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Min-Max Long Run Marginal Cost to Allocate Transmission Tariffs for Transmission Users

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Abstract

The dispersion and volatility of transmission tariffs can provide an unsafe environment for generation investors in electrical systems, which are constantly growing. Dispersion and volatility occur, for example, in Brazil, where the Long Run Marginal Costs (LRMC) method is applied to calculate transmission tariffs. To solve this problem, this paper proposes a new Transmission Tariff Computation (TTC) approach based on the LRMC method and the min-max optimization technique.

The proposed method uses the LRMC approach and the min-max optimization technique to seek less-dispersed transmission tariffs. The proposed modified LRMC method can be employed to optimize tariffs for generators and loads jointly or separately. This choice should be based on the network topology. The results are presented for a 6-bus and the IEEE 118-bus systems. The modified LRMC method is compared with the traditional LRMC method, currently in use in Brazil, and the classical Pro rata technique. Finally, some conclusions are presented.

Keywords: Long Run Marginal Costs; Nodal method; Min-Max technique; Linear Programming; Transmission Tariff Computation.

I. NOTATION

Constants

β Power flow sensitivity matrix due to the nodal injection of active power.

$\boldsymbol{P_G}$ Nodal power generation vector (MW).

P_{Gi} Power generated at bus *i* (MW).

$\boldsymbol{P_D}$ Nodal power demand vector (MW).

P_{Dj} Power demand at bus *j* (MW).

P_i Net power injection at bus *i* (MW).

C_ℓ Cost of line ℓ ($).

c_ℓ Unitary cost of line ℓ ($/MW).

F_ℓ Power flow through line ℓ (MW).

F_ℓ^{max} Maximum power flow of line ℓ (MW). In this paper, the maximum active power flow of line ℓ is considered equal to the transmission capacity of this line.

F_ℓ^{min} Minimum power flow of line ℓ (MW). In this paper, active power flows lower than the minimum power flow is not considered to compute transmission tariffs.

$Fpond_\ell$ Weighting factor of line ℓ. This factor represents the utilization factor of line ℓ.

Variables

α_{ij} Percentage of power injected at bus *i* to feed a unitary load at bus *j*.

$\pi_{G_i}^L$ Locational tariff of generator *i* ($/MW).

$\pi_{D_j}^L$ Locational tariff of load *j* ($/MW).

Δ_G Postage stamp for generators ($/MW).

Δ_D Postage stamp for loads ($/MW).

π_{Gi} Transmission usage tariff for generator *i* ($/MW).

π_{Dj} Transmission usage tariff for load *j* ($/MW).

Sets

Ω_G Set of generators.

Ω_D Set of loads.

Ω_L Set of transmission lines.

G_k Set of generators that are not optimized by the modified LRMC method up to iteration k.

D_k Set of loads that are not optimized by the modified LRMC method up to iteration k.

M_{Gk} Set of generators that have been optimized by the modified LRMC method up to iteration k.

M_{Dk} Set of loads that have been optimized by the modified LRMC method up to iteration k.

S_k Set of tariffs optimized up to iteration k.

II. INTRODUCTION

The transmission system is responsible for connecting the generation plants, which are generally dispersed, to the load centers. According to the Brazilian National Electric Energy Agency (ANEEL), the transmission cost that should be recovered every year in the basic network (a network with 230 kV or above) is currently greater than 13 billion Reais (R$[1]) [1]. The revenue accrued by transmission usage is applied to recover the operation, maintenance and expansion planning costs. It must be paid by generators and loads, which are the transmission users. Notwithstanding, transmission networks are huge infrastructure that aims to provide not only a path between generation and load. It also plays other roles such as ensure reliability and supply adequacy. Such features demand the installation of additional capacity to circumvent contingencies, uncertainty and to meet quality standards. As a result, the transmission system cost is generally not recoverable by transmission cost allocation methods supported on the players' usage. One of the main challenges is how to allocate these costs to generators and loads.

In recent years, several studies have been proposed to allocate transmission costs. The Pro rata method [2] allocates transmission costs to generators and loads in proportion to their respective generations and consumptions. The cost allocated to each generator and/or load is independent of the network configuration.

Other more complex methods allocate transmission costs to generators and loads based on the active power flow participation of generators and loads through transmission lines [3]-[7]. To identify the responsibility of the power flow through each line due to generators or loads, the proportional sharing principle is used in [3]-[6]. To apply the proportional sharing principle, it is necessary to define (a priori) how much of the total transmission cost should be allocated to the generators and the loads of the system. Generally, a 50/50 allocation rule is applied. In [7] the cited responsibility is defined, in the most general manner, by generation shift factors applied to predefined wheeling transactions formed by generators and loads in the system.

In addition, another representative method is the Zbus method [8]. This method considers the current injections into system buses, the impedance matrix (Zbus), and other electrical parameters to allocate the transmission cost. The main characteristic of this method is that is highly dependent on the network topology.

Long Run Marginal Cost (LRMC) methods [9]-[18] are employed in countries such as England,

[1] 1 US$ is approximately 2 R$.

Colombia and Brazil. LRMC methods consider the marginal participation of each generator or load to increase the future investments in the transmission system through the bus-to-line active power sensitivity matrix [19]. Because the method should reflect users' responsibility for this increase, a set of hypothesis are needed to address LRMC methods:

- There is an "ideal minimum cost network" required to supply the demand for existing routes, and this network has the same topology and impedances of the existing network (with expansions under determinative expansion planning);
- The ideal network is defined assuming peak demand conditions of each load;
- To supply the demand, the generators are dispatched proportionally by considering their capacity registered (pro-rata).The last both assumptions are applied to try to achieve the maximum transmission system stress, which according to [16] is not always guaranteed;
- Is assumed that the transmission capacity of each line and transformer coincides with the ideal power flow verified in the element for the demand condition considered;
- It will be considered that the expansion of the transmission system should be built by the existing routes. It means that marginal increments on the power flow in transmission lines would result in additional charges over the tariff, simulating the future real investments in transmission system, which occurs discontinuously with the entrance of new ventures.

In [9], the LRMC technique is applied with the Equivalent Bilateral Exchanges (EBE) method. In this method, an EBE has pre-defined nodal exchange factors (NEFs) that represent the percentage of the generation in bus i that feeds load bus j. The transmission cost is allocated based on the amount of power flow that each EBE produces and is transported through transmission lines. In [10] a Modified Equivalent Bilateral Exchange (MEBE) is proposed by considering the features of EBE method and the losses in the network.

Similar to the EBE method, the LRMC method applied in Brazil [12], [13] and [15], which is also called Nodal LRMC, computes transmission tariffs by considering the impact of a variation in the power injection at bus i and an equivalent compensation of this variation at the slack bus. Because the solution is dependent on the choice of the slack bus, a slight modification is proposed to create a slack-bus-independent method. The details about the method are described in subsection III.A.

For practical purposes, a tariff-based approach is used to allocate transmission costs. The transmission tariff should be multiplied by the maximum generation or demand in a time period (usually a month) to charge the transmission users. The approach based on tariffs is well accepted because, in general, transmission systems have a relatively well-defined power consumption and generation dispatch compared with distribution systems, for example. Thus, the power flow (base case) established can be considered representative. Moreover, the tariffs must change only if the system changes (new lines or new generators are installed). Thus, the tariffs computed are more stable and predictable over time.

There is no consensus yet on the best method to allocate transmission costs. In [20], an analysis of different methods is performed to highlight the advantages and disadvantages of each one. According to [9], the positive characteristics that should be considered are independence from the choice of slack bus, the satisfaction of both laws of Kirchhoff, location-dependent tariffs, a low temporal volatility of transmission tariffs and the allocation of nonzero tariffs to all network users.

