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# A
# Paper apresentado no $67^{th}$ ABM International Congress

# FATIGUE SPECIMENS DESIGNED TO INDUCE NON-PROPAGATING SHORT CRACKS[1]

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**Abstract**

Most structural components are designed against fatigue crack initiation, by procedures which do not recognize cracks. Large cracks may be easily detected and dealt with, but small cracks may pass unnoticed even in careful inspections, if they are smaller than the detection threshold of the inspection method used to identify them. Thus, structural components designed for very long fatigue lives should be designed to avoid fatigue crack initiation and to be tolerant to undetectable short cracks. But this self-evident requirement is still not used in fatigue design routines, which just intend to maintain the loading at the structural component critical point below its fatigue limit. Nevertheless, most long-life designs work just fine, which means that they are somehow tolerant to undetectable or to functionally admissible short cracks. But the question "how much tolerant" cannot be answered by SN procedures alone. This important problem can only be solved by adding a proper fatigue crack propagation threshold requirement to the so-called "infinite" life design criterion, which must include appropriate short crack corrections to be reliable. This paper presents a methodology to design notched fatigue test specimens specially conceived to verify the accuracy of the various theories proposed to evaluate the tolerance to short cracks.

**Key words**: Short cracks; Non-propagating cracks; Fatigue life prediction.

## CORPOS DE PROVA PROJETADOS PARA INDUZIR TRINCAS PEQUENAS NÃO-PROPAGANTES POR FADIGA

**Resumo**

A maioria dos componentes estruturais é projetada para resistir à iniciação de trincas por fadiga por métodos que não reconhecem trincas. Trincas longas podem ser detectadas e modeladas com facilidade, mas trincas curtas podem passar despercebidas mesmo em inspeções cuidadosas, se forem menores do que o limiar de detecção do método de inspeção. Assim, componentes estruturais projetados para vidas muito longas devem resistir à iniciação e serem tolerantes às pequenas trincas que possam passar despercebidas na prática. Mas esta idéia ainda não é usada nas rotinas de projeto à fadiga, que visam manter tensões no ponto crítico abaixo do limite de fadiga. Porém, a maioria de projetos de vida longa funciona muito bem, o que significa que eles são tolerantes às trincas curtas, indetectáveis ou funcionalmente admissíveis. Mas a pergunta "quão tolerante" não pode ser respondida apenas pelas rotinas SN. Este problema importante só pode ser resolvido adicionando um requisito adequado de limiar de propagação de trinca por fadiga ao chamado projeto para "vida infinita", que deve incluir correções de trincas curtas apropriadas para ser confiável. Este trabalho apresenta uma metodologia para projetar corpos de prova entalhados de fadiga especialmente concebidos para verificar a precisão das várias teorias propostas para avaliar a tolerância às trincas curtas.

**Palavras-chave:** Trincas curtas; Trincas não-propagantes; Previsão de vida a fadiga.

## 1 INTRODUCTION

Most structural components must have notches like holes, fillets, slots, grooves, keyways, shoulders, corners, threads etc., to perform their intended functions. Such notches act as local stress raisers which modify the component fatigue resistance in relation to the material fatigue strength measured in standard smooth and polished specimens. In fact, to correctly predict the fatigue resistance of notched structural components has been a major goal for structural designers since Wöhler times.[1]

Following this tradition, this work proposes a methodology to find the nominal stress range $\Delta\sigma_n$ and maximum stress $\sigma_{max}$ combinations that initiate and propagate for a while short cracks from the notch of specifically designed specimens, until they arrest and become non-propagating under fixed loading conditions. These are the conditions that actually define the fatigue resistance of notched structural components. Indeed, when designed for very long fatigue lives, they should be able to avoid fatigue crack initiation and to tolerate undetectable or functionally admissible short cracks. To properly design specific specimens is the only way to check if a given loading condition can in fact be reliably and robustly associated with predictable non-propagating cracks in notched components designed for "infinite" lives.

According to Frost, Marsh e Pook,[2] fatigue cracks initiate and propagate under pulsating loading conditions if $\Delta\sigma_n \geq \Delta S_0/K_t$, where $K_t$ is the elastic stress concentration factor (SCF) and $\Delta S_0$ is the fatigue limit of smooth specimens under $R = \sigma_{min}/\sigma_{max} = 0$; and non-propagating short cracks form at the notch root if $\Delta S_0/K_t \leq \Delta\sigma_n \leq \Delta S_0/K_f$, where $K_f$ is the fatigue SCF for the notch. Short cracks must behave differently from long cracks, since their fatigue crack propagation (FCP) threshold must be smaller than the long crack threshold $\Delta K_{th}(R)$, otherwise the nominal stress range $\Delta\sigma_n$ required to propagate them would be higher than the material fatigue limit $\Delta S_L(R)$.[3,4] Indeed, assuming that the FCP process is primarily controlled by the stress intensity factor (SIF) range, $\Delta K \propto \Delta\sigma\sqrt{(\pi a)}$, if short cracks with $a \rightarrow 0$ had the same $\Delta K_{th}(R)$ threshold of long cracks, their propagation by fatigue would require $\Delta\sigma \rightarrow \infty$, a physical non-sense. The FCP threshold of short fatigue cracks under pulsating loads $\Delta K_{th}(a, R = 0)$ can be modeled using El Haddad-Topper-Smith (ETS) characteristic size $a_0$, which is estimated from $\Delta S_0 = \Delta S_L(R = 0)$ and $\Delta K_0 = \Delta K_{th}(R = 0)$.[5] This clever trick reproduces the Kitagawa-Takahashi plot trend[6] (Figure 1), using a modified SIF range $\Delta K'$ to describe the fatigue propagation of any crack, short or long.

$$\Delta K' = \Delta\sigma\sqrt{\pi(a + a_0)}, \text{ where } a_0 = (1/\pi)(\Delta K_0/\Delta S_0)^2 \tag{1}$$
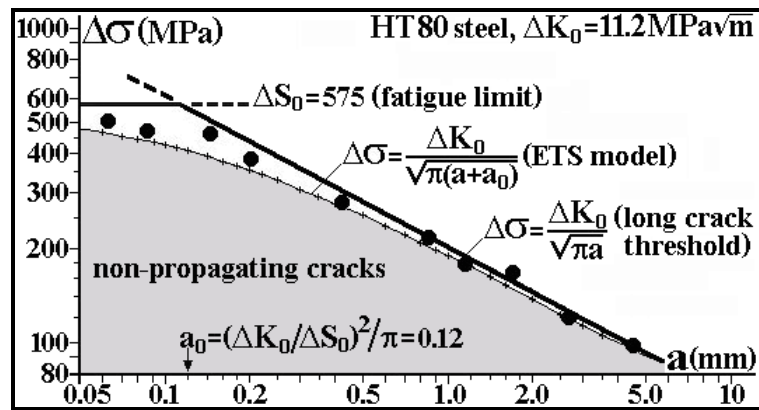
**Figure 1.** Kitagawa-Takahashi plot describing the fatigue propagation of short and long cracks under pulsating loads ($R = 0$) in a HT80 steel with $\Delta K_0 = 11.2 MPa\sqrt{m}$ and $\Delta S_0 = 575 MPa$.

