1 Introduction

Unconstrained Binary Quadratic Programming (UBQP) is widely studied. It is a powerful modeling tool and its associate problem is $\mathcal{NP}$-hard.

It can represent problems from a wide range of sources, including but not limited to: Scheduling [alidaee1994], Combinatorial Optimization [alidaee2005], Molecular Conformation [phillips1994] and Physics [liers2003]. Some classic $\mathcal{NP}$-hard Combinatorial Optimization problems, like max-cut and max-clique, can be trivially modeled as UBQP.

Because of its importance, a wide number of approaches were created to try to solve UBQP. Amongst them there are exact approaches [rendl2010, billionnet2007], approximation algorithms [goemans1995] and heuristics [glover2002, merz2004, merz2002, lodi1999].

In this work two new methods are introduced, which can be used to build a powerful exact algorithm, as well as a heuristic. Also, the fundamental idea behind them can be used in an even wider family of problems.

This exact algorithm derived from the new method is highly parallelizable, which is a desired feature nowadays, when the cloud computing is a reality. For reasonably large instances of UBQP, the new method can parallelize to hundreds, or even thousands, of cores easily, with a near-linear speedup. So, the time of computation is (almost) inversely proportional to the budget available for the project.

The remaining of this work is organized as follows: In chapter 2 the UBQP problem is presented, and its relation with max cut is proved. In chapter 3 related works are exhibited, including the state of the art. In chapter 4 we present the Column Generation Improvement for Heuristics. In chapter 5 the new approach is outlined. In chapter 6 the new approach is used in a branch
and bound scheme. In chapter 7 there are some numerical results, showing the performance of this new method. In chapter 8 a brief conclusion is made.