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***Abstract*— Since early 2000, long-term forward contracts or power purchase agreements (PPA) auctions have been the main mechanisms to ensure long-run supply adequacy in many growing economies, specially in Latin American, such as, Brazil, Chile, etc. With this framework, two issues are of special concern to Government agencies and market agents: (i) testing the design of the auction and its impacts on the power sector and (ii) the definition of bidding strategies by generators companies (Gencos) in these auctions to maximize their operation net revenue adjusted by the risk profile during the whole contract period. This work concentrates in (ii) and a strategic bidding model that takes into account the main uncertainties factors and the long-run Gencos' risk profile will be presented to assess the Willing-to-Supply curve. The agent risk profile is characterized by means of a piecewise linear utility function and an intuitive approach, based on the most relevant financial company's parameters, will be introduced to determine it. In addition, a probability dependent utility function representation for the CVaR coherent risk measure is provided in order to compare both risk attitudes. A case study with realistic data from the Brazilian Power System will be presented to illustrate the applicability of the model.**

***Index Terms* -- Power system economics, forward contracts, contract auctions, portfolio optimization, utility function, Conditional Value-at-Risk.**

I. INTRODUCTION

OPEN auctions of long-term power purchase agreements (PPAs) between generators and distribution companies have been the main mechanism for purchasing and selling energy in many growing economies since early 2000. In Latin American (LA) countries the power system reforms are fully related to long-term PPA auctions, which are to provide the long-run system's expansion marginal cost signal [13], reduce uncertainty in Genco's future cash flow and mitigate short-term market power [15].

In the case of Chile, that was the pioneer in deregulating the energy market, the PPA auctions were only adopted after the natural gas "crisis" in 2004-05, in order to provide incentives to new investments and to ensure supply adequacy. As in Chile, in the Brazilian power system, due to its hydraulic predominance and the complexity of determining the water values [10][14] to operate the reservoirs in a "secure" way, market design is still centralized, and the generation dispatch order and the short term spot price are driven by the National System Operator (SO). The Brazilian system's reform started after the 2002 supply crisis. The "new" regulatory framework, stated in March 2004 and detailed in July, consolidated the guideline for the distribution companies (Discos) PPA purchase auctions.

In this new regulatory model, every Genco is limited to sell through bilateral forward contracts the total Firm Energy Certificates (FEC) that each of its owned Power Generator Unities (PGU) possess. The yearly FEC are issued by the regulator for each PGU and reflect their firm energy production capacity in dry years [17]. In addition, it was stated that 100% of all consumers load should be covered by bilateral forward contracts backed up by FEC, providing the link between load growth and the investment in new capacity. Finally, in order to provide an efficient and transparent capacity expansion, it was also stated that all Discos' bilateral agreements could only be negotiated through public PPA open auctions, which has triggered the competition environment on the Brazilian generation segment and the need of new developments on strategic bidding for long-term contracts.

In spite of the aforementioned benefits, the long-term bilateral contracts also create a set of relevant challenges for Gencos, such as how to take into account the main long-term uncertainties on the bidding process and how to express the long-term risk preference during a multi-product bidding.

Strategic bidding in forward markets is, in some sense, a recent research area in the literature, since it is a step forward on the market liberalization process, which has been taking place since 90's. There is a worldwide consensus that forward contracts reduce uncertainty and there are several works providing substantial evidences on their hedging potential and how to incorporate them into the day-to-day energy trading (see [2][3] and [4][5] for recent studies on the Spanish market). But since the optimal forward consumption of bilateral agreements is highly dependent on the specific market's characteristics, the relevant effects and risk factors that should be taken into account would also be case dependent. The so called Financial Transmission Rights (see [6]) is an example of financial hedge due to cross-zone bilateral agreements. This type of contract deals with the Transmission risk due to lines congestion when trading

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bilateral agreements thought different price zones and aims to refund Gencos exposed to the differences on the locational marginal short-term prices.

