7
Referências bibliográficas

ADHIKARI S. **Damping Models for Structural Vibration**. Dissertation for the Degree of Doctor of Philosophy, Cambridge University, 2000.

ARGYRIS J.H.; STRAUB K. Static and Dynamic Stability of Nonlinear Elastic Systems under Nonconservative Forces-Natural Approach. **Computer Methods in Applied Mechanics and Engineering**, vol. 32, pp. 59–83, 1982.

BATHE, K.J. **Finite Element Procedures**. Prentice Hall, 1996.

BAŽANT, Z.P.; WU, S.T. Dirichlet series creep function for aging concrete, **Journal of the Engineering Mechanics Division**, ASCE, (99), 367-387, 1973.

BELYTSCHKO, T.; LIU, W.K.; MORAN, B. **Nonlinear Finite Elements for Continua and Structures**. John Wiley and Sons, 2000.

BOLOTIN, V.V. **Nonconservative Problems of the Theory of Elastic Stability**. Pergamon Press, 1963.

BOLOTIN, V.V.; ZHINZHER N.I. Effects of Damping on Stability of Elastic Systems Subjected to Nonconservative Forces. **International Journal of Solids and Structures**, v. 5, n. 9, pp. 965–989, 1969.

CLOUGH R.; PENZIEN, J. **Dynamics of Structures**. Computers and Structures, 2010.

COLEMAN, B.D.; NOLL, W. Foundations of Linear Viscoelasticity, **Reviews of Modern Physics**, v. 33, p. 239-249, 1961.

COOK R.D.; MALKUS D.S.; PLESHA M.E.; WITT R.J., **Concepts and Applications of Finite Element Analysis**. John Wiley and Sons, 2002.

FLÜGGE, W., **Viscoelasticity**. Springer-Verlag, 1975.

FOSS, K.A. Coordinates which Uncouple the Equations of Motion of Damped Linear Dynamic Systems. **Journal of Applied Mechanics**, v. 25, pp. 361–364, 1958.

GHARZEDDINE, F.; IBRAHIMBEGOVIC, A. Incompatible Mode Method for Finite Deformation Quasi-incompressible Elasticity. **Computational Mechanics**, v. 24, pp. 419–425, 2000.

HAAN, Y.M.; SLUIMER, G.M., Standard linear solid model for dynamic and time dependent behavior of building materials. **Hero**n, vol. 46, pp. 49–76, 2001.

IBRAHIMBEGOVIC, A. **Nonlinear Solid Mechanics, Theoretical Formulations and Finite Element Solution Methods**, Springer, 2009.

KALISKE, M.; ROTHERT, H., Formulation and Implementation of Three-Dimensional Viscoelasticity at Small and Finite Strains, **Computational Mechanics**, v. 19, p. 228-239, 1997.

KAR, R.C., Stability of a Non-uniform Cantilever Subjected to Dissipative and Non-conservative Forces, **Computers & Structures**, v. 11, p. 175-180, 1980.

KIRILLOV, O.N. Destabilization Paradox, **Doklady Physics**, v. 49, n. 4, p. 239–245, 2004.

KIRILLOV, O.N. Destabilization Paradox due to Breaking the Hamiltonian and Reversible Symmetry, **International Journal of Non-Linear Mechanics**, v. 42, n. 1, p. 71–87, 2007.

KIRILLOV, O.N.; SEYRANIAN A.P. Effect of Small Internal and External Damping on the Stability of Continuous Non-conservative Systems, **ENOC-2005, Eindhoven, Netherlands,** 7-12 August 2005.

KIRILLOV, O.N.; VERHULST F. Paradoxes of Dissipation-Induced Destabilization or who Opened Whitney's Umbrella?, **ZAMM Journal of Applied Mathematics and Mechanics**, v. 90, n. 6, p. 462–488, 2010.

KOTEN, H., Structural Damping, **Heron**, v. 22, p. 4-74, 1977.

LAKES, R. **Viscoelastic Materials**. Cambridge University Press, 2009.

MÜLLER, M.; GROß, M.; BETSCH, P. Dynamic Finite Deformation Viscoelasticity in Principal Stretches: Energy-Consistent Time Integration Using Mixed Finite Elements, **Conference on Computational Methods in Structural Dynamics and Earthquake Engineering**, Rhodes, Greece, 2009.

