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Energy and Reserve Scheduling under a Joint Generation and Transmission Security Criterion: An Adjustable Robust Optimization Approach

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***Abstract*—This paper presents a new approach for energy and reserve scheduling in electricity markets subject to transmission flow limits. Security is imposed by guaranteeing power balance under each contingency state including both generation and transmission assets. The model is general enough to embody a joint generation and transmission $n-K$ security criterion and its variants. An adjustable robust optimization approach is presented to circumvent the tractability issues associated with conventional contingency-constrained methods relying on explicitly modeling the whole contingency set. The adjustable robust model is formulated as a trilevel programming problem. The upper-level problem aims at minimizing total costs of energy and reserves while ensuring that the system is able to withstand each contingency. The middle-level problem identifies, for a given pre-contingency schedule, the contingency state leading to maximum power imbalance if any. Finally, the lower-level problem models the operator's best reaction for a given contingency by minimizing the system power imbalance. The proposed trilevel problem is solved by a Benders decomposition approach. For computation purposes, a tighter formulation for the master problem is proposed. Our approach is finitely convergent to the optimal solution and provides a measure of the distance to the optimum. Simulation results show the superiority of the proposed methodology over conventional contingency-constrained models.**

***Index Terms*—Adjustable Robust Optimization, Benders Decomposition, Energy and Reserve Scheduling, Generation and Transmission Security Criterion, Trilevel Programming.**

NOMENCLATURE

This section lists the main notation used throughout the paper. Additional symbols with superscripts "(j)" and "(m)" are used to indicate the value of a specific variable at iterations j and m, respectively.

A. Functions

$C_i^P(\cdot)$ Energy cost function offered by generator i.

$\boldsymbol{f}(\cdot)$ Vector of functions defining the security criterion.

B. Constants

$\bar{\gamma}_i$ Bounding parameter for dual variable γ_i.

$\bar{\chi}_i$ Bounding parameter for dual variable χ_i.

$\bar{\omega}_l$ Bounding parameter for dual variable ω_l.

A_i^k Availability parameter that is equal to 0 if generator i is unavailable under contingency state k, being 1 otherwise.

A_l^k Availability parameter that is equal to 0 if line l is unavailable under contingency state k, being 1 otherwise.

C_i^D Cost rate offered by generator i to provide down-spinning reserve.

C^I Power-imbalance cost coefficient.

C_i^U Cost rate offered by generator i to provide up-spinning reserve.

D_b Demand at bus b.

$\overline{F}_l$ Power flow capacity of line l.

$fr(l)$ Sending or origin bus of line l.

K, K^G, K^L Number of unavailable system components, generators, and transmission lines, respectively.

n Number of system components.

$\overline{P}_i$ Capacity of generator i.

$\underline{P}_i$ Minimum power output of generator i.

$\overline{R}_i^D$ Upper bound for the down-spinning reserve contribution of generator i.

$\overline{R}_i^U$ Upper bound for the up-spinning reserve contribution of generator i.

$to(l)$ Receiving or destination bus of line l.

x_l Reactance of line l.

C. Decision Variables

α Approximation of the system power imbalance in the master problem.

δ^{wc} Auxiliary variable representing the worst-case system power imbalance.

ΔD^{wc} System power imbalance under the worst-case contingency.

ΔN_b^{wc} Auxiliary variable used in the linearization of the absolute value of the power imbalance at bus b under the worst-case contingency.

ΔP_b^{wc} Auxiliary variable used in the linearization of the absolute value of the power imbalance at bus b under the worst-case contingency.

θ_b Phase angle at bus b in the pre-contingency state.

θ_b^k Phase angle at bus b under contingency k.

θ_b^{wc} Phase angle at bus b under the worst-case contingency.

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a_i^G Binary variable that is equal to 0 if generator i is unavailable under the worst-case contingency, being 1 otherwise.
a_l^L Binary variable that is equal to 0 if line l is unavailable under the worst-case contingency, being 1 otherwise.
f_l Power flow of line l in the pre-contingency state.
f_l^k Power flow of line l under contingency k.
f_l^{wc} Power flow of line l under the worst-case contingency.
h_i Variable equal to the product $\chi_i a_i^G$.
p_i Power output of generator i in the pre-contingency state.
p_i^k Power output of generator i under contingency k.
p_i^{wc} Power output of generator i under the worst-case contingency.
r_i^D Down-spinning reserve provided by generator i.
r_i^U Up-spinning reserve provided by generator i.
v_i Binary variable that is equal to 1 if generator i is scheduled in the pre-contingency state, being 0 otherwise.
y_l Variable equal to the product $\omega_l a_l^L$.
z_i Variable equal to the product $\gamma_i a_i^G$.

D. Dual Variables

β_b Dual variable associated with the power balance equation at bus b under the worst-case contingency.
γ_i Dual variable associated with the lower bound for p_i^{wc}.
λ Dual variable associated with the $n-K^G$ security constraint in the robust approach for energy and reserve scheduling under a generation security criterion.
ξ_i Dual variable associated with the upper bound for generator i availability in the robust approach for energy and reserve scheduling under a generation security criterion.
π_l Dual variable associated with the lower bound for f_l^{wc}.
σ_l Dual variable associated with the upper bound for f_l^{wc}.
χ_i Dual variable associated with the upper bound for p_i^{wc}.
ω_l Dual variable associated with the equation relating power flow and phase angles for line l under the worst-case contingency.

E. Sets

$\mathcal{C}$ Set of contingency indexes.
I Set of generator indexes.
I_b Set of indexes of generators connected to bus b.
$\mathcal{L}$ Set of transmission line indexes.
N Set of bus indexes.

I. Introduction

SECURITY has been one of the main issues in power system operation and planning in the last two decades. There are basically two main approaches to deal with system component availability in such problems: (i) stochastic approaches (see [1] and references therein), and (ii) deterministic approaches [2]-[10]. Most power systems worldwide are currently operated under the well-known $n-1$ and $n-2$ security criteria, which in industry practice are implemented as deterministic approaches [2], [11].

Based on current industry practice, the present work addresses the application of deterministic security criteria in power system operation. Deterministic security criteria require a power system to withstand a set of credible contingencies.

Generation scheduling problems have traditionally incorporated deterministic security criteria by contingency-constrained models [3]-[8]. For the sake of computational tractability, such models explicitly represent the operation of the power system under a reduced set of credible contingencies. This limitation is stressed in the current context where recent blackouts involving the loss of more than two components [12], [13] suggest that tighter security levels comprising multiple outages should be considered. Examples of extended security criteria are the $n-K$ criterion [9], [10], [14], [15], by which the system is able to withstand the simultaneous loss of up to K system components, and the $n-K^G-K^L$ criterion [16], which considers the simultaneous loss of up to K^G generators and up to K^L transmission lines.

To overcome the dimensionality curse observed in conventional contingency-constrained models, Street *et al.* [9], [10] recently proposed robust optimization [17], [18] to schedule energy and reserves under a deterministic $n-K$ security criterion. In both works, the effect of the transmission network was neglected and only generator outages were considered. In [9], a single-period setting was used to illustrate the effectiveness of robust optimization to implicitly consider the whole contingency set. In [10], the approach was extended by analyzing a multiperiod setting and adding non-spinning reserves to the problem formulation.

This paper presents a new approach to incorporate a deterministic security criterion in the co-optimization of energy and reserves [3]-[6], [9], [10]. The salient feature of the proposed model over [9], [10] is the consideration of the transmission network. This modeling novelty is motivated by (i) current industry practice worldwide in the framework of the operation of electricity markets, and (ii) the need to consider line outages in power system operation problems to properly account for standard security criteria. Network constraints are needed to define locational reserves and their deliverability under a given security criterion (see [19] and references therein). Moreover, the second motivation is deemed as crucial given the recent major blackouts where failures in the transmission network played a key role [20]. It should be noted that single-bus models available in the technical literature [9], [10] are not suitable to address both aspects. Hence, new models such as the one proposed in this paper are required.