As ETS $\Delta K'$ has been deduced using the Griffith's plate SIF, $\Delta K = \Delta\sigma\sqrt{(\pi a)}$, it is important to use the non-dimensional geometry factor $g(a/w)$ of the general SIF expression $\Delta K = \Delta\sigma\sqrt{(\pi a)} \cdot g(a/w)$ to deal with other geometries, re-defining as Equation 2.

$$\Delta K' = g(a/w) \cdot \Delta\sigma\sqrt{\pi(a+a_0)} \text{ , where } a_0 = (1/\pi)\left[\Delta K_0/(g(a/w) \cdot \Delta S_0)\right]^2 \quad (2)$$

But the tolerable stress range $\Delta\sigma$ under pulsating loads tends to the fatigue limit $\Delta S_0$ when $a \to 0$ only if $\Delta\sigma$ is the notch root (instead of the nominal) stress range. However, the geometry factors found in SIF tables usually include the notch SCF, because they use instead of $\Delta\sigma_n$ as the nominal stress. A clearer way to define $a_0$ when the short crack departs from a notch root is to explicitly recognize this practice, separating the geometry factor $g(a/w)$ into two parts: $g(a/w) = \eta \cdot \varphi(a)$, where $\varphi(a)$ describes the stress gradient ahead of the notch tip, which tends to the SCF as the crack length $a \to 0$, whereas $\eta$ encompasses all the remaining terms, such as the free surface correction (Equation 3).

$$\Delta K' = \eta \cdot \varphi(a) \cdot \Delta\sigma\sqrt{\pi(a+a_0)} \text{ , where } a_0 = (1/\pi)\left[\Delta K_0/(\eta \cdot \Delta S_0)\right]^2 \quad (3)$$

Operationally, the short crack problem can be better and easier treated by letting the SIF range $\Delta K$ retain its original equation, while the FCP threshold expression (under pulsating loads) is modified to become a function of the crack length $a$, namely $\Delta K_0(a)$, resulting in the Equation 4.[7-9]

$$\Delta K_0(a) = \Delta K_0 \cdot \sqrt{a/(a+a_0)} \quad (4)$$

The ETS equation can be seen as one possible asymptotic match between the short and long crack behaviors. Following Bazant's[10] reasoning, a more general equation can be used introducing an adjustable parameter $\gamma$ to fit experimental data.

$$\Delta K_0(a) = \Delta K_0 \cdot \left[1 + (a_0/a)^{\gamma/2}\right]^{-1/\gamma} \quad (5)$$

Equations 1 to 4 result from Equation 5 if $\gamma = 2.0$. The bi-linear limit, $\Delta\sigma(a \leq a_0) = \Delta S_0$ for short cracks, and $\Delta K_0(a \geq a_0) = \Delta K_0$ for long ones, is obtained if $g(a/w) = \eta \cdot \varphi(a) = 1$ and $\gamma \to \infty$. Most short crack FCP data is fitted by $\Delta K_0(a)$ curves with $1.5 \leq \gamma \leq 8$, but $\gamma = 6$ better reproduces classical $q$-plots based on data measured by testing

semi-circular notched fatigue TS.[7] Using Equation 5 as the FCP threshold, then any crack departing from a notch under pulsating loads should propagate if:

$$\Delta K = \eta \cdot \varphi(a/\rho) \cdot \Delta\sigma\sqrt{\pi a} > \Delta K_0(a) = \Delta K_0 \cdot \left[1 + (a_0/a)^{\gamma/2}\right]^{-1/\gamma} \tag{6}$$

Where $\eta = 1.12$ is the free surface correction. As fatigue damage depends on two driving forces, $\Delta\sigma$ and $\sigma_{max}$, Equation 6 must be extended to consider $\sigma_{max}$ (indirectly modeled by the $R$-ratio) influence in short crack behavior. First, the short crack characteristic size $a_R$ is defined using the FCP threshold for long cracks $\Delta K_R = \Delta K_{th}(a >> a_R, R)$ and the fatigue limit $\Delta S_R$, both measured or properly estimated at the desired $R$-ratio, where $\Delta K_R$ is short crack FCP threshold at the required R-ratio. Then it can be stated that cracks are stable under fixed ($\Delta K$, R) loading conditions if $\Delta K < \Delta K_R(a)$, where:

$$a_R = (1/\pi)\left[\Delta K_R/(1.12 \cdot \Delta S_R)\right]^2 \text{ and } \Delta K_R(a) = \Delta K_R \cdot \left[1 + (a_R/a)^{\gamma/2}\right]^{-1/\gamma} \tag{7}$$
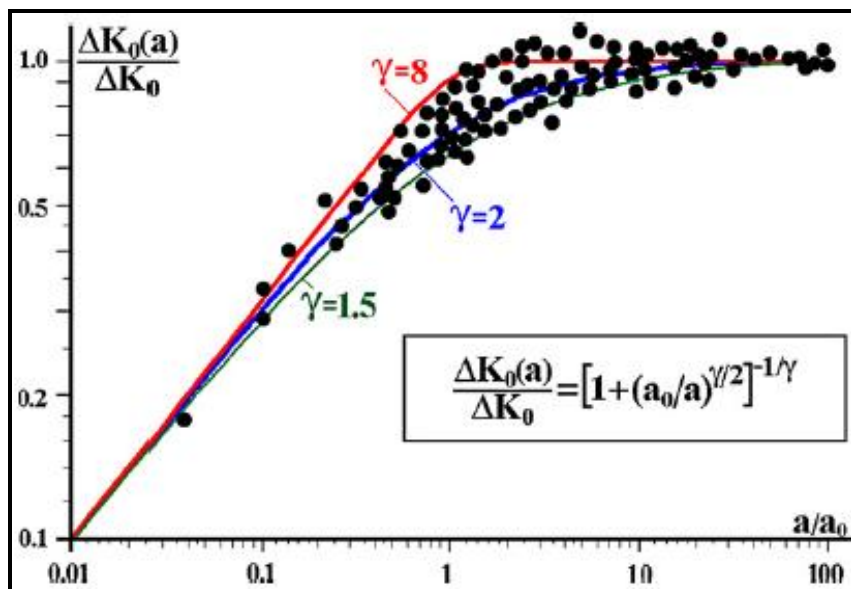


