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A Velocidades e Coeficientes das Redes D2Q9 e D3Q19

Wolf-Gladrow (Wolf-Gladrow, 2000) apresenta os valores de velocidades e coeficientes para as redes D2Q9 e D3Q19. Para a rede D2Q9 (A.1) as velocidades são as seguintes:

$$\begin{aligned}
 \mathbf{v}_0 &= (0, 0) \\
 \mathbf{v}_1 &= (1, 0)c \quad \mathbf{v}_2 = (0, 1)c \quad \mathbf{v}_3 = (-1, 0)c \quad \mathbf{v}_4 = (0, -1)c \\
 \mathbf{v}_5 &= (1, 1)c \quad \mathbf{v}_6 = (-1, 1)c \quad \mathbf{v}_7 = (-1, -1)c \quad \mathbf{v}_8 = (1, -1)c
 \end{aligned} \tag{A.1}$$

sendo $c = \frac{\Delta t}{\Delta x}$, e os coeficientes são dados por:

$$\begin{aligned}
 w_0 &= \frac{4}{9} \\
 w_\alpha &= \frac{1}{9} \quad \alpha = 1, 2, 3, 4 \\
 w_\alpha &= \frac{1}{36} \quad \alpha = 5, 6, 7, 8
 \end{aligned} \tag{A.2}$$

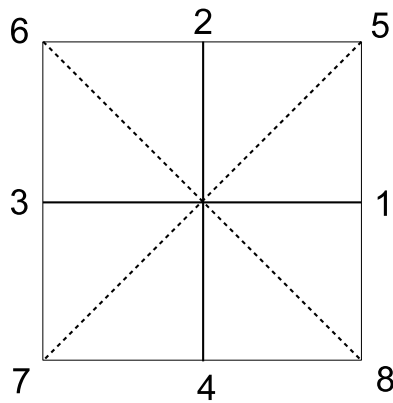


Figura A.1: Direção das velocidades discretas da rede D2Q9

Para a rede D3Q19 (fig. (A.2)) as velocidades e os coeficientes são os seguintes:

$$\begin{aligned}
 \mathbf{v}_0 &= (0, 0, 0) \\
 \mathbf{v}_1 &= (1, 0, 0)c & \mathbf{v}_2 &= (0, 1, 0)c & \mathbf{v}_3 &= (0, 0, 1)c \\
 \mathbf{v}_4 &= (-1, 0, 0)c & \mathbf{v}_5 &= (0, -1, 0)c & \mathbf{v}_6 &= (0, 0, -1)c \\
 \mathbf{v}_7 &= (1, 1, 0)c & \mathbf{v}_8 &= (0, 1, 1)c & \mathbf{v}_9 &= (1, 0, 1)c \\
 \mathbf{v}_{10} &= (-1, -1, 0)c & \mathbf{v}_{11} &= (0, -1, -1)c & \mathbf{v}_{12} &= (-1, 0, -1)c \\
 \mathbf{v}_{13} &= (-1, 1, 0)c & \mathbf{v}_{14} &= (0, -1, 1)c & \mathbf{v}_{15} &= (-1, 0, 1)c \\
 \mathbf{v}_{16} &= (1, -1, 0)c & \mathbf{v}_{17} &= (0, 1, -1)c & \mathbf{v}_{18} &= (1, 0, -1)c
 \end{aligned}
 \tag{A.3}$$

$$\begin{aligned}
 w_0 &= \frac{1}{3} \\
 w_\alpha &= \frac{1}{18} \quad \alpha = 1, 2, 3, 4, 5, 6 \\
 w_\alpha &= \frac{1}{36} \quad \alpha = 7, \dots, 18
 \end{aligned}
 \tag{A.4}$$

