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AIRCRAFT STABILITY AND CONTROL ANALYSIS

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"Life is about passions, thank you for sharing mine" – Michael Schumacher, best driver in the history of Formula 1.

Abstract

Many of the parameters of an airplane design are empirical and based on accumulated knowledge. Some others are acquired using CFD or another finite element analysis software. The experimental data has a strong influence in an aircraft design.

Stability and control analysis is an important discipline to consider when designing an aircraft. An appropriate analysis can overcome deficiencies enforced by others disciplines, such as aerodynamics and loads. The efficiency of the control surfaces can also be maximized by a proper analysis.

In this work, all the requirements of static and dynamic aircraft stability will be analyzed. This two topics are divided into longitudinal, lateral and directional modes.

Another important element in stability analysis is the static margin for free and fixed stick. This margin is a crucial parameter that determines certain behaviors related to aircraft's maneuverability.

This work has the objective to analyze the stability and control of an aircraft designed to compete in SAE Aerodesign Brasil 2013.

Keywords: Aircraft, Stability, Control

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1. Introduction

The stability and control engineer studies how well an aircraft can fly and how easy it is to be controlled. We mean by stability, the tendency of an aircraft to return to its equilibrium point after a disturbance is applied on it. These disturbances may be an input of the pilot or an atmospheric phenomenon, such as: wind gusts, wind gradients and turbulence.

An stable aircraft must to perform a flight where the pilot does not have to control it all the time, in order to correct any deviation caused by small disturbances. Aircrafts with no inherent aerodynamic stability are unsafe to fly, however, there are artificial stability electronic systems that makes the operation of such aircraft possible.

To perform a mission safely, an aircraft has to be able to perform a stable flight and have the capability to maneuver, in a large range of altitudes and velocities. The efficiency of the control surfaces of an airplane is also a topic studied by stability and control engineers.

There are the modes of motion to analyse in order to fulfill the requirements of stability. The aircraft must have longitudinal, lateral and directional stable motions. The stability concept is basically divided in two topics, that will be explained below.

1.1.Static Stability

The static stability is the initial tendency of an aircraft to return to its equilibrium state after a disturbance. A good example of this kind of stability is shown in the next figure, if some external agent moves the ball to any place in the first condition, it has the tendency to return to the bottom of the curved surface, the gravity performed a restoring force to take the ball to its equilibrium point, we denote this kind of system stable. For the second condition, any force applied in the ball will make it move away of the theoretic equilibrium point, this condition makes the system unstable. For the third situation, the ball will stay in any position it is left, characterizing a neutral system.

Figure 1: Stable, unstable and neutral systems behaviors

1.2. Dynamic stability

The main concern of the dynamic stability is to know about the behavior of the aircraft in the time between the disturbance and de equilibrium point.

An aircraft can be statically stable, but dynamically unstable. However, to be dynamically stable, the aircraft must to fulfill the requirements of static stability analysis.

The mathematical model of an aircraft can be described using differential equations. One method to solve this equations and know the system's time response is shown below:

For the following differential equation:

$$a \frac{\partial y(t)}{\partial t} + b \frac{\partial^2 y(t)}{\partial t^2} + \dots + n \frac{\partial^n y(t)}{\partial t^n} = c \frac{\partial x(t)}{\partial t} + d \frac{\partial^2 x(t)}{\partial t^2} + \dots + m \frac{\partial^m x(t)}{\partial t^m} \quad (1.1)$$

Applying the Laplace Transform is yields:

$$aY(S)S + bY(S)S^2 + \dots + nY(S)S^n = cX(S)S + dX(S)S^2 + \dots + mX(S)S^m \quad (1.2)$$

Grouping the terms in X(S) and Y(S) it yields:

$$(aS + bS^2 + \dots + nS^n)Y(s) = (cS + dS^2 + \dots + mS^m)X(s) \quad (1.3)$$

Once the coefficients are separated in two groups, it is possible to write them as a transfer function, a quotient that relates the input and the output of a system:

$$H(S) = \frac{aS + bS^2 + \dots + nS^n}{cS + dS^2 + \dots + mS^m} \quad (1.4)$$

The system equations at the frequency domain can be written as:

$$Y(S) = H(S)X(S) \quad (1.5)$$

The transfer function is composed by two polynomials, the roots of the denominator are known as system zeros and the numerator roots are the system poles. As said before, the poles are the main indicators of the system time response.

Rewriting the numerator polynomial in factored form it yields:

$$H(S) = \frac{aS + bS^2 + \dots + nS^n}{(S - p_1)(S - p_2) \dots (S - p_n)} \quad (1.6)$$

Where $p_1, p_2 \dots p_n$ are the system poles, it is possible to apply the partial fractions method and write the same transfer function as:

$$H(S) = \frac{A}{S - p_1} + \frac{B}{S - p_2} + \dots + \frac{N}{S - p_n} \quad (1.7)$$

The method of partial fractions turns the transfer function into a series of terms with known Laplace transforms, for example:

$$\mathcal{L}^{-1}\left(\frac{A}{S - p_1}\right) = Ae^{p_1 t} \quad (1.8)$$

The time response of the system is composed by exponential modes, the poles can be real or imaginary. The possibilities for system poles and its associated behaviors are explained below:

Real and positive poles: A positive pole will result in a growing exponential mode, what implies in an unstable system, tending to move away from the equilibrium point when a disturbance is applied. The system's behavior is illustrated below:

Figure 2: Time response for a real and positive pole

Real and negative poles: A negative pole results in a damping exponential mode, the expected behavior for stable systems, when a disturbance is applied, the aircraft will tend to return at its equilibrium point, as can be seen in the next figure:

Figure 3: Time response for a real and negative pole

Imaginary poles: The imaginary part of a pole indicates an oscillatory mode. Imaginary poles with real positive parts results in unstable systems, while imaginary poles with negative real parts results in stable systems. The time response for this kind of poles is composed by an oscillatory behavior modulated by an exponential that will grow or decrease the oscillations amplitude according the real part of the pole.

Figure 4: Oscillatory unstable mode

Figure 5: Oscillatory stable mode

Systems with null real parts are neutral, as explained in the theory for static stability. When a pole is complex with null real part, the will oscillate around the equilibrium point with constant amplitude.

The reduction of the disturbance with the time indicates that energy is being dissipated. This condition is called positive damping, in this condition, the forces and moments will oppose the motion of the aircraft and dump the disturbance.

An unstable aircraft has negative aerodynamic damping. It is possible to fly such aircraft, but a customized SAS (Stability Augmented System) will be needed. A SAS is basically an electronic system with sensors and control laws which moves the control surfaces to dump the disturbances. Small corrections are done with the time, but the pilot commands are not influenced by the system.

2. Forces and Moments

2.1. Thrust

Thrust is a reaction force described quantitatively by Newton's second and third laws. When a system expels or accelerates mass in one direction, the accelerated mass will cause a force of equal magnitude but opposite direction on that system. The responsible for this force is the propulsion system, it can use regular engines with propellers, turbine, jet and etc.

The thrust force a relation to the maximum payload the aircraft can carry, but it just accelerates the airplane forward, what increases the relative speed between the aircraft and the wind and increase the lift force.