In [7], the methodology introduced for the optimal short/medium-term forward contracting in [5] is extended to comprise the unit failure risk effect. Assessments of the optimal contracting strategy deviations for a CVaR maximizer Genco are shown for different levels of forced outage rates in an hourly granularity case for a one to three months contract time horizon.

In [1] the optimal consumption allocation between firm and flexible gas supply contracts is introduced. This work presents a medium/long-term risk-constrained optimization model whose objective is to select the optimal percentage of the gas demand of an industrial consumer that should be contracted through firm contracts, with no interruptions in the supply, and flexible supply contracts, for which the supply is conditioned to the thermoelectric unities fuel usage. A monthly based case study is presented considering contracts with three years of duration.

According to the aforementioned literature, bilateral contracting imposes many technical challenges that both buyers and sellers should deal with in order to establish their strategic positions. Different approaches should be considered in order to build the optimal contracting strategy, and they will depend on: (i) the time horizon; (ii) the environment in which such contracts are been negotiated, e.g., auctions or free bilateral trading; (iii) the risk factors that affect the future contract outcomes and; (iv) the agents' risk profile.

Due to the technical challenges that a forward contract auction process imposes and the enormous impact that a long-term bilateral contract may cause on Gencos' wealth, the objective of this work is to determine the optimal Willing-to-Supply (WtS) curve for a risk-averse Genco to be used during a long-term forward contracting auction process, or during a free bilateral negotiation, as a bidding rule. In this sense, an intuitive way of representing the long-run risk-averseness of Gencos was developed based on the association of the most relevant financial companies' indexes to the Piecewise Linear Utility Functions (PLUF) parameters. The piecewise linear form has shown to provide nice properties and flexibilities on its practical applicability and specification, as will be further shown. Correspondences to a coherent commonly used risk measure, the Conditioned Value-at-Risk (CVaR), will also be provided.

The next sections of this work are organized as follows. Section II provides an overview of the Brazilian physical and regulatory framework in order to contextualize the environment in which Gencos are embedded and to explain the methodologies used to model some of the uncertainty factors that will be taken into account in the Genco's revenue expression. In section III Genco's risk profile is introduced and the methodology to determine the piecewise linear utility function parameters is presented, together with a short discussion about the correspondences with the CVaR risk measure. Section IV presents the strategic bidding model that provides the WtS curve. Finally, section V presents some computational results for realistic data of the Brazilian power system and section VI concludes this work.

II. Overview of the Brazilian Power Sector

Current regulation establishes that all federal and state-owned generation companies (Gencos) must sell their energy through an "open and competitive process", which is translated in an auction. Moreover, current regulation also establishes that all Discos should contract its energy needs by means of public auctions. The underlying actions that form the backbone of the energy contracting in Brazil are as follows:

- Every load – both captive and eligible consumers - must be 100% contracted at all times. This obligation has two main objectives: (i) commercially induce generation expansion – the reason is that, as discussed in [13], the Brazilian spot prices are too volatile and do not provide correct signals for system expansion, making bilateral contracts a key element in the market design; (ii) assure the system security of supply: although bilateral contracts are financial instruments, they should be "anchored" by a physical generation capacity. This means that if 100% of the demand is contracted, there is physical generation capacity to supply the correspondent load even under low systems resources (dry periods);
- Mandatory open PPA purchase auctions for Discos – distribution utilities must contract their energy through open PPA auctions, with standardized rules and contracts, designed to allocate risks between generators, distributors and consumers, and to promote efficient energy purchases. There are two main types of regulated auctions: (i) "New Energy" [11], carried out five and three years in advance; the contracts are intended for the construction of new capacity, that will cover the forecasted load increase; and (ii) "Existing Energy" (EE) , which are intended to cover the existing load. Both auctions (i) and (ii) are carried out for the "sum" of the loads of all Discos, which means that auction winners must sign bilateral contracts with all Discos in proportion to their energy needs. This work will concentrates in developing bidding strategies for EE contract auctions.

These two regulatory requirements, 100% of contract obligation (with physical backing) and the use of auctions as the mechanism for bilateral trading, form the backbone of the system to induce the security of supply, tariff adequacy to final consumers and to achieve the long-run efficiency.

