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Apêndice

A.
Códigos em MATLAB®

Programa principal "bolhafim.m"

```
%% Programa Principal - Em funcao da fase continua
% Mestrando: Flavio Bormann - 0812212
% Orientador: Marcio Carvalho
% Data da ultima modificao: 03/09/2010

%% Limpeza do Sistema

clear all;
close all;
clc;

%% Inicializacao das Variaveis

Rg = 0.05*10^-3;   % raio da garganta [m]
Rc = 0.1*10^-3;     % raio do poro - cavidade [m]
sigma = 0.06;       % tensao interfacial [N/m]
mi_1 = 1e-3;       % viscosidade do fluido 1 -interno- fase
dispersa [Pa.s]
mi_2 = 10e-3;          % viscosidade do fluido 2 -externo-
fase continua [Pa.s]
L = 3*10^-3;        % comprimento do poro [m]
T = L/3;            % tamanho da estriccao [m]
c1 = 1/(8*mi_1);    % Constante 1
c2 = 1/(2*mi_2);    % Constante 2
delta = 0.3*Rg;       % Camada de filme do fluido 2 - fase
continua em [m]
P0 = 0;             % Pressao inicial [Pa]
```

```
alpha = 1;          % Parametro de relaxacao

%% Inicializacao dos nos

N = input ('Entre com os números de nós: ');
tmax = input ('Digite o tempo máximo, em segundos, das
iterações: ');
Dt = input ('Escolha, em segundos, o intervalo de tempo: ');

%% Distribuicao dos nos

DZ = L/(N-1);

z(1) = -L/2;

for i=2:N
    z(i) = z(i-1) + DZ; % Funcao de distribuicao dos nos -
distribuicao uniforme
end

%% Definicao da forma do poro

for i=1:length(z)
    if z(i)<-T/2 || z(i)>T/2
        r(i)=Rc;
    else
        r(i) = (Rc-Rg)/2 * cos(2*pi/T * z(i) + pi) +
(Rc+Rg)/2;
    end
end

%% Condicao Inicial

j = 1;
t(j) = 0;

for i=1:N
```

```
    Rpas(i) = r(i) - delta;             % Considerando que no
tempo inicial as fases tem a mesma forma do poro. Vazao
nula.
    Ppas(i) = P0 - (sigma/(Rpas(i)));     % Distribuicao de
pressao inicial.
    Pci(i) = sigma/(Rpas(i));
end

%% ERRO e Iteracao

ERRO_MAX = 1e-7;   % Erro maximo admissivel

ITERACAO_MAX = 1000; % Numero maximo de iteracoes

%% Loop no Tempo

fla = [];
R_min(1) = min(Rpas);

while (t(j) < tmax)

    j = j + 1
    t(j) = t(j-1) + Dt;
    R = Rpas; % Chute inicial para o raio da interface
    P = Ppas; % Chute inicial para a pressão
    c = [Ppas Rpas];

%% Calculo do vetor residuo inicial

    res = residuo11a (P,R,Rpas,N,j,t,c1,c2,sigma,z,r,mi_2);