2.2. Drag

The drag is the opposite force in relation of the thrust, it depends on the square of the velocity and determines what will be the top speed of the aircraft. At the contrary of the thrust force, the drag force is ease to determine, using the following equation:

$$D = \frac{1}{2} \cdot C_d \cdot \rho \cdot S \cdot v^2 \quad (2.1)$$

Where ρ is the air density, S is the wing area, v is the relative speed and C_d will be discussed later.

2.3. Lift

The lift force is generated by the difference of airspeeds between the top and the bottom of the wing. The main responsible for the lift characteristics of an aircraft it the airfoil, its geometry will be decisive to the value of lift and moment coefficients, what will be discussed soon.

The equation for the amount of lift generated by an aircraft is defined by:

$$L = \frac{1}{2} \cdot C_l \cdot \rho \cdot S \cdot v^2 \quad (2.2)$$

The term C_l is analogue to C_d , and will be also discussed later. This force is at the opposite direction of the weight, for this reason, the lift is the force which makes the aircraft takeoff.

2.4. Moments

The standard notation for describing the motion of, and the aerodynamic forces and moments acting upon a flight vehicle are indicated above.

Figure 6: Forces and moments acting in aircraft axis

Virtually all the notation consists of consecutive alphabetic triads:

The variables x, y, z represent coordinates, with origin at the center of mass of the vehicle. The x -axis lies in the symmetry plane of the vehicle¹ and points toward the nose of the vehicle. (The precise direction will be discussed later.) The z -axis also is taken to lie in the plane of symmetry, perpendicular to the x -axis, and pointing approximately down. The y axis completes a right-handed orthogonal system, pointing approximately out the right wing.

The variables u, v, w represent the instantaneous components of linear velocity in the directions of the x, y , and z axes, respectively.

The variables X, Y, Z represent the components of aerodynamic force in the directions of the x, y , and z axes, respectively.

The variables p, q, r represent the instantaneous components of rotational velocity about the x, y , and z axes, respectively.

The variables L, M, N represent the components of aerodynamic moments about the x, y and z axes, respectively.

Although not indicated in the figure, the variables ϕ, θ and ψ represent the angular rotations, relative to the equilibrium state, about the x, y , and z axes, respectively. Thus, $p = \dot{\phi}$, $q = \dot{\theta}$ and $r = \dot{\psi}$, where the dots represent time derivatives.

The velocity components of the vehicle often are represented as angles, as indicated in figure above. The component w can be interpreted as the angle of attack, while the component v can be interpreted by the sideslip angle.

$$\alpha \equiv \tan^{-1} \frac{w}{u} \quad (2.3)$$

$$\beta \equiv \tan^{-1} \frac{v}{V} \quad (2.4)$$

Figure 7: Velocity components

3. Airfoil

The airfoil design is the first step to develop an aircraft, its shape will determine the behavior of the lift, drag and moment coefficients curves.

3.1.Lift Coefficient (C_l)

The lift coefficient is a nondimensional number used to model all of the complex dependencies of shape, inclination and some flow conditions on aircrafts lift. Making a rearrangement of the lift equation where we solve for the lift coefficient in terms of the other variables. The demonstration follows above:

$$L = \frac{1}{2} \cdot C_l \cdot \rho \cdot S \cdot v^2 \rightarrow C_l = \frac{L}{(\frac{1}{2} \cdot \rho \cdot S \cdot v^2)} \quad (3.1)$$

The quantity one half the density times the velocity squared is called the dynamic pressure (Q). The lift coefficient then expresses the ratio of the lift force to the force produced by the dynamic pressure times the area.

3.2.Drag Coefficient (C_d)

The drag force is governed by a the same equation used to calculate the lift force, so the drag coefficient is analogous of the lift coefficient, but related due to drag force. The equation is shown above again:

$$D = \frac{1}{2} \cdot C_d \cdot \rho \cdot S \cdot v^2 \rightarrow C_d = \frac{D}{(\frac{1}{2} \cdot \rho \cdot S \cdot v^2)} \quad (3.2)$$

3.3.Moment Coefficient (C_m)

The moment coefficient is obtained by the same way of the drag and lift coefficients, but instead of a force, a moment is the result of the aerodynamic equation. For most of the analysis in this work, the pitch moment coefficient (C_m) will be used, but in some steps during the stability analysis it will be necessary to use the yaw moment coefficient (C_n).

3.4.Airfoil analysis

As soon as the concepts of Lift, Drag and moment coefficients were understood, an analysis to determine how this coefficients vary with the aircraft's angle of attack is required, not only to know the absolute forces acting on the plane according the angle of attack, but also to determine the derivatives of the lift coefficient with respect of the angle of attack (α). This variation is linear over a range of values, becoming nonlinear to another track. Using the software XFLR5, the graph below was generated from a range of Reynolds number from 10^5 to $3 \cdot 10^5$:

Figure 8: Variation of C_l in different angles of attack and Reynolds number

4. Aircraft Overview

4.1. Conceptual Project

The airplane in question was developed to fulfill the requirements of SAE Aerodesign Brasil, the Brazilian national aerodesign competition. The main objective of the competition is to develop an aircraft that can carry as much water as possible and with the minimum empty weight that the team can build.

4.2. Fuselage

The fuselage carries the hopper (water tank) of the aircraft, supports the tail boom from behind, the engine connected in the firewall and also supports the wing, responsible to sustain the weight of the plane during the flight.

Another component of the fuselage is the main landing gear and the brakes, responsible not also by the plane's movement at the ground, but also to absorb the loads in the landing, the magnitude of this loads were calculated, but it is not the main subject of this work.

The fuselage was built using light and resistant materials, mostly of carbon fiber and Divinycell as can be seen at the next picture:

Figure 9: Bottom view of the fuselage

The hopper is also an important component to observe. Made by a mix of carbon fiber and Kevlar, it is the heaviest component of the aircraft, it directly influences the inertia of the Aircraft, an important parameter when the dynamic stability is analyzed.

Another important parameter that depends on the hopper is the aircraft center of mass. The aircraft's empty weight is 5.2kg, when the hopper is filled the total mass increases to 25.2kg, furthermore, there is a rule in the competition which requires that the airplane center of mass must be in the center of the hopper, it doesn't matter if it is full or empty.

The great mass of water causes some problems, the movement of the liquid inside the hopper can destabilize the aircraft in flight because the mass of the liquid is around five times bigger than the mass of the empty plane, this phenomenon is called slosh. Another problem occurs when the plane tilts and the water goes to the back of the hopper, what will change the original position of the center of mass of the plane. One of the functions of the stability and control analysis is to evaluate if this offset in the

position of the center of mass will not affect the stability of the plane. Thinking about it, the hopper was designed as shown in the picture:

Figure 10: Hopper's Divisions

To prevent Slosh and an offset in the center of mass position, the hopper was divided in five parts, the possible flight configurations are shown in "A", "B", "C" and "D", flying with 100% filled and symmetrical compartments, the water will behave as a rigid body and it will avoid the center of mass displacement and slosh.

4.3.Wing

The aircraft in question has a polyhedral wing, with 4.5m of span, 12.83m of aspect ratio and mean aerodynamic chord of 354mm, responsible to generate a lift force of approximately 280N.