In developing countries, such as Brazil, fast generation and transmission expansion triggered by a sharp demand growth can cause volatility in transmission tariffs. These undesirable effects generate an unsafe environment for new generation investments, mostly for the ones that utilizes renewable resources. In addition, different from conventional sources (gas, oil, etc.), most renewable sources cannot freely select the connection bus in transmission system. In this case, renewable resources availability and quality are the most important variables. There are several examples where renewable sources are far from load centers. For instance, Brazilian hydro basins in Amazon and wind in the middle of US are some representative examples. In some cases, transmission tariff obtained by traditional methods can make new investments in renewables unattractive. To tackle this problem, a new paradigm based on the robust min-max optimization technique is proposed to set transmission tariffs [21]-[23]. The idea behind the min-max technique is that, for a given steady state operation point, the agents with the worst tariffs should have priority in the tariff optimization process to minimize tariffs. The main motivation for this approach relies on the fact that volatile tariffs bring uncertainty to the investors' future cash flows, which ultimately can be seen as an entrance barrier for new investments.

Inspired by the EBE concept and the nodal tariffs technique applied in Brazil, the NEFs are endogenously defined by our method to reduce the dispersion of transmission tariffs through a min-max optimization procedure. As a result, the volatility of the tariffs over time is also reduced. Because those factors (NEFs) are not observable in practice and they are arbitrarily chosen in the EBE method, we consider them as optimization variables in our methodology. In contrast to the Nodal LRMC method, which assumes that all generation or load variations are compensated by the slack bus, our method takes into account the contribution of all buses to compensate nodal variations through NEFs.

The proposed approach optimizes the worst-case tariff by an iterative-based method providing Pareto-optimal tariffs [24]. This method optimizes the remaining tariffs iteratively using the optimality results of previous iterations. Thus, the dispersion of the tariffs is reduced, and, as a consequence, the volatility of the tariffs over the time is also reduced when new transmission or generation assets are incorporated into the system.

Accordingly, the contributions of this work are two-fold:

(i) a LRMC-based method with endogenously defined NEFs through a min-max optimization procedure to reduce the dispersion of tariffs;

(ii) an iterative and computationally efficient procedure to set the NEFs in the proposed method based on a linear programming optimization model.

To achieve these goals, this paper is organized as follows. Section III is divided into three parts: in the first part, the traditional Nodal LRMC method is revisited; in the second part, the min-max optimization technique and its combination with the Nodal LRMC and NEF are presented in a model where the tariffs of the generators and the loads are jointly optimized; and the third part is a slight variation of the latter, defining two separate models for the generators and the loads. In Section IV, a 6-bus system is used to illustrate the proposed joint model. In addition, the IEEE 118-bus system is used to analyze the proposed method capability to reduce the dispersion and volatility of tariffs among agents using two separate models for generators and loads. Comparative analyses are performed with respect to other methods to highlight the properties and applicability of the proposed method. Finally, the conclusions and final considerations are given in Section V.

III. LRMC Methods and Min-Max Technique

The following subsections present the Nodal LRMC method and its modification using a min-max optimization technique. In addition, the joint and separate models are introduced.

A. Nodal LRMC Method

The concept behind the Nodal LRMC method indicates how much an injection/extraction of additional power at a bus increases the marginal transmission cost. Thus, the transmission tariff calculated for a generator (or load) indicates how much a generator (or load) increases or decreases the need for new transmission investments. In this framework, a representative system dispatch (nodal net injections/extractions and their respective network flows) is required (see [12], [13] , [15] and [20]). To compute the tariffs, a fictitious power flow solution is performed by considering the capacity of the generators and the maximum demand by the loads. In this way, the network can have a high load, so that any additional power flow through the transmission lines can represent an increase in the transmission costs.

To represent the effect on tariffs of the Nodal LRMC method, the sensitivity of the power flow through the lines due to the power injected at each bus and the corresponding extraction at the slack bus are considered. These values are calculated using the sensitivity matrix (β) [19]. The term $\beta_{\ell i}$ in β represents the infinitesimal active power flow variation in line ℓ due to an infinitesimal power injection at bus *i*, in the neighborhood of a given operating point. Thus,

$$\beta_{\ell i} = \frac{\partial F_\ell}{\partial P_i} \tag{1}$$

For this case, the corresponding variation in the power injected at the slack bus is implicit. To determine the locational transmission tariff, two other parameters are considered ([12], [13] and [15]): (i) the unitary cost of the line (annual transmission cost of the line divided by its capacity) and (ii) the weighting factor of the transmission line. The cost of a line (C_ℓ) is defined every year by the system regulator and is beyond the scope of this paper. For our purposes, this cost is considered as an input parameter. The unitary cost (c_ℓ) of a line can be obtained as

$$c_\ell = \frac{C_\ell}{F_\ell^{max}} \tag{2}$$

The weighting factor of a given transmission line ($Fpond_\ell$) is a measure that determines the usage factor of each line. The rule to determine the usage factor is

$$Fpond_\ell = \begin{cases} 0 & if \quad F_\ell < F_\ell^{min} \\ \dfrac{F_\ell}{F_\ell^{max}} & if \quad F_\ell^{min} \le F_\ell \le F_\ell^{max} \\ 1 & if \quad F_\ell > F_\ell^{max} \end{cases} \tag{3}$$

In (3), if the power flow through line (ℓ) (performed by a DC method) is lower than the minimum power flow established for this line, the weighting factor is equal to 0, which occurs because a marginal increase does not imply a transmission cost variation for low power flow levels. However, if the power flow (ℓ) is greater than the maximum pre-defined power flow for this line, the weighting factor is equal to 1, which establishes that the line capacity is completely used. For the sake of simplicity, the minimum power flow is considered equal to zero for all lines, and the maximum one is given by the transmission capacity of the lines, which is related to their thermal limits. Thus, the locational transmission tariffs for generators and loads, given a 50/50 allocation rule, can be obtained by (4) and (5), respectively:

$$\pi_{G_i}^{L} = \sum_{\ell \in \Omega_L} \beta_{\ell i} \cdot \frac{c_\ell}{2} \cdot Fpond_\ell \ , \forall i \in \Omega_G \tag{4}$$

$$\pi_{D_j}^{L} = \sum_{\ell \in \Omega_L} -\beta_{\ell j} \cdot \frac{c_\ell}{2} \cdot Fpond_\ell \ , \forall j \in \Omega_D \tag{5}$$

The 50/50 rule is an intuitive and arbitrary choice. However, it is important to note that the proposed method can be easily changed to consider any other proportion. The discussion of this issue is beyond the scope of this work.

Because loads correspond to negative power injections, the $\beta_{\ell j}$ element in (5) is preceded by a negative sign. Because the weighting factor ($Fpond_\ell$) can vary between 0 and 1, the tariff defined in (4) and (5) multiplied by the generation or consumption does not guarantee that the total transmission cost is recovered. Thus, an additional complementary charge is needed to recover the difference. To obtain the total cost, an additional term, called the postage stamp, should be added to the tariff. Therefore, the postage stamp term of the final tariff applied to the generators and the loads can be obtained by the following expressions:

$$\Delta_G = \frac{\sum_{\ell \in \Omega_L} \frac{c_\ell}{2} - \sum_{i \in \Omega_G} P_{Gi} \cdot \pi_{G_i}^{L}}{\sum_{i \in \Omega_G} P_{Gi}} \tag{6}$$

$$\Delta_D = \frac{\sum_{\ell \in \Omega_L} \frac{c_\ell}{2} - \sum_{j \in \Omega_D} P_{Dj} \cdot \pi_{D_j}^{L}}{\sum_{j \in \Omega_D} P_{Dj}} \tag{7}$$

In this way, the final tariffs (π_{G_i} and π_{D_j}) applied to generators and loads are as follows:

$$\pi_{Gi} = \pi_{G_i}^{L} + \Delta_G, \forall i \in \Omega_G \tag{8}$$

$$\pi_{Dj} = \pi_{D_j}^{L} + \Delta_D, \forall j \in \Omega_D \tag{9}$$

The tariffs in (8) and (9) are multiplied by the power generated and consumed to recover the overall system transmission cost.

B. MIN-MAX LRMC METHOD – JOINT MODEL

Robust optimization methods [25] are based on the min-max approach. In particular, a min-max optimization technique is applied to solve the transmission cost allocation problem in [21]-[23]. This optimization approach is equivalent to the minimization of the supreme (maximum) value and is commonly used whenever dispersion and variability among a set of objectives is undesirable. The max operator identifies the worst-case tariff during the optimization process, and the min operator minimizes this tariff. As a result, the final optimal tariff computation presents a stable pattern.

