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Energy and Reserve Scheduling under Correlated Nodal Demand Uncertainty: An Adjustable Robust Optimization Approach

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***Abstract*— This paper presents a nonparametric approach based on adjustable robust optimization to consider correlated nodal demand uncertainty in a joint energy and reserve scheduling model with security constraints. In this model, up- and down-spinning reserves provided by generators are endogenously defined as a result of the optimization problem. Adjustable robust optimization is used to model the worst-case load variation under a given user-defined uncertainty set. This paper generalizes recent previous work in two respects: (i) nonparametric correlations between nodal demands are accounted for in the uncertainty set, and (ii) based on the binary expansion linearization approach, a mixed-integer linear model is provided for the optimization related to the worst-case demand. The resulting problem is formulated as a trilevel optimization problem and solved by means of Benders decomposition. Preliminary empirical results suggest that the incorporation of nodal correlations can be captured by the robust scheduling model.**

***Index Terms*—Adjustable Robust Optimization, Benders Decomposition, Binary Expansion Linearization, Energy and Reserve Scheduling, Correlated Nodal Demand Uncertainty.**

NOMENCLATURE

A. Functions

$C_i^P(\cdot)$ Energy cost function offered by generator i.

B. Constants

Γ Conservativeness parameter.

A_i^k Availability parameter that is equal to 0 if generator i is unavailable under contingency state k, being 1 otherwise.

A_l^k Availability parameter that is equal to 0 if line l is unavailable under contingency state k, being 1 otherwise.

C_i^D Cost rate offered by generator i to provide down-spinning reserve.

C^I Cost of power imbalance.

C_i^U Cost rate offered by generator i to provide up-spinning reserve.

c_i^f Fixed cost to turn on generator i.

c_i^v Cost rate offered by generator i to provide power.

$\widehat{D}_b$ Nominal demand at bus b.

D_b^1 Minimum demand level at bus b.

$fr(l)$ Sending or origin bus of line l.

$\overline{F}_l$ Power flow capacity of line l.

h_q Bound of the q-th general polyhedral constraint.

J_b Number of binary variables used in the discretization of D_b.

$L_{bb'}$ Element (b, b') of the Cholesky decomposition of the nodal-demand covariance matrix.

M Big number used in the disjunctive constraints.

n Number of system components.

$\overline{P}_i$ Capacity of generator i.

$\underline{P}_i$ Minimum power output of generator i.

$\overline{R}_i^D$ Upper bound for the down-spinning reserve contribution of generator i.

$\overline{R}_i^U$ Upper bound for the up-spinning reserve contribution of generator i.

s_b Discretization step for D_b.

$to(l)$ Receiving or destination bus of line l.

W_{bq} Element (b, q) of the matrix representing a general polyhedral constraint to bound the demand.

x_l Reactance of line l.

z Scaling factor for the Cholesky matrix.

C. Decision Variables

α Approximation of the system power imbalance in the Benders master problem.

δ^{wc} Auxiliary variable representing the worst-case system power imbalance.

Δ^{wc} System power imbalance under the worst-case contingency.

ΔD_b^k Auxiliary variable used in the linearization of the absolute value of the power imbalance at bus b under contingency k.

ΔP_b^k Auxiliary variable used in the linearization of the absolute value of the power imbalance at bus b under contingency k.

δD_b Auxiliary variable used in the linearization of the absolute value of the power imbalance at bus b under the worst-case contingency.

δP_b Auxiliary variable used in the linearization of the absolute value of the power imbalance at bus b under the worst-case contingency.

θ_b Phase angle at bus b in the pre-contingency state.