% Define o vetor residuo

    norm_res = 1;

    iteracao = 1;          % Inicializacao da contagem das
iteracoes
```

```
%% Solucao por Metodo de Newton

    while (norm_res > ERRO_MAX) && (iteracao < ITERACAO_MAX)

        [J]                    =                    jacobiana11a
(P,R,Rpas,z,N,j,t,c1,c2,sigma,r,mi_2);

        DC = pinv(J) * (-res)';

        c = c + alpha*DC';

        P = c(1:N);

        R = c(N+1:2*N);

        res                    =                    residuo11a
(P,R,Rpas,N,j,t,c1,c2,sigma,z,r,mi_2);

        fla = [fla;j iteracao norm_res];

        iteracao = iteracao + 1; % Incrementa a contagem das
iteracoes

        norm_res = norm(res)/(2*N);     % Calcula a norma do
novo vetor residuo

    end

    fla = [fla;j iteracao norm_res];

    if (iteracao == ITERACAO_MAX)
        disp  ('EXTRAPOLOU  O  NUMERO  MAXIMO  DE  ITERACOES
PERMITIDAS');
        disp ('ITERACOES PERMITIDAS:');
        disp (iteracao);
        break
    else
```

```
        R_min(j) = min(R);
        Rpas = R;
        Ppas = P;

        for i=1:N
            sg(i) = sigma/R(i);
            Pfd(i) = P(i) + sg(i);
        end

        R_hist801 {j} = R;
        Pfc_hist{j} = P;
        Pfd_hist{j} = Pfd;

        if (min(R) <= 0)
            disp ('OCORREU QUEBRA DE GOTA EM:');
            disp (t(j));
            break
        end

    end

end

%% Apresentação dos resultados

set (gcf,'Color', 'White');
figure(1)
plot(t,R_min,'-k','LineWidth',2);hold on
grid on;
xlabel('Tempo [s]');
ylabel('Raio da interface no ponto central da garganta
[m]');

set (gcf,'Color', 'White');
figure(2)
plot(z,r,'-k'); hold on  % Plota a forma do poro
plot(z,(Rg-delta)*ones(length(z)),'-.k');
```

```
plot(z,R,'--b');hold off % Plota a interfece dos fluidos em
funcao do comprimento do poro
xlabel('Comprimento [m]');
ylabel('Geometria do Poro [m] / Interface [m]');
axis ([-L/2 L/2 0 Rc]);

set (gcf,'Color', 'White');
figure (3)
plot(z,Pfc_hist{1,(j-1)},'-r','LineWidth',2);hold on
plot(z,Pfd_hist{1,(j-1)},'-k','LineWidth',2);hold off
grid on;
xlabel('z [m]');
ylabel('Distribuições de Pressão das Fases Dispersa e
Contínua & Razão das Distribuições [Pa & -]');
```

Função auxiliar "residuo11a.m"

```