To combine the LRMC method with the min-max procedure, we slightly modify the Nodal LRMC method to consider a generalization of the Equivalent Bilateral Exchange method [9]. Because NEFs are arbitrarily pre-defined in [9], we consider the contribution (or percentage) of a unitary power injection at bus i to feed a load at bus j as an optimization variable, namely, α_{ij}. This method can be seen as a generalization of the EBE method because the contribution from each generator to each load can assume different values depending on the method's objective. Thus, α_{ij} is selected to minimize the highest tariff subject to the system dispatch and the network flows through the lines. This approach can be seen as a robust LRMC-type method because it maintains the LRMC method features, yet allocates tariffs in a robust fashion, i.e., reducing the dispersion of tariffs among buses.

Because the problem being solved is a tariff allocation problem, the minimization of the worst tariff value is achieved by increasing the value of the other ones. Thus, the total transmission cost of the system is always recovered. However, the worst-case minimization does not imply a Pareto-efficient [24] tariff computation for the remaining tariffs. An iterative process is proposed to solve this problem: the min-max method is applied for the remaining tariffs, and the previous worst-case tariff is constrained to its previously found optimal value. This procedure is repeated until the set of remaining tariffs is empty and all tariffs are minimized.

To obtain the new tariffs, expressions (4) and (5) are slightly modified, according to (10) and (11), to incorporate NEFs to optimize the supply contributions between the generators and the loads. The postage stamp terms, (6) and (7), and the final tariffs, (8) and (9), remain the same.

$$\pi_{G_i}^{L} = \sum_{\ell \in \Omega_L} \frac{c_\ell}{2} \cdot Fpond_\ell \cdot \left[\sum_{j \in \Omega_D} \left(\beta_{\ell i} - \beta_{\ell j} \right) \alpha_{ij} \right], \forall i \in \Omega_G \tag{10}$$

$$\pi_{D_j}^{L} = - \sum_{\ell \in \Omega_L} \frac{c_\ell}{2} \cdot Fpond_\ell \cdot \left[\sum_{i \in \Omega_G} \left(\beta_{\ell j} - \beta_{\ell i} \right) \alpha_{ij} \right], \forall j \in \Omega_D \tag{11}$$

In (10), $\beta_{\ell i} - \beta_{\ell j}$ represents the power flow sensitivity in line ℓ, which is related to the extraction at bus j of $\alpha_{ij} \cdot 100\%$ of the power injection at bus i. Thus, the inner summation in (10) computes the total net power flow contribution in a given line ℓ related to the share of a unitary injection at a given generation bus i for all of the load buses.

Expression (11) follows the same idea explained for expression (10) but considers a load bus j: the inner summation accounts for the total assigned extractions at load bus j given the injections spread throughout all of the generation buses. It is important to notice that the flow sensitivities in (10) and (11) are given by the difference between the elements of different columns of the sensitivity matrix, β; therefore, the tariffs do not depend on the choice of the slack bus. A similar idea is presented in the Equivalent Bilateral Exchange (EBE) approach in [9], where a proof showing that the method does not depend on the choice of the slack bus is shown.

The iterative tariff computation process is controlled by the sets G_k, D_k, M_{Gk} and M_{Dk}. As mentioned before, an iterative process based on the worst-case minimization procedure is employed to obtain a Pareto-efficient solution. In each step, the previous worst-case tariff is saved, and the minimization procedure continues with the remaining tariffs. The Min-Max LRMC method is as follows:

Min-Max LRMC

Initialization: $k = 1$, $G_0 = \Omega_G$, $D_0 = \Omega_D$, $M_{G0} = \{\emptyset\}$, $M_{D0} = \{\emptyset\}$. $\boldsymbol{P_G}$ and $\boldsymbol{P_D}$ are given by a linear DC power flow.

While $G_{k-1} \neq \{\emptyset\}$ or $D_{k-1} \neq \{\emptyset\}$, do

1) Solve

$$\underset{\alpha_{ij}, \forall i \in \Omega_G, \forall j \in \Omega_D}{\text{minimize}} \; z_k \tag{12}$$

subject to

$$\sum_{j \in \Omega_D} \alpha_{ij} = 1, \forall i \in \Omega_G \tag{13}$$

$$0 \leq \alpha_{ij} \leq 1, \forall i \in \Omega_G, \forall j \in \Omega_D \tag{14}$$

$$P_{Dj} = \sum_{i \in \Omega_G} \alpha_{ij} P_{Gi}, \forall\, j \in \Omega_D \tag{15}$$

$$z_k \geq \sum_{\ell \in \Omega_L} \frac{c_\ell}{2} \cdot Fpond_\ell \cdot \left[\sum_{j \in \Omega_D} \left(\beta_{\ell i} - \beta_{\ell j}\right) \alpha_{ij} \right], \forall\, i \in G_{k-1} \tag{16}$$

$$z_k \geq -\sum_{\ell \in \Omega_L} \frac{c_\ell}{2} \cdot Fpond_\ell \cdot \left[\sum_{i \in \Omega_G} \left(\beta_{\ell j} - \beta_{\ell i}\right) \alpha_{ij} \right], \forall\, j \in D_{k-1} \tag{17}$$

$$\pi_{G_i}^{L} \geq \sum_{\ell \in \Omega_L} \frac{c_\ell}{2} \cdot Fpond_\ell \cdot \left[\sum_{j \in \Omega_D} \left(\beta_{\ell i} - \beta_{\ell j}\right) \alpha_{ij} \right], \forall\, i \in M_{Gk-1} \tag{18}$$

$$\pi_{D_j}^{L} \geq -\sum_{\ell \in \Omega_L} \frac{c_\ell}{2} \cdot Fpond_\ell \cdot \left[\sum_{i \in \Omega_G} \left(\beta_{\ell j} - \beta_{\ell i}\right) \alpha_{ij} \right], \forall\, j \in M_{Dk-1} \tag{19}$$

2) Calculate locational tariffs

$$\pi_{G_i}^{L} = \sum_{\ell \in \Omega_\ell} \frac{c_\ell}{2} \cdot Fpond_\ell \cdot \left[\sum_{j \in \Omega_D} \left(\beta_{\ell i} - \beta_{\ell j}\right) \alpha_{ij} \right], \forall i \in \Omega_G \tag{20}$$

$$\pi_{D_j}^{L} = -\sum_{\ell \in \Omega_\ell} \frac{c_\ell}{2} \cdot Fpond_\ell \cdot \left[\sum_{i \in \Omega_G} \left(\beta_{\ell j} - \beta_{\ell i}\right) \alpha_{ij} \right], \forall j \in \Omega_D \tag{21}$$

3) Obtain the values of the dual variables

$$y_{Gi}^{*}, \textit{Dual variable of constraint } i \textit{ in (16)}, \forall i \in G_{k-1} \tag{22}$$

$$y_{Dj}^{*}, \textit{Dual variable of constraint } j \textit{ in (17)} \; \forall j \in D_{k-1} \tag{23}$$

4) Update sets

$$G_k = G_{k-1} - \{i \in G_{k-1} |\, y_{Gi}^{*} \neq 0\} \tag{24}$$

$$D_k = D_{k-1} - \left\{j \in D_{k-1} |\; y_{Dj}^{*} \neq 0\right\} \tag{25}$$

$$M_{Gk} = M_{Gk-1} \cup \{i \in G_{k-1} |\; y_{Gi}^{*} \neq 0\} \tag{26}$$

$$M_{Dk} = M_{Dk-1} \cup \left\{j \in D_{k-1} |\; y_{Dj}^{*} \neq 0\right\} \tag{27}$$

5) $k = k + 1$

End do

Calculate postage stamp terms

$$\Delta_G = \frac{\sum_{\ell \in \Omega_L} \frac{C_\ell}{2} - \sum_{i \epsilon \Omega_G} P_{Gi} \cdot \pi_{G_i}^{L}}{\sum_{i \epsilon \Omega_G} P_{Gi}} \tag{28}$$

$$\Delta_D = \frac{\sum_{\ell \in \Omega_L} \frac{C_\ell}{2} - \sum_{j \epsilon \Omega_D} P_{Dj} \cdot \pi_{D_j}^{L}}{\sum_{j \epsilon \Omega_D} P_{Dj}} \tag{29}$$

Calculate final tariffs

$$\pi_{Gi} = \pi_{G_i}^{L} + \Delta_G, \qquad \forall i \in \Omega_G \tag{30}$$

$$\pi_{Dj} = \pi_{D_j}^{L} + \Delta_D, \qquad \forall j \in \Omega_D \tag{31}$$

The mathematical program presented in the first step of the algorithm belongs to the class of linear programming problems. The objective function (12) minimizes an auxiliary variable, z_k, that, due to constraints (16) and (17), is greater than or equal to the highest tariff among all generators and loads. Thus, minimizing z_k is equivalent to minimizing the highest tariff value that has not been minimized until iteration *k*.