Figure 11: 3D view of wing

The image above shows a CAD model of the wing. The dihedral of 6° was chosen in order to provide more lateral and directional stability. A study of wing's natural frequencies resulted in the design of the compensators installed in the ailerons to avoid flutter.

4.4.Tail

The tail is the main responsible to compensate the pitching moment generated by the wing and the fuselage, its arrangement is defined by **Raymer** as conventional. The initial estimation of tail size depends

on two volume coefficients, C_{VT} and C_{HT} for horizontal and vertical tail respectively. These coefficients are used to determine a possible the area for the control surfaces, once we have:

$$V_V = \frac{L_{VT}S_{VT}}{b_w S_w} \quad (4.1)$$

$$V_H = \frac{L_{HT}S_{HT}}{\bar{C}_w S_w} \quad (4.2)$$

Where "L" is the moment arm for vertical (VT) or horizontal tail (HT), "S" the area of the wing or tail, "b" is the span and " $\bar{C}$ " is the mean chord. **Raymer** also defines a guideline for an initial sizing of the tail using this parameters:

Table 1: volume coefficients for initial sizing

	Horizontal (V_H)	Vertical (V_V)
Sailplane	0.5	0.02
Homebuilt	0.5	0.04
General (single engine)	0.7	0.04
General (twin engine)	0.8	0.07
Agricultural	0.5	0.04
Twin turboprop	0.9	0.08
Flying boat	0.7	0.06
Jet trainer	0.7	0.06
Jet fighter	0.4	0.07
Military cargo / bomber	1.00	0.08
Jet transport	1.00	0.09

The aircraft in study has characteristics of an sailplane, so the initial values were $C_{HT} = 0.5$ and $C_{VT} = 0.02$, this two coefficients can be changed later depending on the stability and control analysis, to improve the effectiveness of the rudder and elevator.

With this information, it's possible to estimate the areas of rudder (S_{VT}) and elevator (S_{HT}), that will be used later in the stability and control analysis.

5. Static Stability And Control

5.1. Static Margin

For any aircraft there is a CG location that provides no change in pitching moment with a variation in the angle of attack, this point is called neutral point (X_{NP}). The static margin is a distance measured in percent of MAC (mean aerodynamic chord) from the center of gravity to the neutral point.

If the CG is located ahead the Neutral Point (positive static margin), the pitching-moment derivative is negative, and the aircraft is stable. For a tailless aircraft ("flying wing"), the neutral point must be ahead the CG, this configuration will make the pitching-moment derivative negative. According **Raymer** a typical transport aircraft has a positive static margin of approximately 5-10% of the MAC.

The negative static margin (0 to -15%) is also used in fighters like the F-16 and F-22, this project concept is known as "relaxed static stability" and uses a control system to deflect the elevator, providing an artificial stability.

The neutral point for fixed stick can be obtained as follows:

$$\frac{X_{NP_{fixed}}}{\bar{c}} = \frac{X_{ac}}{\bar{c}} - \frac{C_{m_{\alpha_f}}}{C_{L_{\alpha_w}}} + V_H \cdot \eta \cdot \frac{C_{L_{\alpha_t}}}{C_{L_{\alpha_w}}} \left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) \quad (5.1)$$

The difference between the stick fixed and stick free neutral point can be determined by:

$$\frac{X_{NP_{fixed}}}{\bar{c}} = \frac{X_{NP_{fixed}}}{\bar{c}} - ((1-f) \cdot V_H \cdot \eta \cdot \frac{C_{L_{\alpha_t}}}{C_{L_{\alpha_w}}} \cdot \left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)) \quad (5.2)$$

Where f is the parameter which will determine if the stick free neutral point is forward or aft the stick fixed neutral point. This parameter was deducted by **Nelson**, and is a function of: the equation of the elevator hinge moment coefficient, effects of angle of attack, elevator deflection and tab angle. The equation can be expressed by:

$$C_{h_e} = C_{h_0} + C_{h_{\alpha_t}} \alpha_t + C_{h_{\delta_e}} \delta_e + C_{h_{\delta_t}} \delta_t \quad (5.3)$$

Where:

$$C_{h_{\alpha_t}} = \frac{dC_h}{d\alpha_t}; \quad C_{h_{\delta_e}} = \frac{dC_h}{d\delta_e}; \quad C_{h_{\delta_t}} = \frac{dC_h}{d\delta_t};$$

C_{h_e} is analogue to the terms C_l , C_d and C_m explained in the beginning of this work. Using the following equation for the hinge moment:

$$H_e = \frac{1}{2} \cdot C_{h_e} \cdot \rho \cdot S_e \cdot v^2 \cdot C_e \quad (5.4)$$

Where C_e is the chord, measured from the hinge line to trailing edge and S_e is the area aft of the hinge line. According **Nelson**, the term f is equal to $\left(1 - \frac{C_{L_{\delta_e}} C_{h_{\alpha_t}}}{C_{L_{\alpha_t}} C_{h_{\delta_e}}}\right)$, and it can be greater or less than the unity, depending on the sign of the hinge coefficients. The derivative of downwash angle with respect to the angle of attack ($\frac{\partial \varepsilon}{\partial \alpha}$) can be simplified by:

$$\frac{\partial \varepsilon}{\partial \alpha} = \frac{2C_{L_{0w}}}{\pi AR} \quad (5.5)$$

Downwash is the chance in direction of the air deflected by the aerodynamic action of an airfoil, wing or helicopter blade in motion, as part of the process of producing lift.

Figure 12: Downwash angle

Once all the parameters are known, we can solve the equations for $X_{NP_{fixed}}$ and $X_{NP_{free}}$ and finally find the static margin that is given by:

$$\text{Stick fixed static margin} = \frac{X_{NP_{fixed}}}{\bar{c}} - \frac{X_{cg}}{\bar{c}} \quad (5.6)$$

$$\text{Stick-free static margin} = \frac{X_{NP_{free}}}{\bar{c}} - \frac{X_{cg}}{\bar{c}} \quad (5.7)$$

The values found for the aircraft in study were 14.4% for stick fixed and 12.9% for stick-free, what can be considered higher than the limit established by Raymer, but as said before, it qualifies an typical transport aircraft. The airplane in question was designed to carry much more weight than a simple transport aircraft. According the University of Stanford Stability and Control guidelines, a static margin between 5-15% of MAC is considered acceptable.

5.2. Longitudinal Stability

We denote by longitudinally stable the aircraft that receives a disturbance and tends to generate a moment in the opposite direction, in other words, the longitudinal stability is achieved when $\frac{\partial C_m}{\partial \alpha} < 0$

Each component of the aircraft contributes to the resulting pitching moment generated. This contributions are represented by a constant and a variable part that depends on the angle of attack.

For the wing, this two terms are

$$C_{m_{0w}} = C_{m_{acw}} + C_{L_{0w}} \left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}} \right) \quad (5.8)$$

$$C_{m_{\alpha w}} = C_{L_{\alpha w}} \cdot \left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}} \right) \quad (5.9)$$

The wing contribution for the aircraft pitching moment is equal to $C_{m_{\alpha w}} \alpha + C_{m_{0w}}$.