%% Funcao Residuo - em funcao da fase continua
% Mestrando: Flavio Bormann - 0812212
% Orientador: Marcio Carvalho
% Data da ultima modificao: 03/09/2010

%% Chamada da funcao residuo

function res = residuo11a
(P,R,Rpas,N,j,t,c1,c2,sigma,z,r,mi_2)


res(1) = P(1) + (sigma/R(1));     % Primeira condicao de
contorno da equacao 1

res(N+1) = R(2) - R(1);     % Primeira condicao de
contorno da equacao 2

for i=2:N-1

        res(i) = 1/8*((  (((P(i+1)-P(i))/(z(i+1)-z(i))) -
((P(i)-P(i-1))/(z(i)-z(i-1))))    /    (((z(i)+z(i+1))/2)    -
((z(i)+z(i-1))/2))    )*r(i)^5-2*(    (((P(i+1)-P(i))/(z(i+1)-
z(i))) - ((P(i)-P(i-1))/(z(i)-z(i-1)))) / (((z(i)+z(i+1))/2)
-    ((z(i)+z(i-1))/2))    )*R(i)^2*r(i)^3+r(i)*(    (((P(i+1)-
P(i))/(z(i+1)-z(i)))    -    ((P(i)-P(i-1))/(z(i)-z(i-1))))    /
(((z(i)+z(i+1))/2) - ((z(i)+z(i-1))/2)) )*R(i)^4-4*((R(i+1)-
R(i-1))/(z(i+1)-z(i-1)))*sigma*r(i)^2*((r(i+1)-r(i-
1))/(z(i+1)-z(i-1)))+4*sigma*(       (((R(i+1)-R(i))/(z(i+1)-
z(i))) - ((R(i)-R(i-1))/(z(i)-z(i-1)))) / (((z(i)+z(i+1))/2)
-                                        ((z(i)+z(i-1))/2))
)*R(i)^2*r(i)*log(r(i)/R(i))+8*sigma*((R(i+1)-R(i-
1))/(z(i+1)-z(i-1)))^2*R(i)*log(r(i)/R(i))*r(i)-2*sigma*(
(((R(i+1)-R(i))/(z(i+1)-z(i)))    -    ((R(i)-R(i-1))/(z(i)-z(i-
1))))     /     (((z(i)+z(i+1))/2)     -     ((z(i)+z(i-1))/2))
)*r(i)^3+2*sigma*(  (((R(i+1)-R(i))/(z(i+1)-z(i)))  -  ((R(i)-
```

```
R(i-1))/(z(i)-z(i-1)))) / (((z(i)+z(i+1))/2) - ((z(i)+z(i-
1))/2)) )*R(i)^2*r(i)-4*R(i)^2*r(i)^2*((P(i+1)-P(i-
1))/(z(i+1)-z(i-1)))*((r(i+1)-r(i-1))/(z(i+1)-z(i-1)))-
4*R(i)*r(i)^3*((P(i+1)-P(i-1))/(z(i+1)-z(i-1)))*((R(i+1)-
R(i-1))/(z(i+1)-z(i-1)))+4*r(i)^4*((P(i+1)-P(i-1))/(z(i+1)-
z(i-1)))*((r(i+1)-r(i-1))/(z(i+1)-z(i-
1)))+4*R(i)^3*r(i)*((P(i+1)-P(i-1))/(z(i+1)-z(i-
1)))*((R(i+1)-R(i-1))/(z(i+1)-z(i-1)))+4*((R(i+1)-R(i-
1))/(z(i+1)-z(i-1)))*sigma*R(i)^2*((r(i+1)-r(i-1))/(z(i+1)-
z(i-1))))/(mi_2*r(i)) +... % Primeiro termo da equacao 1
            2*R(i)*( (R(i)-Rpas(i)) / (t(j)-t(j-1)) ); %
Termo da igualdade da equacao 1, levado ao lado esquerdo da
equacao


        res(i+N) = (-c1*(R(i)^4) +
((c2/2)*(R(i)^2)*(R(i)^2-r(i)^2)))*(((P(i+1)-P(i-
1))/(z(i+1)-z(i-1)))-((sigma/(R(i)^2))*((R(i+1)-R(i-
1))/(z(i+1)-z(i-1))))) + ((sigma*c2)*((R(i+1)-R(i-