From the modeling perspective, the consideration of the transmission network gives rise to two major modifications with respect to the problem formulation presented in [9], [10]: (i) down reserves are required to characterize the operation under contingency, and (ii) line outages are addressed thus allowing the consideration of transmission assets in the security criteria.

This paper also differs from [9], [10] from the methodological perspective. The consideration of down reserves requires explicitly modeling the operation under contingency. Therefore, the robust optimization framework based on bilevel programming of [9], [10] is not readily applicable when the effect of the transmission network is accounted for. As a distinctive feature, the proposed approach is based on adjustable robust optimization (ARO) [21], [22]. Similar to robust optimization, ARO is suitable to model optimization problems where the optimal solution must remain feasible for the worst-case parameter variation in a user-defined set, denoted as uncertainty set [17], [18]. In contrast, ARO allows incorporating the flexibility of adjustable decisions, also known as recourse actions, in robust counterparts [21], [22]. In this setting, ARO involves a trilevel optimization process [21]-[23]. The upper level determines optimal non-adjustable decisions, i.e., decisions that must be feasible for every deviation of the uncertain parameters. The middle level identifies the worst-case parameter values leading to maximum feasibility damage of the upper-level decisions. Finally, the lower level aims at finding the best reaction, by means of adjustable variables, that minimizes the upper-level infeasibility. Two recent examples of successful application of ARO and trilevel programming in power systems can be found in [24], [25], where the unit commitment problem was solved considering uncertain nodal injections. In [24], uncertainty was associated with wind power generation and recourse variables modeled the operation of pumped-storage hydro units. In [25], uncertain nodal demand was also considered.

In the proposed ARO-based approach for generation scheduling under a joint generation and transmission security criterion, the parameters allowed to vary represent the availability of system components under the contingency states. In addition, adjustable decisions are post-contingency operation variables such as generation levels and line flows. Similar to [24], [25], the adjustable robust counterpart is formulated as a trilevel mixed-integer program that is solved by a Benders decomposition approach involving bilinear terms and the iterative solution of a master problem and a subproblem. It should be noted that the presence of binary variables in the middle level of the proposed trilevel program does not allow its transformation to a single-level equivalent, as done in [9], [10]. Two methods have been proposed in the technical literature to deal with those bilinear terms: (i) a linearization scheme based on disjunctive constraints [24], which has also been widely used in the application of bilevel programming in power system planning (see [26] and references therein); and (ii) an outer approximation technique based on an iterative heuristic procedure [25]. In this paper, the former method is used. Hence, the subproblem is formulated as a bilevel programming problem that is equivalently recast as a mixed-integer linear program. The master problem is a mixed-integer linear program that provides an approximation of the original trilevel problem. In order to improve the performance of the decomposition procedure, two sets of valid constraints are added to the master problem.

The main contributions of this paper are as follows:

1. A new model is presented for the contingency-constrained energy and reserve scheduling problem under a deterministic security criterion. Unlike previously reported works, this model allows examining the effect of the transmission network while also considering line failures.
2. Adjustable robust optimization with a combinatorial uncertainty set is proposed as a suitable solution framework. The resulting problem is formulated as a trilevel programming problem.
3. A solution methodology based on Benders decomposition is proposed. The performance of the proposed approach is improved by adding two sets of valid constraints that provide a tighter formulation. The superiority of the proposed method is backed by its faster performance and its ability to solve cases for which conventional contingency-constrained models are unable to find a feasible solution.
4. The proposed tool allows the system operator to assess the impact of tighter security criteria than currently used $n-1$ and $n-2$. In addition, the proposed methodology is flexible enough to comprise a wide range of security criteria such as separate criteria for generation and transmission, as well as specific criteria for subsets of system components. Finally, since the proposed model relies on the co-optimization of energy and reserves, it also constitutes a suitable methodology to define locational reserve requirements needed to implement a deterministic security criterion considering the effect of the transmission network.

The rest of this paper is organized as follows. Section II presents the conventional contingency-constrained formulation for the energy and reserve scheduling problem under a joint generation and transmission security criterion. In Section III, the trilevel ARO counterpart is provided. Section IV describes the proposed solution algorithm. In Section V, two case studies are analyzed. In Section VI, conclusions are drawn, and ongoing as well as future research topics are presented. Finally, the robust scheduling problem from which valid generation outage constraints are derived is formulated in the Appendix.

II. Conventional Contingency-Constrained Problem Formulation

The contingency-constrained generation scheduling problem determines the optimal generation schedule and reserve allocation so that the power balance is ensured under both normal and contingency states. Here we propose the explicit consideration of a joint generation and transmission security criterion. For expository purposes, we use a contingency-dependent network-constrained model based on that of [4], where a single period is analyzed and the focus is placed on synchronized reserves, specifically up-spinning and down-spinning. This model is simple to describe and analyze, yet bringing out the main features of contingency dependence. The multiperiod model would require extra indices denoting time periods and the inclusion of inter-temporal constraints such as minimum up and down times, ramping limits, and storage management. An example of multiperiod scheduling with non-synchronized reserves can be found in [10], where network constraints were neglected. The network-constrained contingency-dependent scheduling problem is formulated as:

$$\underset{\theta_b,\theta_b^k,f_l,f_l^k,p_i,p_i^k,r_i^D,r_i^U,v_i}{\text{Minimize}} \sum_{i\in I} \left(C_i^P(p_i,v_i) + C_i^U r_i^U + C_i^D r_i^D\right) \quad (1)$$

subject to:

$$\sum_{i\in I_b} p_i + \sum_{l\in\mathcal{L}|to(l)=b} f_l - \sum_{l\in\mathcal{L}|fr(l)=b} f_l = D_b; \forall\, b\in N \quad (2)$$

$$f_l = \frac{1}{x_l}\left(\theta_{fr(l)} - \theta_{to(l)}\right); \forall\, l\in\mathcal{L} \quad (3)$$

$$-\overline{F}_l \le f_l \le \overline{F}_l; \forall\, l\in\mathcal{L} \quad (4)$$

$$\underline{P}_i v_i \le p_i \le \overline{P}_i v_i; \forall\, i\in I \quad (5)$$

$$p_i + r_i^U \le \overline{P}_i v_i; \forall\, i\in I \quad (6)$$

$$p_i - r_i^D \ge \underline{P}_i v_i; \forall\, i\in I \quad (7)$$

$$0 \le r_i^U \le \overline{R}_i^U v_i; \forall\, i\in I \quad (8)$$

$$0 \le r_i^D \le \overline{R}_i^D v_i; \forall\, i\in I \quad (9)$$

$$v_i\in\{0,1\}; \forall\, i\in I \quad (10)$$

$$\sum_{i\in I_b} p_i^k + \sum_{l\in\mathcal{L}|to(l)=b} f_l^k - \sum_{l\in\mathcal{L}|fr(l)=b} f_l^k = D_b; \quad \forall\, b\in N, \forall\, k\in\mathcal{C} \quad (11)$$

$$f_l^k = \frac{A_l^k}{x_l}\left(\theta_{fr(l)}^k - \theta_{to(l)}^k\right); \forall\, l\in\mathcal{L}, \forall\, k\in\mathcal{C} \quad (12)$$

$$-\overline{F}_l \le f_l^k \le \overline{F}_l; \forall\, l\in\mathcal{L}, \forall\, k\in\mathcal{C} \quad (13)$$

$$A_i^k(p_i - r_i^D) \le p_i^k \le A_i^k(p_i + r_i^U); \forall\, i\in I, \forall\, k\in\mathcal{C}. \quad (14)$$

The objective function to be minimized (1) consists of the sum of the offered cost functions for generating energy plus the cost of all up- and down-spinning reserves offered by the generators.