**Figure 2.** Ratio between short and long crack propagation thresholds as a function of $a/a_0$.

The test specimens presented here are designed to check whether the predictions about the behavior of short cracks that depart from notches under any fixed loading conditions based on the stress gradient (SG) ahead of the notch tip are accurate, and to compare them with an alternative approach that deals with the tolerance to short cracks by the so-called Theory of Critical Distance (TCD),[11] as explained below.

## 2 SHORT CRACKS THAT DEPART FROM SEMI-ELLIPTICAL NOTCHES

Consider a specimen containing a single semi-elliptical notch with semi-axes $b$ and $c$ (where $b$ is in the same direction as $a$), loaded by a nominal tensile stress range $\Delta\sigma_n$, applied normal to the crack propagation direction. The SIF range for a crack $a$ emanating from the root such notches can be written as Equation 9.

$$\Delta K_I(a) = \eta \cdot f(a/b, c/b) \cdot \Delta\sigma_n\sqrt{\pi a} \tag{9}$$

Where the geometry factor *f(a/b,c/b)* has been calculated using the Quebra2D program[12] and then are fitted within 3% by Equation 10.[7]

$$f\left(\frac{a}{b},\frac{c}{b}\right) \equiv f(K_t,a) = \sqrt{\left[1-exp\left(-K_t^2\cdot\frac{a}{a+b}\right)\right]\cdot\left(\frac{a+b}{a}\right)} \qquad \text{for } c \leq b \qquad (10)$$

Where $K_t = (1 + 2b/c) [1 + 0.12/(1 + c/b)^{2.5}]$ and $c^2 = \rho\cdot b$. As by definition the crack propagation threshold $\Delta K_{th}(a)$ limits the propagation and non-propagation conditions for short cracks, they propagate if their SIF range overcomes $\Delta K_{th}(a)$, therefore:

$$\Delta K_I(a) = \eta\cdot f(K_t,a)\cdot\Delta\sigma_n\sqrt{\pi a} > \Delta K_{th}(a) = \Delta K_0\cdot\left[1+(a_0/a)^{\gamma/2}\right]^{-1/\gamma} \qquad (11)$$

After some algebraic manipulation, the Equation 11 results in Equation 12.

$$f(K_t,a) > g\left(a,\frac{\Delta K_0}{\Delta S_0},\frac{\Delta S_0}{\Delta\sigma_n},\gamma\right) = \left(\frac{\Delta K_0}{\Delta S_0}\right)\cdot\left(\frac{\Delta S_0}{\Delta\sigma_n}\right)\cdot\left[\left(\eta\cdot\sqrt{\pi a}\right)^\gamma + \left(\frac{\Delta K_0}{\Delta S_0}\right)^\gamma\right]^{-1/\gamma} \qquad (12)$$

Following the analysis of Meggiolaro, Miranda e Castro,[7] there is a single value of crack size $a_{max}$ that limits the condition of propagating and non-propagating cracks, satisfying Equation 12 for a given material/notch pair. The stress range that can cause crack initiation and propagation without arrest is associated with the fatigue limit of such a pair. So, the relation $\Delta S_0/\Delta\sigma_n$ corresponding to $a_{max}$ is equal to the fatigue SCF for the notch $K_f$. Therefore, those $a_{max}$ and $K_f$ values can be determined by solving (Equation 13).

$$\begin{cases} f(K_t,a_{max}) = g(a_{max},K_f,\Delta K_0/\Delta S_0,\gamma) \\ \dfrac{\partial}{\partial a}f(K_t,a_{max}) = \dfrac{\partial}{\partial a}g(a_{max},K_f,\Delta K_0/\Delta S_0,\gamma) \end{cases} \qquad (13)$$

## 3 THE TCD MODEL

Investigations related with critical distances principles began with Neuber (1936) and Peterson (1938).[11] The so-called Theory of Critical Distances (TCD) is a group of methods based on a characteristic material length parameter called the critical distance *L*. This group includes the Point Method (PM), the Line Method (LM), the Area Method (AM), and the Volume Method (VM), which is the most general one. To make predictions, the TCD requires that the elastic stress range (in the loading direction) to be known as a function of its distance *x* from the notch tip, $\Delta\sigma(x)$. In addition, two material parameters are also needed: the fatigue limit of smooth specimen $\Delta S_0$ and the critical distance *L*, calculated as Equation 14.

$$L = (1/\pi)[\Delta K_0/\Delta S_0]^2 \qquad (14)$$

Note that the above expression is similar to ETS's $a_0$ (Equation 1), except that it does not include the free surface correction $\eta$. So, the critical distance can also be calculated as $L = a_0\cdot\eta^2$.

### 3.1 The Point Method (PM)

The PM is the simplest form of the TCD. In this approach, the criterion for crack propagation (fatigue limit) is that the local stress at a distance $x = L/2$ equals to the smooth specimen fatigue limit $\Delta S_0$. It can be expressed mathematically as Equation 15.

$$\Delta\sigma(L/2) = \Delta S_0$$

(15)

### 3.2 The Line Method (LM)

The LM uses an average stress over a distance $x = 2L$ from the notch root rather a stress at particular point as in the PM. For the fatigue limit it is required that such average stress equals to the fatigue limit of smooth specimens $\Delta S_0$. Mathematically, it can be expressed as Equation 16.

$$\frac{1}{2L}\int_0^{2L}\Delta\sigma(x)\,dx = \Delta S_0$$

(16)

### 3.3 The Area (AM) and Volume (VM) Methods

The AM involves an average stress over some area in the vicinity of the notch, whilst the VM makes use of a volume average. Considering a semicircular area, or a hemispherical volume in the VM, centered on the notch root, Bellet et al.[13] showed that the radius of the semicircular area is $1.32 \cdot L$ and that of the hemispherical volume is $1.54 \cdot L$. However, the PM and LM are more used because they are easier to apply. For simplicity, a PM method is used here to determine the fatigue limit of the notched specimen (Figure 3). The stress field at the notch root for that configuration is calculated as Equation 17.

$$\Delta\sigma(x) = f(K_t,a)\cdot\Delta\sigma_n$$

(17)

Where the geometry factor $f(K_t,a)$ is assumed to be equal to that showed in Equation 5.