For the elevator, the computation of this terms is:

$$C_{m_{0t}} = \eta \cdot S_{HT} + C_{L\alpha_t} \left(\frac{2 \cdot C_{L\alpha_w}}{\pi \cdot AR_w} + i_w + i_t \right) \quad (5.10)$$

$$C_{m_{\alpha_t}} = \eta \cdot S_{HT} + C_{L\alpha_t} \left(1 - \frac{2 \cdot C_{L\alpha_w}}{\pi \cdot AR_w} \right) \quad (5.11)$$

Resulting in an analogue expression that says the elevator contribution to pitching moment is:

$$C_{m_{\alpha_t}} \alpha + C_{m_{0t}} \quad (5.12)$$

The fuselage has a small contribution to the pitching moment, depending on its size, it can assumed as zero. This contribution is expressed by:

$$C_{m_{\alpha_f}} \alpha \quad (5.13)$$

Where $C_{m_{\alpha_f}} = \frac{K_f \cdot W_f^2 \cdot l_f}{\bar{c} \cdot S_w}$ depends on empiric and geometric fuselage parameters, better explained by Etkin.

The next figure shows the behavior of the aircraft computing all the pitching moment contributions.

Figure 13: Aircraft behavior for longitudinal static stability

The figure shows that the aircraft is stable for this kind of movement and flies in a desirable condition.

The trim angle is by definition, the angle of attack where the moment coefficient is null. A negative trim angle says that without any disturbance the airplane will fly with its nose down, what is undesirable.

As said in the beginning of this work, the development process of an aircraft is interactive, and the stability and control analysis frequently find inconsistencies. There are some ways to correct an unacceptable behavior in this kind of movement, some considerable solutions are: increasing the wing incidence angle (i_w), decreasing the elevator trim angle or incidence angle (i_t).

5.3. Longitudinal Control

The main responsible to stabilize the aircraft longitudinally is the elevator. Its efficiency is an important factor for the control of the plane.

The term to be analyzed is $C_{m\delta_e}$, which shows the variation of the moment generated by the tail when the elevator is deflected by an angle δ . The larger is the value of $C_{m\delta_e}$, the more effective is the generation pitching moment for the stabilization. For the airplane in question, the value of $C_{m\delta_e}$ is 1.02 rad^{-1} , there are no specified ranges for this value, but one way to analyze if the elevator is well scaled is the Trim Abacus.

Considering the moment coefficient as:

$$C_m = C_{m\delta_e} \cdot \delta_e + C_{m\alpha} \cdot \alpha + C_{m0} \quad (5.14)$$

Where:

- C_{m0} is the moment coefficient for $\delta = 0$ and $\alpha = 0$.
- $C_{m\alpha}$ is the derivative of moment coefficient with respect to the angle of attack.
- $C_{m\delta_e}$ is the derivative of moment coefficient with respect to the elevator deflection.
- δ_e is the deflection of the elevator.

The Trim Abacus for this aircraft is shown in figure below:

Figure 14: Trim abacus

The Trim Abacus is also used to determine the default angle of attack in flight, this angle can be chosen using different concepts, the plane can be trimmed, for example, to fly at $\alpha = 0$, this aircraft flies at 3 degree of angle of attack, which maximizes the wing power factor.

5.4. Lateral and Directional stability

The lateral and directional stability analysis deal with the yawing and rolling moments. The first one to be analyzed will be the yawing moment, according the theory explained by **Nelson**, an aircraft is directionally stable if the derivative of yawing moment coefficient with respect to the side slip angle (C_{n_β}) is positive, what will generate a restoring moment when the side slip angle is increased, bringing the aircraft to its natural position.

To have a stable rolling mode, the aircraft's derivative of the rolling moment with respect with the side slip angle (C_{l_β}) must be negative. This situation also generates the restoring moment to level the aircraft when a disturbance makes it roll.

C_{n_β} is composed by two factors: the wing-fuselage and the rudder contributions.

$$C_{n_{\beta_{wf}}} = - \frac{k_n \cdot k_{RL} \cdot S_{fs} \cdot l_f}{S_w \cdot b} \quad (5.15)$$

$$C_{n_{\beta_v}} = S_{VT} \cdot C_{L\alpha_v} \cdot \eta_v \cdot \left(1 + \frac{\partial \sigma}{\partial \beta}\right) \quad (5.16)$$

Where:

- k_n is an empirical wing-body interference factor that is function of fuselage geometry.
- k_{RL} is an empirical correction factor that is a function of the fuselage Reynolds number.
- S_{fs} is the projected side area of the fuselage.
- l_f is the length of the fuselage.
- $\frac{\partial \sigma}{\partial \beta}$ is the derivative of the sidewash angle with respect of the sideslip angle.

The sidewash angle is analogous to the downwash angle for the horizontal tail plane. It is caused by the flow field distortion due to the wing and fuselage.

Figure 15: Sidewash vortices

The major contributor for the coefficient C_{l_β} is the wing dihedral angle Γ . The dihedral angle is defined as the spanwise inclination of the wing with respect to the horizontal. The following graph shows

the variation of C_n and C_l with respect to the sideslip angle. Observing the values of the derivatives, it can be concluded that the aircraft is laterally stable.

Figure 16: Moment coefficient values for lateral mode

*Despite of the same representation, the moment coefficient C_l has no relation with the Lift coefficient.

5.5. Directional control

The size of the rudder is determined by the directional control requirements. The rudder control power must be enough to accomplish the requirements listed in the table above:

Table 2: Directional control requirements.

Rudder Requirements	Result in design
Adverse Yaw	When the aircraft makes a turn maneuver, the ailerons create a yawing moment that opposes the turn, it is called adverse yaw. The rudder must have sufficient power to overcome the adverse yaw.
Crosswind land	To maintain the aircraft in a straight way during a crosswind landing, the pilot must fly the aircraft at a sideslip angle. The rudder also may have power to trim the aircraft in this condition.
Asymmetric power condition	This cases occurs in multiple engines aircraft when one of the engines stops working. The rudder may be able to generate yawing moment to correct the moment generated by the asymmetric power condition.
Spin Recovery	The rudder may be able to oppose the spin rotation.

The rudder control effectiveness is the rate of change of yawing moment with rudder deflection angle:

$$\frac{\partial C_{L_v}}{\partial \delta_r} = C_{L_v} \tau \quad (5.17)$$

Where C_{L_v} is the "Lift coefficient" for the vertical tail and τ is a factor that is function of the reason of the surface area and the lifting surfacing area defined by **Nelson**.

5.6. Roll Control

The responsible surfaces to perform the roll control are the ailerons. Located at the wing, when the pilot gives a roll command, the ailerons deflects to opposite directions modifying the lift distribution at the wing and making the aircraft roll, as can be seen in the figure below:

Figure 17: Aileron deflection effect

A simple way to calculate the roll power control for an aileron is to solve an integral to compute all the moment it can generate. The roll control power $C_{l_{\delta_a}}$ can be calculated as follows:

$$C_{l_{\delta_a}} = \frac{2C_{L_{\alpha_w}}\tau}{Sb} \int_{Aileron\ Span} cydy \quad (5.18)$$

Where:

- c is the chord of the aileron at the point.
- y is the aileron span.
- y_1 and y_2 are the beginning and the end of the aileron.
- S is the wing area.
- b is the wingspan.
- τ is also a factor that is function of the reason of the surface area and the lifting surfacing area defined by **Nelson**.