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θ_b^k Phase angle at bus b under contingency k.
μ_b^k Variable equal to the product $\beta_b^k D_b$.
ξ_{jb}^k Variable equal to the product $\beta_b^k u_{jb}$.
D_b Demand at bus b.
$e_b^{(+)}$ Positive error on the demand of bus b.
$e_b^{(-)}$ Negative error on the demand of bus b.
f_l Power flow of line l in the pre-contingency state.
f_l^k Power flow of line l under contingency k.
p_i Power output of generator i in the pre-contingency state.
p_i^k Power output of generator i under contingency k.
r_i^D Down-spinning reserve provided by generator i.
r_i^U Up-spinning reserve provided by generator i.
u_{jb} Binary variable used in the discretization of D_b.
v_i Binary variable that is equal to 1 if generator i is scheduled in the pre-contingency state, being 0 otherwise.

D. Dual Variables

β_b^k Dual variable associated with the power balance equation at bus b under contingency k.
$\psi_b^{(D)k}$ Dual variable associated with the lower bound for δD_b under contingency k.
$\psi_b^{(P)k}$ Dual variable associated with the lower bound for δP_b under contingency k.
γ_i^k Dual variable associated with the lower bound for p_i^k.
π_l^k Dual variable associated with the lower bound for f_l^k.
σ_l^k Dual variable associated with the upper bound for f_l^k.
χ_i^k Dual variable associated with the upper bound for p_i^k.
ω_l^k Dual variable associated with the equation relating power flow and phase angles for line l under contingency k.

E. Sets

$\mathcal{C}$ Set of contingency indexes.
I Set of generator indexes.
I_b Set of indexes of generators connected to bus b.
$\mathcal{L}$ Set of transmission line indexes.
N Set of bus indexes.
Q Set of polyhedral constraints to bound the demand.

I. Introduction

THE determination of adequate levels of reserves is an issue of major concern in power system operation with high impact on power system security and energy prices [1]. Spinning reserves are part of the ancillary services that provide the system with the ability to withstand load variations as well as the most relevant contingencies [2]. The joint schedule of energy and reserves is one way to capture the interactions between both commodities [2], [3].

Most power systems worldwide operate under the well-known deterministic security criteria $n-1$ and $n-2$ [4]. Deterministic contingency-constrained models, which explicitly represent the operation under each credible contingency, are generally used to define optimal levels of reserves. Relevant applications of such models to co-optimize energy and reserves can be found in [2], [3], [5].

Stochastic models [6] are also used in generation scheduling. They aim to capture probabilistic structures present in the underlying uncertainty process, e.g., correlations between nodal demand and renewable injections, by means of scenarios and their probabilities. Their main goal is to optimize the level of resources, e.g., energy and reserves, taking advantage of the uncertainty structure while ensuring the system security in a probabilistic fashion.

Robust optimization models for generation scheduling have recently drawn a great deal of attention [5], [7], [8]. As a distinctive feature, uncertainty is characterized in an endogenous way, thereby avoiding the need for modeling the system operation under each contingency or scenario. To that end, robust counterparts are formulated as multi-level optimization programs. Robust models find a solution that is feasible for all possible realizations of the uncertainty in a given polyhedral uncertainty set [9]. The polyhedral uncertainty set allows controlling the conservativeness level of the model by means of a user-defined parameter. Such parameter constrains the number of uncertainty coefficients that can deviate from their nominal value.

Recent works [7], [8] employ the two-stage or adjustable robust optimization (ARO) approach to deal with nodal-injection uncertainty in unit commitment. In [7], a Benders decomposition procedure is proposed to solve the resulting bilevel program. In addition, Monte Carlo sampling is used to assess the quality of the solutions. In [8], a more general model is also addressed by Benders decomposition. However, upper bounds for the Benders procedure are obtained through an iterative heuristic algorithm. Hence, global optimality is not guaranteed. Both works, [7], [8], make use of a polyhedral uncertainty set disregarding the possibility of considering the correlation effect between nodal demands or injections.

In the present work, we extend the uncertainty set described in [8] in order to consider a nonparametric correlation between nodal demands. To that end, the Cholesky decomposition of the nodal-demand covariance matrix is used without changing the model complexity. Furthermore, bilinear products of continuous variables are linearized within a user-defined precision through the binary expansion approach proposed in [10]. Finally, we present an ARO model capable of capturing the effect of nonparametric correlations between nodal demands to determine the least-cost schedule of energy and up- and down-spinning reserves. The joint effect of contingencies and demand uncertainty is also examined through the consideration of a deterministic security criterion.