## 4 SPECIMENS DESIGNED TO INDUCE NON-PROPAGATING SHORT CRACKS

### 4.1 Material

The material for the specimen designed to induce non-propagating cracks is 1020 steel, with mechanical properties measured by Durán, Castro e Payão Filho:[14]

- $E = 205MPa$;
- $S_U = 491MPa$;
- $S_Y = 285MPa$;
- $\Delta K_{th} = 11.6MPa\sqrt{m}$.

## 4.2 Specimen

To verify the accuracy of both the SG and the TCD models to predict tolerance to short fatigue cracks, a methodology is proposed to design notched test specimens specially conceived to induce non-propagating short cracks under prescribed loading conditions. It uses a modified SE(T) specimen with a machined notch which ends in a circular hole with radius $\rho$, (Figure 3b, where the line $a$ represents the length of the crack that departs from the notch). The externals dimensions are shown in Figure 3a. The notch stress concentration factor (SCF) can be easily controlled by properly choosing the $b$ and $\rho$ combination. Moreover, such a specimen can be used to test several $b/\rho$ combinations and be used for repeated tests, just by incrementing the notch size.



(a)                                                                                          (b)
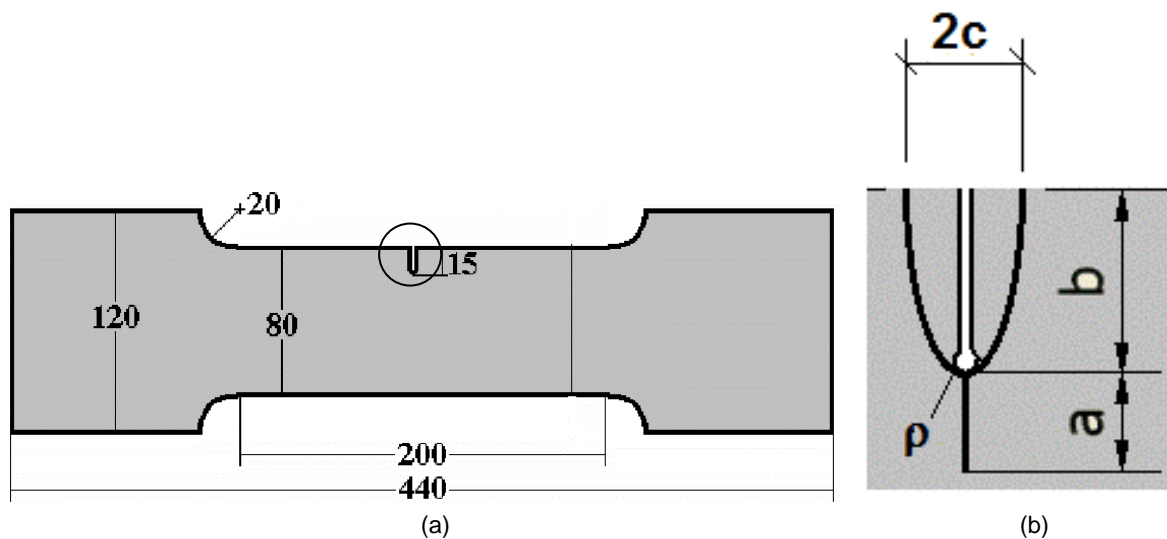
**Figure 3.** (a) Modified SE(T) specimen, dimensions in mm; and (b) approximation by a semi-elliptical notch.

For simplified analyses, the specimen notch can be approximated by a semi-ellipsis with semi-axes $b$ e $c$ (Figure 3b), where $c$ is a function of the notch root radius $\rho = c^2/b$. From an elastic stress analysis, the nominal stress range applied at the notch can be calculated as Equation 18.

$$\Delta\sigma_n = \frac{\Delta P}{t \cdot (w - b)} \tag{18}$$

Therefore, after the value of $\Delta\sigma_n$ is determined by solving the system of Equation 13, the load $\Delta P$ to be applied to the specimen can be easily found. Considering the specimen configuration shown in Figure 3, the 1020 steel properties listed in section 5, and a Bazant's parameter $\gamma = 6$, as suggested by [8], the behavior of such a test specimen is explored as follows.

## 4.3 Numerical Results

Assuming as usual the fatigue limit of smooth specimen as $S'_L = 0.5 \cdot S_u$, for a load ratio $R = -1$ (fully reversed loading), by Goodman it can be estimated that the fatigue limit for pulsating loading conditions ($R = 0$) is $\Delta S_0 = 2 \cdot S_u/3$.
Following Frost's statement, it is the difference between $K_t$ and $K_f$ that defines the generation of non-propagating fatigue cracks. Numerical results for such a fatigue

