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An Improved Exact Method for the UBQP

Dissertação de Mestrado

Dissertation presented to the Postgraduate Program in Informatics of the Departamento de Informática do Centro Técnico Científico da PUC-Rio, as partial fulfillment of the requirements for the degree of Mestre.

Advisor: Prof. Marcus Vinicius Soledade Poggi de Aragão

Rio de Janeiro
August 2010

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Bibliographic data

Fleischman, Daniel

An improved exact method for the UBQP / Daniel Fleischman; adviser: Marcus Vinicius Soledade Poggi de Aragão. — 2010.

56 f; il. (color.); 29,7 cm

Dissertação (mestrado) – Pontifícia Universidade Católica do Rio de Janeiro, Departamento de Informática, 2010.

Inclui bibliografia.

1. Informática – Teses. 2. Programação semidefinida. 3. Branch-and-bound. 4. Programação não linear. 5. Programação quadrática binária irrestrita. 6. Condições de otimalidade. 7. Geração de colunas. I. Soledade Poggi de Aragão, Marcus Vinicius. II. Pontifícia Universidade Católica do Rio de Janeiro. Departamento de Informática. III. Título.

CDD: 004

Acknowledgments

To my adviser Professor Marcus Poggi, for his dedication. He has been a great source of knowledge and inspiration.

To my parents, my brother, my grandparents, and all my family for the support and the interest on my work, and for the patience.

To my girlfriend for being with me, even in the days that I could not be with her.

To the While True group. It is always good to have some friends who can laugh at the same jokes you do.

To my colleagues in the ICPC team, Fábio Dias and Roberto Cavalcante, for pushing me further. Thanks for the great years practicing together, and for the trips to San Antonio and Tokyo.

To the members of the teams I coached for the great time, and for the high level discussions. Special thanks to the Dynasty of Samba team, Caio Valentim, Eduardo Cardoso and Guilherme De Napoli for taking me to Harbin.

To my Professors in PUC-Rio, for teaching me with passion and depth.

To the Department of Informatics staff, for the efficiency and the commitment to their work.

To CNPq and PUC-Rio, for the financial support.

Abstract

Fleischman, Daniel; Soledade Poggi de Aragão, Marcus Vinicius.
An Improved Exact Method for the UBQP. Rio de Janeiro,
2010. 56p. M.Sc. Dissertation — Departamento de Informática,
Pontifícia Universidade Católica do Rio de Janeiro.

Unconstrained Binary Quadratic Programming (UBQP) is widely studied. It is a powerful modeling tool and its associate problem is \mathcal{NP} -hard. In this work a new approach is introduced, which can be used to build an exact algorithm. Also, the fundamental idea behind it can be used in an even wider family of problems. This exact algorithm derived from the new method is highly parallelizable, which is a desired feature nowadays, when the *cloud computing* is a reality. For reasonably large instances of UBQP, the new method can parallelize to hundreds, or even thousands, of cores easily, with a near-linear speedup.

Keywords

Semidefinite programming. Branch-and-bound. Nonlinear programming. Unconstrained binary quadratic programming. Optimality conditions. Column generation.

Resumo

Fleischman, Daniel; Soledade Poggi de Aragão, Marcus Vinicius. **Um Método Exato Melhorado para o UBQP**. Rio de Janeiro, 2010. 56p. Dissertação de Mestrado — Departamento de Informática, Pontifícia Universidade Católica do Rio de Janeiro.

A Programação Quadrática Binária Irrestrita (UBQP) é amplamente estudada. Trata-se de uma ferramenta de modelagem poderosa, mas otimizar de um problema \mathcal{NP} -difícil. Neste trabalho uma nova abordagem é apresentada, que pode ser usada para construir um algoritmo exato. Além disso, a ideia básica que fundamenta o trabalho pode ser usado em um espectro ainda mais amplo de problemas. O algoritmo exato derivado do novo método é altamente paralelizável, o que é uma característica desejável nos dias de hoje em que *cloud computing* já é uma realidade. Para instâncias razoavelmente grandes do UBQP, o novo método pode paralelizar a centenas, ou até milhares, de núcleos com facilidade, com um aumento de desempenho quase linear.

Palavras-chave

Programação semidefinida. Branch-and-bound. Programação não linear. Programação quadrática binária irrestrita. Condições de otimalidade. Geração de colunas.

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*Hofstadter's Law: It always takes longer than
you expect, even when you take into account
Hofstadter's Law.*

Douglas Hofstadter, *Gödel, Escher, Bach: An Eternal Golden Braid.*