While the system relies on competition for contracts to achieve the market efficiency, in Brazil the system dispatch is cost-based[1] and carried out in a centralized way by the system operator. Hydro plants are dispatched based on their expected opportunity costs ("water values"), which are computed by a multi-stage stochastic optimization model that takes into account a detailed representation of hydro plant operation and inflow uncertainties (see [12][14]). As a consequence, the system operator (SO) dispatch model needs to simulate the future system optimal dispatch, under many possible resources conditions (hydrology), in order to obtain the today's water value and optimal system schedule. Thus, a sub-product of such simulation is the future generation of each PGU in the system and the related optimal system's water values (spot prices) in each future hydrological simulated scenario. These scenarios constitute the basis to simulate the future contracts outcomes, as will be shown in C.

The establishment of the mandatory contracting and the PPA auctions create important challenges for both bidders and the regulator. Differently from typical day-ahead spot market auctions, the products involved in a long-term PPA auction are bilateral contracts, whose maturity can be of several years. Consequently, a bad outcome from the auction can have negative impacts on the Genco's future cash flow, since a bad contracting strategy affects the company's future economic results.

A. *Energy spot price volatility*

Spot prices are very volatile and negatively correlated with system's hydrological conditions, as Fig. 1 shows. This happens because a hydro based system is designed to supply the load under very adverse inflow conditions, which do not occur frequently. As a result, most of the time (when inflows are in their "normal" pattern) demand is covered by hydro generation and the marginal demand cost or the spot price is very low. But in contrast, when the system's future reliability is in danger, the water value increases very fast and the marginal cost can reach its price cap in a period shorter than a month.

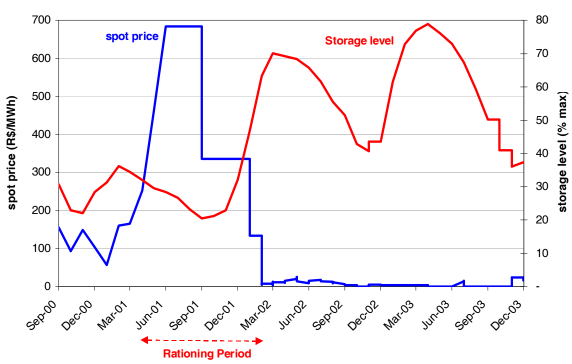

Fig. 1 – Spot Prices vs. Storage Levels during 2000 to 2003

B. *Forward Contract Revenue*

As commented at the beginning of this section, the focus of this work is on the EE forward contract auctions, in which the negotiated contracts are purely financial instruments named "quantity contracts" with typical duration of 5 to 10 years. This type of contract is close related to the so called two-side forward contract for differences (see [8] for more details).

The energy delivery risk belongs to generators (Gencos), which are not obliged to physically produce the contracted amount, but must clear the differences between energy production and total contracted amount in the spot market. Thus, there is a market clearing at the end of each period (monthly in the Brazilian case) in which for every Genco the total generation is accounted and subtracted from the total contracted amount. Thus, the future net revenue of a Genco forward contracted with a Disco will be composed of three terms: a fixed component (deterministic), which is due to the Disco payments for the energy supply rights and two variable components (stochastic) due to the spot differences clearing and generation costs. The spot clearing variable component will provide positive values for the production surplus scenarios (due to the surplus sales in the spot market) and negative outcomes on deficit scenarios (due to the deficits purchases in the same market).

The future revenue for each contracting period t and simulated scenario s of a Genco owing n_u PGU possessing n_c forward contracts follows as expression (1):

$$R_{t,s} = \Sigma_i P_i \cdot E_{i,t} \cdot h_t + (\Sigma_j G_{j,t,s} - \Sigma_i E_{i,t} \cdot h_t) \cdot \pi_{t,s} - \Sigma_j G_{j,t,s} \cdot c_{j,t} \qquad (1)$$

where,

- i is the contract index and belongs to the set of contracts $\{1,\ldots, n_c\}$,
- j is the units index and belongs to the set of units $\{1,\ldots,n_u\}$,
- s is the scenario index and belongs to the set $\{1,\ldots,S\}$,
- $G_{i,t,s}$ is the generation of unit i in each period t and simulated scenario s (in MWh),
- $E_{i,t}$ is the energy amount of contract i at perio t (in avgMW),
- P_i is the price of contract i (in \$/MWh) and
- c_t is the average operational cost of unit i in period t (in \$/MWh).