notch factor $K_f$ calculated by both the SG and TCD models are plotted in Figure 4 as a function of the notch root radius $\rho$, assuming $b = 15\ mm$ in Figure 3. Figure 4 also shows the stress concentration factor $K_t$ and how its value tends to $K_f$ as the notch root radius $\rho$ increases. Therefore, for this material and specimen configuration, notches with root radii $\rho < \approx 1.5\ mm$ will be able to generate non-propagating cracks.
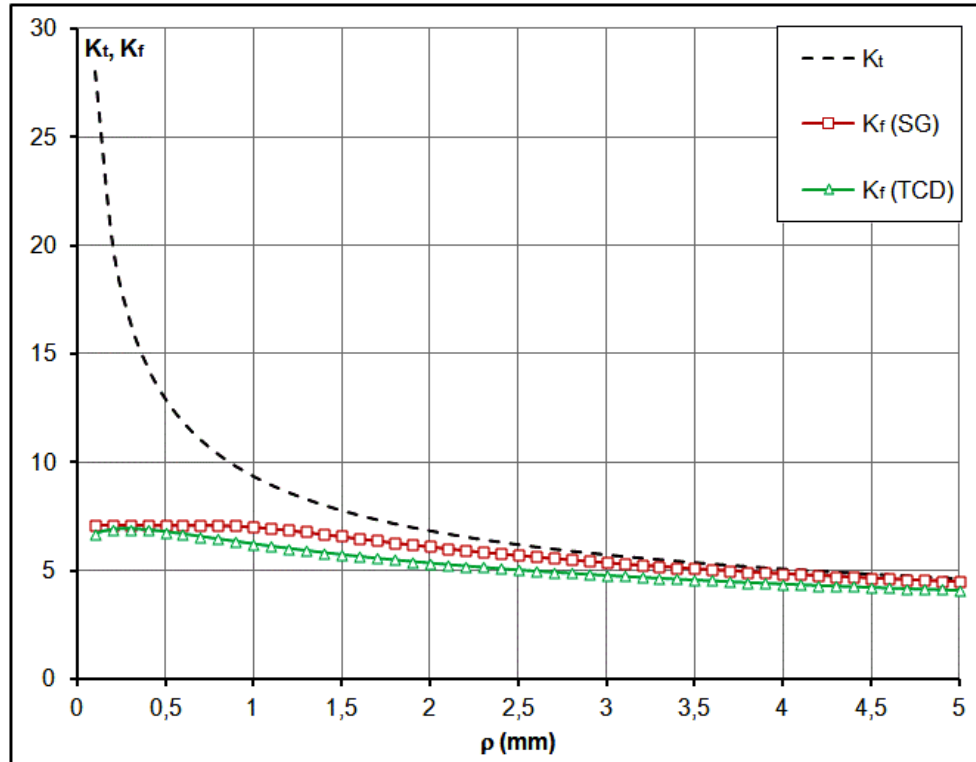


**Figure 4.** Comparison of predictions of the notch fatigue factor $K_f$ from the SG and TCD models with the stress concentration $K_t$ as a function of the notch root radius $\rho$.

As indicated in Section 3, in addition to $K_f$, the SG model also allows the largest non-propagating crack $a_{max}$ that can arise from fatigue alone to be calculated. Figure 5 shows the value of $a_{max}$ as a function of the notch root radius $\rho$. Ideally, it would be better to deal with higher values of $\rho$ because they are easier to machine at the notch tip. In the other hand, the smaller the notch root radius $\rho$, the greater the maximum non-propagating crack $a_{max}$ is, and, consequently, the more reliably the method can be applied to predict non-propagating cracks that can be robustly measured. According to the numerical results shown in Figure 5, for $\rho \leq \approx 1.5\ mm$ it can be expected that the maximum non-propagating crack size should be $0.83\ mm < a_{max} < 0.309\ mm$. Those values can be easily measured by an optical microscope. Thus, for Figure 3 specimen configuration the notches should have e.g. root radii $\rho = 0.5$, 1.0, and 1.5 mm, and be machined using reamers to obtain accurate dimensions.
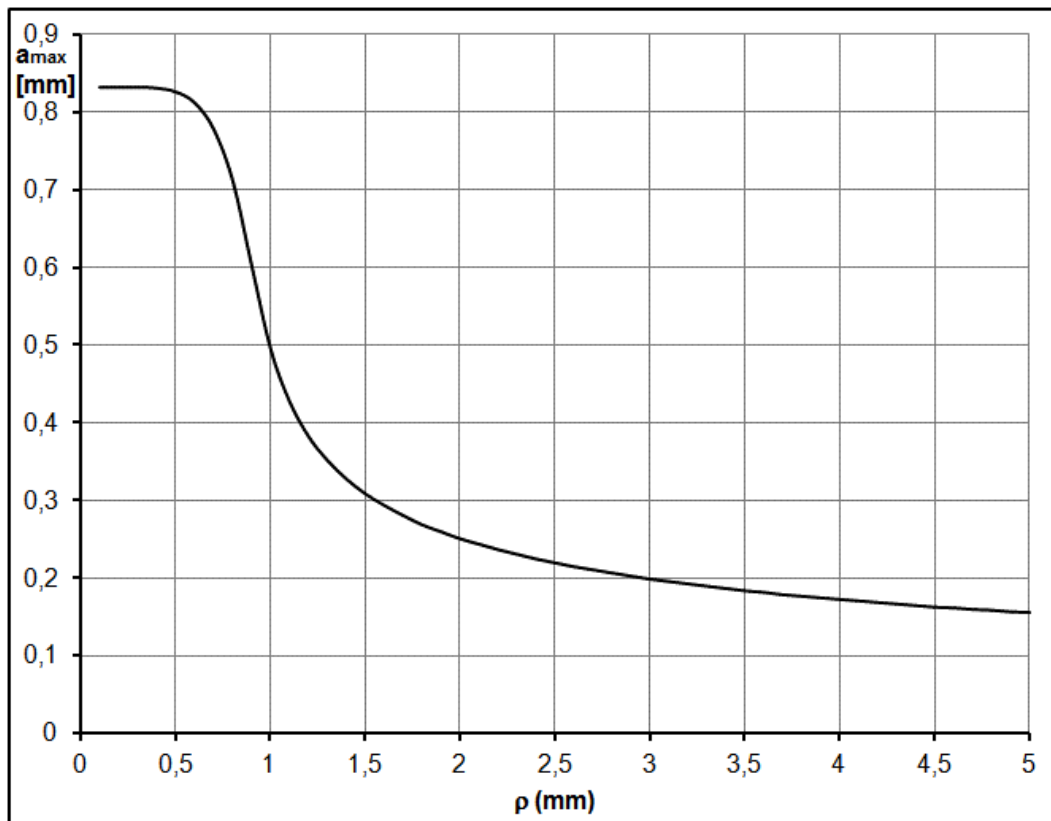
**Figure 5.** Predictions of the maximum non-propagating crack $a_{max}$ in function of the notch root radius $\rho$.

## 5 SENSITIVITY ANALYSIS OF THE SG MODEL

The SG model is based entirely on sound mechanical principles, which do not require an arbitrary choice of some critical distance parameter. Under fixed ($\Delta\sigma$, R) loading conditions, it is a function of the long crack fatigue propagation threshold $\Delta K_R$ and of the fatigue limit $\Delta S_R$, both well defined but relatively disperse mechanical properties. However, it also uses Bazant's data-fitting parameter $\gamma$, which certainly can improve the ETS description of short crack fatigue propagation using Kitagawa-Takahashi or similar diagrams, which are difficult to obtain in practice. Therefore, it is worth to evaluate how such parameters can influence the SG predictions.