1))/(z(i+1)-z(i-1)))*((((R(i)^2)-(r(i)^2))/2)-
((R(i)^2)*(log(R(i)/r(i)))))) + ... %Primeiro termo da
equacao 2
            (-(c2/4)*((r(i)^2-R(i)^2)^2)*(((P(i+1)-P(i-
1))/(z(i+1)-z(i-1)))-((sigma/(R(i)^2))*((R(i+1)-R(i-
1))/(z(i+1)-z(i-1))))))  - (((sigma*2*c2)*((R(i+1)-R(i-
1))/(z(i+1)-z(i-1))))*( (((r(i)^2-R(i)^2)^2)/(8*R(i)^2)) +
(((R(i)^2)/2)*log(r(i)/R(i))) - (((r(i)^2)-(R(i)^2))/4) ));
%Segundo termo da equacao 2

    end


    res(N) = P(N) + (sigma/R(N)); % Segunda condicao de
contorno da equacao 1

    res(2*N) = R(N) - R(N-1); % Segunda condicao de
contorno da equacao 2
```

Função auxiliar "jacobiana11a.m"

```
%% Funcao Jacobiana - em funcao da fase continua
% Mestrando: Flavio Bormann - 0812212
% Orientador: Marcio Carvalho
% Data da ultima modificao: 03/09/2010

%% Chamada da funcao jacobiana

function [J] = jacobiana11a
(P,R,Rpas,z,N,j,t,c1,c2,sigma,r,mi_2)

%% Montagem da Matriz

J(1,1) = 1;
J(1,N+1) = -sigma/(R(1)^2);


for i=2:N-1
    a11 = (1/8/(z(i+1)-z(i))/(1/2*z(i+1)-1/2*z(i-
1))*r(i)^5-1/4/(z(i+1)-z(i))/(1/2*z(i+1)-1/2*z(i-
1))*R(i)^2*r(i)^3+1/8*r(i)/(z(i+1)-z(i))/(1/2*z(i+1)-
1/2*z(i-1))*R(i)^4-1/2*R(i)^2*r(i)^2/(z(i+1)-z(i-
1))^2*(r(i+1)-r(i-1))-1/2*R(i)*r(i)^3/(z(i+1)-z(i-
1))^2*(R(i+1)-R(i-1))+1/2*r(i)^4/(z(i+1)-z(i-1))^2*(r(i+1)-
r(i-1))+1/2*R(i)^3*r(i)/(z(i+1)-z(i-1))^2*(R(i+1)-R(i-
1)))/mi_2/r(i); % Pi+1
    a12 = (1/8*(-1/(z(i+1)-z(i))-1/(z(i)-z(i-
1)))/(1/2*z(i+1)-1/2*z(i-1))*r(i)^5-1/4*(-1/(z(i+1)-z(i))-
1/(z(i)-z(i-1)))/(1/2*z(i+1)-1/2*z(i-
1))*R(i)^2*r(i)^3+1/8*r(i)*(-1/(z(i+1)-z(i))-1/(z(i)-z(i-
1)))/(1/2*z(i+1)-1/2*z(i-1))*R(i)^4)/mi_2/r(i); % Pi
    a13 = (1/8/(z(i)-z(i-1))/(1/2*z(i+1)-1/2*z(i-
1))*r(i)^5-1/4/(z(i)-z(i-1))/(1/2*z(i+1)-1/2*z(i-
1))*R(i)^2*r(i)^3+1/8*r(i)/(z(i)-z(i-1))/(1/2*z(i+1)-
1/2*z(i-1))*R(i)^4+1/2*R(i)^2*r(i)^2/(z(i+1)-z(i-
1))^2*(r(i+1)-r(i-1))+1/2*R(i)*r(i)^3/(z(i+1)-z(i-
1))^2*(R(i+1)-R(i-1))-1/2*r(i)^4/(z(i+1)-z(i-1))^2*(r(i+1)-
```

```
r(i-1))-1/2*R(i)^3*r(i)/(z(i+1)-z(i-1))^2*(R(i+1)-R(i-
1)))/mi_2/r(i); % Pi-1
        a14  =  (-1/2/(z(i+1)-z(i-1))^2*sigma*r(i)^2*(r(i+1)-
r(i-1))+1/2*sigma/(z(i+1)-z(i))/(1/2*z(i+1)-1/2*z(i-
1))*R(i)^2*r(i)*log(r(i)/R(i))+2*sigma*(R(i+1)-R(i-
1))/(z(i+1)-z(i-1))^2*R(i)*log(r(i)/R(i))*r(i)-
1/4*sigma/(z(i+1)-z(i))/(1/2*z(i+1)-1/2*z(i-