In expression (1) each contract can have different initial dates, duration and energy seasonality. Such differences can be addressed by means of the $E_{i,t}$ quantity variable, which should cover the whole analyzed time horizon for each contract, assuming zero value at periods in which contract i is not available. Fig. 2 shows the case of two different contracts in which contract 1 has a seasonal energy clause and contract 2 is a flat supply agreement. We assume in this work that the energy profile is fixed and pre-determined. In this sense the energy profile $E_{i,t}$ can be addressed by means of a seasonal coefficient $q_{i,t}$, which should sum one during the whole contract time horizon, multiplied by the average nominal contract amount e_i (in avgMW). Thus, Gencos decisions consist on defining how much of each contract should be signed in terms of nominal average energy (vector $\mathbf{e}=[e_1,\ldots,e_{n_c}]^{\mathbf{T}}$). This is the main idea of this work and will be introduced on section IV.

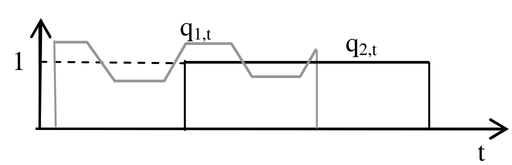

Fig. 2 – Contracts energy profile

III. Risk-Aversion in Long-Term Portfolio Problems

In this work, the risk preference of the decision maker is represented through a classical von-Neumann Expected

Utility (EU) functional. EU functionals takes into account the whole range of scenarios by "translating" monetary revenues into "utility units" and express agents preferences through the expectation of the resultant risk-adjusted utility scenarios [9]. The objective of a rational agent is to maximize the expected utility searching into a pre-specified set of probability distribution functions the one that provides the greatest EU index.

The agent risk profile is characterized by the form of the utility function, e.g. , a risk-neutral Genco would have a linear UF. This means that a revenue increase has the same impact, in magnitude, as a decrease. Instead, a risk-averse agent would have a concave UF, as shown in Fig. 3. In this case, the loss from a "bad" outcome is not "compensated" by the gain from a "good" one with the same magnitude: for each revenue outcome, the UF have different marginal increasing rates. Fig. 3 shows that a decrease (-d) from a reference value (R_o) results in a utility variation of DU^{dw} that is greater than the increase of utility DU^{up} due to increase of the same amount (+d) on R_o. This is the main characteristic of a risk-averse agent. Theoretically, agents can be risk-averse, risk-neutral or even risk-taker, but in the corporative segment, companies' commitments with shareholders and sponsors implies in a risk-averse profile. Therefore, we will assume that Gencos are all risk-averse agents in this work.

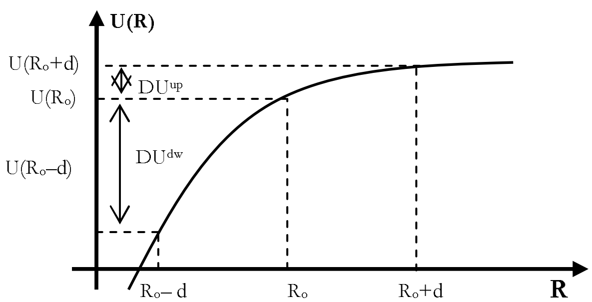

Fig. 3 – Concave UF representing a risk-averse profile.

A. *Piecewise linear representation of utility functions*

Risk-averse utility functions can be represented as concave functions, such as the logarithmic and negative exponential functions [9]. This work adopts a Piecewise Linear representation of Utility Function (PLUF) as shown in Fig.5, which is flexible (adjustments in the breakpoints and slopes define different risk-aversive behaviors) and avoids the need of nonlinear curves and algorithms.