MURAVSKII, G.B., On Frequency Independent Damping, **Journal of Sound and Vibration**, v. 274, p. 653-668, 2004.

MUSCOLINO, G.; PALMERI, A.; RICCIARDELLI, F., Time-domain Response of Linear Hysteretic Systems to Deterministic and Random Excitations, **Earthquake Engineering and Structural Dynamics**, v. 34, p. 1129-1147, 2005.

NEUMARK, S., Concept of Complex Stiffness Applied to Problems of Oscillations with Viscous and Hysteretic Damping, **Aeronautical Research Council Reports and Memoranda**, n. 3269, 1957.

PIERSOL, A.G.; PAEZ, T.L. **Harris' Shock and Vibration Handbook**. McGraw-Hill, 2010.

RESEE, S.; GOVINDJEE, S. A Theory of Finite Viscoelasticity and Numerical Aspects, **International Journal of Solids and Structures**, v. 35, p. 3455-3482, 1998.

SIMO, J.C.; ARMERO, F. Geometrically Non-linear Enhanced Strain Mixed Methods and the Method of Incompatible Modes, **International Journal for Numerical Methods in Engineering**, v. 33, p. 1413-1449, 1992.

SIMO, J.C.; ARMERO, F.; TAYLOR R.L. Improved Versions of Assumed Enhanced Strain Tri-Linear Elements for 3D Finite Deformation Problems, **Computer Methods in Applied Mechanics and Engineering**, v. 110, p. 359-386, 1993.

SIMO, J.C.; HUGHES, T.J.R., **Computational Inelasticity**. Springer, 1998.

SIMO, J.C.; RIFAI, M.S. A Class of Mixed Assumed Strain Methods and the Method of Incompatible Modes, **International Journal for Numerical Methods in Engineering**, v. 29, p. 1595-1638, 1990.

SNOWDON, J.C., **Vibration and Shock in Damped Mechanical Systems**, John Wiley and Sons, New York, 1968.

SUANNO R.L. **Análise da Estabilidade de Estruturas sob a Ação de Cargas Não Conservativas**. Dissertação de Mestrado, Pontifícia Universidade Católica do Rio de Janeiro, 1988.

SURGULADZE, T. A. On Certain Applications of Fractional Calculus to Viscoelasticity. **Journal of Mathematical Sciences**, v. 112, p. 4517-4557, 2002.

TAYLOR, R.L.; PISTER, K.S.; GOUDREAU, G.L. Thermomechanical Analysis of Viscoelastic Solids, **International Journal for Numerical Methods in Engineering**, v. 2, p. 45-59, 1970.

TRINDADE, M.A.; BENJEDDOU, A.; OHAYON, R. Modeling of Frequency-Dependent Viscoelastic Materials for Active-Passive Vibration Damping, **Journal of Vibration and Acoustics**, v. 122, p. 169-174, 2000.

VASQUES, C.M.A.; MOREIRA, R.A.S.; DIAS RODRIGUES, J.; Viscoelastic Damping Technologies - Part I: Modeling and Finite Element Implementation. **Journal of Advanced Research in Mechanical Engineering**, v. 1, p. 76-95, 2010.

WILSON, E.L., **Three-Dimensional Static and Dynamic Analysis of Structures**. Computers and Structures, Inc., 2002.

WRIGGERS, P., **Nonlinear Finite Element Methods**. Springer, 2010.

ZIEGLER, H. **Principles of Structural Stability**. Blaisdell Publishing Company, 1968.

8
Anexo

8.1.
Matrizes para o elemento viga de Timoshenko

$$\mathbf{K}_{s1} = \frac{E\,I}{40\,L^e}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 148 & 0 & -189 & 0 & 54 & 0 & -13 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -189 & 0 & 432 & 0 & -297 & 0 & 54 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 54 & 0 & -297 & 0 & 432 & 0 & -189 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -13 & 0 & 54 & 0 & -189 & 0 & 148 \end{bmatrix} \tag{A.1}$$