II. An Adjustable Robust Optimization Formulation for the Energy and Reserve Scheduling Problem

The proposed problem determines the optimal generation schedule and reserve allocation so that the uncertain power demand is supplied under both normal and contingency states. Unlike [7] and [8], spatial correlation among nodal demands is explicitly modeled. In addition, contingencies are associated with a deterministic security criterion. For expository purposes, a single period is considered. The extension to a multiperiod framework can be achieved based on the findings of [8]. The joint scheduling of energy and reserves can be formulated as the following trilevel optimization model:

$$\underset{\substack{\Delta^{wc},\theta_b,f_l,\\ p_i,r_i^D,r_i^U,v_i}}{\text{Minimize}} \sum_{i\in I}\left(C_i^P(p_i,v_i)+C_i^U r_i^U+C_i^D r_i^D\right)+C^I\Delta^{wc} \quad (1)$$

subject to:

$$\sum_{i\in I_b} p_i + \sum_{l\in\mathcal{L}|to(l)=b}\frac{1}{x_l}\left(\theta_{fr(l)}-\theta_{to(l)}\right) - \sum_{l\in\mathcal{L}|fr(l)=b}\frac{1}{x_l}\left(\theta_{fr(l)}-\theta_{to(l)}\right) = \widehat{D}_b; \forall\, b\in N \quad (2)$$

$$-\overline{F}_l \le \frac{1}{x_l}\left(\theta_{fr(l)}-\theta_{to(l)}\right) \le \overline{F}_l; \forall\, l\in\mathcal{L} \quad (3)$$

$$\underline{P}_i v_i \le p_i \le \overline{P}_i v_i; \forall\, i\in I \quad (4)$$

$$p_i + r_i^U \le \overline{P}_i v_i; \forall\, i\in I \quad (5)$$

$$p_i - r_i^D \ge \underline{P}_i v_i; \forall\, i\in I \quad (6)$$

$$0 \le r_i^U \le \overline{R}_i^U v_i; \forall\, i\in I \quad (7)$$

$$0 \le r_i^D \le \overline{R}_i^D v_i; \forall\, i\in I \quad (8)$$

$$v_i \in \{0,1\}; \forall\, i\in I \quad (9)$$

$$\Delta^{wc} = \max_{D_b,e_b^{(+)},e_b^{(-)}} \Big\{\delta^{wc} \quad (10)$$

subject to:

$$D_b = \widehat{D}_b + z\sum_{b'\in N|b'\le b} L_{bb'}\left(e_{\boldsymbol{b'}}^{(+)}-e_{\boldsymbol{b'}}^{(-)}\right); \forall\, b\in N \quad (11)$$

$$\sum_{b\in N} W_{bq} D_b \le h_q; \forall\, q\in Q \quad (12)$$

$$\sum_{b\in N}\left(e_b^{(+)}+e_b^{(-)}\right) \le \Gamma \quad (13)$$

$$0 \le e_b^{(+)} \le 1; \forall\, b\in N \quad (14)$$

$$0 \le e_b^{(-)} \le 1; \forall\, b\in N \quad (15)$$

$$\delta^{wc} = \min_{\substack{\Delta D_b^k,\delta D_b,\Delta P_b^k,\delta P_b,\\ f_l^k,p_i^k,\theta_b^k}} \Bigg[\sum_{b\in N}(\delta D_b+\delta P_b) \quad (16)$$

subject to:

$$\sum_{i\in I_b} p_i^k + \sum_{l\in\mathcal{L}|to(l)=b} f_l^k - \sum_{l\in\mathcal{L}|fr(l)=b} f_l^k - \Delta D_b^k + \Delta P_b^k = D_b : (\beta_b^k); \forall\, b\in N, \forall\, k\in\mathcal{C} \quad (17)$$