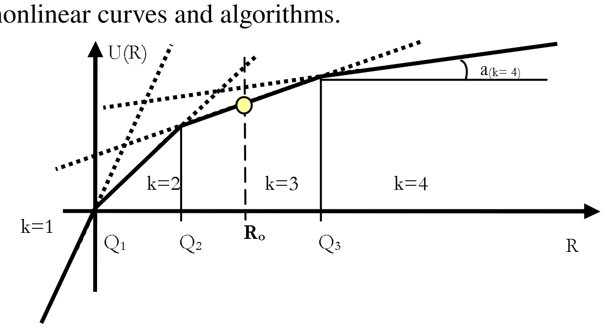

Fig. 4 – Risk aversion profile through the PLUF.

The motivations for the use of PLUF comes from two reasons: (i) any concave non linear UF can be well represented through a concave PLUF by choosing an adequate number of linear segments (see Fig. 4); (ii) in a long-term (yearly base) framework decision makers are, generally, not interested in specifying the risk aversion for the entire revenue's domain, but only for the specifics points that may chance Genco's status quo. The latter, is supported on the fact that Gencos' investors are generally local risk-neutral, but global risk-averse, and in the long-term such aversion occurs when decisions make Gencos' wealth to cross a critical revenue point, such as the operational breakeven, providing a change in the annual financial reports. Thus, for small revenue variations, between such critical points, Gencos' decision makers should behave as risk-neutral agents (linear UF).

The chosen piecewise linear form for the utility representation, together with the fact that we are interesting in building a company risk profile and not to estimate a "person" risk attitude, which is known to exhibit a few rationality paradoxes [15][16], provides a interesting utility specification approach based on corporative financial parameters. The proposed approach explores the nature and the magnitude of such long-term contract decisions, which, in opposite to the well kwon short-term day-ahead selling ones, can provide a large impact in the future company's financial performance. In this sense, the decision maker should take into account the "benefit" of being in each revenue segment, translate it in terms of marginal utility and then maximize the total Genco expected utility.

B. *Specification of the PLUF*

Many different approaches can be used to estimate an agent UF [9], but as argued before, a Genco is a company that should not act as a person. The proposed approach is to construct the PLUF of a Genco in order to express the benefit of achieving the different possible income results. In this sense, we first need to collect a set of relevant revenue points based on a Genco investors' board and risk department consensus, together with the marginal utility coefficients (since it is piecewise linear) – one for each segment delimited by revenue points (see Fig. 4). These coefficients can be set to express many different risk perceptions of being in each revenue segment, but an interesting one corresponds to the market risk spread rate for loan capital.

In this context, the proposed approach should capture the market credit rate for each possible revenue segment, e.g., in a very bad scenario, in which the operational breakeven is not achieved, the market will rate (classify) this company as a potential swindler and then, will ask an additional spread rate in order to lend an extra capital to compensate the default risk, which would not be necessary in a health situation. In this context, each utility coefficient (slopes) can be chosen in order to express the market risk perception through the risk spread on interest rates. Thus, a Genco only needs to specify the relevant revenue points and the corresponding market spread rates for each segment in order to fully determine its annual utility function.

The construction of a Genco risk profile is important in order to avoid human errors and ambiguous risk-attitudes during non-trivial decisions, which involve many possible feasible solutions with different stochastic attributes. In these cases, such as bidding in a multi-product contract auctions or a bilateral negotiation involving contracts with different properties and risks characteristics, an optimization model taking into account the company's risk profile is a decision supporting tool of great interest and relevance, which will certainly follow the pre-specified preference.

The EU of a discrete random revenue (R) defined by a set of S outcomes and associated probabilities $\{R_s, p_s\}_{s=1,\ldots,S}$ can be assessed through the following linear programming (LP) problem:

$$E[U(R)] = \text{Maximize}_{(u)} \textstyle\sum_s u_s \cdot p_s \qquad (2)$$

Subjected to:

$$u_s \leq a_k \cdot R_s + b_k \quad \forall\, k, s \qquad (2.1)$$

$$u_s \in \Re \quad \forall\, s \qquad (2.2)$$

Where,

- k represents PLUF segment index and belongs to $\{1,\ldots,K\}$,
- a_k represents the angular coefficient (slope) of the k^{-th} segment,
- b_k represents the linear coefficient of the k-th segment and
- u_s is a decision variable that plays the role of UF for each revenue scenario s.