Constraints (2)-(10), hereinafter referred to as pre-contingency scheduling constraints, impose the feasibility of the pre-contingency state schedule. Constraints (2) represent the nodal power balance equations. Using a dc load flow model, constraints (3) express the line flows in terms of the nodal phase angles, while constraints (4) enforce the corresponding line flow capacity limits. As is customary in generation scheduling in electricity markets [2], [4]-[8], a dc load flow model is used to characterize the behavior of the network, recognizing that the use of such a simplified model leads to results that may be optimistic and that a complete study of the scheduling problem under a joint generation and transmission security criterion should also consider the effect of reactive power. This generalization would, however, render the problem essentially intractable. This modeling limitation notwithstanding, the solution of the energy and reserve scheduling problem based on the dc load flow is acceptable for the purposes of the operation of electricity markets [2], [4]-[8] and provides the system operator with a first estimate of a secure generation scheme.

Constraints (5) set the generation limits. Constraints (6) and (7) respectively relate the up- and down-spinning reserve contributions to the power levels produced under the pre-contingency state. Constraints (8)-(9) provide the bounds for the up- and down-spinning reserve contributions, respectively. Finally, the binary nature of scheduling variables is expressed in (10).

In (11)-(14), a feasible post-contingency redispatch is ensured. Analogous to (2)-(4), expressions (11)-(13) are the network constraints under contingency. Generation limits for the contingency states are set in (14). In (12) and (14), the statuses of system components are characterized by the generator and line availability binary parameters, A_i^k and A_l^k, respectively.

The dimension of model (1)-(14), in terms of the number of variables and constraints, and hence its computational tractability, both depend on the cardinality of $\mathcal{C}$. For the case of a joint generation and transmission security criterion, the contingency set $\mathcal{C}$ can be modeled in a compact way as:

$$\boldsymbol{f}\left(\{A_i^k\}_{i\in I}, \{A_l^k\}_{l\in\mathcal{L}}\right) \ge \mathbf{0};\ \forall\, k\in\mathcal{C}, \quad (15)$$

where $\boldsymbol{f}(\cdot)$ is a vector function. Typical joint generation and transmission security criteria can be modeled by a linear form of $\boldsymbol{f}(\cdot)$. For an $n-K$ criterion, the formulation of (15) would be $\sum_{i\in I} A_i^k + \sum_{l\in\mathcal{L}} A_l^k \ge n-K;\ \forall\, k\in\mathcal{C}$, where $n=|I|+|\mathcal{L}|$. Variants of such criterion such as the $n-K^G-K^L$ can also be considered in a similar fashion. Under such criteria, the size of problem (1)-(14) presents an exponential dependence with K, K^G, and K^L, which may lead to intractability even for low values of those parameters.

III. Adjustable Robust Optimization Approach

Problem (1)-(14) finds the least-cost schedule of power and reserves able to circumvent the contingency states included in $\mathcal{C}$. In other words, the power imbalance is explicitly set to zero for all contingencies considered in the contingency-dependent formulation. This problem can be viewed as a particular instance of ARO [21], [22] wherein the parameters allowed to vary are A_i^k and A_l^k. Under this framework, the decisions modeling the reaction of the system operator against the occurrence of contingencies, i.e., decision variables with superscript k, are denoted as recourse actions or adjustable decisions [21]. Hence, the proposed ARO-based model belongs to the class of contingency-constrained generation scheduling problems, but differs from (1)-(14) in the way the operation under contingency is accounted for.

Next, the ARO-based modeling framework is described, the formulation of the proposed robust counterpart is provided, its equivalence with the original contingency-dependent model is discussed, and a simple illustrative example is analyzed.

A. ARO-Based Modeling Framework

The proposed ARO-based approach is characterized as a trilevel program [23], as shown in Fig. 1, which is based on the following rationale: for a given upper-level decision, the middle level problem searches in the contingency set the most damaging subset of K elements in terms of power imbalance, given the best redispatch provided by the lower level within the scheduled reserves and the remaining network and generators after contingency.

The upper level determines the cheapest pre-contingency schedule for power and reserves. In order for the pre-contingency schedule to be feasible, the system power imbalance should be equal to zero for all contingencies. Based on robust optimization [21], [22], [24], [25], pre-contingency feasibility can be modeled as a worst-case analysis requiring two additional optimization levels. For a given upper-level pre-contingency schedule, the middle level maximizes the system power imbalance over all contingencies characterizing the $n-K$ security criterion. Finally, the lower level models the operator's reaction against the contingency identified by the middle level. This reaction comprises some corrective measures, namely the adjustable decisions, to minimize the system power imbalance. Adjustable decisions include generation redispatch within the scheduled power and reserves for the available units. In each level, an objective function is optimized subject to the reaction of the subsequent level.

It should be noted that the role of the two lowermost optimizations is the identification of the contingency leading to the largest system power imbalance for each pre-contingency schedule considered in the upper level. Thus, rather than considering a single worst-case contingency associated with a base-case schedule, this framework implicitly considers all contingencies characterizing the $n-K$ security criterion for each pre-contingency schedule. It is worth mentioning that for all power and reserve schedules compliant with the security criterion, i.e., able to circumvent the loss of up to K elements, the two lowermost problems return zero system power imbalance. In other words, the worst-case system power imbalance is equal to zero since the system power imbalance is also zero for every contingency.

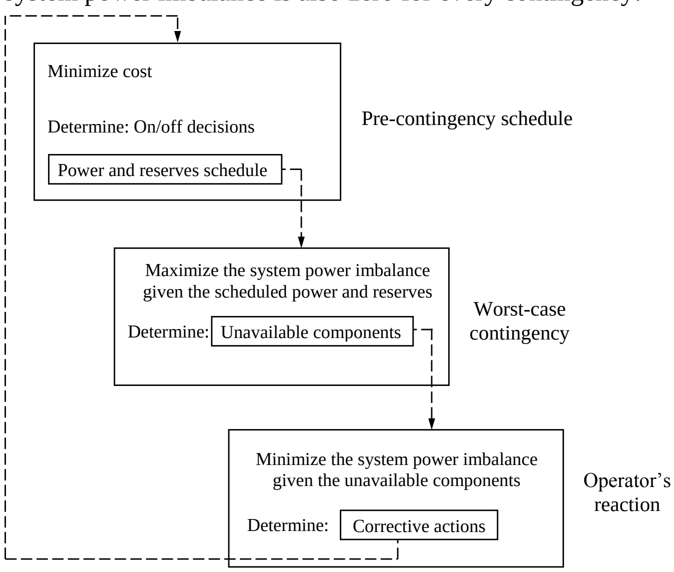

Fig. 1. Trilevel model for the adjustable robust optimization approach.