The value found for this aircraft was 0.036 rad^{-1} . The bigger the power roll control, the better the aircraft will behave, but once again, there is no specific rule to measure if this value is good or not. The better way to know if the aileron is well sized or not is to compare the achieved value with historical guidelines.

The information of the values for all the control power coefficients are one of the secrets of the project of an aircraft. For this reason, all the values obtained for this aircraft are compared to the values achieved in past projects of the team. All the values for longitudinal, directional and roll control were considered good compared with the past aircrafts developed by the team.

6. Dynamic Stability

The calculations for dynamic stability modes of motion analysis certifies that the longitudinal, lateral and directional poles for the dynamic stability are acceptable.

6.1. Aerodynamic force and moment representation

The method of representing the aerodynamic forces and moments by stability coefficients was introduced over three-quarters of a century ago. The technique assumes that the aerodynamic forces and moments can be expressed as a function of the instantaneous values of the disturbance variables. This variables represents the changes from reference conditions of the translational and angular velocities, control surfaces deflection and their derivatives.

Assuming we can express the aerodynamic forces and moments as a Taylor series expansion of the perturbation variables about the reference equilibrium condition. For example, the change in force in the x direction can be expressed as:

$$\Delta X(u, \dot{u}, w, \dot{w}, \dots, \delta_e, \dot{\delta}_e) = \frac{\partial X}{\partial u} \cdot \Delta u + \frac{\partial X}{\partial \dot{u}} \cdot \Delta \dot{u} + \dots + \frac{\partial X}{\partial \delta_e} \cdot \Delta \delta_e + (\text{High order terms}) \quad (6.1)$$

The term $\frac{\partial X}{\partial u}$, the stability derivative, is evaluated at the reference flight condition. The effect of changing the velocity u in the force in x is $\frac{\partial X}{\partial u} \cdot \Delta u$, this derivative can also be expressed in terms of the stability coefficient C_{x_u} :

$$\frac{\partial X}{\partial u} = C_{x_u} \frac{1}{u_0} QS \quad (6.2)$$

Where C_x is the force coefficient in the x axis and $C_{x_u} = \frac{\partial C_x}{\partial (u/u_0)}$ is the stability coefficient that relates the force variation in x and the velocity u. Note that the stability derivative has dimensions, but the coefficient is nondimensional.

The same idea is valid for the aerodynamic moments, for example, to calculate the pitching moment in terms of the perturbation variables:

$$M(u, v, w, \dot{u}, \dot{v}, \dot{w}, p, q, r, \delta_e, \delta_r) = \frac{\partial M}{\partial u} \cdot \Delta u + \frac{\partial M}{\partial v} \cdot \Delta v + \dots + \frac{\partial M}{\partial p} \cdot \Delta p + \dots \quad (6.3)$$

It is easy to understand that terms such as $\frac{\partial M}{\partial v} \cdot \Delta v$ and $\frac{\partial M}{\partial p} \cdot \Delta p$ are not significant for an airplane and can be neglected. It's important to notice that all this assumptions are valid considering the small disturbance theory, which assume that the motion of an aircraft consists of small deviations about a steady flight condition, it allows to linearize the equations of motion and use only the first term of a Taylor series of each disturbance variable.

The next topics will show how to calculate the most important derivatives and stability coefficients for an aircraft. Some of them will not be deducted, but it will be listed in the respective tables.

6.2. Derivatives due change in forward speed

The aerodynamic forces acting in the x axis are the drag (D) force and thrust (T), both are function of the forward speed (u). The change in the X force, due the change in forward speed can be expressed as:

$$\frac{\partial X}{\partial u} = -\frac{\partial D}{\partial u} + \frac{\partial T}{\partial u} \quad (6.4)$$

This derivative is called speed damping derivative, and can be rewritten as:

$$\frac{\partial X}{\partial u} = -\frac{\rho S}{2} \left(u_0^2 \frac{\partial C_D}{\partial u} + 2u_0 C_{D_0} \right) + \frac{\partial T}{\partial u} \quad (6.5)$$

Where the subscript 0 indicates de reference condition. The respective stability coefficient related to this derivative is C_{x_u} and yields:

$$C_{x_u} = -(C_{D_u} + 2C_{D_0}) + C_{T_u} \quad (6.6)$$

Where $C_{D_u} = \frac{\partial C_D}{\partial (u/u_0)}$ and $C_{T_u} = \frac{\partial C_T}{\partial (u/u_0)}$ are the changes in the Drag and Thrust coefficients with forward speed. This coefficients becomes nondimensional by differentiating with respect (u/u_0) . The coefficient C_{D_0} can be estimated analyzing the variation of the Drag coefficient with the Mach number:

$$C_{D_u} = M \frac{\partial C_D}{\partial M} \quad (6.7)$$

The Mach number is the ratio of speed of an object and the speed of sound. The Thrust term C_{T_u} can be assumed as 0 for a gliding flight. It also is a good approximation for jet powered aircrafts. For the aircraft in question (powered by propeller), it can be considered equal to the negative of the reference Drag coefficient (i.e. $C_{T_u} = -C_{D_0}$).

The change in Z force with the forward speed can be calculated as:

$$\frac{\partial Z}{\partial u} = -\frac{1}{2} \rho S u_0 (C_{L_u} + 2C_{L_0}) \quad (6.8)$$

And the respective stability coefficient is:

$$C_{Z_u} = -(C_{L_u} + 2C_{L_0}) \quad (6.9)$$

The coefficient C_{L_u} arises with the change in lift coefficient with the Mach number. C_{L_u} can be estimated as:

$$C_{L_u} = M \frac{\partial C_L}{\partial M} \quad (6.10)$$

$$C_{L_u} = \frac{M^2}{1 - M^2} C_{L_0} \quad (6.11)$$

Following the same path, the change in the pitching moment due the variations in the forward speed can be written as:

$$\frac{\partial M}{\partial u} = C_{m_u} \rho S \bar{c} u_0 \quad (6.12)$$

The coefficient C_{m_u} can be expressed as follows:

$$C_{m_u} = M \frac{\partial C_{m_u}}{\partial M} \quad (6.13)$$

6.3. Derivatives due to the pitching velocity

Following the same logic of the derivatives due change in forward speed, the one with respect the variation of pitching velocity (q) will be calculated at the same way.

The change in the Z force and pitching moment coefficients are respectively C_{z_q} and C_{m_q} . The pitching moment of the aircraft affects the wing and vertical tail behaviors. The coefficients are expressed as follows:

$$C_{z_q} = -2C_{L_{\alpha_t}}\eta V_H \quad (6.14)$$

$$C_{m_q} = \frac{-2C_{L_{\alpha_t}}\eta V_H l_t}{\bar{c}} \quad (6.15)$$

6.4. Derivatives due the time rate of change in the angle of attack

As the wing angle of attack changes, the circulation around it will be altered. The lag in the wing downwash to arrive at the tail is the main responsible to arise the stability coefficients $C_{m_{\dot{\alpha}}}$ and $C_{z_{\dot{\alpha}}}$. The change in circulation alters the downwash at the tail, however, it takes a finite time to happen. If the aircraft is traveling with a velocity u_0 , the trailing vortex will take the increment time $\Delta t = l_t/u_0$ to arrive at the tail.