The right hand side of constraints (16) and (17) are expressions (10) and (11), respectively. Constraints (18) and (19) bind the tariffs already minimized in previous iterations to values equal to or less than those found before. Because $M_{G0} = M_{D0} = \{\emptyset\}$ in the first iteration, those constraints are not employed. Finally, expression (15) constrains NEFs to reproduce the power balance equations. Thus, the power flow solution is the same during the entire process.

In step two, the locational tariffs are calculated according to the α_{ij} values determined by the optimization. These values are used in the next iteration in constraints (18) and (19). Because more than one generator/load may assume the worst-case tariff, i.e., a tariff equal to the optimal value, z_k, which one(s) are constraining the minimization processes and which ones could still be optimized must be defined, which is the case of degenerate solutions. In [22], this problem is solved using the Lagrange multipliers associated with constraints (14) and (15). In this paper, the same idea is employed. In any iteration, the dual variables of constraints (16) and (17) are obtained, as shown in the third step, according to (22) and (23). If those variables have zero values, the corresponding generators/demands should be moved for the next iteration because their optimal tariff values are not the ones that constrain the worst-case minimization process. Following this steps, a Pareto-efficient solution is found [24].

Given the optimal value obtained, the sets in (24)-(27) are updated in the fourth step of the iterative process. In the iterative process, the counter k is incremented, and the process restarts until all of the generators and loads tariffs are optimized, i.e., $G_{k-1} = \{\emptyset\}$ and $D_{k-1} = \{\emptyset\}$. With all of the locational tariffs optimized, the postage stamp terms for the generators and the loads are obtained in (28) and (29), respectively. Then, the final tariffs can be obtained in (30) and (31).

To complement the understanding of the algorithm, in Fig. 1 is presented a flowchart that describes the dynamics of the process of calculating tariffs through Min-Max LRMC method.

(FIGURE 1)

It is worth mentioning that the proposed method is based on the solution of at most the sum of the cardinalities of Ω_G and Ω_D linear programs, for which there are polynomial-time algorithms embedded into off-the-shelf solvers capable to efficiently solve large-scale problems [26]. Therefore, computational effort should not be seen as a barrier or challenge for the proposed method.

C. MIN-MAX LRMC METHOD – SEPARATE MODELS

Some electrical systems are characterized by demands that have better interconnections to the rest of the system than most generators, which is the case of the transmission systems in countries with a large territory and with renewable energy sources, such as Brazil.

In many cases, loads are dispersed, and the generators are concentrated in an area. From the point of view of the generators, their options of using transmission lines are limited, and, consequently, the reduction of the dispersion of tariffs is smaller compared to the case of the loads. In this type of system, the model presented in subsection III.B should be changed by considering generators and loads in separate models. This recommendation relies on the fact that, in a joint model, the generators that are weakly interconnected to the system can distort the locational signal of the loads, resulting in tariffs that do not give the desired economic signals.

Having two separate models follows the same idea of the joint model presented in subsection III.B. The difference is that the constraints and ex-post calculations of the tariffs are referenced only to generators in the optimization of the tariffs of the generators and only to loads in the optimization of the tariffs of the loads. Constraints (13)-(15) that refer to NEFs and the power flow balance must be present in both tariff optimization models (for generators and loads).

IV. CASE STUDIES

In this section, the results of the proposed method are presented in three subsections. In the first case, a 6-bus system is used to improve the understanding of the proposed method by applying the joint model. In the second case, the IEEE 118-bus system is used with the separate models, and the dispersion, and the locational signals of the tariffs are discussed. Finally, the third case addresses the issue of the volatility of the tariffs in the IEEE 118-bus system. The proposed algorithms presented in Section III were implemented in MATLAB [27]. The optimization problem was implemented in Xpress [26].

A. 6-BUS SYSTEM

The 6-bus system is presented in Fig. 2. For each line, the cost and the active power flow through the line are shown. For this system, the results of the first two iterations of the joint model of the proposed method are presented.

(FIGURE 2)

In the first iteration ($k = 1$), the sets are initialized as follows: G_0 = {1,3,5}, D_0 = {2,4,6}, $M_{G0} = M_{D0} = \{\emptyset\}$. Constraints (18) and (19) are not active in the first iteration. The optimized tariff in the first iteration is z_1 = 1.15 with the α_{ij} values shown in Table 1.

(TABLE 1)

Thus, the locational tariffs calculated by (20) and (21) are [0.89 0.37 1.15] for generators 1, 3 and 5 and [0.68 1.11 0.62] for loads 2, 4 and 6, which are all given in $/MW. To verify which generator/load should receive the optimal tariff, the dual variables of constraints (16) and (17) are obtained, as shown in (22) and (23). The dual variable is not zero for generator at bus 5, and this agent receives the optimized tariff. After applying this result to update the sets presented in (24)-(27), the new sets will be G_1= {1,3}, D_1 = {2,4,6}, M_{G1} = {5} and M_{D1}= {Ø}. The updated sets will be used in the second iteration ($k = 2$).

In the second iteration, constraint (18) is active because M_{G1}= {5}. For this iteration, the optimal tariff is z_2 = 0.95 with the α_{ij} values shown in Table 2.

(TABLE 2)

Some α_{ij} values were changed to optimize the second-iteration tariff. However, constraints (15) ensure that the results are still in accordance with the requirements of the system, and constraint (18) ensures that the generator at bus 5 retains its optimized tariff obtained in the first iteration. The locational tariffs are computed with the new α_{ij} values. Using (20) and (21), the locational tariffs for generators 1, 3 and 5 are [0.95 0.27 1.15], and for loads 2, 4 and 6, they are [0.80 0.95 0.62], which are all reported in $/MW. Thus, the dual variables of constraints (16) and (17) are computed. The dual variable is not zero for the generator at bus 1 and for the load at bus 4, and these agents receive the tariff.

At the end of the second iteration, the sets are updated to G_1 = {3}, D_1 = {2,6}, M_{G1} = {1,5} and M_{D1} = {4}. These sets are used in the next iteration. For the next iteration, all of the constraints are active because none of the sets is empty.

The iterative process continues until all generators and loads receive optimized locational tariffs. For the system shown in Fig. 2, the final tariffs (locational tariffs plus postage stamps) are [6.98 6.30 7.18] for generators 1, 3 and 5 and [6.92 7.07 7.04] for loads 2, 4 and 6, which are all in $/MW. The results of the power flow tracing (associated with the optimal final solution) for each pair ij is presented in (32):

$$\left(\beta_{\ell i} - \beta_{\ell j}\right) \cdot \alpha_{ij} \cdot P_{Gi} \tag{32}$$

Expression (32) represents the portion of the power flow through line ℓ caused by the pair ij for each line ℓ and pair ij. Due to lack of space, this analysis is performed only for the pair (5,6). Table 3 presents results to pair (5,6) for all lines. The generator 5 is the first one to receive an optimal tariff. Thus, it has the freedom to choose all routes in the system, bounded by the constraints.

(TABLE 3)

The tariffs can be minimized by taking advantage of the counter-flows, as occurs in lines 1-4, 2-3 and 3-4. Because (32) is present in (10) and (11) (excluding the generation term, P_{Gi}), it represents a reduction in the locational tariff.

B. IEEE 118-Bus System

In this subsection, the original system and a modified IEEE 118-bus system [28], depicted in Fig. 3, are used to present the tariff results of the Min-Max LRMC method compared with the results provided by other methods. The modified system is created by eliminating some transmission lines that are indicated as darker dashed lines in Fig. 3. The objective is to analyze the dispersion and the locational signals of the tariffs in both systems. To do so, we present the computation of the tariff using the Nodal LRMC, Min-Max LRMC and Pro rata methods.