B. Problem Formulation

The robust trilevel counterpart for problem (1)-(14) is formulated as follows:

$$\underset{\substack{\Delta D^{wc},\theta_b,f_l,\\ p_i,r_i^D,r_i^U,v_i}}{\text{Minimize}} \sum_{i\in I}\left(C_i^P(p_i,v_i)+C_i^U r_i^U+C_i^D r_i^D\right)+C^I\Delta D^{wc} \tag{16}$$

subject to:

Pre-contingency scheduling constraints (2)-(10) (17)

$$\Delta D^{wc} = \max_{\delta^{wc},a_i^G,a_l^L}\Big\{\delta^{wc} \tag{18}$$

subject to:

$$\boldsymbol{f}(\{a_i^G\}_{i\in I},\{a_l^L\}_{l\in\mathcal{L}})\geq \mathbf{0} \tag{19}$$

$$a_i^G\in\{0,1\};\forall\, i\in I \tag{20}$$

$$a_l^L\in\{0,1\};\forall\, l\in\mathcal{L} \tag{21}$$

$$\delta^{wc} = \min_{\Delta N_b^{wc},\Delta P_b^{wc},\theta_b^{wc},f_l^{wc},p_i^{wc}}\left[\sum_{b\in N}\left(\Delta N_b^{wc}+\Delta P_b^{wc}\right)\right. \tag{22}$$

subject to:

$$\sum_{i\in I_b}p_i^{wc}+\sum_{l\in\mathcal{L}|to(l)=b}f_l^{wc}-\sum_{l\in\mathcal{L}|fr(l)=b}f_l^{wc} = D_b-\Delta N_b^{wc}+\Delta P_b^{wc}:(\beta_b);\forall\, b\in N \tag{23}$$

$$f_l^{wc}=\frac{a_l^L}{x_l}\left(\theta_{fr(l)}^{wc}-\theta_{to(l)}^{wc}\right):(\omega_l);\forall\, l\in\mathcal{L} \tag{24}$$

$$-\overline{F}_l\leq f_l^{wc}\leq\overline{F}_l:(\pi_l,\sigma_l);\forall\, l\in\mathcal{L} \tag{25}$$

$$a_i^G(p_i-r_i^D)\leq p_i^{wc}\leq a_i^G(p_i+r_i^U):(\gamma_i,\chi_i);\ \forall\, i\in I \tag{26}$$

$$\left.\Delta N_b^{wc}\geq 0,\Delta P_b^{wc}\geq 0;\forall\, b\in N\right]\Big\}. \tag{27}$$

Problem (16)-(27) comprises three optimization levels: (i) the upper level (16)-(17), which is associated with the pre-contingency schedule; (ii) the middle level (18)-(21), characterizing the worst-case contingency for the pre-contingency schedule; and (iii) the lower level (22)-(27), corresponding to the reaction of the system operator against the worst-case contingency. Dual variables associated with the lower-level problem are in parentheses. Note that the lower level is parameterized in terms of upper-level variables (p_i, r_i^D, r_i^U) and middle-level variables (a_i^G, a_l^L).

The objective of the upper-level problem is identical to that of the contingency-dependent model (1) except for the last term, which penalizes the system power imbalance. A sufficiently large value for C^I ensures the feasibility of the pre-contingency schedule, which requires the largest system power imbalance, due to the worst-case contingency, to be zero. The upper-level minimization is subject to the set of pre-contingency constraints (2)-(10).

The middle-level problem (18)-(21) determines the maximum system power imbalance by the definition of new binary decision variables a_i^G and a_l^L associated with the worst-case contingency. Constraint (19) imposes the security criterion, whereas constraints (20) and (21) respectively set the integrality of variables a_i^G and a_l^L. The feasibility space associated with those binary variables includes all contingencies characterizing the $n-K$ security criterion. It should be noted that constraints (19)-(21) define the combinatorial (discrete) uncertainty set, which is an extension of that used in [9], [10] by also characterizing the availability of transmission lines.

In the lower-level problem (22)-(27), the reaction of the system operator is modeled by an optimal power flow where the system power imbalance is minimized (22). The system power imbalance is defined as the sum over all buses of the absolute value of nodal power balance violations. The absolute value is modeled in a linear fashion by two sets of nonnegative variables ΔN_b^{wc} and ΔP_b^{wc}. Network constraints include nodal power balances (23), line flows (24), and line flow limits (25). Constraints (26) set the generation limits considering the reserves allocated by the upper level. Finally, constraints (27) impose the nonnegativity of nodal power-imbalance variables. It is worth mentioning that the lower-level problem (22)-(27) is always feasible and provides the upper level with a non-zero penalty when the scheduled power and reserves lead to nodal balance violations under the worst-case contingency.

In addition, once the optimal solution to the robust problem (16)-(27) is obtained, the operation under each contingency can be straightforwardly obtained by solving the lower-level problem (22)-(27) for the optimal values of the upper-level variables, p_i^*, r_i^{D*}, and r_i^{U*}, and for the values of a_i^G and a_l^L characterizing the contingency under consideration.

C. Equivalence between the Trilevel Model and the Original Contingency-Dependent Formulation

Similar to the contingency-dependent model (1)-(14), the robust trilevel counterpart (16)-(27) accounts for all contingencies characterizing the $n-K$ security criterion. In contrast, problem (16)-(27) differs from the original contingency-dependent model (1)-(14) in two aspects: (i) power-imbalance terms are included, and (ii) the operation under each contingency is not explicitly modeled. As a consequence, the feasibility search spaces of both models are different. In the original model (1)-(14), power balance under both normal and contingency states is explicitly imposed and hence pre-contingency schedules satisfying the $n-K$ security criterion are only dealt with. On the other hand, pre-contingency schedules handled by problem (16)-(27) may violate the security criterion, thereby resulting in system power imbalance. In other words, the feasibility space of the trilevel model comprises pre-contingency schedules characterized by optimal solutions to the two lowermost optimization levels (18)-(27) with system power imbalance greater than 0 MW.

In mathematical programming [27], constraint violations are customarily accounted for by including a penalty function in the objective function. Here, the penalty function is the worst-case power-imbalance cost $C^I \Delta D^{wc}$. In this framework, constraints (11)-(14) are relaxed in the robust counterpart (16)-(27) and the worst-case violation, i.e., the largest system power imbalance among all contingency states, is penalized in the objective function (16). Thus, the equivalence between the trilevel model and the original contingency-dependent formulation is guaranteed by the selection of a sufficiently large value for the power-imbalance cost coefficient C^I, so that a distinction is made between solutions complying with the $n-K$ security criterion and solutions violating such criterion. In other words, for a suitable value of C^I, and assuming that the system is able to withstand all contingencies without nodal balance violations, the optimal solution to (16)-(27) is identical to that of (1)-(14) in terms of system cost. Therefore, both models determine the lowest system cost incurred to meet the pre-specified security criterion defining the contingency set $\mathcal{C}$ with no power-imbalance cost. When the system is unable to meet the security criterion, the original contingency-dependent problem is infeasible, whereas the robust counterpart flags such infeasibility by attaining an optimal solution with a power-imbalance cost greater than zero.

D. Illustrative Example

The performance of the proposed trilevel methodology under an $n-1$ security criterion is illustrated with the following example with two load pockets, denoted as LPA and LPB. The contingency set comprises two line outages, namely those associated with the tie lines into LPA and LPB, respectively. For notational consistency, a_{LPA} and a_{LPB} respectively represent the binary variables associated with the availability of those lines (1 if available and 0 if unavailable).

Let us assume that three possible pre-contingency schedules can be implemented. Schedule 1 circumvents the loss of the tie line into LPA but leads to system power imbalance for the LPB contingency due to the limited transmission capacity. Schedule 2 withstands the loss of the tie line into LPB but does not cover the outage of the tie line into LPA due to network limitations. Moreover, it is assumed that the system power imbalance associated with Schedule 2 is larger than that of Schedule 1. Finally, Schedule 3 is compliant with the $n-1$ security criterion and hence guards against the loss of any single tie line. Thus, the proposed trilevel model is expected to select Schedule 3 since both Schedules 1 and 2 are infeasible for the contingency-dependent model.