$$\mathbf{K}_{s2}$$

$$= \frac{G\,k_s\,A}{40\,L^e}\begin{bmatrix} 148 & 20L & -189 & 57L/2 & 54 & -12L & -13 & 7L/2 \\ 20L & 64L^2/21 & -57L/2 & 33L^2/14 & 12L & -6L^2/7 & -7L/2 & 19L^2/4 \\ -189 & -57L/2 & 432 & 0 & -297 & 81L/2 & 54 & -12L \\ 57L/2 & 33L^2/14 & 0 & 108L^2/7 & -81L/2 & -27L^2/14 & 12L & -6L^2/ \\ 54 & 12L & -297 & -81L/2 & 432 & 0 & -189 & 57L/ \\ -12L & -6L^2/7 & 81L/2 & -27L^2/14 & 0 & 108L^2/7 & -57L/2 & 33L^2/ \\ -13 & -7L/2 & 54 & 12L & -189 & -57L/2 & 148 & -20L \\ 7L/2 & 19L^2/42 & -12L & -6L^2/7 & 57L/2 & 33L^2/14 & -20L & 64L^2/ \end{bmatrix} \tag{A.2}$$

$$\mathbf{M} = \frac{\bar{m}\, L^e}{40}\begin{bmatrix} 64/21 & 0 & 33/14 & 0 & -6/7 & 0 & 19/42 & 0 \\ 0 & 16h^2/63 & 0 & 11h^2/56 & 0 & -h^2/14 & 0 & 19h^2/504 \\ 33/14 & 0 & 108/7 & 0 & -27/14 & 0 & -6/7 & 0 \\ 0 & 11h^2/56 & 0 & 9h^2/7 & 0 & -9h^2/56 & 0 & -h^2/14 \\ -6/7 & 0 & -27/14 & 0 & 108/7 & 0 & 33/14 & 0 \\ 0 & -h^2/14 & 0 & -9h^2/56 & 0 & 9h^2/7 & 0 & 11h^2/56 \\ 19/42 & 0 & -6/7 & 0 & 33/14 & 0 & 64/21 & 0 \\ 0 & 19h^2/504 & 0 & -h^2/14 & 0 & 11h^2/56 & 0 & 16h^2/63 \end{bmatrix} \tag{A.3}$$

$$\mathbf{K}_G = \frac{P}{40\, L^e}\begin{bmatrix} 148 & 0 & -189 & 0 & 54 & 0 & -13 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -189 & 0 & 432 & 0 & -297 & 0 & 54 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 54 & 0 & -297 & 0 & 432 & 0 & -189 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -13 & 0 & 54 & 0 & -189 & 0 & 148 & 0 \\ 0 & 0 & -12L & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{A.4}$$

8.2. Funções de forma do elemento tridimensional de oito nós

$$h_1 = \frac{1}{8}(1+\hat{r})(1+\hat{s})(1+\hat{t})$$
$$h_2 = \frac{1}{8}(1-\hat{r})(1+\hat{s})(1+\hat{t})$$
$$h_3 = \frac{1}{8}(1-\hat{r})(1-\hat{s})(1+\hat{t})$$
$$h_4 = \frac{1}{8}(1+\hat{r})(1-\hat{s})(1+\hat{t})$$
$$h_5 = \frac{1}{8}(1+\hat{r})(1+\hat{s})(1-\hat{t})$$
$$h_6 = \frac{1}{8}(1-\hat{r})(1+\hat{s})(1-\hat{t})$$
$$h_7 = \frac{1}{8}(1-\hat{r})(1-\hat{s})(1-\hat{t})$$
$$h_8 = \frac{1}{8}(1+\hat{r})(1-\hat{s})(1-\hat{t}) \tag{A.5}$$

8.3. Matrizes deformação deslocamento elemento tridimensional

$$ {}_0\mathbf{B}_L = {}_0\mathbf{B}_{L0} + {}_0\mathbf{B}_{L1} \tag{A.6}$$

$$ {}_0\mathbf{B}_{L0} = \begin{bmatrix} {}_0h_{1,1} & 0 & 0 & {}_0h_{2,1} & 0 & \cdots & 0 \\ 0 & {}_0h_{1,2} & 0 & 0 & {}_0h_{2,2} & \cdots & 0 \\ 0 & 0 & {}_0h_{1,3} & 0 & 0 & \cdots & {}_0h_{8,3} \\ {}_0h_{1,2} & {}_0h_{1,1} & 0 & {}_0h_{2,2} & {}_0h_{2,1} & \cdots & 0 \\ 0 & {}_0h_{1,3} & {}_0h_{1,2} & 0 & 0 & \cdots & {}_0h_{8,2} \\ {}_0h_{1,3} & 0 & {}_0h_{1,1} & {}_0h_{2,3} & 0 & \cdots & {}_0h_{8,1} \end{bmatrix}_{[6\times 24]} \tag{A.7}$$