$$f_l^k = \frac{A_l^k}{x_l}\left(\theta_{fr(l)}^k-\theta_{to(l)}^k\right) : \left(\omega_l^k\right); \forall\, l\in\mathcal{L}, k\in\mathcal{C} \quad (18)$$

$$-\overline{F}_l \le f_l^k \le \overline{F}_l : \left(\pi_l^k,\sigma_l^k\right); \forall\, l\in\mathcal{L}, \forall\, k\in\mathcal{C} \quad (19)$$

$$A_i^k(p_i-r_i^D) \le p_i^k \le A_i^k(p_i+r_i^U) : (\gamma_i^k,\chi_i^k); \forall\, i\in I, \forall\, k\in\mathcal{C} \quad (20)$$

$$\delta D_b - \Delta D_b^k \ge 0 : \left(\psi_b^{(D)k}\right); \forall\, b\in N, \forall\, k\in\mathcal{C} \quad (21)$$

$$\delta P_b - \Delta P_b^k \ge 0 : \left(\psi_b^{(P)k}\right); \forall\, b\in N, \forall\, k\in\mathcal{C} \quad (22)$$

$$\Delta D_b^k \ge 0, \Delta P_b^k \ge 0; \forall\, b\in N, \forall\, k\in\mathcal{C} \quad \Big]\Big\}. \quad (23)$$

The goal of the upper-level problem (1)-(9) is to minimize the total cost including the production cost, up- and down-spinning reserve costs, and the system imbalance cost, for which a sufficiently large imbalance penalty cost is used (1). Based on the formulation presented in [2], expressions (2)-(9) model energy and reserve scheduling for the pre-contingency state. Expressions (2) and (3) define a dc-power flow model, (4)-(6) ensure that the levels of energy and reserves lie in the feasible generation region of each scheduled unit, and (7) and (8) set the reserve limits. Finally, the binary nature of the scheduling on/off variables is imposed in (9).

The middle-level problem (10)-(15) represents the worst-case demand scenario that maximizes the system load imbalance (10). Similar to [8], nodal demands are middle-level decision variables lying in a given user-defined polyhedral region. Unlike in [8], the positive and negative nodal-error vectors go through a linear transformation $\boldsymbol{L}$, which can be found by means of the Cholesky decomposition (lower triangular matrix) of the estimated covariance matrix $\boldsymbol{\Sigma}$ [11]. A scaling factor z is added to allow the possibility of enlarging the error variability if needed. This is justified in cases where observed data exhibit well-known correlated patterns. Expressions (12) are general polyhedral constraints used to characterize bounds and other types of constraints. Expressions (13)-(15) limit the number of demand deviations among buses to a given user-defined uncertainty budget Γ, also known as the conservativeness parameter [9]. Note that if $\boldsymbol{\Sigma}$ is diagonal and $z = 1$, the correlation of nodal demands is neglected and each nodal demand may deviate, at most, one standard deviation around its nominal value. In this setting, the model is reduced to that presented in [8].

The lower-level problem (16)-(23) identifies a new feasible dispatch satisfying (17)-(19) for all contingencies defining the security criterion, and within the energy and reserves scheduled in the upper-level (20). The goal of the lower-level is to minimize the system power imbalance (16) for the worst-case demand realization given by the middle-level problem. The system power imbalance is defined as the sum over all buses of the largest absolute value of the nodal power imbalances for all contingencies considered. Constraints (21)-(23) are related to the definition of the system power imbalance.

This model is general and nonparametric, i.e., it can be used in different ways without associating $\boldsymbol{L}$ with a probabilistic or statistical model. Notwithstanding, it can still be used assuming a parametric multivariate Gaussian noise, in which case z can be interpreted as the quantile function for a given confidence level.

III. Solution Methodology

The solution methodology proposed to address the mixed-integer trilevel optimization (1)-(23) comprises two stages: (i) the transformation of the original problem into a bilevel program, and (ii) the subsequent application of Benders decomposition.