1))*r(i)^3+1/4*sigma/(z(i+1)-z(i))/(1/2*z(i+1)-1/2*z(i-
1))*R(i)^2*r(i)-1/2*R(i)*r(i)^3*(P(i+1)-P(i-1))/(z(i+1)-z(i-
1))^2+1/2*R(i)^3*r(i)*(P(i+1)-P(i-1))/(z(i+1)-z(i-
1))^2+1/2/(z(i+1)-z(i-1))^2*sigma*R(i)^2*(r(i+1)-r(i-
1)))/mi_2/r(i); % R(i)+1
        a15  =  (-1/2*((P(i+1)-P(i))/(z(i+1)-z(i))-(P(i)-P(i-
1))/(z(i)-z(i-1)))/(1/2*z(i+1)-1/2*z(i-
1))*R(i)*r(i)^3+1/2*r(i)*((P(i+1)-P(i))/(z(i+1)-z(i))-(P(i)-
P(i-1))/(z(i)-z(i-1)))/(1/2*z(i+1)-1/2*z(i-
1))*R(i)^3+1/2*sigma*(-1/(z(i+1)-z(i))-1/(z(i)-z(i-
1)))/(1/2*z(i+1)-1/2*z(i-
1))*R(i)^2*r(i)*log(r(i)/R(i))+sigma*((R(i+1)-R(i))/(z(i+1)-
z(i))-(R(i)-R(i-1))/(z(i)-z(i-1)))/(1/2*z(i+1)-1/2*z(i-
1))*R(i)*r(i)*log(r(i)/R(i))+sigma*(R(i+1)-R(i-
1))^2/(z(i+1)-z(i-1))^2*log(r(i)/R(i))*r(i)-sigma*(R(i+1)-
R(i-1))^2/(z(i+1)-z(i-1))^2*r(i)-1/4*sigma*(-1/(z(i+1)-
z(i))-1/(z(i)-z(i-1)))/(1/2*z(i+1)-1/2*z(i-
1))*r(i)^3+1/4*sigma*(-1/(z(i+1)-z(i))-1/(z(i)-z(i-
1)))/(1/2*z(i+1)-1/2*z(i-1))*R(i)^2*r(i)-
R(i)*r(i)^2*(P(i+1)-P(i-1))/(z(i+1)-z(i-1))^2*(r(i+1)-r(i-
1))-1/2*r(i)^3*(P(i+1)-P(i-1))/(z(i+1)-z(i-1))^2*(R(i+1)-
R(i-1))+3/2*R(i)^2*r(i)*(P(i+1)-P(i-1))/(z(i+1)-z(i-
1))^2*(R(i+1)-R(i-1))+(R(i+1)-R(i-1))/(z(i+1)-z(i-
1))^2*sigma*R(i)*(r(i+1)-r(i-1)))/mi_2/r(i); % R(i)
        a16  =  (1/2/(z(i+1)-z(i-1))^2*sigma*r(i)^2*(r(i+1)-
r(i-1))+1/2*sigma/(z(i)-z(i-1))/(1/2*z(i+1)-1/2*z(i-
1))*R(i)^2*r(i)*log(r(i)/R(i))-2*sigma*(R(i+1)-R(i-
1))/(z(i+1)-z(i-1))^2*R(i)*log(r(i)/R(i))*r(i)-
1/4*sigma/(z(i)-z(i-1))/(1/2*z(i+1)-1/2*z(i-
1))*r(i)^3+1/4*sigma/(z(i)-z(i-1))/(1/2*z(i+1)-1/2*z(i-
```

```
1))*R(i)^2*r(i)+1/2*R(i)*r(i)^3*(P(i+1)-P(i-1))/(z(i+1)-z(i-
1))^2-1/2*R(i)^3*r(i)*(P(i+1)-P(i-1))/(z(i+1)-z(i-1))^2-
1/2/(z(i+1)-z(i-1))^2*sigma*R(i)^2*(r(i+1)-r(i-
1)))/mi_2/r(i); % R(i)-1

        e11 = 0; % Pi+1
        e12 = 0; % Pi
        e13 = 0; % Pi-1
        e14 = 0; % R(i)+1
        e15  =  2*(R(i)-Rpas(i))/(t(j)-t(j-1))+2*R(i)/(t(j)-
t(j-1)); % R(i)
        e16 = 0; % R(i)-1

        J(i,i-1) = a13 + e13;     % Pi-1
        J(i,i) = a12 + e12;   % Pi
        J(i,i+1) = a11 + e11;   % Pi+1
        J(i,N+i-1) = a16 + e16;   % R(i)-1
        J(i,N+i) = a15 + e15; % R(i)
        J(i,N+i+1) = a14 + e14; % R(i)+1
    end

    J(N,2*N) = -sigma/(R(N)^2);
    J(N,N) = 1;