Fig. 4 illustrates a PLUF with four segments, with Q_k representing a break point, where the marginal utility changes from a_k to a_{k+1}.

C. Connection between PLUFs and risk measures (CVaR)

The Conditional Value at Risk (CVaR) measure roughly corresponds to the expected value of the worst (1-α) net revenue or profit scenarios. Both theoretical and practical features have made the use of the CVaR widespread in portfolio allocation problems and management science applications [1][4][5][20]. It combines a set of virtues such as an intuitive parameter specification process, all needed coherence properties [17][20], in which sub-additivity is included, the advantage of capturing the averseness to high-impact with low probability losses, and also can be incorporated into a linear programming problems as a set of linear constraints [19].

Mathematically, the CVaR of a random variable R (net revenue or operative profit) with cumulative probability function F_R is given by:

$$\text{CVaR}_\alpha(R) = E(R \mid \Psi) \qquad (3)$$

Ψ is the set of net revenue values below the associated Values-at-Risk, $\text{VaR}_\alpha(R)$, defined as $\text{VaR}_\alpha(R) = F_R^{-1}(1-\alpha) = \inf_{(r)}\{r \in \text{support}(R) \mid F_R(r) \geq 1-\alpha\}$. As shown in [19], the CVaR of a random variable may also be written as a linear programming (LP) problem and can be easily inserted into a linear profit maximization problem by inserting a set of linear constraints. Thus, according to [19] expression (3) can be driven by the maximum expression

$$\text{CVaR}_\alpha(R) = \text{Maximize}_{(z)}\{z - \textstyle\sum_s (z - R_s)|^+ \cdot p_s/(1-\alpha)\} \qquad (4)$$

and assessed through the following LP problem:

$$\text{CVaR}_\alpha(R) = \text{Maximize}_{(u)}\; z - \textstyle\sum_s \delta_s \cdot p_s/(1-\alpha) \qquad (5)$$

Subjected to:

$$\delta_s \geq z - R_s \quad \forall\, s \qquad (5.1)$$

$$\delta_s \in \Re_+ \quad \forall\, s \qquad (5.2)$$

In expression (4), the truncate function $(.)|^+ = \max\{0, .\}$ accounts only the positive revenue deviations from the z variable. Then, the conditioned expectation of such deviations are taken and shifted of z, which maximizes the whole expression at point $z^* = \text{VaR}_\alpha(R)$.

As proposed in [21] an alternative view point for the CVaR risk measure can be found on its probability dependent utility functional form. A Probability Dependent Utility Function (PDUF) is a function that depends on both the support of the random variable ($\{R_s\}_s$ possible outcomes) and the probability distribution ($\{p_s\}_s$) of the assessed random variable (R).

A CVaR maximizer agent is the one that seeks in a set of feasible set of random variables the one that maximizes the CVaR measure. The CVaR associated PDUF can be found after a careful look on expression (4) by substituting the optimal value z^* and extending the expected operator for all components. Expression (6) shows the expected PDUF form of a CVaR maximizer agent.

$$E[U_\alpha(R, F_R)] = E[F_R^{-1}(1-\alpha) - (z - R)|^+/(1-\alpha)] \qquad (6)$$

In (6) the PDUF can be straightforward identified inside the expect value operator and in Fig. 5 such utility is shown for a general continuous random variable. The CVaR maximizer associated PDUF is a two segments piecewise linear function with a fixed point on the (1–α) quantile ($\text{VaR}_\alpha(R)$). Thus, depending on the probability distribution of the assessed random variable (R) such utility will be translated, sliding its unique kink over the identity function according to the random variable VaR_α.