A. Transformation to a bilevel program

The two lowermost levels (10)-(23) can be reformulated as an equivalent single-level mixed-integer linear program (S-MILP), leading to a bilevel programming problem. This transformation consists of the following steps:

Step 1) Based on [9], the middle-level objective function δ^{wc} is replaced in (10) by the dual lower-level objective function, and the lower-level problem (16)-(23) is replaced by its dual feasibility constraints. The two lowermost levels are recast as:

$$\Delta^{wc} = \max_{\substack{D_b, e_b^{(+)}, e_b^{(-)}, \\ \beta_b^k, \gamma_i^k, \pi_l^k, \sigma_l^k, \chi_i^k, \omega_l^k, \\ \psi_b^{(D)k}, \psi_b^{(P)k}}} \left\{ \sum_{k \in \mathcal{C}} \left[\sum_{b \in N} \beta_b^k D_b - \sum_{l \in \mathcal{L}} \pi_l^k \overline{F}_l - \sum_{l \in \mathcal{L}} \sigma_l^k \overline{F}_l + \sum_{i \in I} \gamma_i^k A_i^k (p_i - r_i^D) - \sum_{i \in I} \chi_i^k A_i^k (p_i + r_i^U) \right] \right. \quad (24)$$

subject to:

$$\text{Constraints (11)-(15)} \quad (25)$$

$$\beta_b^k + \gamma_i^k - \chi_i^k \leq 0; \forall\, b \in N, \forall\, i \in I_b, \forall\, k \in \mathcal{C} \quad (26)$$

$$\beta_{to(l)}^k - \beta_{fr(l)}^k + \omega_l^k + \pi_l^k - \sigma_l^k = 0; \forall\, l \in \mathcal{L}, \forall\, k \in \mathcal{C} \quad (27)$$

$$\sum_{l \in \mathcal{L} \,|\, to(l)=b} \frac{A_l^k}{x_l} \omega_l^k - \sum_{l \in \mathcal{L} \,|\, fr(l)=b} \frac{A_l^k}{x_l} \omega_l^k = 0;\ \forall\, b \in N \quad (28)$$

$$\beta_b^k + \psi_b^{(D)k} \geq 0; \forall\, b \in N, \forall\, k \in \mathcal{C} \quad (29)$$

$$\beta_b^k - \psi_b^{(P)k} \leq 0; \forall\, b \in N, \forall\, k \in \mathcal{C} \quad (30)$$

$$\sum_{k \in \mathcal{C}} \psi_b^{(D)k} \leq 1; \forall\, b \in N \quad (31)$$

$$\sum_{k \in \mathcal{C}} \psi_b^{(P)k} \leq 1; \forall\, b \in N \quad (32)$$

$$\gamma_i^k \geq 0, \chi_i^k \geq 0; \forall\, i \in I, \forall\, k \in \mathcal{C} \quad (33)$$

$$\left. \psi_b^{(D)k} \geq 0, \psi_b^{(P)k} \geq 0; \forall\, b \in N, \forall\, k \in \mathcal{C} \right\}. \quad (34)$$

Step 2) Bilinear terms $\beta_b^k D_b$ in (24) are linearized through the binary expansion approach described in [10]. First, one set of the variables is discretized using equally-sized levels. Such discretization is then represented as the sum of binary variables, which can reproduce all of the discretization levels.