Typical fatigue crack propagation thresholds under pulsating loads ($R = 0$) for long cracks in steels are in the range $6 \leq \Delta K_0 \leq 12 \ MPa\sqrt{m}$.[1] Figure 6 shows values of $K_f$ predicted by SG procedures for $\Delta K_0 = 6, 8, and\ 11.6 \ MPa\sqrt{m}$. Note that the smaller $\Delta K_0$ is, the greater the value of $K_f$, and, therefore, smaller its difference to $K_t$.

A variation of $\pm 10\%$ in the fatigue limit $\Delta S_0$ is considered in Figure 7. The smaller $\Delta S_0$ is, the smaller is the value predicted for $K_f$, thus larger is its difference from $K_t$. This means that less strong steels should be more tolerant to short cracks because they should be able to tolerate larger short cracks than high-strength steels, a prediction that seems vary reasonable, since high strength is usually associated to more sensitivity to defects.

Finally, the fitting parameter $\gamma$ can typically vary between $1.5 \leq \gamma \leq 8$.[1] Figure 8 shows how the predicted notch fatigue factor $K_f$ varies for $\gamma = 3, 6$ and $8$. Even considering that $\gamma$ is only a fitting parameter, it may have a large influence on the $K_f$ values predicted by the SG method.
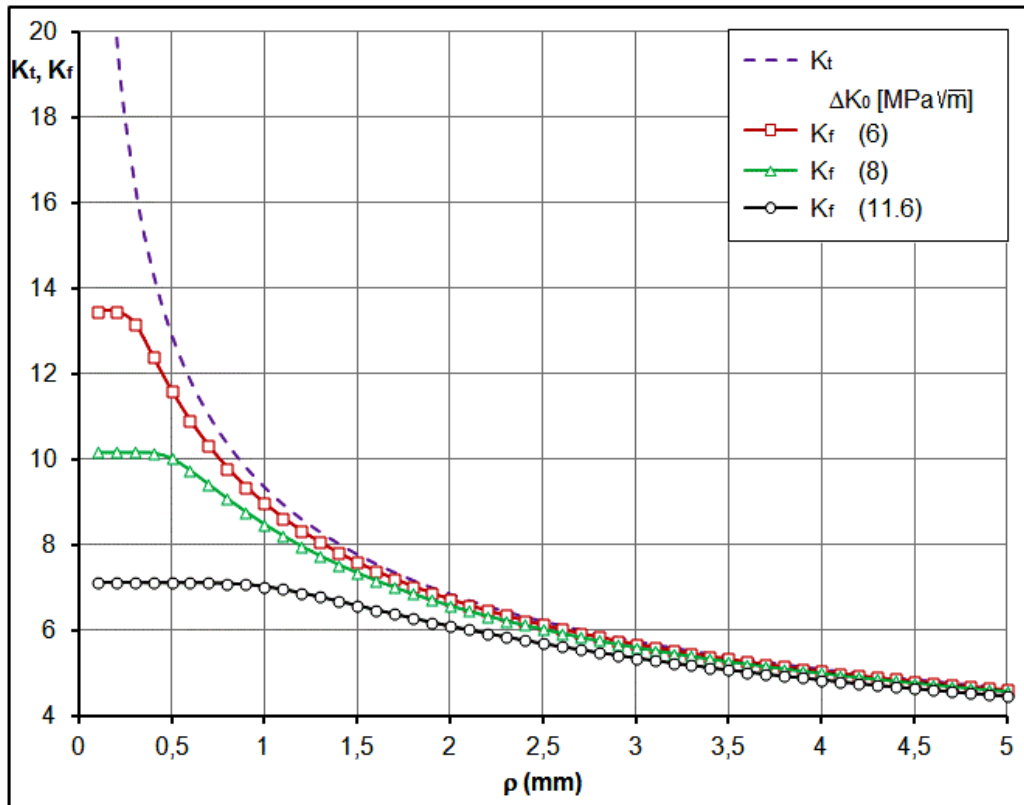
**Figure 6.** Influence of the crack propagation threshold $\Delta K_0$ in the predictions of the notch fatigue factor $K_f$ from the SG model.
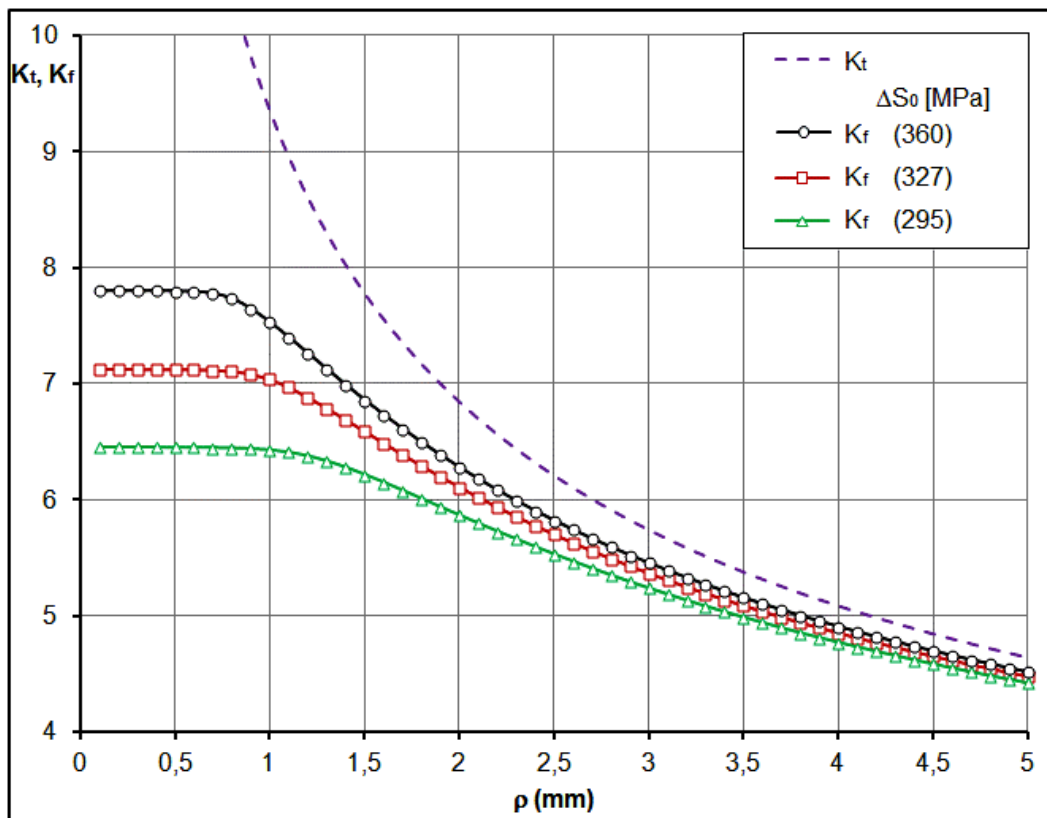