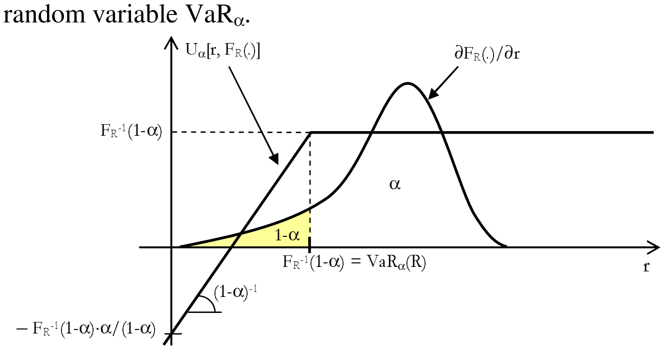

Fig. 5 – PDUF of a CVaR maximizer

"*An interesting interpretation for such probability dependent utility may rise from the investment under uncertainty context, in which investors may only regret if a given specified project (with F_R distribution) provides an* *improbable downsize realization R_{s_o}, with $F_R(R_{s_o}) \leq 1 - \alpha$, based on their previous estimated probability function F_R. In this sense, $F_R^{-1}(1 - \alpha)$ – or $VaR_\alpha(R)$ – turns to be the critical and generally pessimistic point for which the project is dimensioned. Thus, surplus realizations scenarios are not differently accounted, if compared to the critical value, but on another hand, deficit scenarios are penalized with $(1 - \alpha)^{-1}$ per \$ (revenue unit) of violation.*" (extracted from [21]).

The connection made between the CVaR and a probability dependent utility functional is a theoretical result and should be used in order to provide a better understanding of a CVaR maximizer preference and properties instead of been used in practice as an alternative way to assess such measure. In this sense, [19] distinguishes behaviors between a CVaR maximizer and a classical (von-Neumann-Morgenstern) expected utility agent through illustrative examples and based on important consequent results such as the associated Certainty Equivalent (CE). In the aforementioned work the results established for a CVaR maximizer are extended for a convex combination between this measure and the expected value in order to extend the connection made for a widespread used optimization metric (see [4] and [5] for examples on the usage of this metric). Besides, the associated PDUF of such metric slightly differs from the pure CVaR PDUF shown in Fig. 5. This function should exhibit a non zero slope for outcomes greater than VaR_α (we refer to [21] for a further explanations).

An interesting result for an agent who optimizes a measure composed of a convex combination between the CVaR and the expected value is that the associated expected utility functional index is equal the associated agent CE, which in the multi-period context turns to be a interesting property, since the discount factor will discount money instead of utility (see [24] for a detailed discussion). Finally, [22] provides a close related connection between CVaR risk constrained problems and utility maximization by means of the dual lagrangean relaxation of the risk constraint.

IV. Long-term Portfolio maximization problem: the Willing to Supply Curve

The WtS is a bidding curve to be used during bilateral negotiations or iterative price clock auctions. The main idea of this curve is to generate a map that, for each possible contract price (P), it provides the optimal quantity bid (e^*). A more general case deals with simultaneous multi-product bidding strategies, which for the optimization procedure is a straightforward extension.

In this sense, the WtS depends on the uncertainties (spot prices and future generation), on the risk profile of the decision maker, on the products price vector $\mathbf{P}=[P_1, \ldots, P_{n_c}]^T$, and on the characteristics of each product such as initial date and duration. Thus, given a set of n_c prices, the optimal quantity bids for a Genco owing n_u PGU should be limited by its maximum amount of annual average FEC. The following LP model provides the mathematical formulation for the optimal bidding strategy under uncertainty for an expected PLUF maximizer, which is a parametric LP problem that describes the WtS curve.

$$\mathbf{e}^*(\mathbf{P}) = \text{argmax}_{(u,e)} \textstyle\sum_t \sum_s u_{t,s} \cdot p_s \cdot (1+J)^{-t} \tag{7}$$