In order to show that the trilevel model works as expected, the three schedules are examined on an individual basis as follows:

1. For Schedule 1, the optimal solution to the two lowermost optimization levels (18)-(27) would be $a_{LPA}=1$, $a_{LPB}=0$, $\Delta D^{wc}=\delta^{wc}>0$ MW. In other words, the worst-case contingency for the schedule guarding against the loss of the tie line into LPA is precisely the loss of the tie line into LPB. Therefore, Schedule 1 would yield a system power imbalance greater than 0 MW and the value of the objective function (16) is denoted as C1.
2. For Schedule 2, the optimal solution to the two lowermost optimization levels (18)-(27) would be $a_{LPA}=0$, $a_{LPB}=1$, $\Delta D^{wc}=\delta^{wc}>0$ MW. In other words, the worst-case contingency for the schedule guarding against the loss of the tie line into LPB is precisely the loss of the tie line into LPA. Therefore, Schedule 2 would also lead to a system power imbalance greater than 0 MW, being C2 the corresponding value of the objective function (16). Under the aforementioned assumption on the severity of the system power imbalance associated with Schedules 1 and 2, C2 is greater than C1.
3. For Schedule 3, all feasible combinations of binary variables a_{LPA} and a_{LPB} would be the optimum to the two lowermost optimization levels (18)-(27), namely (i) $a_{LPA}=1$, $a_{LPB}=1$; (ii) $a_{LPA}=1$, $a_{LPB}=0$, and (iii) $a_{LPA}=0$, $a_{LPB}=1$. Note that all combinations yield a value of the system power imbalance ΔD^{wc} equal to 0 MW since Schedule 3 meets the $n-1$ security criterion. Therefore, any of those

combinations would represent the worst-case contingency for Schedule 3. Furthermore, since $\Delta D^{wc} = 0$ MW, the power-imbalance cost of Schedule 3 is \$0 and the value of the objective function (16) is denoted as C3.

Therefore, the three schedules constitute feasible solutions for the trilevel model, being two actually infeasible for the original contingency-dependent model since they lead to system power imbalance. The choice of a sufficiently large value for C^I would yield the following relation among the values of the objective function (16) for the three schedules considered: C3 << C1 < C2. Since the trilevel model is a minimization problem, the optimal solution would be Schedule 3, as desired. Moreover, if there were additional schedules compliant with the security criterion, the same rationale would be applied and the trilevel model would select, among those with $\Delta D^{wc} = 0$ MW, the one with the least energy and reserve cost.

IV. Solution Methodology

Problem (16)-(27) is a mixed-integer linear trilevel program. This class of multilevel optimization is a strongly NP-hard problem [23]. As will be explained later, ΔD^{wc} is a convex function of the upper-level variables p_i, r_i^D, and r_i^U. Therefore, it can be described by an outer approximation algorithm [28]. Here, we propose a Benders decomposition approach [29], referred to as BP, that comprises the iterative solution of a master problem and a subproblem. The master problem is an approximation of the original trilevel problem where in each iteration a cutting plane or Benders cut is added to locally characterize ΔD^{wc}. The subproblem is associated with the middle- and lower-level problems for specific values of the upper-level decision variables as determined by the previous master problem. In each iteration, the solution to the subproblem provides relevant information, such as the value of ΔD^{wc} and its subgradient, to generate an additional cutting plane for the master problem.

Next, we present the mathematical formulation of the subproblem and the master problem resulting from the application of Benders decomposition to problem (16)-(27). In addition, two sets of valid constraints are provided to improve the performance of the proposed procedure.

A. Subproblem

At each iteration j, the subproblem determines the worst-case contingency for the pre-contingency schedule for power and reserves identified by the previous master problem. Mathematically, the subproblem is a mixed-integer linear max-min problem comprising the two lowermost optimization levels (18)-(27) for given values of the upper-level decision variables $p_i^{(j)}$, $r_i^{D(j)}$, and $r_i^{U(j)}$. This particular instance of bilevel programming can be recast as an equivalent single-level mixed-integer linear problem suitable for efficient off-the-shelf software based on the branch-and-cut algorithm [30].

This transformation comprises two steps:

Step 1) Based on its linearity, the lower-level problem can be replaced by its dual. Thus, the original max-min subproblem is converted into a max-max problem. Moreover, since the same objective function is optimized at both levels of the original max-min problem, the strong duality theorem can be applied. As a consequence, the max-max problem becomes a single joint maximization problem in the coupled primal and dual spaces of the middle and lower levels, respectively. Hence, this step consists in replacing (i) the lower-level problem by its dual feasibility constraints, and (ii) the middle-level objective function by the dual lower-level objective function. For further details on this transformation, the interested reader is referred to [18] and the references therein.

The single-level equivalent is formulated as:

$$\Delta D^{wc} = \max_{\substack{\beta_b,\gamma_i,\pi_l,\sigma_l,\chi_i,\omega_l,\\ a_i^G,a_l^L}} \sum_{b\in N} \beta_b D_b - \sum_{l\in\mathcal{L}} \pi_l \overline{F}_l - \sum_{l\in\mathcal{L}} \sigma_l \overline{F}_l + \sum_{i\in I} \gamma_i a_i^G \left(p_i^{(j)} - r_i^{D(j)}\right) - \sum_{i\in I} \chi_i a_i^G \left(p_i^{(j)} + r_i^{U(j)}\right) \quad (28)$$

subject to:

$$\boldsymbol{f}(\{a_i^G\}_{i\in I}, \{a_l^L\}_{l\in\mathcal{L}}) \geq \boldsymbol{0} \quad (29)$$

$$a_i^G \in \{0,1\}; \forall\, i \in I \quad (30)$$

$$a_l^L \in \{0,1\}; \forall\, l \in \mathcal{L} \quad (31)$$

$$\beta_b + \gamma_i - \chi_i \leq 0; \forall\, b \in N, \forall\, i \in I_b \quad (32)$$

$$\beta_{to(l)} - \beta_{fr(l)} + \omega_l + \pi_l - \sigma_l = 0; \forall\, l \in \mathcal{L} \quad (33)$$

$$-1 \leq \beta_b \leq 1; \forall\, b \in N \quad (34)$$

$$\sum_{l\in\mathcal{L}\,|\,fr(l)=b} \frac{\omega_l a_l^L}{x_l} - \sum_{l\in\mathcal{L}\,|\,to(l)=b} \frac{\omega_l a_l^L}{x_l} = 0;\ \forall\, b \in N \quad (35)$$

$$\pi_l \geq 0, \sigma_l \geq 0; \forall\, l \in \mathcal{L} \quad (36)$$

$$\gamma_i \geq 0, \chi_i \geq 0; \forall\, i \in I. \quad (37)$$

In (28), the worst-case system power imbalance ΔD^{wc} is determined by the maximization of the dual objective function of the lower-level problem (22)-(27). This optimization is subject to constraints (29)-(31), which are respectively the same as (19)-(21); and to constraints (32)-(37), which are the dual feasibility constraints of the lower-level problem.