**Figure 7.** Influence of the smooth specimen fatigue limit $\Delta S_0$ in the notch fatigue factor $K_f$ predicted by the SG model.
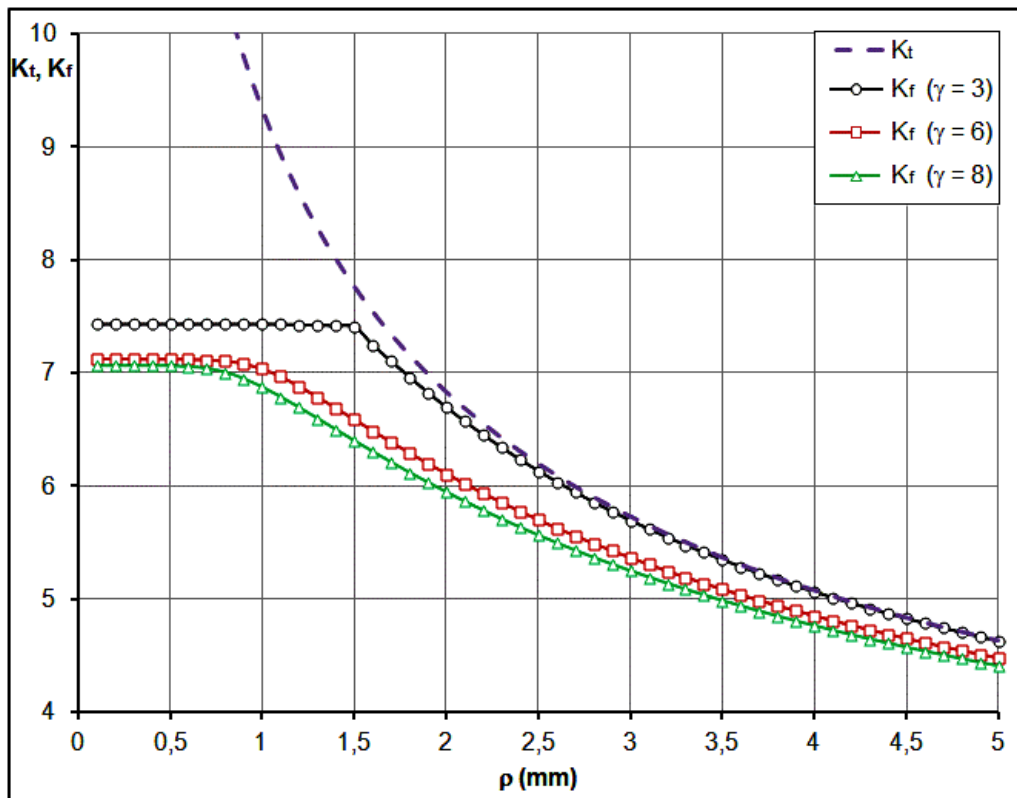
**Figure 8.** Influence of Bazant's parameter $\gamma$ in the notch fatigue factor $K_f$ predicted by the SG model.

The largest non-propagating crack $a_{max}$ predicted by the SG method is also influenced by the three parameters mentioned above, as expected. Figures 9 to 11 show the influence of $\Delta K_0$, $\Delta S_0$, and $\gamma$, respectively, on the $a_{max}$ value.
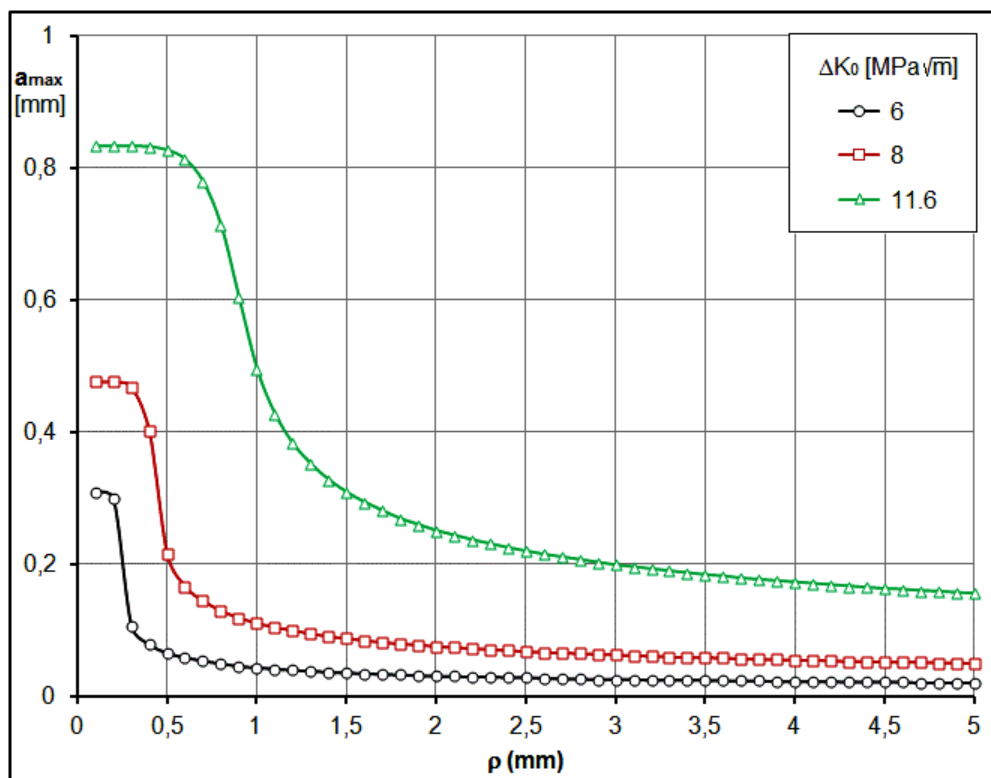


