I
Introduction

Routing problems have long been a field of interest in the operations research community, a rather fundamental one, could be said. One of the most well known combinatorial optimization problems is the Traveling Salesman Problem (TSP). The TSP is concerned with the determination of the shortest route that passes exactly once through every of n given points. The origins of the TSP are not clear. Its first documented appearances are back from the decades of 1920's to 1940's (APPLEGATE *et al.*, 1998). It was the mathematician Merrill Flood that popularized this problem by the name of TSP (FLOOD, 1956).

The Vehicle Routing Problem (VRP) is another combinatorial optimization problem that has been widely studied in the literature. It can be described as the problem of supplying geographically dispersed customers with goods using a number of vehicles operating from a common depot. The goal is to find the best set of routes, according to a given objective function, that serves all customers and complies with given vehicles operating constraints. The objective function usually aims to minimize the total distance traveled, minimize the number of vehicles used or a combination of both.

The VRP was first introduced by (DANTZIG; RAMSER, 1959) in the context of a real world application concerning the delivery of gasoline to service stations. The VRP is closely related to the TSP, in fact, it is a generalization of the TSP (TOTH; VIGO, 2002) where the capacity of the carrier needs to be explicitly taken into account.

Among other reasons, VRPs are of great interest to the operations research area due to the difficulty that represents to solve them. The VRP belongs to the class of $\mathcal{NP}$-hard problems (SOLOMON, 1987), for which no polynomial-time algorithm is known.

It is important to notice that some $\mathcal{NP}$-hard problems present a *nice structure* that can be exploited to efficiently solve them exactly or heuristically. An example of such a problem is the 0-1 knapsack problem, which is known to be $\mathcal{NP}$-hard in the weak sense (PISINGER, 2005). Pisinger presents in his work pseudo-polynomial dynamic programming algorithms that easily solve the classical benchmark instances.

Yet that is not the case for the VRP. The VRP is a step further in terms of complexity as it is considered to be $\mathcal{NP}$-hard in the strong sense. This comes from the fact that it generalizes the TSP, which is well known to be $\mathcal{NP}$-hard in the strong sense as well (GAREY; JOHNSON, 1979).

In addition to its rich combinatorial structure, another fact that makes the VRP an interesting field of research is its wide range of applications. Vehicle Routing Problems naturally arise in various fields of transportation, distribution and logistics. Typical applications of this type are, for instance, solid waste collection, street cleaning and newspaper delivery.

Side constraints typically found in many other real world applications impose new conditions that give rise to all kinds of variants to the VRP. For an overview of the different VRP variants see (TOTH; VIGO, 2002).

One well known variant of the classical VRP is the Vehicle Routing Problem with Time Windows (VRPTW). In this variant, each customer has an associated time interval defined by a release date and a due date, indicating the time periods when the customer can start to be served. With the inclusion of the time dimension, a wider range of applications can be addressed. Specfic examples include bank deliveries, postal deliveries, dial-a-ride systems and school bus routing and scheduling (SOLOMON, 1987).

This dissertation addresses a variant of the VRP known as the Vehicle Routing Problem with Synchronization Constraints. In the classical Vehicle Routing Problem, synchronization is needed between vehicles so as to determine which vehicle visits which customer. In the Vehicle Routing Problem with Synchronization Constraints more than one vehicle may or must be used to fulfill a task, therefore further synchronization constraints are required.

The VRP with Synchronization Constraints is currently a hot topic in the VRP research field, most of the literature regarding this problem was published not more than 5 years ago (DREXL, 2012). Drexl introduces in his work a classification of the different types of synchronization. We review this classification scheme and embrace it to present the Vehicle Routing Problem with Time Windows and Exact Operation Synchronization (VRPTWEOS), the problem that this dissertation focuses on.

The VRPTWEOS is a generalization of the VRP where customers may require two or more vehicles of different types to perform a task collaboratively. Each customer has an associated time window within which it can be attended. The required vehicles may still arrive at the customer location at different times but they must begin to operate at the same time and remain together during the processing time of the task. Similar problems were previously addressed in the literature, like The Vehicle Routing Problem with Temporal Constraints (DOHN *et al.*, 2011) and The Synchronized Vehicle Dispatching Problem (ROUSSEAU *et al.*, 2003).

We first ran into the VRPTWEOS while working in a project for a major Brazilian mining company. The problem we had to solve was to plan the weekly routing and scheduling of heavy duty vehicles that were required at different locations to perform joint tasks. This type of application is in part the motivation to this work.

This dissertation provides a formal definition to the VRPTWEOS. Integer programming models are proposed and analyzed. Furthermore, we propose a solution method based on the Dantzig-Wolfe decomposition for which exact and aproximated resolution algorithms are described. In order to test the performance of those algorithms, we created a group of benchmark instances for the VRPTWEOS. This instances were created, on top of the Solomon benchmark for the VRPTW, using an adaptation method that we describe in detail. Such a method can be further extended to create a greater group of larger instances. Computational experiments were executed over the mentioned set of instances. Those computational experiments allowed us to observe the increased level of difficulty brought to VRPs by synchronization constraints. We defend the idea that this increased dificulty relies on a property known as interdependency. Moreover, the results of the computational experiments showed that the column generation solution approach is a promising alternative for solving the VRPTWEOS.

I.1 Dissertation Outline

This work is organized as follows:

- Chapter II introduces the VRP and a variant relevant to this dissertation, the VRPTW.
- Chapter III describes the VRP with Synchronization Constraints, shows a specific case (VRPTWEOS) and present a literature review.
- Chapter IV shows mathematical formulations for the Vehicle Routing Problem with Exact Operation Synchronization.
- Chapter V describes an algorithmic approach for the VRPTWEOS.
- Chapter VI presents results of computational experiments over the proposed approach.
- Chapter VII contains the conclusions of this work.