In contrast to lower-level dual variables β_b^k, variables D_b are not contingency-dependent, being the appropriate choice for discretization. As a result, dual sub-optimality is avoided while keeping the model with the minimum number of binary variables. Hence, assuming that, for each bus b, the nodal demand D_b is discretized into H_b equally-sized levels (with step size s_b), the binary representation of D_b requires at least $J_b = \lceil \log_2 H_b \rceil$ binary variables. Thus, the discretization of D_b can be represented as follows:

$$D_b = D_b^1 + s_b \sum_{j=1}^{J_b} 2^{j-1} u_{jb}. \quad (35)$$

The new D_b yields products between continuous variables β_b^k and binary variables u_{jb}, which are subsequently linearized by using disjunctive inequalities [10]. Thus, a new variable, namely μ_b^k, replaces $\beta_b^k D_b$ in (24) and the following set of disjunctive constraints is added to (24)-(34) (see [10] for an equivalent linearization procedure):

$$\mu_b^k = \beta_b^k D_b^1 + s_b \sum_{j=1}^{J_b} 2^{j-1} \xi_{jb}^k \quad (36)$$

$$-M u_{jb} \leq \xi_{jb}^k \leq M u_{jb};\ \forall\, b \in N, \forall\, k \in \mathcal{C}, \forall\, j = 1, \dots, J_b \quad (37)$$

$$-M(1 - u_{jb}) \leq \xi_{jb}^k - \beta_b^k \leq M(1 - u_{jb}); \quad \forall\, b \in N, \forall\, k \in \mathcal{C}, \forall\, j = 1, \dots, J_b. \quad (38)$$

B. Benders decomposition procedure

The worst-case imbalance variable Δ^{wc} can be viewed as a function of the upper-level variables. Moreover, from (24), Δ^{wc} is the maximum of affine functions within the middle-level feasibility set. Therefore, it is a convex function of the upper-level decision variables (see [12], item 3.2.3). Under convexity, such function can be approximated from below by means of a standard Benders procedure [13] that finitely converges to a near-global optimal solution within a user-defined tolerance level.

In the proposed Benders decomposition procedure, an upper bound UB is provided by the master problem, which comprises (1)-(9), where Δ^{wc} is replaced by a new decision variable α that represents the maximum within a set of affine functions, i.e., the Benders cuts. In addition, the subproblem, namely S-MILP, provides a lower bound LB. The proposed methodology works as follows (symbols with superscript (t) are used to indicate the optimal value of a specific variable at iteration t):

1) *Initialization*.
   - Initialize the iteration counter: $t \leftarrow 1$;
   - Solve the master problem without cuts. This step provides $p_i^{(t)}$, $r_i^{D(t)}$, $r_i^{U(t)}$, and a lower bound for the optimal cost $LB = \sum_{i \in I} \left(C_i^P \left(p_i^{(t)}, v_i^{(t)}\right) + C_i^U r_i^{U(t)} + C_i^D r_i^{D(t)} \right)$.
2) *Subproblem solution*. Solve the subproblem for the given $p_i^{(t)}, r_i^{D(t)}$, and $r_i^{U(t)}$. This step provides an upper bound for the optimal cost $UB = \sum_{i \in I} \left(C_i^P \left(p_i^{(t)}, v_i^{(t)}\right) + C_i^U r_i^{U(t)} + C_i^D r_i^{D(t)} \right) + C^I \Delta^{wc(t)}$ and a new Benders cut for the next iteration lower bound $\alpha \geq \sum_{k \in \mathcal{C}} \left[\sum_{b \in N} \beta_b^{k(t)} D_b^{(t)} - \sum_{l \in \mathcal{L}} \pi_l^{k(t)} \overline{F}_l - \sum_{l \in \mathcal{L}} \sigma_l^{k(t)} \overline{F}_l + \sum_{i \in I} \gamma_i^{k(t)} A_i^k (p_i - r_i^D) - \sum_{i \in I} \chi_i^{k(t)} A_i^k (p_i + r_i^U) \right]$.
3) *Iteration counter updating*. $t \leftarrow t + 1$.
4) *Master problem solution*. Solve the full master problem. This step provides $p_i^{(t)}, r_i^{D(t)}$, $r_i^{U(t)}$, $\alpha^{(j)}$, and a lower bound for the optimal cost $LB = \sum_{i \in I} \left(C_i^P \left(p_i^{(t)}, v_i^{(t)}\right) + C_i^U r_i^{U(t)} + C_i^D r_i^{D(t)} \right) + C^I \alpha^{(t)}$.
5) *Convergence checking*. If a solution with a level of accuracy ϵ has been found, i.e., $\frac{(UB - LB)}{LB} \leq \epsilon$, then stop, otherwise go to step 2.