**Figure 9.** Influence of the crack propagation threshold $\Delta K_0$ in the SG predictions for the largest non-propagating short crack $a_{max}$ that can be tolerated at the $b = 15\ mm$ notch tip.
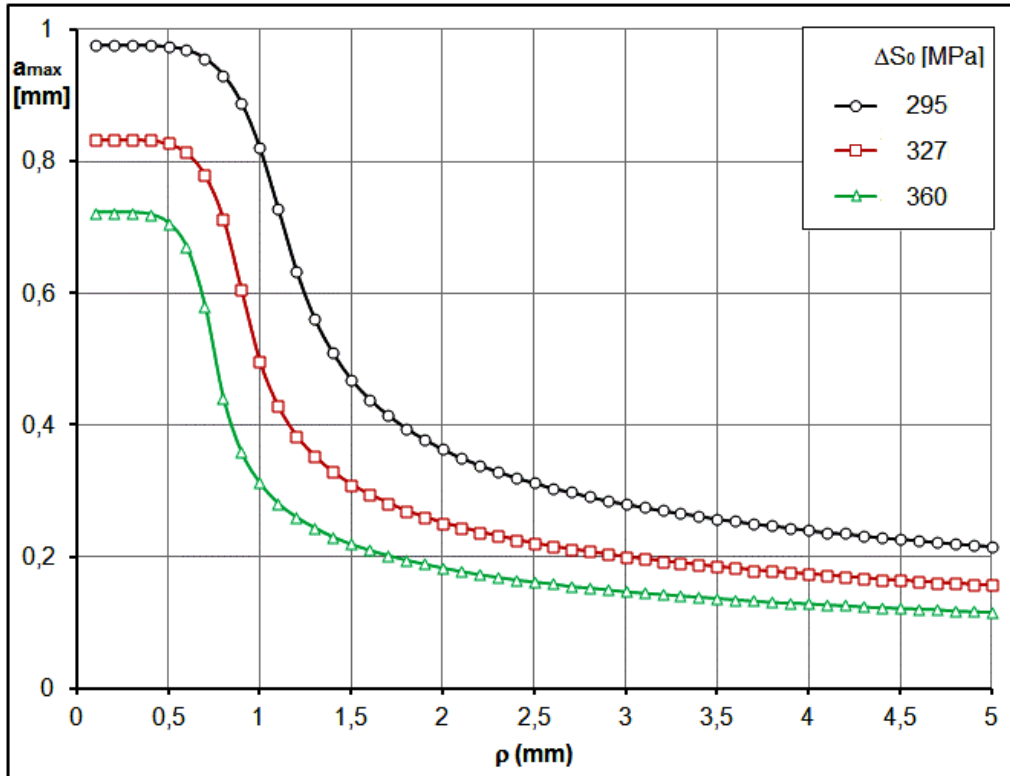
**Figure 10.** Influence of the smooth specimen fatigue limit $\Delta S_0$ in the SG predictions for the largest non-propagating short crack $a_{max}$ that can be tolerated at the $b = 15\ mm$ notch tip.
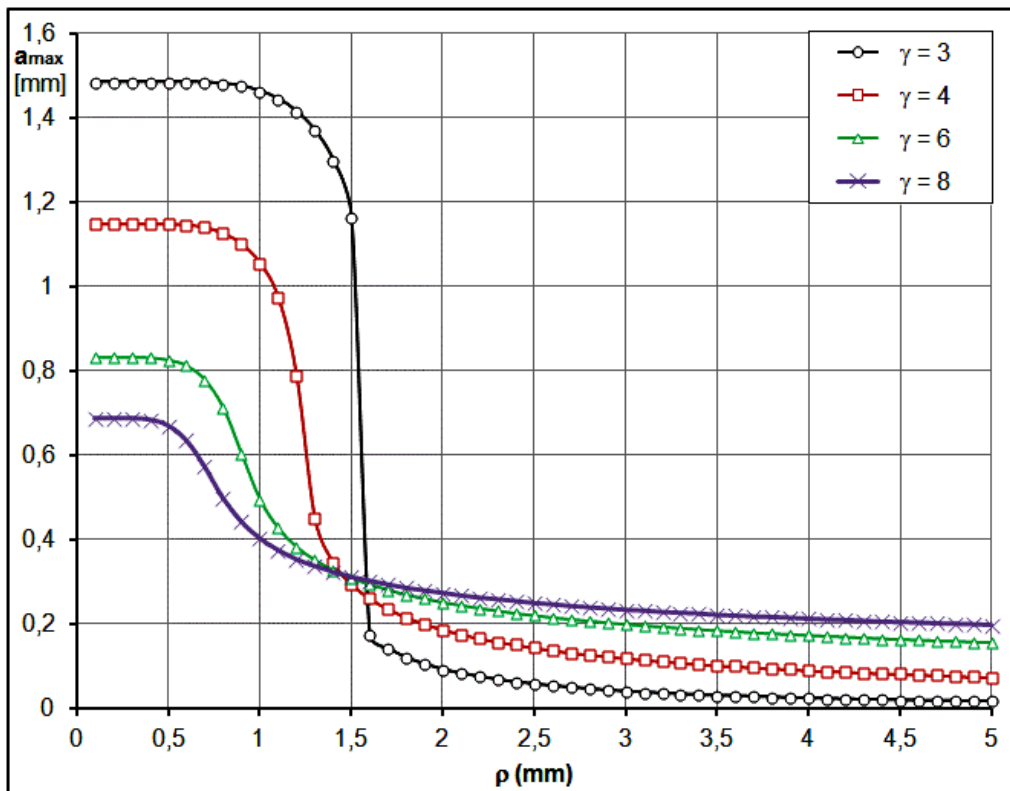


**Figure 11.** Influence of the Bazant's parameter $\gamma$ in the SG predictions for the largest non-propagating short crack $a_{max}$ that can be tolerated at the $b = 15\ mm$ notch tip.

This $a_{max}$ sensibility to the parameters of the SG model are measurable and can certainly be used to verify the accuracy of their predictions.

## 6 CONCLUSIONS

The stress gradient (SG) model, which is based on sound mechanical principles, was used to predict fatigue notch stress concentration factors $K_f$ and the short crack tolerance of a notched specimen designed to induce such non-propagating cracks at the notch root. Some of its predictions were compared with an alternative Theory of the Critical Distance (TCD), which assumes that the fatigue limit is related with a material-dependent length parameter. It was shown that the designed specimen can indeed be used to verify experimentally the accuracy of such predictions.

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