    J(N+1,N+1) = -1;
    J(N+1,N+2) = 1;

    for k=N+2:2*N-1
        i=k-N;
        a21       =       (-c1*R(i)^4+1/2*c2*R(i)^2*(R(i)^2-
r(i)^2))/(z(i+1)-z(i-1)); % Pi+1
        a22 = 0; %Pi
        a23       =       -(-c1*R(i)^4+1/2*c2*R(i)^2*(R(i)^2-
r(i)^2))/(z(i+1)-z(i-1)); % Pi-1
        a24       =       -(-c1*R(i)^4+1/2*c2*R(i)^2*(R(i)^2-
r(i)^2))*sigma/R(i)^2/(z(i+1)-z(i-1))+sigma*c2/(z(i+1)-z(i-
1))*(1/2*R(i)^2-1/2*r(i)^2-R(i)^2*log(R(i)/r(i))); % R(i)+1
```

```
        a25         =         (-4*c1*R(i)^3+c2*R(i)*(R(i)^2-
r(i)^2)+c2*R(i)^3)*((P(i+1)-P(i-1))/(z(i+1)-z(i-1))-
sigma/R(i)^2*(R(i+1)-R(i-1))/(z(i+1)-z(i-1)))+2*(-
c1*R(i)^4+1/2*c2*R(i)^2*(R(i)^2-
r(i)^2))*sigma/R(i)^3*(R(i+1)-R(i-1))/(z(i+1)-z(i-1))-
2*sigma*c2*(R(i+1)-R(i-1))/(z(i+1)-z(i-
1))*R(i)*log(R(i)/r(i)); % R(i)
        a26       =       (-c1*R(i)^4+1/2*c2*R(i)^2*(R(i)^2-
r(i)^2))*sigma/R(i)^2/(z(i+1)-z(i-1))-sigma*c2/(z(i+1)-z(i-
1))*(1/2*R(i)^2-1/2*r(i)^2-R(i)^2*log(R(i)/r(i))); % R(i)-1

        b21  =  -1/4*c2*(r(i)^2-R(i)^2)^2/(z(i+1)-z(i-1));  %
Pi+1
        b22 = 0; % Pi
        b23  =  1/4*c2*(r(i)^2-R(i)^2)^2/(z(i+1)-z(i-1));  %
Pi-1
        b24                 =                 1/4*c2*(r(i)^2-
R(i)^2)^2*sigma/R(i)^2/(z(i+1)-z(i-1))-2*sigma*c2/(z(i+1)-
z(i-1))*(1/8*(r(i)^2-
R(i)^2)^2/R(i)^2+1/2*R(i)^2*log(r(i)/R(i))-
1/4*r(i)^2+1/4*R(i)^2); % R(i)+1
        b25  =  c2*(r(i)^2-R(i)^2)*((P(i+1)-P(i-1))/(z(i+1)-
z(i-1))-sigma/R(i)^2*(R(i+1)-R(i-1))/(z(i+1)-z(i-1)))*R(i)-
1/2*c2*(r(i)^2-R(i)^2)^2*sigma/R(i)^3*(R(i+1)-R(i-
1))/(z(i+1)-z(i-1))-2*sigma*c2*(R(i+1)-R(i-1))/(z(i+1)-z(i-
1))*(-1/2*(r(i)^2-R(i)^2)/R(i)-1/4*(r(i)^2-
R(i)^2)^2/R(i)^3+R(i)*log(r(i)/R(i))); % R(i)
        b26                 =                 -1/4*c2*(r(i)^2-
R(i)^2)^2*sigma/R(i)^2/(z(i+1)-z(i-1))+2*sigma*c2/(z(i+1)-
z(i-1))*(1/8*(r(i)^2-
R(i)^2)^2/R(i)^2+1/2*R(i)^2*log(r(i)/R(i))-
1/4*r(i)^2+1/4*R(i)^2); % R(i)-1


        J(k,i-1) = a23 + b23;    % Pi-1
        J(k,i) = a22 + b22;      % Pi
        J(k,i+1) = a21 + b21;    % Pi+1
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```
        J(k,k-1) = a26 + b26;    % R(i)-1
        J(k,k) = a25 + b25;      % R(i)
        J(k,k+1) = a24 + b24;    % R(i)+1
    end

    J(2*N,2*N-1) = -1;
    J(2*N,2*N) = 1
```