This lag can be written as:

$$\Delta\alpha_t = \frac{d\varepsilon}{dt}\Delta t \quad (6.16)$$

Where $\Delta t = \frac{l_t}{u_0}$ and so:

$$\Delta\alpha_t = \frac{d\varepsilon}{d\alpha}\dot{\alpha}\frac{l_t}{u_0} \quad (6.17)$$

The change in Z force coefficient can be expressed as:

$$\Delta C_z = \frac{\Delta L_t}{QS} \quad (6.18)$$

The change in the Lift force at the tail can be expressed as:

$$\Delta L_t = C_{L_{\alpha_t}}\Delta\alpha_t Q_t S_t \quad (6.19)$$

The equations can be group and the fact that the reason of the dynamic pressures Q/Q_t can be expressed as a factor η :

$$\Delta C_z = C_{L_{\alpha_t}} \frac{d\varepsilon}{d\alpha} \eta \frac{l_t}{u_0} \frac{S_t}{S} \dot{\alpha} \quad (6.20)$$

$$C_{z_{\dot{\alpha}}} \equiv \frac{\partial C_z}{\partial \left(\frac{\dot{\alpha} \bar{c}}{2u_0} \right)} = \frac{2u_0}{\bar{c}} \frac{\partial C_z}{\partial \dot{\alpha}} \quad (6.21)$$

$$C_{z_{\dot{\alpha}}} = -2V_H \eta C_{L_{\alpha_t}} \frac{\partial \varepsilon}{\partial \alpha} \quad (6.22)$$

The same calculations can be done for the pitching moment variation due the lag in downwash field:

$$C_{m_{\dot{\alpha}}} \equiv \frac{\partial C_m}{\partial \left(\frac{\dot{\alpha} \bar{c}}{2u_0} \right)} = \frac{2u_0}{\bar{c}} \frac{\partial C_m}{\partial \dot{\alpha}} \quad (6.23)$$

$$C_{m_{\dot{\alpha}}} = -2V_H \eta \frac{l_t}{\bar{c}} \frac{\partial \varepsilon}{\partial \alpha} \quad (6.24)$$

The tail contribution for this kind of motion is much higher than the other plane parts, for this reason, to obtain an estimate for the complete airplane contribution, these coefficients are increase by 10%.

The next table resumes the expressions for the longitudinal stability coefficients:

Table 3: Equations for longitudinal stability coefficients

	X-Force Derivatives	Z-Force Derivatives	Pitching moment Derivatives
u	$C_{x_u} = -(C_{D_u} + 2C_{D_0}) + C_{T_u}$	$C_{z_u} = -(\frac{M^2}{1-M^2} C_{L_0} + 2C_{L_0})$	$C_{m_u} = M \frac{\partial C_{m_u}}{\partial M}$
α	$C_{x_\alpha} = C_{L_0} - \frac{2C_{L_0} C_{L_\alpha}}{\pi e AR}$	$C_{z_\alpha} = -(C_{L_\alpha} + C_{D_0})$	$C_{m_\alpha} = C_{m_{\alpha_t}} + C_{m_{\alpha_w}} + C_{m_{\alpha_{fus}}}$
$\dot{\alpha}$	0	$C_{z_{\dot{\alpha}}} = -2V_H \eta C_{L_{\alpha_t}} \frac{\partial \varepsilon}{\partial \alpha}$	$C_{m_{\dot{\alpha}}} = -2V_H \eta \frac{l_t}{\bar{c}} \frac{\partial \varepsilon}{\partial \alpha}$
q	0	$C_{z_q} = -2C_{L_{\alpha_t}} \eta V_H$	$C_{m_q} = \frac{-2C_{L_{\alpha_t}} \eta V_H l_t}{\bar{c}}$
α_e	0	$C_{z_{\delta_e}} = -2V_H \eta \frac{l_t}{\bar{c}} \frac{\partial \varepsilon}{\partial \alpha}$	$C_{m_{\delta_e}} = -V_H \eta \frac{dC_{L_t}}{d\delta_e}$

6.5. Derivatives due to the rolling rate

The stability coefficients C_{y_p} , C_{n_p} and C_{l_p} arise due to the rolling angular velocity (p). When the aircraft rolls, the motion causes a linear velocity distribution over the vertical, horizontal and wing surfaces, causing a local change in the Lift distribution and the moment around the center of gravity. This parameters have strong influence of the aircraft geometry and are deducted by **Caughey** as:

$$C_{y_p} = C_L \frac{AR \cos \Lambda}{AR + 4 \cos \Lambda} \tan \Lambda \quad (6.25)$$

Where Λ is the wing sweep angle.

$$C_{n_p} = -\frac{C_L}{8} \quad (6.26)$$

And

$$C_{l_p} = \frac{C_{L_\alpha}}{12} \frac{1 + 3\lambda}{1 + \lambda} \quad (6.27)$$

Where λ is the taper ratio, a reason between the tip chord and the root chord.

6.6. Derivative due the yawing rate

The Yawing angular velocity stability coefficients C_{y_r} , C_{n_r} and C_{l_r} are analogous to the rolling coefficients C_{y_p} , C_{n_p} and C_{l_p} . A positive Yaw variation causes a negative sideslip angle on the vertical tail. The side force acting on the aircraft can be expressed as:

$$Y = -C_{L_{\alpha_v}} \Delta \beta Q_v S_v \quad (6.28)$$

Where $\beta = -\frac{r l_v}{u_0}$ for a positive yawing rate. Substituting this term in the equation:

$$C_y = \frac{C_{L_{\alpha_v}} Q_v S_v \frac{r l_v}{u_0}}{Q S} \quad (6.29)$$

And again the quotient between the dynamic pressures can be written as η_v , for the vertical tail.

$$C_y = \frac{C_{L_{\alpha_v}} S_v \frac{r l_v}{u_0}}{S} \eta_v \quad (6.30)$$

The respective stability coefficient is defined by:

$$C_{y_r} \equiv \frac{\partial C_y}{\partial (r^b/2u_0)} \quad (6.31)$$

Solving the derivative:

$$C_{y_r} = 2 C_{L_{\alpha_v}} \frac{S_v l_v}{S b} \eta_v \quad (6.32)$$

The term C_{n_r} is calculated by a similar way. This coefficient represents a change in yawing moment caused by a nondimensional yaw rate ($r^b/2u_0$). This yaw moment caused by the yaw rate is a result of the sideslip angle induced on the vertical tail. A positive force causes a negative yawing moment.

$$N = C_{L_{\alpha_v}} \Delta \beta Q_v l_v \quad (6.33)$$

Remembering that $\beta = -\frac{r l_v}{u_0}$, the quotient between the dynamic pressures is η_v :

$$C_{n_r} = C_{L_{\alpha_v}} \frac{S_v l_v}{S b} \eta_v \quad (6.34)$$

The factor $\frac{S_v l_v}{S b}$ is known as the vertical tail volume ratio (V_v), explained in the beginning of this work. All the coefficients for lateral stability can be found in the next table:

Table 4: Equations for lateral stability coefficients

	Y force derivatives	Yawing moment derivatives	Rolling moment derivatives
β	$C_{y_\beta} = C_{L_{\alpha_v}} \eta \frac{S_t}{S} \left(\frac{d\sigma}{d\beta} \right)$	Calculated for static stability	Calculated for static stability
p	$C_{y_p} = C_L \frac{AR \cos \Lambda}{AR + 4 \cos \Lambda} \tan \Lambda$	$C_{n_p} = -\frac{C_L}{8}$	$C_{l_p} = \frac{C_{L_\alpha}}{12} \frac{1 + 3\lambda}{1 + \lambda}$
r	$C_{y_r} = 2 C_{L_{\alpha_v}} \frac{S_v l_v}{S b} \eta_v$	$C_{n_r} = C_{L_{\alpha_v}} \frac{S_v l_v}{S b} \eta_v$	$C_{y_r} = \frac{C_L}{4} - 2 \frac{l_v z_v}{b^2} C_{y_{\beta tail}}$
δ_a	0	$C_{y_\beta} = 2 K C_{L_0} C_{l_{\delta a}}$	$C_{l_{\delta a}} = \frac{2 C_{L_{\alpha_w}} \tau}{S b} \int_{Aileron Span} c y dy$
δ_r	$C_{n_r} = C_{L_{\alpha_v}} \frac{S_v}{S} \tau$	$C_{n_{\delta_r}} = C_{L_{\alpha_v}} \eta_v V_v \tau$	$C_{l_{\delta_r}} = C_{L_{\alpha_v}} \frac{S_v z_v}{S b} \tau$

* The parameter K is an empirical factor, function of the aspect ratio, the curves to estimate its value can be found at **Nelson**.

The next two tables explicit the longitudinal and lateral derivatives:

Table 5: Longitudinal derivatives

$X_u = \frac{-(C_{Du} + 2C_{D0})QS}{mu_0} [s^{-1}]$	$X_w = \frac{-(C_{Du} + C_{L0})QS}{mu_0} [s^{-1}]$
$Z_u = \frac{-(C_{Lu} + 2C_{L0})QS}{mu_0} [s^{-1}]$	
$Z_w = \frac{-(C_{Lw} + 2C_{D0})QS}{mu_0} [s^{-1}]$	$Z_{\dot{w}} = c_{z\dot{w}} \frac{c}{2u_0} \frac{QS}{mu_0}$
$Z_{\dot{\alpha}} = u_0 Z_w \left[\frac{m}{s^2} \right]$	$Z_{\dot{\alpha}} = u_0 Z_{\dot{w}} \left[\frac{m}{s} \right]$
$Z_q = c_{zq} \frac{c}{2u_0} \frac{QS}{m} \left[\frac{m}{s^2} \right]$	$Z_{\delta_e} = c_{z\delta_e} \frac{QS}{m} \left[\frac{m}{s^2} \right]$
$M_u = c_{mu} \frac{QSc}{u_0 I_y} [m^{-1}s^{-1}]$	
$M_w = c_{mw} \frac{QSc}{u_0 I_y} [m^{-1}s^{-1}]$	$M_{\dot{w}} = c_{m\dot{w}} \frac{\bar{c}}{2u_0} \frac{QSc}{u_0 I_y} [m^{-1}]$
$M_{\dot{\alpha}} = u_0 M_w [s^{-2}]$	$M_{\dot{\alpha}} = u_0 M_{\dot{w}} [s^{-1}]$
$M_q = c_{mq} \frac{\bar{c}}{2u_0} [s^{-1}]$	$M_{\delta_e} = c_{m\delta_e} \frac{QSc}{I_y} [s^{-2}]$

Table 6: Lateral derivatives

$Y_{\beta} = \frac{QSc_{y\beta}}{m} \left[\frac{m}{s^2} \right]$	$N_{\beta} = \frac{QSc_{n\beta}}{I_z} [s^{-2}]$	$L_{\beta} = \frac{QSc_{l\beta}}{I_x} [s^{-2}]$
$Y_p = \frac{QSc_{yp}}{2mu_0} \left[\frac{m}{s} \right]$	$N_p = \frac{QSc_{np}}{2u_0 I_z} [s^{-1}]$	$L_p = \frac{QSc_{lp}}{2u_0 I_x} [s^{-1}]$
$Y_r = \frac{QSc_{yr}}{2mu_0} \left[\frac{m}{s} \right]$	$N_r = \frac{QSc_{nr}}{2u_0 I_z} [s^{-1}]$	$L_r = \frac{QSc_{lr}}{2u_0 I_x} [s^{-1}]$
$Y_{\delta_a} = \frac{QSc_{y\delta_a}}{m} \left[\frac{m}{s^2} \right]$	$Y_{\delta_r} = \frac{QSc_{y\delta_r}}{m} \left[\frac{m}{s^2} \right]$	
$N_{\delta_a} = \frac{QSc_{n\delta_a}}{I_z} [s^{-2}]$	$N_{\delta_r} = \frac{QSc_{n\delta_r}}{I_z} [s^{-2}]$	
$L_{\delta_a} = \frac{QSc_{l\delta_a}}{I_x} [s^{-2}]$	$L_{\delta_r} = \frac{QSc_{l\delta_r}}{I_x} [s^{-2}]$	

6.7. Longitudinal motion

The longitudinal motion analysis for dynamic stability covers two modes of motion, the short period and the long period (Phugoid). The first one analyses the aircraft behavior when a disturbance in the angle of attack is applied, the second one studies if the plane tends to return to the equilibrium point when a disturbance in the altitude is applied.

Nelson uses state variable representation to approximate the equations of motion for this modes. This method groups the equations of motion in vectors and matrixes as shown below:

$$\dot{X} = AX + BU \quad (6.35)$$

Where X is the state vector, U is the control vector, and the matrixes A (state matrix) and B contain the aircraft stability derivatives. The model for the longitudinal modes of motion can be represented as:

$$\begin{bmatrix} \Delta \ddot{u} \\ \Delta \ddot{w} \\ \Delta \dot{q} \\ \Delta \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_w & 0 & -g \\ Z_u & Z_w & u_0 & 0 \\ M_u + M_{\dot{w}}Z_w & M_w + M_{\dot{w}}Z_w & M_q + M_{\dot{w}}u_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta \theta \end{bmatrix} + \begin{bmatrix} X_{\delta} & X_{\delta_t} \\ Z_{\delta} & Z_{\delta_t} \\ M_{\delta} + M_{\dot{w}}Z_{\delta} & M_{\delta_t} + M_{\dot{w}}Z_{\delta_t} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \delta_t \end{bmatrix} \quad (6.36)$$

The force derivatives Z_q and $Z_{\dot{w}}$ were neglected because they contribute very little for this modes of motion. The terms $\Delta \delta$ and $\Delta \delta_t$ are the aerodynamic and propulsive controls. To infer the behavior of the system modelled by this equations, the first action is to discover the poles, which can be calculated extracting the eigenvalues of the State Matrix. Another way to extract the poles is to calculate the natural frequencies and the damping factors of the system. Once this two items are known the poles can be calculated as follows:

$$\lambda = -\xi \omega_n \pm (\omega_n \cdot \sqrt{1 - \xi^2}) \quad (6.37)$$

For short period mode, the expressions for the natural frequency and damping ratio are:

$$\omega_n = \sqrt{\frac{Z_{\alpha} M_q}{u_0} - M_{\alpha}} \quad (6.38)$$

$$\xi = -\frac{M_q + M_{\dot{\alpha}} + \frac{Z_{\alpha}}{u_0}}{2\omega_n} \quad (6.40)$$

The short period poles for the aircraft in question were: $-2,66 \pm 2,71$. The natural frequency and damping ratio were: $\omega_n = 3.8$ rad/s and $\xi = 0.7$. According the short period flying qualities defined in **Etkin** and shown below:

Figure 18: Flying qualities for short period mode

The time response for short period for a 3 degree disturbance in the angle of attack can be seen as follows:

Figure 19: Short period time response

The time response shows a highly damped behavior, what is totally desirable for this mode of motion. Once the aircraft receives a disturbance in the angle of attack, it must return at natural the trim angle as soon as possible, inside the delimiters fixed by the flying qualities.