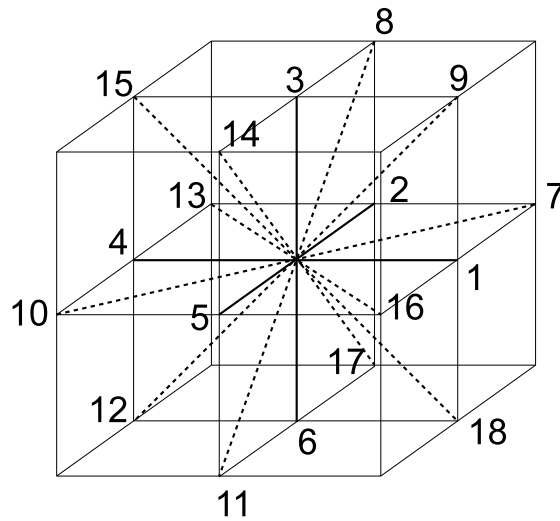


Figura A.2: Direção das velocidades discretas da rede D3Q19

B

Condição de Contorno de Pressão Prescrita

A condição de contorno de pressão prescrita implementada neste trabalho segue o esquema proposto por Zou e He (Zou e He, 1997). Considerando uma rede D2Q9, nos nós da entrada na figura (B.1) são impostos os valores $p = p_{in}$ e $u_y = 0$. Após a etapa de propagação são conhecidos os seguintes valores, nos nós da entrada: $p_0, p_2, p_3, p_4, p_6, p_7$. É necessário estabelecer as expressões para u_x, p_1, p_5, p_8 . A partir das equações (2.48) e (2.49) tem-se:

$$p_{in} = p_0 + p_1 + p_2 + p_3 + p_4 + p_5 + p_6 + p_7 + p_8 \quad (\text{B.1})$$

$$p_{in}u_x = c[p_1 + p_5 + p_8 - (p_3 + p_6 + p_7)] \quad (\text{B.2})$$

$$0 = [p_2 + p_5 + p_6 - (p_4 + p_7 + p_8)] \quad (\text{B.3})$$

Das equações (B.1) e (B.2) obtém-se:

$$u_x = c - \frac{c[p_0 + p_2 + p_4 + 2(p_3 + p_6 + p_7)]}{p_{in}} \quad (\text{B.4})$$

O método proposto por Zou e He (Zou e He, 1997) admite que a regra de retorno (“bounce-back rule”) é válida para a parte de não-equilíbrio: $p_1 - p_1^{eq} = p_3 - p_3^{eq}$. O que permite obter o valor de p_1 :

$$p_1 = p_3 + \frac{2}{3}p_{in} \frac{u_x}{c} \quad (\text{B.5})$$

Rearranjando as equações (B.2) e (B.3):

$$p_8 = p_{in} \frac{u_x}{c} + p_3 + p_6 + p_7 - p_1 - p_5 \quad (\text{B.6})$$

$$p_5 = p_8 + p_4 + p_7 - p_2 - p_6 \quad (\text{B.7})$$

Usando a equação (B.5) nas equações acima obtém-se as expressões para p_5 e p_8 :

$$p_5 = p_7 - \frac{1}{2}(p_2 - p_4) + \frac{1}{6}p_{in} \frac{u_x}{c} \quad (B.8)$$

$$p_8 = p_6 + \frac{1}{2}(p_2 - p_4) + \frac{1}{6}p_{in} \frac{u_x}{c} \quad (B.9)$$

As equações (B.5), (B.8) e (B.9) são as necessárias para a imposição da pressão prescrita na entrada. A mesma metodologia é aplicada para se obter a condição de pressão prescrita na saída e para a condição de velocidade prescrita na entrada e saída.

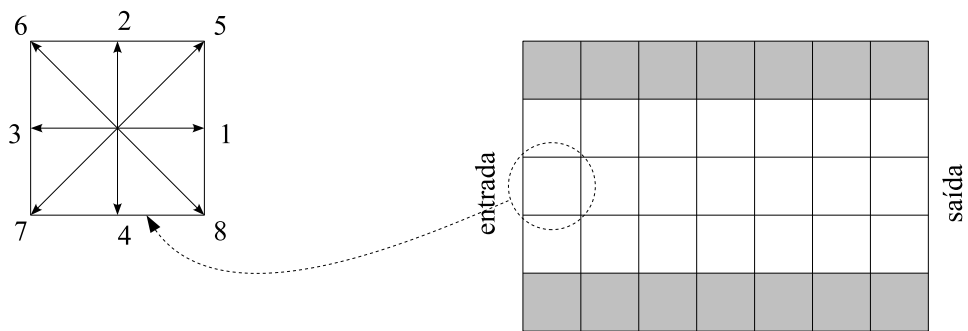


Figura B.1: Rede D2Q9