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A
Primeiro Apêndice

A.1
Cálculo dos termos de tensor de tensões

Em coordenadas cilíndricas, as componentes do tensor de tensões $\bar{\bar{T}}$ são definidas como:

$$\begin{aligned} T_{zz} &= -p + 2\mu\frac{\partial v_z}{\partial z} \\ T_{zr} &= \mu(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}) \\ T_{rr} &= -p + 2\mu\frac{\partial v_r}{\partial r} \\ T_{\theta\theta} &= -p + 2\mu\frac{v_r}{r} \end{aligned} \tag{A-1}$$

Considerando a viscosidade como uma propriedade constante em cada uma das fases (óleo e água) tem-se,

$$\frac{\partial T_{rr}}{\partial V_{Rj}} = 2\mu\frac{\partial \phi_j}{\partial r}; j = 1, \cdots, 9 \tag{A-2}$$

$$\frac{\partial T_{zr}}{\partial V_{Rj}} = \mu\frac{\partial \phi_j}{\partial z}; j = 1, \cdots, 9 \tag{A-3}$$

$$\frac{\partial T_{\theta\theta}}{\partial V_{Rj}} = 2\mu\frac{\phi_j}{r}; j = 1, \cdots, 9 \tag{A-4}$$

$$\frac{\partial T_{zr}}{\partial V_{Zj}} = \mu\frac{\partial \phi_j}{\partial r}; j = 1, \cdots, 9 \tag{A-5}$$

$$\frac{\partial T_{zz}}{\partial V_{Zj}} = 2\mu\frac{\partial \phi_j}{\partial z}; j = 1, \cdots, 9 \tag{A-6}$$

$$\frac{\partial T_{rr}}{\partial C_j} = 2\frac{\partial \mu}{\partial C_j}\frac{\partial v_r}{\partial r}; j = 1, \cdots, 9 \tag{A-7}$$

$$\frac{\partial T_{zr}}{\partial C_j} = (\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z})\frac{\partial \mu}{\partial C_j}; j = 1, \cdots, 9 \tag{A-8}$$

$$\frac{\partial T_{zz}}{\partial C_j} = 2\frac{\partial \mu}{\partial C_j}\frac{\partial v_z}{\partial z}; j = 1, \cdots, 9 \tag{A-9}$$

$$\frac{\partial T_{\theta\theta}}{\partial C_j} = 2\frac{\partial \mu}{\partial C_j}\frac{v_r}{r}; j = 1, \cdots, 9 \tag{A-10}$$

$$\frac{\partial T_{rr}}{\partial P_j} = -\chi_j; j = 1, \cdots, 3 \tag{A-11}$$

$$\frac{\partial T_{zz}}{\partial P_j} = -\chi_j; j = 1, \cdots, 3 \tag{A-12}$$

$$\frac{\partial T_{\theta\theta}}{\partial P_j} = -\chi_j; j = 1, \cdots, 3 \tag{A-13}$$

B
Segundo Apêndice

Devido aos novos campos c_r e c_z o vetor $\mathbf{S_V}$ tem mudado,

$$\mathbf{S_V}^{**} = \begin{pmatrix} V_{Rj} \\ V_{Zj} \\ C_j \\ P_j \\ C_{Rj} \\ C_{Zj} \end{pmatrix} \tag{B-1}$$

Além disso, o vetor de resíduos $\mathbf{R}$ também é redefinido como,

$$\mathbf{R}^{**} = \begin{pmatrix} R^i_{mr} \\ R^i_{mz} \\ R^i_{c} \\ R^i_{mc} \\ R^i_{c_r} \\ R^i_{c_z} \end{pmatrix} \tag{B-2}$$

Consequentemente a matriz $\mathbf{J_{RP}}$ é dada por,

$$\mathbf{J}^{**}_{\mathbf{RP}} = \begin{pmatrix}
\frac{\partial R^{i*}_{mr}}{\partial V_{Rj}} & \frac{\partial R^{i*}_{mr}}{\partial V_{Zj}} & \frac{\partial R^{i*}_{mr}}{\partial C_j} & \frac{\partial R^{i*}_{mr}}{\partial P_j} & \frac{\partial R^{i*}_{mr}}{\partial C_{Rj}} & \frac{\partial R^{i*}_{mr}}{\partial C_{Zj}} \\
\frac{\partial R^{i*}_{mz}}{\partial V_{Rj}} & \frac{\partial R^{i*}_{mz}}{\partial V_{Zj}} & \frac{\partial R^{i*}_{mz}}{\partial C_j} & \frac{\partial R^{i*}_{mz}}{\partial P_j} & \frac{\partial R^{i*}_{mz}}{\partial C_{Rj}} & \frac{\partial R^{i*}_{mz}}{\partial C_{Zj}} \\
\frac{\partial R^{i*}_{c}}{\partial V_{Rj}} & \frac{\partial R^{i*}_{c}}{\partial V_{Zj}} & \frac{\partial R^{i*}_{c}}{\partial C_j} & \frac{\partial R^{i*}_{c}}{\partial P_j} & \frac{\partial R^{i*}_{c}}{\partial C_{Rj}} & \frac{\partial R^{i*}_{c}}{\partial C_{Zj}} \\
\frac{\partial R^{i*}_{mc}}{\partial V_{Rj}} & \frac{\partial R^{i*}_{mc}}{\partial V_{Zj}} & \frac{\partial R^{i*}_{mc}}{\partial C_j} & \frac{\partial R^{i*}_{mc}}{\partial P_j} & \frac{\partial R^{i*}_{mc}}{\partial C_{Rj}} & \frac{\partial R^{i*}_{mc}}{\partial C_{Zj}} \\
\frac{\partial R^{i}_{c_r}}{\partial V_{Rj}} & \frac{\partial R^{i}_{c_r}}{\partial V_{Zj}} & \frac{\partial R^{i}_{c_r}}{\partial C_j} & \frac{\partial R^{i}_{c_r}}{\partial P_j} & \frac{\partial R^{i}_{c_r}}{\partial C_{Rj}} & \frac{\partial R^{i}_{c_r}}{\partial C_{Zj}} \\
\frac{\partial R^{i}_{c_z}}{\partial V_{Rj}} & \frac{\partial R^{i}_{c_z}}{\partial V_{Zj}} & \frac{\partial R^{i}_{c_z}}{\partial C_j} & \frac{\partial R^{i}_{c_z}}{\partial P_j} & \frac{\partial R^{i}_{c_z}}{\partial C_{Rj}} & \frac{\partial R^{i}_{c_z}}{\partial C_{Zj}}
\end{pmatrix} \tag{B-3}$$

