synchronization is strongly $\mathcal{NP}$-hard, as it generalizes the VRP. If time windows $[0, +\infty]$ are defined for each customer, and there is only one equipment type, then the problem is identical to the VRP.

III.3 Previous Work on VRP with Exact Operation Synchronization

Previous work done in the field of VRP with exact operation synchronization show a variety of interesting applications. Those works more relevant to this dissertation are briefly reviewed in this section. For a comprehensive survey of the literature in VRPs with Synchronization Constraints, please refer to (DREXL, 2012).

(ROUSSEAU *et al.*, 2003) introduce the special case of a dial-a-ride problem that was used as an example at the beginning of this chapter. As previously mentioned, this problem refer to the transportation of elderly or disabled patients which require special assistance that must be provided some time before the transportation. The authors suggested that a Constraint Programming approach is well suited to address the interdependency problem introduced by synchronization constraints. They propose a method where customers are iteratively introduced as they request service. Local search methods are applied between requests. Computational experiments were executed on the well known Solomon benchmark instances (SOLOMON, 1987).The authors concluded that best insertion methods performed better than first insertion methods. They also found that, for their problem, simple local search was as effective as guided local search.

A similar application on home-care-staff scheduling was addressed by (BREDSTRÖM; RÖNNQVIST, 2008). The authors suggested that similar synchronization requirements arise in the problem of planning of security guards, in which guards need to be teamed up to make nightly inspections to buildings. These synchronization requirements also arise in forest operations, where cranes and lorries have to work together to collect felled trees. The authors developed a MIP model for which they used a standard MIP solver. They concluded that the developed model is not significantly harder to solve than standard VRPTW without synchronization constraints and that the inclusion of more synchronization constraints does not make the model any harder to solve.

(LI *et al.*, 2005) introduces the manpower allocation problem with time windows and job-teaming constraints. This problem arose from a real world application in the port of Singapore. The problem is described by a set of jobs situated at different locations each requiring a team of workers with different qualifications. All workers are dispatched from a depot and each job has a time window in which it can start to be attended. All workers have to be present in order to start the task execution. A job is satisfied if the required team works for the necessary duration within the job's time window. The authors presented a MIP formulation based on a network with one vertex per job, and described two constructive procedures and a simulated annealing based heuristic to solve the problem. Computational experiments over a set of instances developed by the authors showed that the results obtained with the heuristics were close to the lower bounds obtained when solving the model with a standard MIP solver.

(DOHN *et al.*, 2011) introduces the Vehicle Routing Problem with Time Window and Temporal Dependencies. The problem is presented as a generalization of the VRPTW with synchronization modeled as generic time precedence constraints between customers. They present various formulations, one of them being a time indexed formulation. They concluded that this formulation was the strongest one as it delivered the best lower bounds. Nevertheless, that formulation has a great number of constraints and for that reason they proposed a solution method based in a branch-cut-and-price algorithm. The authors claim that their model is general and therefore can be applied to various practical problems where synchronization can be modeled as precedence relations.