(FIGURE 3)

An analysis of the system presented in Fig. 3 clearly shows that loads have better interconnections around them than most generators, which illustrates the situation described in Section III.C. Thus, the two separate models are used for all of the tests using the IEEE 118-bus system.

The Min-Max LRMC results are obtained for the original and modified systems. The objective of the tests is to analyze the reduction of the dispersion of the tariffs compared to the ones given by the Nodal LRMC method. In addition, the behavior of the locational signals is shown for two topologies: a system that offers few transmission routes for generators far from an area of concentrated loads (modified system) and a system that offers more routes for these generators (original system).

To do so, some lines are removed from the original system to isolate some generators as much as possible in the modified system. The expected result is that the values of the tariffs in the modified system will be closer to the ones obtained with the Nodal LRMC method. However, in the original system, the values of the tariffs will be closer to the ones obtained with the Pro rata method. The results obtained for generators and loads are shown in Fig. 4 and Fig. 5, respectively.

(FIGURE 4)

(FIGURE 5)

The results presented in Fig. 4 for the original and modified systems show a reduction in the dispersion of the tariffs using the Min-Max LRMC methods when compared to the Nodal LRMC method. The Min-Max LRMC method offers a better environment for new investors, mainly for dispersed energy sources, without losing adequate locational signals for investment. The highest tariffs, corresponding to generators far from load concentration areas, that are present in the traditional Nodal LRMC method for generators 10 and 61, are attenuated in the Min-Max LRMC method. Nevertheless, these tariffs are still among the highest ones, preserving the expected investment signals. The better-located generators (111, 103, 80, 87, 89 and 100) for both methods receive the lowest tariffs, reinforcing the idea that the Min-Max LRMC method preserves the locational signals for investment.

As mentioned in [16] the negative tariffs for some generators presented by LRMC Nodal method can be understood as a reduction in the need of future transmission investments due to a marginal generation capacity installed in the related buses. For this reason, a marginal power injection in those buses would avoid future investments in the transmission system and therefore, generators. This result can happen for any method based on LRMC Nodal factors.

The tariffs obtained with the Min-Max LRMC method have stronger locational signals in the modified system, and they are closer to the ones obtained with the Pro rata method in the original system. In particular, the increase of the strength of the locational signals occurs with generators 69, 25, 26, 46 and 49. These results are coherent with the expected behavior of the proposed method. The removal of several lines reduces the available transmission routes, making locational signals stronger. As a consequence, the tariffs follow the pattern of the Nodal LRMC method.

According to an analysis of the load results, the dispersion of the tariffs is reduced for both systems when comparing the Min-Max LRMC method with the Nodal LRMC method. As in the case of generators, the highest tariffs obtained using the Min-Max LRMC method are attenuated. However, in contrast to generators, almost all of the tariffs are equal to the tariffs given by the Pro rata method in the original and modified systems, which is justified by the structure of the network available to the loads.

In the original system, the concentration of loads has a well-connected set of lines available, which allows the Min-Max LRMC method to reduce the dispersion of the tariffs until almost all tariffs are equal to the ones of the Pro rata method. When the system is modified, the removal of lines (shown in Fig. 3) does not change the situation of the loads described for the original system dramatically. Once again, the system structure allows the proposed method to reduce the dispersion until the results are very close to those of the Pro rata method. An exception to the latter is only seen in the loads that receive the lowest tariffs in the Nodal LRMC and Min-Max LRMC methods. For these loads, a tariff allocation with a smooth locational signal occurs, which is explained by the proximity of the loads to the some of the largest generators in the system (generators on buses 59, 54, 61, 65 and 66). This proximity contributes to a lower usage of the network by the loads. Consequently, they receive lower tariffs compared to the other methods.

C. Volatility Analysis

To complete the simulations performed to the IEEE 118-bus system, a volatility analysis is presented. The objective of the analysis is to simulate a system expansion by gradually adding transmission lines, staring with the modified system and concluding the expansion in original system, both presented on Fig.3. The groups of lines added at each step (1 to 6) are signalized in Fig.3 by the number of step next to the darker dashed lines. The system obtained in step 6 of the expansion is the original system.

In this test, the Min-Max LRMC and LRMC Nodal methods were compared to observe the benefits of the proposed method in terms of reducing the volatility of the tariffs. To complete the comparison, results of Pro rata method are also presented. Two results for the proposed simulation are analyzed: first, (i) tariffs for one generator of the system considering the motivations for the development of the proposed method, and then (ii) standard deviations of tariffs during the expansion process for all generators in the system.

As mentioned in the introduction of this paper (Section II), one of the main motivations to the Min-Max LRMC method is to reduce the uncertainty in investments for generators located far from load centers, or loads far from generation centers. This simulation can reproduce the situation of a new generator that is far from loads and wishes to predict its tariff, based on a planned expansion of the transmission system. Thus, the volatility analysis will focus in one generator that represents this context in IEEE 118-bus system. The generator chosen was the one on the bus 59.

The tariffs for each step of the simulated expansion of the system are presented in Fig. 6 for generator 59. Also, the percentage variations of tariffs (related to the tariff of the modified system) are given in the discussion of results.

(FIGURE 6)

In Fig. 6, the Min-Max LRMC method presents less tariff variation through the process of expansion. In Step 2, with the addition of the second group of lines, a significant value of the tariff for LRMC Nodal method takes place. In the same step, Min-Max LRMC follows the tendency of increase. The difference stands in the intensity of the impact caused in the tariff by the change in the system, which is reduced in the Min-Max LRMC method compared to the LRMC Nodal method. This is an expected effect for the proposed method, and the result that favors the investments in generation. In relation to Pro rata method, for the generator 59, the Min-Max LRMC method has a similar tendency in variation of the tariff. As Pro rata is traditionally a method that proportionate low variation in tariffs, this proximity reinforces the tendency of the proposed method to provide tariffs with reduced volatility when changes in the transmission system occur.

Comparing the higher variations of the tariffs of each method in Fig.6, it is verified in step 2, for LRMC Nodal, a percentage variation (to the tariff in step 1) equal to 46.4%. To the Min-Max LRMC method, the higher variation occurs in the same step with a percentage variation equal to 15.4%. The occurrence of the higher variations in the same step reinforces the existence of a similar tendency between both methods. In Pro rata method the maximum percentage variation is also in step 2 and is equal to 15.9%. It is also important to observe the initial (modified system) and final (original system) tariffs in the three methods. In LRMC Nodal is verified a difference of 33.7% between both tariffs, against 12.3% in Min-Max LRMC. In Pro rata is verified a variation of 24.8%. These results show that the proposed method is capable of provide less volatile tariffs along a process of expansion of the transmission system. This is an important and desirable feature for new investors in generation that look for an economic safety through more stable and predictable transmission usage costs.

To the other generators and demands in the system, is verified a maximum variation of 182.7% in generator 12 and 823.8% in demand 51, for the LRMC Nodal method. In the Min-Max LRMC method the maximum variations occur in generator 12 with a 96.1% and 19.9% for demand 62. In Pro rata method the higher variations are of 15.9% for all generators and demands in step 2. These results complete the first ones presented, and confirm the expectations of less volatility for the proposed method compared with the traditional LRMC Nodal.

The last analysis of volatility involves the standard deviations of tariffs for each one of all generators in the system in the simulated expansion. The objective is to extend the analysis of volatility to all generators and compare the methods. The results are presented in Fig. 7.

(FIGURE 7)

It can be seen that, for most of the generators in the system, Min-Max LRMC method provides lower standard deviation when compared to LRMC Nodal method. In particular, generators 46, 49 and 59, which have the higher deviations on LRMC Nodal method, have a reduced deviation on Min-Max LRMC method. It is also observed that, for many results, the proposed method has a proximity to Pro rata, indicating a tendency of low volatility.

It is worth observing the standard deviations obtained for the generators 12, 25 and 26 on Min-Max LRMC method. For these generators, is verified a higher tariff deviation on Min-Max LRMC in comparison with the LRMC Nodal. This result is justified by the fact that the mentioned generators received more increments in the process described in Section III.B. However, it is highlighted that the same process provides significant reduction in the deviations of other generators (especially in generators 46. 49 and 59), which keeps the proposed method as a valuable alternative to reduce volatility of the tariffs.