IV. Case Study

The three-bus system shown in Fig. 1 is used to illustrate how the proposed model is able to capture the economic effect of considering the correlation between nodal demands. To that end, we run the model for different correlation levels between nodal demands. Generators offer linear cost functions of the form $C_i^P(p_i, v_i) = C_i^f v_i + C_i^v p_i$. Nodal demands may deviate from the nominal value in the range between plus and minus one standard deviation, which is set equal to 0.31 pu. Thus, $z = 1$ and the entries of the main diagonal of $\mathbf{\Sigma}$ are set to 0.31^2. The discretization step for nodal demands is $s_b = 0.01$

pu. The model has been implemented using Xpress-MP 7.2 under MOSEL [14].

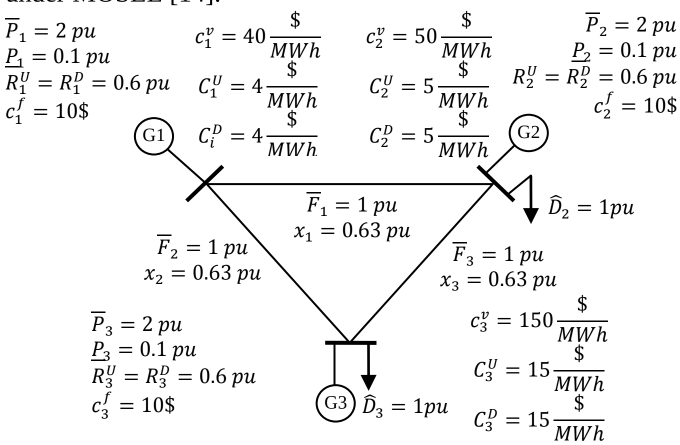

Fig. 1. Three-bus system.

Results are shown in Table I considering both no security criterion and an $n-1$ criterion. The overall energy and reserve costs are presented in percentage of the reference case, where the correlation between nodal demands (listed in the first column) is assumed to be zero. The system imbalance is provided in percentage of the system load. Since there are only two uncertainty factors, the conservativeness parameter Γ is set to 1.

TABLE I
SCHEDULING COSTS AND LOAD SHED

	With no security criterion		With an $n-1$ security criterion		
Correlation	Energy Cost	Reserve Cost	Energy Cost	Reserve Cost	System Imbalance (% load)
-1	100%	46%	87%	85%	0%
-0.5	100%	82%	91%	99%	0%
0	*8,120.0	*384.0	*11,340.0	*1,564.0	0%
0.5	100%	114%	128%	104%	0%
1.0	103%	143%	141%	106%	4%

*Reference values shown in $.

As shown in Table 1, the joint scheduling of energy and reserves is valuable in the presence of correlated nodal demands. For the case where no security criterion is enforced, reserves are the resources that mostly compensate the load variability. However, in the security-constrained case where reserves are required to cope with credible contingencies, the energy schedule presents a higher dependence upon the demand correlation, backing up the ultimate goal of efficiently co-optimizing energy and reserves.

V. CONCLUSIONS AND FUTURE RESEARCH

The consideration of some relevant information about the uncertainty modeling in generation scheduling via robust optimization models is not explored yet. In this paper, the correlation between nodal demands is explicitly considered to provide a least-cost schedule of energy and reserves based on adjustable robust optimization. The resulting model is formulated as a trilevel program that is solved by the combined use of Benders decomposition, a binary expansion approach, and a linearization scheme based on disjunctive constraints. Preliminary results suggest that nodal demand correlation might have a relevant impact on the scheduling. Therefore, further research is needed to investigate the effectiveness of robust models to capture this and other effects while achieving practical, economical, and robust schedules of energy and reserves.

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