The long period poles also can be extracted from the state matrix or by calculating the natural frequency and the damping ratio. The equations for this terms in long period mode are:

$$\omega_n = \sqrt{\frac{Z_{u\dot{g}}}{u_0}} \quad (6.41)$$

$$\xi = -\frac{X_u}{2\omega_n} \quad (6.42)$$

The poles for long period mode were: $-0.091 \pm 0.918i$, and the natural frequency and damping ratio were: $\omega_n = 0.922$ rad/s and $\xi = 0.0987$. The aircraft can also be considered stable for this mode, as shown in the figure below:

Figure 20: Long period time response

Other flight qualities associated with longitudinal motion are the period of the oscillations, the time of half amplitude and number of oscillations until the time of half amplitude. This qualities are not much important for the short period mode, because of its typical reduced number of oscillations, but long period mode usually has oscillations with higher amplitudes and low damping factors, in other words, the long period responses can last many seconds and it is interesting to know how much time it will oscillate until the stabilization. The formulas and a table with the information for this aircraft can be found below:

$$t_{\frac{1}{2}} = \frac{0.69}{|Re(\lambda)|} \quad (6.43)$$

$$Period = \frac{2\pi}{\omega} \quad (6.44)$$

$$N_{\frac{1}{2}} = \frac{t_{\frac{1}{2}}}{Period} \quad (6.45)$$

Table 7: Complementary flight qualities

	Long-period	Short-period
$t_{\frac{1}{2}} (s)$	10,6	0,13
Period (s)	6,41	1,22
$N_{\frac{1}{2}}$	1,65	0,11

Nelson also classifies the aircrafts into classes, using categories and levels to give a guideline about typical values of natural frequencies and damping factors for each class of airplane. The problem is that most of this classifications were done using feedbacks given by pilots flying typical aircrafts. It's a very subjective classification what depends on the type of the aircraft and the mission it needs to accomplish.

Aerodesign airplanes are a very specific kind of aircrafts. If a regular person look at an aerodesign plane, it will certainly will say it is a kind of drone, this is a very new kind of aircraft that is not yet qualified in the literature. Another interesting point is that this aircraft was developed to carry around five times its empty weight, there is no plane in the world with this structural efficiency (i.e. the reason between the empty weight and the MTOW). Summarizing, the aerodesign aircrafts cannot be classified as the bigger aircrafts because of its size, and the classifications about drones are also insufficient. The solution found were to compare the actual aircraft with other planes developed by the team.

Concluding, according the calculated poles, the time responses, the flying qualities listed in the literature, the pilot's opinions about the airplane and the knowledge acquired in the development of seven aircrafts to accomplish 3 different kinds of mission, the aircraft shown to be very stable and robust. Considering that having negative poles is not the only requirement a reliable aircraft.

6.8.Lateral-directional motion

The lateral-directional dynamic of an aircraft is composed by three modes: Roll, Dutch Roll and Spiral. The rolling mode is the most easy to notice, it occurs when a disturbance makes the aircraft rotate in the x-axis. The Dutch Roll mode is a combination of rolling and yawing modes, making the plane behave as shown in the next figure:

Figure 21: Dutch Roll mode

Once the Dutch Roll poles are positive, the aircraft's oscillations tends to grow and make the plane unstable, a famous accident caused by positive Dutch Roll poles were the case of the US Air force KC-135, in 2013, when the crew failed to recover the plane from a Dutch Roll and caused a fatal crash.

The spiral mode is the aircraft's tendency to make a curve when it is flying in a straight line. The figure below shows the three possibilities for a spiral behavior:

Figure 22: Spiral mode

At the flight, the aircraft may suffer disturbances at sideslip angle, the first behavior listed above makes the bank angle increase, the sideslip angle also increases, making the aircraft fly in a spiral. This behavior normally is caused by a large value of C_{n_β} or an inadequate value of C_{l_β} . The second one shows a directional divergence, caused by a low value of C_{n_β} . The last one shows a desirable behavior, when a disturbance in sideslip angle is applied, the aircraft tends to return to its equilibrium point. The expected behavior of each mode is:

- The rolling mode is usually heavily damped;
- The spiral mode is only lightly damped, or may even be unstable. The dihedral effect has an important stabilizing influence;
- The Dutch Roll is lightly damped and oscillatory. For this mode, dihedral effect is generally destabilizing.

The state variable model for the Lateral-directional dynamic motion are deducted by **Caughey** and shown below:

$$\begin{bmatrix} \Delta v' \\ \Delta p' \\ \Delta r' \\ \Delta \phi' \end{bmatrix} = \begin{bmatrix} Y_v & 0 & -u_0 & g \\ L_v & L_p & L_r & 0 \\ N_v & N_p & N_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta \phi \end{bmatrix} + \begin{bmatrix} 0 & Y_{\delta_r} \\ L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \delta_a \\ \Delta \delta_r \end{bmatrix} \quad (6.46)$$

As the same way, the poles for lateral-directional modes are the eigenvalues of the state matrix. The poles for lateral-directional motions are shown in the diagram below:

Figure 23: Poles for lateral-directional motions

The diagram shows that all the modes for lateral-directional dynamics are considered stable. The values for each pole were:

- Dutch Roll: $-0.17 \pm 7.13j$
- Spiral: -0.012
- Roll: -36.95

Considering that all the poles have negative real parts and the definitions for each mode of motion done by **Caughey**, the behaviors for lateral-directional modes of motion were considered acceptable.

7. Conclusion

As said in the beginning of this work, the process to develop an aircraft is iterative and most of times based in acquired knowledge. For this reason, when the characteristics of this aircraft are not defined by the literature, the experience and the lessons learned of pasts projects was used as guidelines to determine if the values found are satisfactory.

For the static stability, the aircraft shown to be longitudinally stable, flying at 3 degrees of default angle of attack, this value was considered satisfactory because it maximizes the wing power factor. The lateral and directional static stability analysis were also positive. The combination of this analysis allows to affirm that the aircraft is statically stable.

Analyzing the control power of the surfaces, and considering that the literature does not defines acceptable values for them, the experience of the past 7 aircrafts designed for this competition was used to compare the obtained values and it's possible to say that the aircraft has well sized control surfaces.

For the dynamic stability, according the poles for long and short period, the applicable definitions, the definitions done by **Caughey** and the know-how of the past projects, the aircraft also shown to be dynamically stable.

Considering all the results from the calculations done in this work and the positive pilot's feedbacks, the aircraft in question fulfilled all the requirements of the stability and control analysis and shown to be robust and reliable.

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