onde:

$$ {}_0h_{k,j} = \frac{\partial h_k}{\partial\, {}^0x_j} \tag{A.8}$$

$$\Delta u_j^k = {}^{t+\Delta t}u_j^k - {}^{t+\Delta t}u_j^k \tag{A.9}$$

$$l_{\mathrm{ij}} = \sum_{k=1}^{8} {}_0h_{k,j} \ {}^{t}u_i^k \tag{A.10}$$

$${}_0\mathbf{B}_{NL} = \begin{bmatrix} {}_0\widetilde{\mathbf{B}}_{NL} & \widetilde{\mathbf{0}} & \widetilde{\mathbf{0}} \\ \widetilde{\mathbf{0}} & {}_0\widetilde{\mathbf{B}}_{NL} & \widetilde{\mathbf{0}} \\ \widetilde{\mathbf{0}} & \widetilde{\mathbf{0}} & {}_0\widetilde{\mathbf{B}}_{NL} \end{bmatrix}_{[9\times 24\,]} \tag{A.11}$$

$${}_0\widetilde{\mathbf{B}}_{NL} = \begin{bmatrix} {}_0h_{1,1} & 0 & 0 & {}_0h_{2,1} & \cdots & {}_0h_{8,1} \\ {}_0h_{1,2} & 0 & 0 & {}_0h_{2,2} & \cdots & {}_0h_{8,2} \\ {}_0h_{1,3} & 0 & 0 & {}_0h_{2,3} & \cdots & {}_0h_{8,3} \end{bmatrix}_{[3\times 22\,]} \tag{A.12}$$

$$\widetilde{\mathbf{0}} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_{[3\times 1\,]} \tag{A.13}$$

$${}_0\mathbf{S} = \begin{bmatrix} {}_0\tilde{\mathbf{S}} & \overline{\mathbf{0}} & \overline{\mathbf{0}} \\ \overline{\mathbf{0}} & {}_0\tilde{\mathbf{S}} & \overline{\mathbf{0}} \\ \overline{\mathbf{0}} & \overline{\mathbf{0}} & {}_0\tilde{\mathbf{S}} \end{bmatrix}_{[9\times 9]} \tag{A.14}$$

$${}_0\tilde{\mathbf{S}} = \begin{bmatrix} {}_0S_{11} & {}_0S_{12} & {}_0S_{13} \\ {}_0S_{21} & {}_0S_{22} & {}_0S_{23} \\ {}_0S_{31} & {}_0S_{32} & {}_0S_{33} \end{bmatrix}_{[3\times 3\,]} \tag{A.15}$$

$$\overline{\mathbf{0}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{[3\times 3]} \tag{A.16}$$

$${}_0\hat{\mathbf{S}}^T = \begin{bmatrix} {}_0S_{11} & {}_0S_{22} & {}_0S_{33} & {}_0S_{12} & {}_0S_{23} & {}_0S_{31} \end{bmatrix}_{[1\times 6\,]} \tag{A.17}$$

$$ {}_0\mathbf{G}_L^{\varphi} = {}_0\mathbf{G}_{L0}^{\varphi} + {}_0\mathbf{G}_{L1}^{\varphi} \tag{A.18}$$

$$ {}_0\mathbf{G}_{L0}^{\varphi} = \begin{bmatrix} {}_0\phi_{1,1} & 0 & 0 & {}_0\phi_{2,1} & 0 & \cdots & 0 \\ 0 & {}_0\phi_{1,2} & 0 & 0 & {}_0\phi_{2,2} & \cdots & 0 \\ 0 & 0 & \phi_{1,3} & 0 & 0 & \cdots & {}_0\phi_{3,3} \\ {}_0\phi_{1,2} & {}_0\phi_{1,1} & 0 & {}_0\phi_{2,2} & {}_0\phi_{2,1} & \cdots & 0 \\ 0 & {}_0\phi_{1,3} & {}_0\phi_{1,2} & 0 & {}_0\phi_{2,3} & \cdots & {}_0\phi_{3,2} \\ {}_0\phi_{1,3} & 0 & {}_0\phi_{1,1} & {}_0\phi_{2,3} & 0 & \cdots & {}_0\phi_{3,1} \end{bmatrix}_{[6\times 9\,]} \tag{A.19}$$