Step 2) The resulting single-level equivalent is a mixed-integer nonlinear programming problem. Nonlinearities arise in (28) and (35) due to the products between middle-level binary variables and lower-level dual variables. However, those bilinear terms can be recast into linear expressions using well-known algebra results [31]. The formulation of the resulting mixed-integer linear subproblem at iteration j is as follows:

$$\Delta D^{wc} = \max_{\substack{\beta_b,\gamma_i,\pi_l,\sigma_l,\chi_i,\omega_l,\\ a_i^G,a_l^L,h_i,y_l,z_i}} \sum_{b\in N} \beta_b D_b - \sum_{l\in\mathcal{L}} \pi_l \overline{F}_l - \sum_{l\in\mathcal{L}} \sigma_l \overline{F}_l + \sum_{i\in I} z_i \left(p_i^{(j)} - r_i^{D(j)}\right) - \sum_{i\in I} h_i \left(p_i^{(j)} + r_i^{U(j)}\right) \quad (38)$$

subject to:

$$\boldsymbol{f}(\{a_i^G\}_{i\in I}, \{a_l^L\}_{l\in\mathcal{L}}) \geq \boldsymbol{0} \quad (39)$$

$$a_i^G \in \{0,1\}; \forall\, i \in I \quad (40)$$

$$a_l^L \in \{0,1\}; \forall\, l \in \mathcal{L} \tag{41}$$

$$\beta_b + \gamma_i - \chi_i \le 0; \forall\, b \in N, \forall\, i \in I_b \tag{42}$$

$$\beta_{to(l)} - \beta_{fr(l)} + \omega_l + \pi_l - \sigma_l = 0; \forall\, l \in \mathcal{L} \tag{43}$$

$$-1 \le \beta_b \le 1; \forall\, b \in N \tag{44}$$

$$\sum_{l \in \mathcal{L}\,|\,fr(l)=b} \frac{y_l}{x_l} - \sum_{l \in \mathcal{L}\,|\,to(l)=b} \frac{y_l}{x_l} = 0; \forall\, b \in N \tag{45}$$

$$-(1-a_l^L)\overline{\omega}_l \le \omega_l - y_l \le (1-a_l^L)\overline{\omega}_l; \forall\, l \in \mathcal{L} \tag{46}$$

$$-a_l^L\overline{\omega}_l \le y_l \le a_l^L\overline{\omega}_l; \forall\, l \in \mathcal{L} \tag{47}$$

$$0 \le \gamma_i - z_i \le (1-a_i^G)\bar{\gamma}_i; \forall\, i \in I \tag{48}$$

$$0 \le z_i \le \bar{\gamma}_i a_i^G; \forall\, i \in I \tag{49}$$

$$0 \le \chi_i - h_i \le (1-a_i^G)\bar{\chi}_i; \forall\, i \in I \tag{50}$$

$$0 \le h_i \le \bar{\chi}_i a_i^G; \forall\, i \in I, \tag{51}$$

where h_i, y_l, and z_i are new variables representing the bilinear terms of (28) and (35): $h_i = \chi_i a_i^G$, $y_l = \omega_l a_l^L$, and $z_i = \gamma_i a_i^G$. Parameters $\bar{\gamma}_i$, $\bar{\chi}_i$, and $\overline{\omega}_l$ respectively represent the bounds for γ_i, χ_i, and ω_l. Since the lower level is always a feasible problem, the values of such parameters may be set based on sensitivity analysis. Note that modifying the right-hand side of (24) by an infinitesimal factor, the largest change in the lower-level objective function (22) is limited to such factor multiplied by 2. This occurs because every flow variable f_l^{wc} appears in two nodal power balance constraints respectively corresponding to the sending and receiving buses. Similarly, by perturbing (26), the largest change in the lower-level objective function (22) is limited to the magnitude of such perturbation. Therefore, the upper bounds for the dual variables associated with (24) and (26) can be set to $\overline{\omega}_l = 2$ and $\bar{\gamma}_i = \bar{\chi}_i = 1$.

Expressions (38)-(45) are respectively equivalent to (28)-(35) whereas constraints (46)-(51) represent the linearization of the bilinear products. It should be noted that, in terms of the upper-level variables, ΔD^{wc} is the maximum of affine functions within the middle-level feasibility set. Therefore, it is a convex function of the upper-level decision variables (see [32], item "3.2.3 Pointwise maximum and supremum", for a proof).

B. Master Problem

The master problem at iteration j is:

$$\underset{\substack{\alpha,\theta_b,f_l,\\ p_i,r_i^D,r_i^U,v_i}}{\text{Minimize}} \sum_{i \in I} \left(C_i^P(p_i, v_i) + C_i^U r_i^U + C_i^D r_i^D\right) + C^I \alpha \tag{52}$$

subject to:

Pre-contingency feasibility constraints (2)-(10) (53)

$$\alpha \ge \Delta D^{wc(m)} + \sum_{i \in I} \Big[\big(p_i - p_i^{(m)}\big)\big(z_i^{(m)} - h_i^{(m)}\big) + \big(r_i^D - r_i^{D(m)}\big)\big(-z_i^{(m)}\big) + \big(r_i^U - r_i^{U(m)}\big)\big(-h_i^{(m)}\big)\Big]; \quad m = 1, \dots, j-1 \tag{54}$$

$$\alpha \ge 0. \tag{55}$$

The objective function (52) corresponds to (16), where variable α represents the approximation of ΔD^{wc}. Expressions (53) are identical to (17). At each iteration, the search space is restricted by adding a Benders cut (54). $\Delta D^{wc(m)}$ is obtained from the optimal solution to the subproblem (38)-(51) at iteration m for given values of the upper-level decision variables $p_i^{(m)}$, $r_i^{D(m)}$, and $r_i^{U(m)}$. In addition, coefficients $\big(z_i^{(m)} - h_i^{(m)}\big)$, $\big(-z_i^{(m)}\big)$, and $\big(-h_i^{(m)}\big)$ represent the partial subgradients of $\Delta D^{wc(m)}$ that can be derived from (38). Finally, constraint (55) sets the nonnegativity of α.

C. Valid Constraints

In the proposed Benders decomposition approach, the master problem implements a cutting-plane approximation of function ΔD^{wc}, that is iteratively improved. In addition, it should be noted that reserves are penalized at the objective function of the master problem through their respective cost rates. As a consequence, the first iterations of BP are prone to yield solutions with no scheduled reserves, i.e., infeasible solutions that would lead to nodal power imbalances and thereby violate the security criterion.

Based on the findings of [9] and [25], two sets of valid constraints can be added to the master problem. These constraints provide a tighter formulation that avoids dealing with infeasible solutions, i.e., the search space is narrowed without removing the optimal solution. Thus, the performance of the proposed BP is improved.

a) Generation outage constraints:

In [9], the $n-K$ contingency-constrained problem was addressed by considering only generator outages in a single-bus system. Based on robust optimization theory, a set of linear inequalities (expressions (9.7)-(9.10) of [9]) equivalently represent the effect of the two lowermost optimization levels considered here for the case of generator outages. Therefore, such constraints can be straightforwardly added to the master problem when the considered security criterion embeds the simultaneous loss of up to a specific number of generators, as is the case of the $n-K$ and the $n-K^G-K^L$ security criteria. For quick reference, the set of generation outage constraints is formulated as follows:

$$(|I| - K^G)\lambda - \sum_{i \in I} \xi_i \ge \sum_{b \in N} D_b \tag{56}$$

$$\lambda - \xi_i \le p_i + r_i^U; \forall\, i \in I \tag{57}$$

$$\xi_i \ge 0; \forall\, i \in I \tag{58}$$

$$\lambda \ge 0, \tag{59}$$

where λ and ξ_i are dual variables of the lower-level problem defining the worst-case generation outage in [9]. The derivation of constraints (56)-(59) and the rationale that supports their inclusion into the master problem to yield a tighter formulation are both addressed in the Appendix.

b) Redispatch constraints:

According to [25], the convergence of BP can be accelerated by cutting off the infeasible schedules identified by the subproblem along the iterative process. Thus, at each iteration j, the following redispatch constraints are added to the master problem:

$$\sum_{i \in I_b} p_i^m + \sum_{l \in \mathcal{L}|to(l)=b} f_l^m - \sum_{l \in \mathcal{L}|fr(l)=b} f_l^m = D_b; \quad \forall\, b \in N, m = 1, \dots, j-1 \quad (60)$$

$$f_l^m = \frac{a_l^{L(m)}}{x_l}\left(\theta_{fr(l)}^m - \theta_{to(l)}^m\right); \forall\, l \in \mathcal{L}, m = 1, \dots, j-1 \quad (61)$$

$$-\overline{F}_l \le f_l^m \le \overline{F}_l; \forall\, l \in \mathcal{L}, m = 1, \dots, j-1 \quad (62)$$

$$a_i^{G(m)}(p_i - r_i^D) \le p_i^m \le a_i^{G(m)}(p_i + r_i^U); \quad \forall\, i \in I, m = 1, \dots, j-1, \quad (63)$$

where f_l^m, p_i^m, and θ_b^m constitute decision variables of the tight master problem. These variables model the operation under contingency m as identified by the subproblem at that iteration through $a_i^{G(m)}$ and $a_l^{L(m)}$. Constraints (60)-(63) respectively correspond to post-contingency redispatch constraints (11)-(14), where A_i^k and A_l^k are replaced by $a_i^{G(m)}$ and $a_l^{L(m)}$, respectively.