For all tests, the computational time taken by the proposed method was about 5 min. for the IEEE 118-bus systems. Simulations were performed with an Intel i7 core, 3.40 GHz CPU, and 8 GB of RAM memory, in a 64 bits operational system.

V. CONCLUSION

This paper presents a new transmission tariff computation methodology based on the LRMC method combined with a min-max optimization technique. An iterative algorithm is developed that ensures that the generators/loads with higher tariffs are prioritized to minimize them, maintaining the desirable locational signals. The first objective is to guarantee a lower dispersion of the tariffs, which is made respecting systems` power flow constraints, and considering the availability of routes through transmission lines. The second one, as a consequence, is the reducing of the volatility of the tariffs given the network changes that can occur. Because the transmission tariff should change when the system changes (new generators and/or transmission lines are installed), in developing countries, where huge investments in generation and transmission can take place, the uncertainty created by volatile tariffs is generally addressed through *ad hoc* procedures. The proposed method permits avoiding these procedures, and treat the uncertainty inside the model of the method.

As a conclusion, the proposed method takes into account the network usage, its topology, and the system participants to produce minimal-dispersed tariffs regarding the need of locational signal. The proposed method can be useful for regulators to mitigate uncertainty and create incentives for renewables regarding the network characteristics.

The results presented indicate that the proposed approach has less-dispersed tariffs compared with the traditional Nodal LRMC method but retains its good features. The proposed method, which is capable of reducing the dispersion between tariffs, is consistent with the existence of network constraints and results in appropriate locational signals for generation investment. Also, in the case of a system expansion, tariffs tend to have a more stable pattern with reduced volatility that decrease the economic uncertainty faced by new investors in generation.

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Figure and Table captions

Fig. 1: Flowchart of LRMC Min-Max algorithm.

Fig. 2: Six-bus test system.

Fig. 3: IEEE 118-bus system.

Fig. 4: Tariffs of the generators for the original and modified IEEE 118-bus systems.

Fig. 5: Tariffs of the demands for the original and modified IEEE 118-bus systems.

Fig. 6: Tariffs of generator 59 for a simulated expansion of IEEE 118-bus system.

Fig. 7: Standard deviation of generators' tariffs for a simulated expansion of IEEE 118-bus system.

Table 1. α_{ij} values for the 6-bus system – first iteration.

Table 2. α_{ij} values for the 6-bus system – second iteration.

Table 3. Power flows caused by an exchange between generator 5 and load 6.

Tables (3)

Table 1. α_{ij} values for the 6-bus system – first iteration

Generator 1	*Generator 3*	*Generator 5*
$\alpha_{12} = 0.30$	$\alpha_{32} = 1.00$	$\alpha_{52} = 0.00$
$\alpha_{14} = 0.70$	$\alpha_{34} = 0.00$	$\alpha_{54} = 0.38$
$\alpha_{16} = 0.00$	$\alpha_{36} = 0.00$	$\alpha_{56} = 0.62$

Table 2. α_{ij} values for the 6-bus system – second iteration

Generator 1	*Generator 3*	*Generator 5*
$\alpha_{12} = 0.59$	$\alpha_{32} = 0.51$	$\alpha_{52} = 0.00$
$\alpha_{14} = 0.41$	$\alpha_{34} = 0.49$	$\alpha_{54} = 0.38$
$\alpha_{16} = 0.00$	$\alpha_{36} = 0.00$	$\alpha_{56} = 0.62$

Table 3. Power flows caused by an exchange between generator 5 and load 6

Transmission Line	Power flows caused by pair (5,6)
Line 1-2	1.245 MW
Line 1-4	-8.716 MW
Line 1-6	7.471 MW
Line 2-3	-1.245 MW
Line 3-4	-12.451 MW
Line 3-6	13.697 MW
Line 4-5	21.1673 MW
Line 5-6	18.833 MW

Figures (7)

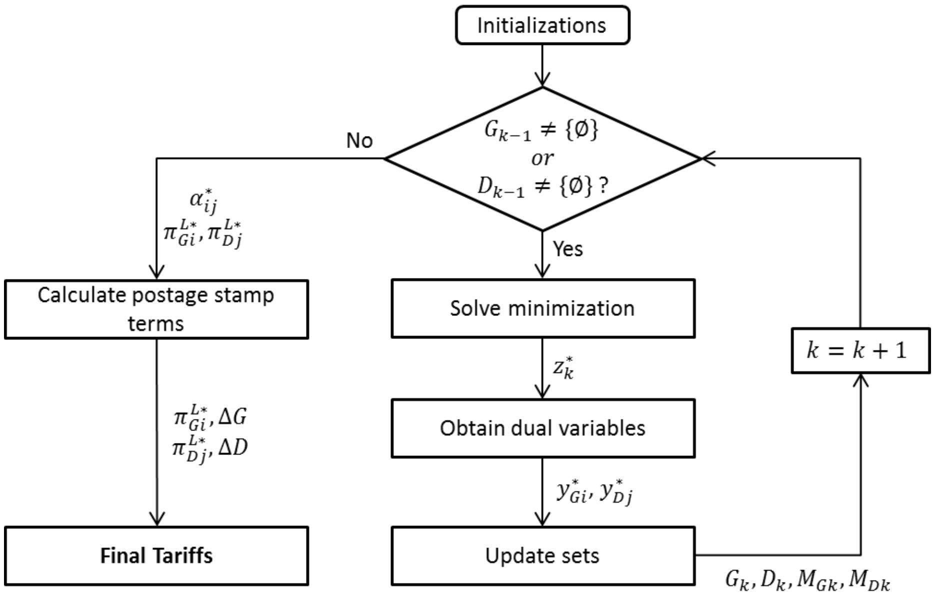

Fig. 1: Flowchart of LRMC Min-Max algorithm.

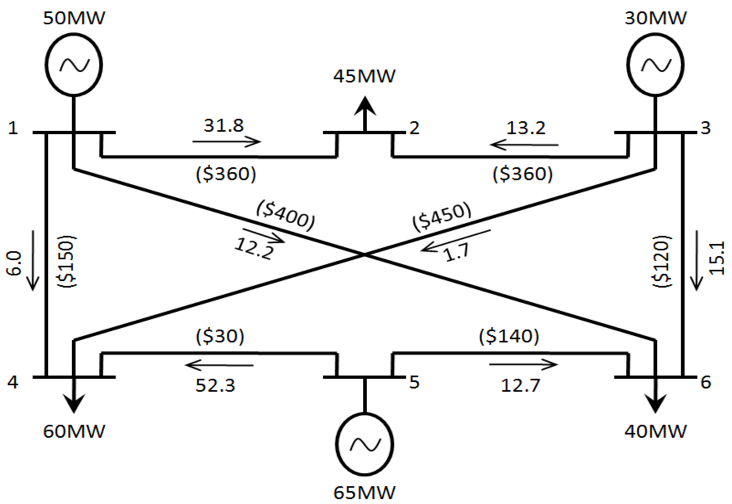

Fig. 2: Six-bus test system.