onde:

$$ {}_0\phi_{k,j} = \frac{\partial \phi_k}{\partial\ {}^0x_j} \tag{A.20}$$

$$ \Delta\tilde{u}_j^k = {}^{t+\Delta t}\tilde{u}_j^k - {}^{t+\Delta t}\tilde{u}_j^k \tag{A.21}$$

$$ {}_0\mathbf{G}_{NL}^{\varphi} = \begin{bmatrix} {}_0\widetilde{\mathbf{G}}_{NL}^{\varphi} & \widetilde{\mathbf{0}} & \widetilde{\mathbf{0}} \\ \widetilde{\mathbf{0}} & {}_0\widetilde{\mathbf{G}}_{NL}^{\varphi} & \widetilde{\mathbf{0}} \\ \widetilde{\mathbf{0}} & \widetilde{\mathbf{0}} & {}_0\widetilde{\mathbf{G}}_{NL}^{\varphi} \end{bmatrix}_{[9\times 9\,]} \tag{A.22}$$

$$ {}_0\widetilde{\mathbf{G}}_{NL}^{\varphi} = \begin{bmatrix} {}_0\phi_{1,1} & 0 & 0 & {}_0\phi_{2,1} & 0 & 0 & {}_0\phi_{8,1} \\ {}_0\phi_{1,2} & 0 & 0 & {}_0\phi_{2,2} & 0 & 0 & {}_0\phi_{8,2} \\ {}_0\phi_{1,3} & 0 & 0 & {}_0\phi_{2,3} & 0 & 0 & {}_0\phi_{8,3} \end{bmatrix}_{[3\times 7\,]} \tag{A.23}$$

$$ \tilde{l}_{\mathrm{ij}} = \sum_{k=1}^{8} {}_0\phi_{k,j}\ \ {}^t\tilde{u}_i^k \tag{A.24}$$

Nas matrizes abaixo:

$$ \widehat{l_{\mathrm{ij}}} = l_{\mathrm{ij}} + \tilde{l}_{\mathrm{ij}} \tag{A.25}$$

$$
{}_0\mathbf{B}_{L1} = \begin{bmatrix}
\widehat{l_{11}}\ {}_0h_{1,1} & \widehat{l_{21}}\ {}_0h_{1,1} & \widehat{l_{31}}\ {}_0h_{1,1} & \widehat{l_{11}}\ {}_0h_{2,1} & \cdots & \widehat{l_{31}}\ {}_0h_{8,1} \\
\widehat{l_{12}}\ {}_0h_{1,2} & \widehat{l_{22}}\ {}_0h_{1,2} & \widehat{l_{32}}\ {}_0h_{1,2} & \widehat{l_{12}}\ {}_0h_{2,2} & \cdots & \widehat{l_{32}}\ {}_0h_{8,2} \\
\widehat{l_{13}}\ {}_0h_{1,3} & \widehat{l_{23}}\ {}_0h_{1,3} & \widehat{l_{33}}\ {}_0h_{1,3} & \widehat{l_{13}}\ {}_0h_{2,3} & \cdots & \widehat{l_{33}}\ {}_0h_{8,3} \\
\widehat{l_{11}}\ {}_0h_{1,2} + \widehat{l_{12}}\ {}_0h_{1,1} & \widehat{l_{21}}\ {}_0h_{1,2} + \widehat{l_{22}}\ {}_0h_{1,1} & \widehat{l_{31}}\ {}_0h_{1,2} + \widehat{l_{32}}\ {}_0h_{1,1} & \widehat{l_{11}}\ {}_0h_{2,2} + \widehat{l_{12}}\ {}_0h_{2,1} & \cdots & \widehat{l_{31}}\ {}_0h_{8,2} + \widehat{l_{32}}\ {}_0h_{8,1} \\
\widehat{l_{12}}\ {}_0h_{1,3} + \widehat{l_{13}}\ {}_0h_{1,2} & \widehat{l_{22}}\ {}_0h_{1,3} + \widehat{l_{23}}\ {}_0h_{1,2} & \widehat{l_{32}}\ {}_0h_{1,3} + \widehat{l_{33}}\ {}_0h_{1,2} & \widehat{l_{12}}\ {}_0h_{2,3} + \widehat{l_{13}}\ {}_0h_{2,2} & \cdots & \widehat{l_{32}}\ {}_0h_{8,3} + \widehat{l_{33}}\ {}_0h_{8,2} \\
\widehat{l_{11}}\ {}_0h_{1,3} + \widehat{l_{13}}\ {}_0h_{1,1} & \widehat{l_{21}}\ {}_0h_{1,3} + \widehat{l_{23}}\ {}_0h_{1,1} & \widehat{l_{31}}\ {}_0h_{1,3} + \widehat{l_{33}}\ {}_0h_{1,1} & \widehat{l_{11}}\ {}_0h_{2,3} + \widehat{l_{13}}\ {}_0h_{2,1} & \cdots & \widehat{l_{31}}\ {}_0h_{8,3} + \widehat{l_{33}}\ {}_0h_{8,1}
\end{bmatrix} \quad (A.26)
$$