D. Algorithm

The proposed methodology works as follows:

1) *Initialization*.
   - Initialize the iteration counter: $j \leftarrow 1$;
   - Solve the master problem without cuts. This step provides $p_i^{(1)}, r_i^{D(1)}, r_i^{U(1)}, \alpha^{(1)}$, and a lower bound for the optimal cost $LB = \sum_{i \in I}\left(C_i^P\left(p_i^{(1)}, v_i^{(1)}\right) + C_i^U r_i^{U(1)} + C_i^D r_i^{D(1)}\right)$.
2) *Subproblem solution*. Solve the subproblem for the given $p_i^{(j)}, r_i^{D(j)}$, and $r_i^{U(j)}$. This step provides $z_i^{(j)}, h_i^{(j)}$, $\Delta D^{wc(j)}$, and an upper bound for the optimal cost $UB = \sum_{i \in I}\left(C_i^P\left(p_i^{(j)}, v_i^{(j)}\right) + C_i^U r_i^{U(j)} + C_i^D r_i^{D(j)}\right) + C^I \Delta D^{wc(j)}$.
3) *Iteration counter updating*. Increase the iteration counter: $j \leftarrow j + 1$.
4) *Master problem solution*. Solve the full master problem. This step provides $p_i^{(j)}, r_i^{D(j)}, r_i^{U(j)}$, $\alpha^{(j)}$, and a lower bound for the optimal cost $LB = \sum_{i \in I}\left(C_i^P\left(p_i^{(j)}, v_i^{(j)}\right) + C_i^U r_i^{U(j)} + C_i^D r_i^{D(j)}\right) + C^I \alpha^{(j)}$.
5) *Convergence checking*. If a solution with a level of accuracy ϵ has been found, i.e., $\frac{(UB-LB)}{LB} \le \epsilon$, then stop, otherwise go to step 2.

Since ΔD^{wc} is a convex function of the upper-level variables p_i, r_i^D, and r_i^U, and the master problem is a mixed-integer linear program, BP finitely converges to optimality. In addition, the upper and lower bounds provide a measure of the distance to the optimum.

V. Case Studies

This section presents results from two test cases based on the 24-bus IEEE Reliability Test System (RTS) [33] and the IEEE 118-bus system [34], respectively. For the sake of simplicity, generators offer linear cost functions of the form $C_i^P(p_i, v_i) = C_i^f v_i + C_i^v p_i$. For all simulations, C^I was set equal to $\$10^6$/MWh. The model has been implemented on an Amazon virtual machine [35] with 32 Intel Xeon Cloud Computing, 2.63-GHz processors with 60.5 GB of RAM, using Xpress-MP 7.2 under MOSEL [30].

A. RTS-Based Case

This case study illustrates the performance of BP under an $n - K$ security criterion. The 24-bus IEEE Reliability Test System [33] comprises 26 generators and 38 transmission assets. The data for the generators can be found in [3]. Coefficients C_i^f and C_i^v respectively correspond to the intercept and the linear coefficient of the cost function provided in [3]. The load profile corresponds to Monday of week 48 at 3:00 a.m. The resilience of the system against multiple contingencies is increased by adding three circuits in line 7-8, and one circuit in lines 1-2, 1-3, 1-5, 2-4, 2-6, 3-9, 3-24, 4-9, 5-10, 6-10, 8-9, 8-10, 11-14, 12-23, 13-23, 14-16, 15-16, 15-24, 16-17, and 16-19. As a consequence, the system is able to be operated under the $n - 3$ security criterion.

This case study has been solved by five approaches: (i) the mixed-integer linear contingency-dependent model (1)-(14), referred to as CD; (ii) the original adjustable robust optimization approach without valid constraints, denoted as O-BP; (iii) the tight robust method with generation outage constraints only, labeled as T(G)-BP; (iv) the tight robust approach with redispatch constraints only, denoted as T(R)-BP; and (v) the tight robust technique with both sets of valid constraints, referred to as T-BP. Tables I and II summarize the results obtained for different values of the security parameter K ranging between 0 and 5. For all simulations, the level of accuracy ϵ was set at 10^{-3}. Given the huge number of constraints that have to be explicitly considered in CD, a time limit of 4 h (14400 s) was set for the execution of Xpress.

Table I
RTS-Based Case: System Costs ($)

K	CD	O-BP	T(G)-BP	T(R)-BP	T-BP
0	2068020	2068020	2068020	2068020	2068020
1	2688760	2688760	2688760	2688760	2688760
2	3751680	3751680	3751680	3751680	3751680
3	Time exceeded	5232740	5232740	5232740	5232740
4	Out of memory	Infeasible	Infeasible	Infeasible	Infeasible
5	Out of memory	Infeasible	Infeasible	Infeasible	Infeasible

Table II
RTS-Based Case: Computing Times (s)

K	CD	O-BP	T(G)-BP	T(R)-BP	T-BP
0	0.30	0.33	0.33	0.58	0.34
1	3.60	1.36	0.72	1.75	0.75
2	334.78	4.96	1.70	5.76	1.79
3	14400.00	48.33	27.14	28.88	16.77
4	Out of memory	371.75	384.37	4.85	1.31
5	Out of memory	1816.71	10.95	4.59	2.45

Table I provides information on the quality of the solutions attained by the proposed adjustable robust approaches in terms of system cost. As can be seen, all methods achieved the same optimal solution identified by CD for values of K up to 2. For an $n - 3$ security criterion, CD was unable to find a feasible solution within the pre-specified 4-h time limit. In contrast, the adjustable robust models attained a feasible solution meeting the $n - 3$ security criterion. As expected, tighter security criteria yield higher system costs. For this case study, imposing an $n-3$ security criterion incurs a 39.5% cost increase over the operation under an $n-2$ security criterion. Tighter security criteria than $n-3$ led to intractable contingency-dependent models that ran out of memory. On the other hand, the adjustable robust approaches converged to infeasible solutions resulting in power imbalances. In other words, the adjustable robust models were able to identify that the system is unable to be operated under such tight security criteria. These results provide the system operator with valuable information on the ability of the power system to withstand multiple contingencies.

Table II presents the computational results for all methods. As above mentioned, the computational burden of CD is prohibitive for more than 2 simultaneous out-of-service components, whereas the adjustable robust approaches converge in moderate computing times. Even for the conventional $n-1$ and $n-2$ security criteria, the robust methods also outperform CD. These results clearly back the superiority of the adjustable robust approaches over the contingency-dependent formulation from a computational viewpoint. Moreover, the results shown in Table II highlight the computational advantage of jointly considering both sets of valid constraints included in T-BP. While these constraints do not affect the quality of the solution in terms of system cost (Table I), they yield large reductions in computing time with respect to O-BP. As can be observed, computing time reductions are particularly significant for tighter security criteria. For these particular cases, redispatch constraints are more effective than generation outage constraints when considered separately.