B.1
Cálculo dos termos adicionais da matriz $\mathbf{J_{RP}}$

– Termo adicionado a $\frac{\partial R^{i*}_{mr}}{\partial C_{Rj}}$:

$$+\int_\Omega \sigma \frac{\partial k(c_r, c_z)}{\partial C_{Rj}} \frac{\partial c}{\partial r} \phi_i \delta(c)\, d\Omega; i = 1, \cdots, 9; j = 1, \cdots, 4 \tag{B-4}$$

– Termo adicionado a $\frac{\partial R^{i*}_{mr}}{\partial C_{Zj}}$:

$$+\int_\Omega \sigma \frac{\partial k(c_r,c_z)}{\partial C_{Zj}}\frac{\partial c}{\partial r}\phi_i \delta(c)\, d\Omega; i=1,\cdots,9; j=1,\cdots,4 \tag{B-5}$$

– Termo adicionado a $\frac{\partial R^{i*}_{mz}}{\partial C_{Rj}}$:

$$+\int_\Omega \sigma \frac{\partial k(c_r,c_z)}{\partial C_{Rj}}\frac{\partial c}{\partial z}\phi_i \delta(c)\, d\Omega; i=1,\cdots,9; j=1,\cdots,4 \tag{B-6}$$

– Termo adicionado a $\frac{\partial R^{i*}_{mz}}{\partial C_{Zj}}$:

$$+\int_\Omega \sigma \frac{\partial k(c_r,c_z)}{\partial C_{Zj}}\frac{\partial c}{\partial z}\phi_i \delta(c)\, d\Omega; i=1,\cdots,9; j=1,\cdots,4 \tag{B-7}$$

B.2
Cálculo das derivadas da curvatura k em função de C_{Rj} e C_{Zj}

$$\begin{aligned}\frac{\partial k}{\partial C_{Rj}} &= \{(c_z^2\frac{\partial \varphi_j}{\partial r} - 2c_z\varphi_j c_{zr} + 2c_r\varphi_j c_{zz})(c_r^2+c_z^2)^{\frac{3}{2}} \\ &\quad -3(c_z^2 c_{rr} - 2c_r c_z c_{zr} + c_r^2 c_{zz})(c_r^2+c_z^2)^{\frac{1}{2}}(c_r\varphi_j)\} \\ &\quad /(c_r^2+c_z^2)^3 + \frac{\varphi_j}{r}[\frac{1}{(c_r^2+c_z^2)^{1/2}} \\ &\quad -\frac{c_r^2}{(c_r^2+c_z^2)^{3/2}}]; j=1,\cdots,4\end{aligned} \tag{B-8}$$

$$\begin{aligned}\frac{\partial k}{\partial C_{Zj}} &= \{(2c_z\varphi_j c_{rr} - 2c_r[\varphi_j c_{zr} + c_r\frac{\partial \varphi_j}{\partial r}] + c_r^2\frac{\partial \varphi_j}{\partial z})(c_r^2+c_z^2)^{\frac{3}{2}} \\ &\quad -3(c_z^2 c_{rr} - 2c_r c_z c_{zr} + c_r^2 c_{zz})(c_r^2+c_z^2)^{\frac{1}{2}}(c_z\varphi_j)\} \\ &\quad /(c_r^2+c_z^2)^3 - \frac{\varphi_j}{r}\frac{c_r c_z}{(c_r^2+c_z^2)^{3/2}}; j=1,\cdots,4\end{aligned} \tag{B-9}$$

B.3
Cálculo dos novos termos da matriz $\mathbf{J}^{**}_{\mathbf{RP}}$

$$\frac{\partial R^i_{c_r}}{\partial C_j} = \int_\Omega \frac{\partial \phi_j}{\partial r}\varphi_i\, d\Omega; i=1,\cdots,4; j=1,\cdots,9 \tag{B-10}$$

$$\frac{\partial R^i_{c_r}}{\partial C_{Rj}} = -\int_\Omega \varphi_j\varphi_i\, d\Omega; i,j=1,\cdots,4 \tag{B-11}$$

$$\frac{\partial R^i_{c_z}}{\partial C_j} = \int_\Omega \frac{\partial \phi_j}{\partial z}\varphi_i\, d\Omega; i=1,\cdots,4; j=1,\cdots,9 \tag{B-12}$$

$$\frac{\partial R^i_{c_z}}{\partial C_{Zj}} = -\int_\Omega \varphi_j\varphi_i\, d\Omega; i,j=1,\cdots,4 \tag{B-13}$$

O resto dos termos não definidos da matriz $\mathbf{J}^{**}_{\mathbf{RP}}$ consideram-se zeros.