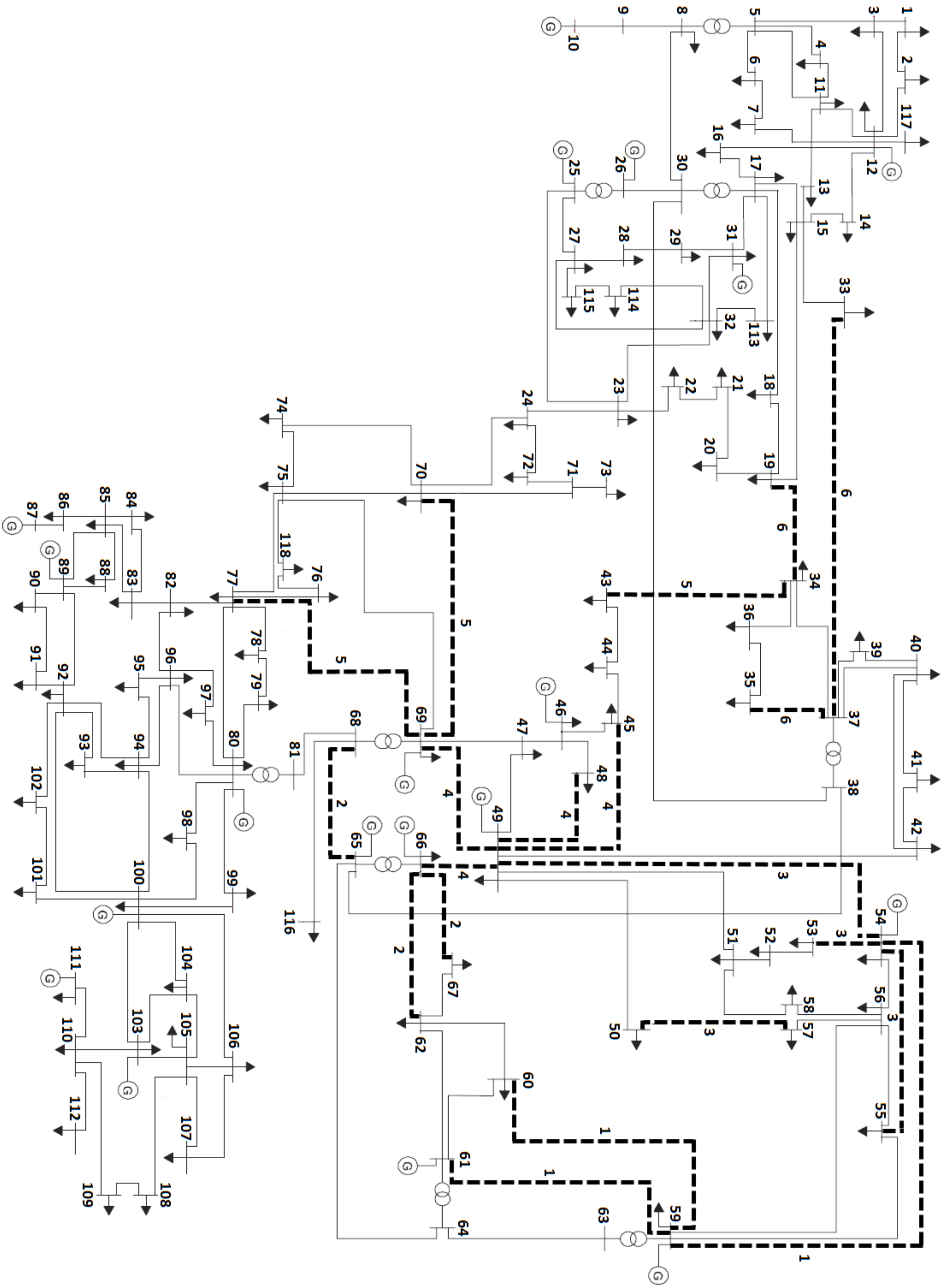

Fig. 3: IEEE 118-bus system.

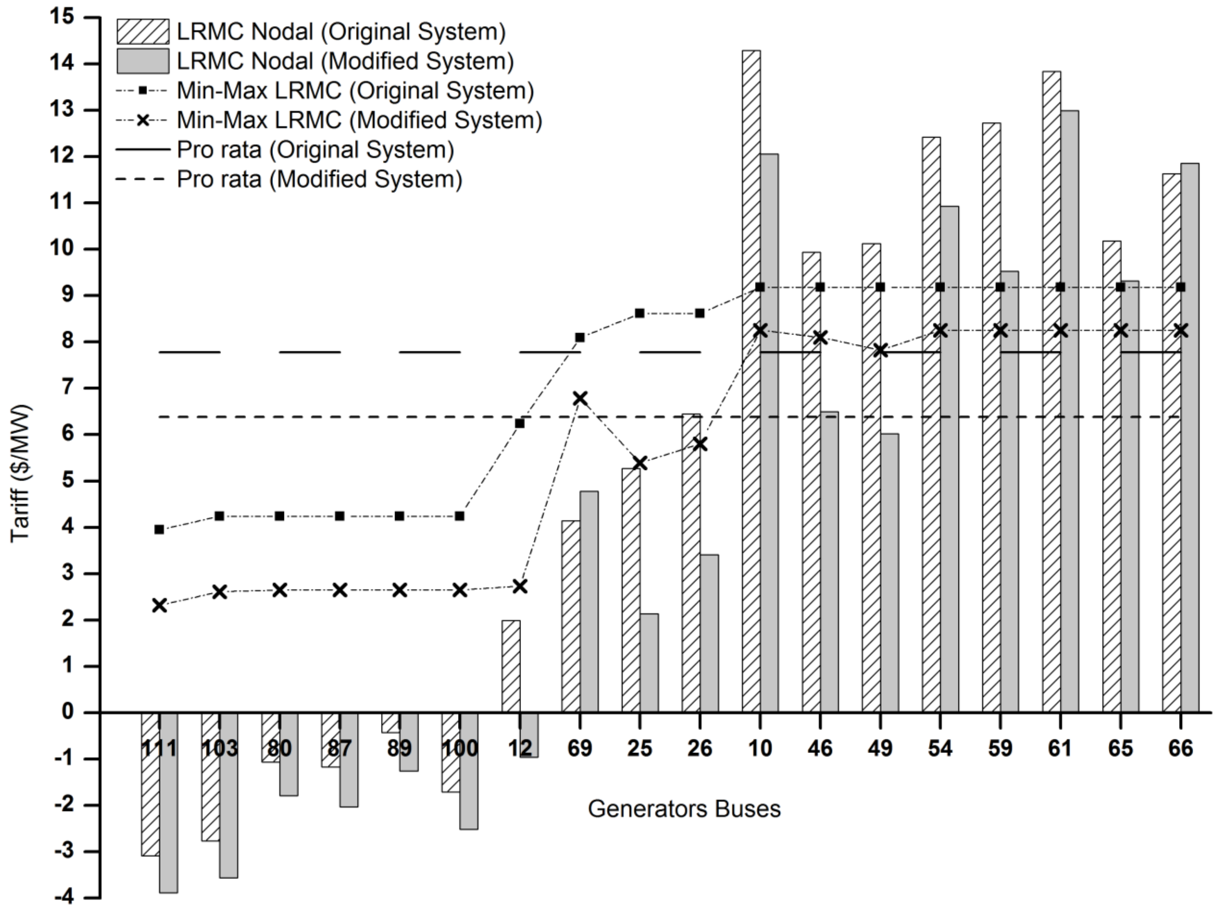

Fig. 4: Tariffs of the generators for the original and modified IEEE 118-bus systems.