$$
{}_0\mathbf{G}_{L1}^{\varphi} = \begin{bmatrix}
\widehat{l_{11}}\ {}_0\phi_{1,1} & \widehat{l_{21}}\ {}_0\phi_{1,1} & \widehat{l_{31}}\ {}_0\phi_{1,1} & \widehat{l_{11}}\ {}_0\phi_{2,1} & \cdots & \widehat{l_{31}}\ {}_0\phi_{8,1} \\
\widehat{l_{12}}\ {}_0\phi_{1,2} & \widehat{l_{22}}\ {}_0\phi_{1,2} & \widehat{l_{32}}\ {}_0\phi_{1,2} & \widehat{l_{12}}\ {}_0\phi_{2,2} & \cdots & \widehat{l_{32}}\ {}_0\phi_{8,2} \\
\widehat{l_{13}}\ {}_0\phi_{1,3} & \widehat{l_{23}}\ {}_0\phi_{1,3} & \widehat{l_{33}}\ {}_0\phi_{1,3} & \widehat{l_{13}}\ {}_0\phi_{2,3} & \cdots & \widehat{l_{33}}\ {}_0\phi_{8,3} \\
\widehat{l_{11}}\ {}_0\phi_{1,2} + \widehat{l_{12}}\ {}_0\phi_{1,1} & \widehat{l_{21}}\ {}_0\phi_{1,2} + \widehat{l_{22}}\ {}_0\phi_{1,1} & \widehat{l_{31}}\ {}_0\phi_{1,2} + \widehat{l_{32}}\ {}_0\phi_{1,1} & \widehat{l_{11}}\ {}_0\phi_{2,2} + \widehat{l_{12}}\ {}_0\phi_{2,1} & \cdots & \widehat{l_{31}}\ {}_0\phi_{8,2} + \widehat{l_{32}}\ {}_0\phi_{8,1} \\
\widehat{l_{12}}\ {}_0\phi_{1,3} + \widehat{l_{13}}\ {}_0\phi_{1,2} & \widehat{l_{22}}\ {}_0\phi_{1,3} + \widehat{l_{23}}\ {}_0\phi_{1,2} & \widehat{l_{32}}\ {}_0\phi_{1,3} + \widehat{l_{33}}\ {}_0\phi_{1,2} & \widehat{l_{12}}\ {}_0\phi_{2,3} + \widehat{l_{13}}\ {}_0\phi_{2,2} & \cdots & \widehat{l_{32}}\ {}_0\phi_{8,3} + \widehat{l_{33}}\ {}_0\phi_{8,2} \\
\widehat{l_{11}}\ {}_0\phi_{1,3} + \widehat{l_{13}}\ {}_0\phi_{1,1} & \widehat{l_{21}}\ {}_0\phi_{1,3} + \widehat{l_{23}}\ {}_0\phi_{1,1} & \widehat{l_{31}}\ {}_0\phi_{1,3} + \widehat{l_{33}}\ {}_0\phi_{1,1} & \widehat{l_{11}}\ {}_0\phi_{2,3} + \widehat{l_{13}}\ {}_0\phi_{2,1} & \cdots & \widehat{l_{31}}\ {}_0\phi_{8,3} + \widehat{l_{33}}\ {}_0\phi_{8,1}
\end{bmatrix} \quad (A.27)
$$