B. IEEE 118-Bus System

This case study shows the behavior of BP under an $n-K^G-K^L$ security criterion. The IEEE 118-bus system consists of 54 thermal generators and 186 transmission lines [34]. Coefficients C_i^f and C_i^v respectively correspond to the intercept and the linear coefficient of the cost function provided in [34]. Nodal peak load data were obtained from [36] and were modulated with the same factors presented in [33]. The load profile corresponds to Monday of week 48 at 10:00 p.m. reduced by 50%. Similar to the RTS-based case, an additional circuit was considered in lines 9-10, 12-117, 68-116, 71-73, 85-86, 86-87, 110-111, and 110-112. Moreover, generator 5 was also duplicated.

Table III compares the performance of T-BP and CD for different values of K^G and K^L and a level of accuracy ϵ equal to 10^{-2}. As can be seen, T-BP attained either the optimum or an ϵ-optimal solution in reasonable times for all cases but one. The $n-5-1$ criterion resulted in the most challenging case from a computational perspective given the vast feasible search space to be explored. For this case, the optimality gap could only be reduced down to 2.4% after 7897.73 s. Note also that T-BP converged to infeasible solutions leading to power imbalances for all cases with $K^L=2$. In other words, T-BP allowed identifying that the system is unable to withstand the loss of more than one transmission line.

In contrast, CD attained the optimal solution in only 4 out of 18 cases. It is worth mentioning that CD could not be loaded into the computer memory when considering security criteria with $K^G+K^L>2$. Moreover, the computing times required by CD to attain optimality were significantly larger than those of T-BP. These results also substantiate the superior performance of T-BP over the contingency-dependent formulation.

TABLE III
RESULTS FOR THE IEEE 118-BUS SYSTEM

		T-BP		CD	
K^G	K^L	System Cost ($)	Time (s)	System Cost ($)	Time (s)
0	0	12826.6	0.17	12826.6	0.81
0	1	12826.6	2.31	12826.6	115.06
0	2	Infeasible	6.55	–	14400.00
1	0	15450.7	0.47	15450.7	15.77
1	1	15643.4	50.91	–	14400.00
1	2	Infeasible	14.13	–	Out of memory
2	0	17507.4	0.61	17507.4	8070.47
2	1	17641.6	276.66	–	Out of memory
2	2	Infeasible	8.42	–	Out of memory
3	0	18503.7	1.05	–	14400.00
3	1	18503.7	38.60	–	Out of memory
3	2	Infeasible	15.37	–	Out of memory
4	0	19356.5	1.22	–	Out of memory
4	1	19881.2	453.62	–	Out of memory
4	2	Infeasible	43.04	–	Out of memory
5	0	20343.5	2.17	–	Out of memory
5	1	21520.5[#]	7897.73	–	Out of memory
5	2	Infeasible	96.47	–	Out of memory

[#] Optimality gap = 2.4%.

VI. CONCLUSIONS

This paper proposes a novel formulation and solution methodology to solve the contingency-constrained scheduling of energy and reserves considering a joint generation and transmission security criterion. The distinctive modeling features are (i) the consideration of the effect of the transmission network, which requires not only up-spinning reserves but also down-spinning reserves, and (ii) the inclusion of transmission outages in the security criterion. The proposed approach is based on adjustable robust optimization by which the original contingency-constrained model is formulated as a trilevel programming problem. In order to solve the resulting mixed-integer linear trilevel program, a Benders decomposition technique is applied. The proposed methodology comprises the iterative resolution of a master problem and a subproblem. Both problems are formulated as mixed-integer linear programs suitable for efficient off-the-shelf branch-and-cut software. Two sets of valid constraints are also proposed to improve the computational performance of the proposed approach.

Numerical results show that the adjustable robust approach is able to attain optimal or high-quality near-optimal solutions with reasonable computational effort. Moreover, the superiority of the proposed methodology over the conventional contingency-constrained formulation is shown.

Although a single-period formulation has been presented in this paper, it should be noted that the main steps used in the proposed solution approach are readily applicable to the multiperiod instance with time-coupling constraints and some additional notation to properly index variables and parameters over the time periods. We recognize that the multiperiod case will require much more computation time and needs further numerical studies.

Ongoing research is focused on the extension of the proposed formulation to a multiperiod setting and the consideration of non-spinning reserves. Further work will explore the computational savings that may be gained from the use of stabilization methods and parallel computation. Other interesting avenues of research are the consideration of non-deterministic security criteria and the modeling of the uncertainty associated with renewable energy sources.

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APPENDIX

Using a notation consistent with that of this paper, the $n-K$ contingency-constrained problem considering only generator outages in a single-bus system, which was presented in [9], is formulated as:

$$\underset{p_i, r_i^U, v_i}{\text{Minimize}} \sum_{i\in I} \left(C_i^P(p_i, v_i) + C_i^U r_i^U\right) \tag{64}$$

subject to:

$$\sum_{i\in I} p_i = \sum_{b\in N} D_b \tag{65}$$

$$\underline{P}_i v_i \le p_i \le \overline{P}_i v_i; \forall\, i \in I \tag{66}$$

$$p_i + r_i^U \le \overline{P}_i v_i; \forall\, i \in I \tag{67}$$

$$0 \le r_i^U \le \overline{R}_i^U v_i; \forall\, i \in I \tag{68}$$

$$v_i \in \{0,1\}; \forall\, i \in I \tag{69}$$

$$D^* \ge \sum_{b\in N} D_b \tag{70}$$

$$D^* = \min_{a_i^G} \sum_{i\in I} a_i^G (p_i + r_i^U) \tag{71}$$

subject to:

$$\sum_{i\in I} a_i^G \ge |I| - K^G : (\lambda) \tag{72}$$

$$0 \le a_i^G \le 1 : (\xi_i); \forall\, i \in I, \tag{73}$$

where D^* denotes the maximum power that can be supplied under the worst-case contingency. The upper-level problem (64)-(70) determines the least-cost schedule for power and up-spinning reserve, whereas the lower-level problem (71)-(73) determines the value of D^* under the $n-K^G$ security criterion.

As described in [9], the left-hand side of (70) and the lower-level problem (71)-(73) can be equivalently replaced by the dual objective function and the dual feasibility constraints associated with (71)-(73). Thus, the resulting single-level equivalent is formulated as:

$$\underset{p_i, r_i^U, v_i}{\text{Minimize}} \sum_{i\in I} \left(C_i^P(p_i, v_i) + C_i^U r_i^U\right) \tag{74}$$

subject to:

Pre-contingency feasibility constraints (65)-(69) (75)

$$(|I| - K^G)\lambda - \sum_{i\in I} \xi_i \ge \sum_{b\in N} D_b \tag{76}$$

$$\lambda - \xi_i \le p_i + r_i^U; \forall\, i \in I \tag{77}$$

$$\xi_i \ge 0; \forall\, i \in I \tag{78}$$

$$\lambda \ge 0. \tag{79}$$

Expressions (70)-(73), and their equivalents (76)-(79), guarantee that enough post-contingency power can be supplied for all generator outages included in the $n-K^G$ criterion. The rationale for these constraints lies in the fact that the loss of any generator is compensated by the remaining available generators. In single-bus models, all available generators increase their respective power outputs over the pre-contingency levels. In dc network-constrained models, although generators may reduce their production under contingency, the sum of the up-spinning reserve contributions of all available generators should at least equal the generation of the out-of-service generators. As a consequence, (76)-(79) become a set of necessary (but insufficient) conditions to ensure a feasible post-contingency schedule in the network-constrained case. Hence, (76)-(79) constitute valid constraints that can be readily added to the original trilevel model (1)-(14) or to its approximation, namely the master problem (52)-(55), when the considered security criterion includes the loss of up to K^G generators.

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