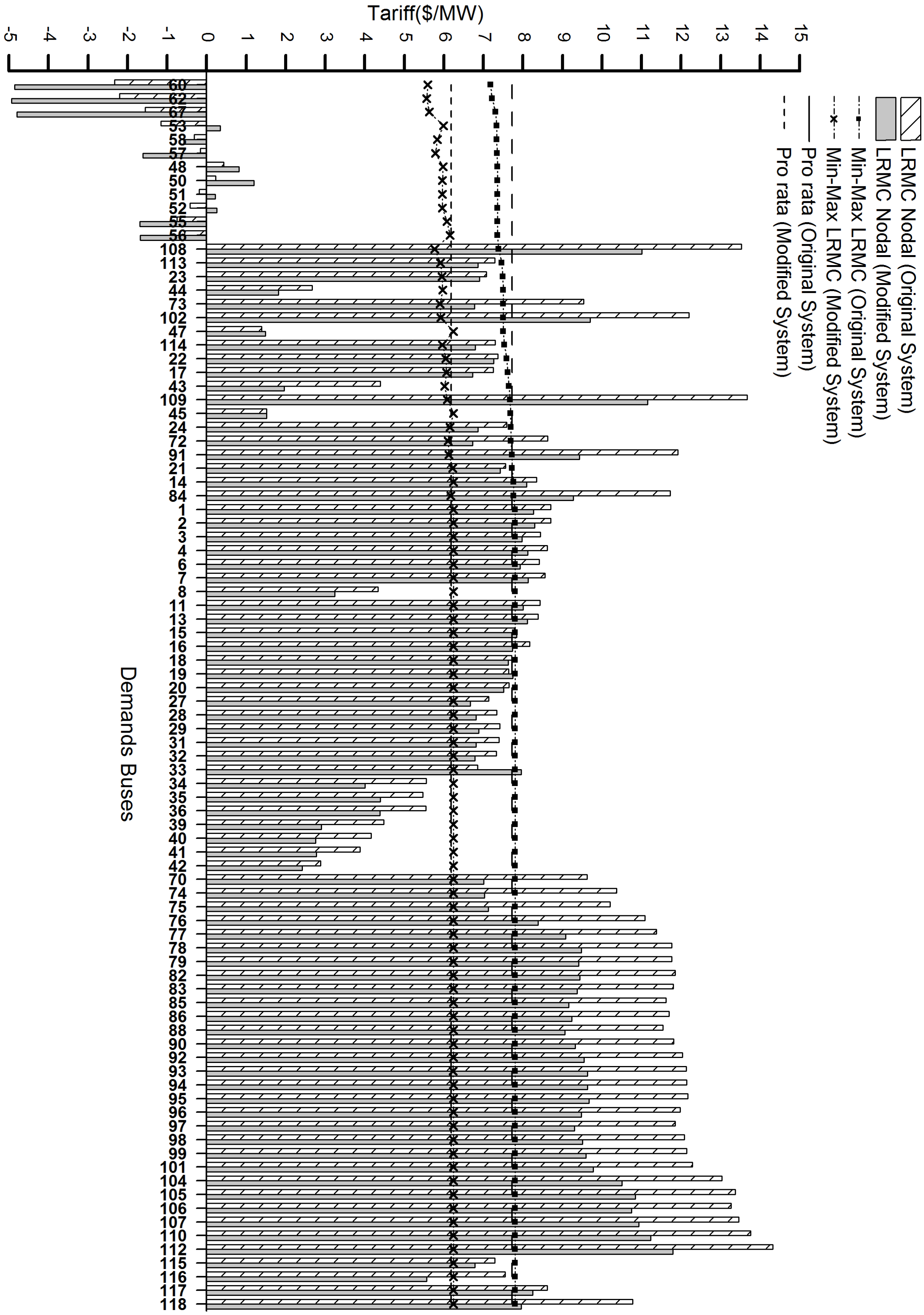

Fig. 5: Tariffs of the demands for the original and modified IEEE 118-bus systems.

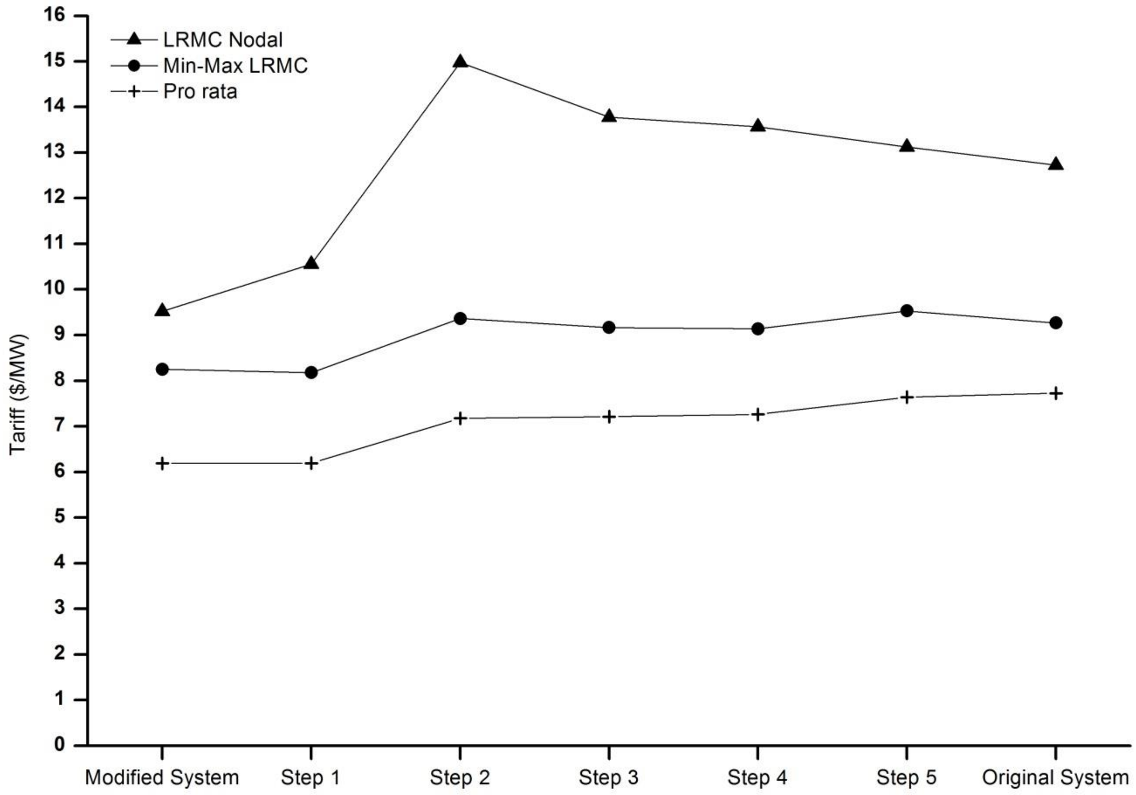

Fig. 6: Tariffs of generator 59 for a simulated expansion of IEEE 118-bus system.

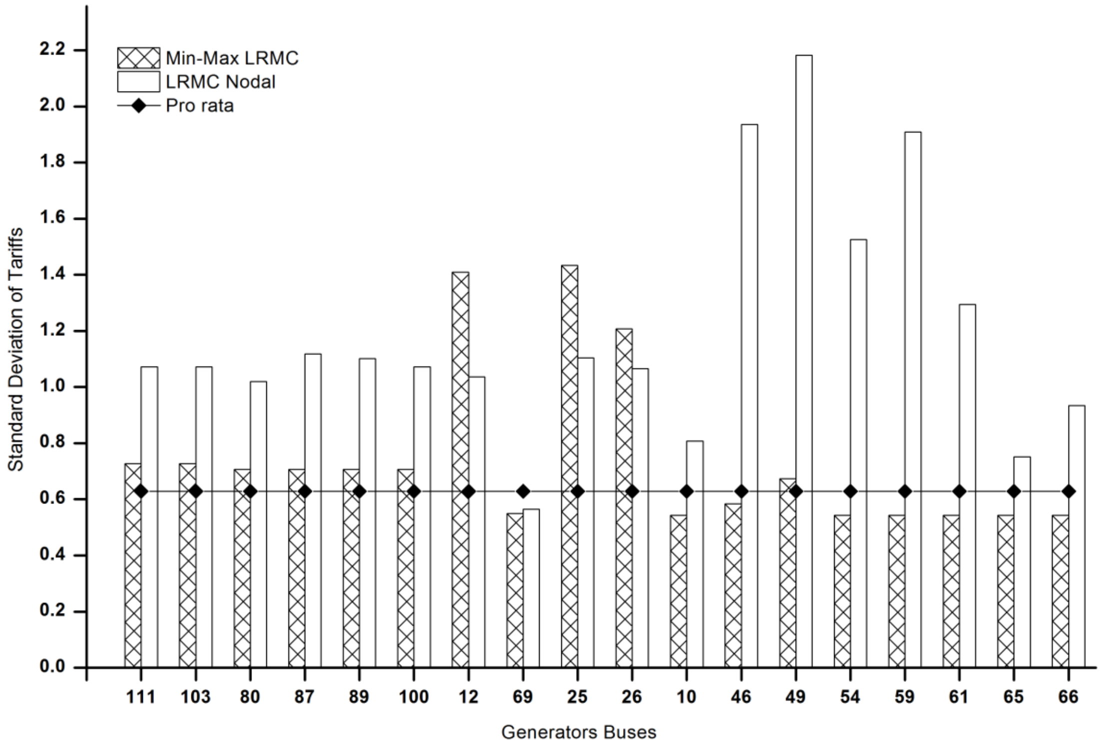

Fig. 7: Standard deviation of generators' tariffs for a simulated expansion of IEEE 118-bus system.