Subjected to:

$$u_{t,s} \leq a_k \cdot R_{t,s} + b_k \quad \forall\ k, t, s \tag{7.1}$$

$$R_{t,s} = \textstyle\sum_i P_i \cdot (e_i \cdot q_{i,t}) \cdot h_t + [\sum_j G_{j,t,s} - \sum_i (e_i \cdot q_{i,t}) \cdot h_t] \cdot \pi_{t,s} - \sum_j G_{j,t,s} \cdot c_{j,t} \quad \forall\ t, s \tag{7.2}$$

$$\textstyle\sum_i e_i \leq FEC \tag{7.3}$$

$$u_{t,s} \in \Re\ \forall\ t,s \ \text{ and } \ e_i \in \Re_+\ \forall\ i \tag{7.4}$$

In model (7)-(7.4), expression (7) assesses the expected utility functional for a separable per-period utility maximizer whose timing preference (inter-period) is accounted through an impatience factor $(1+J)^{-t}$. Expression (7.1) have been introduced before and expression (7.2) meets expression (2) by substituting the per-period contract energy amount ($E_{i,t}$) by its average nominal amount times its seasonality coefficient ($e_i \cdot q_{i,t}$). In addition to that, the existing portfolio can be easily accounted into this model by extending the number of contracts and fixing the associated e_i variables to the known amounts.

As an alternative, a CVaR based bidding model can be obtained following the same idea of problem (7)-(7.4), by substituting the expected utility assessment from expression (7)-(7.1) by the convex combination between the CVaR and the expected value of the revenue function in each period.

$$\mathbf{e}^*(\mathbf{P}) = \text{argmax}_{(\delta,e,z)} \textstyle\sum_t [\lambda \cdot (z_t - \sum_s \delta_{t,s} \cdot p_s) + (1-\lambda) \cdot \sum_s R_{t,s} \cdot p_s] \cdot (1+J)^{-t} \tag{8}$$

Subjected to:

$$\delta_{t,s} \geq z_t - R_{t,s} \quad \forall\ t, s \tag{8.1}$$

$$R_{t,s} = \textstyle\sum_i P_i \cdot (e_i \cdot q_{i,t}) \cdot h_t + [\sum_j G_{j,t,s} - \sum_i (e_i \cdot q_{i,t}) \cdot h_t] \cdot \pi_{t,s} - \sum_j G_{j,t,s} \cdot c_{j,t} \quad \forall\ t, s \tag{8.2}$$

$$\textstyle\sum_i e_i \leq FEC \tag{8.3}$$

$$\delta_{t,s} \in \Re_+\ \forall\ t,s \ \text{ and } \ e_i \in \Re_+\ \forall\ i \tag{8.4}$$

The CVaR implementation follows the idea of (5)-(5.2) and $\lambda \in [0,1]$ (a risk aversion parameter) expresses the importance (weight) of the CVaR on each period in the objective function.

One can note that both proposed models discriminate the risk preference for each period by applying the expected utility (7)-(7.4) or the CVaR based preference functional (8)-(8.4) to the net revenue in each period. This is due to the long-term characteristic of the related contract's cash flow that would imply in a liquidity risk aversion that would not be captured in the case of applying the aforementioned functionals to the net present value[2]. Finally, it is out of the scope of this work to determine or suggest which model (7) or (8) should be used. This is a private choice that can only be made by the decision maker.

V. Case Study

As introduced earlier in section II, the new regulatory framework of the Brazilian Power System implies that every year Discos must contract their loads thought public auctions. These auctions are purchase auctions in which the auctioneer represents Discos by decreasing prices in each round so that the total Gencos bids meets the total Discos' demand. In this sense, Gencos competes by submitting quantity bids (nominal average energy) for each price level determined in each round. The total demand and others bids are not reveled during the auction process and the only information available for Gencos is the contract price at each round. In this context, the WtS curve can be direct applied.

In this case study, for the sake of simplicity we will consider a Genco with one hydroelectric PGU owning 100 avgMW of FEC available for trading at the beginning of 2011. In order to obtain the monthly spot prices and future generation scenarios for a five years contract horizon (from 2011 to 2015) a SDDP based dispatch model [10] was used. The model was implemented in a yearly based step, with a pre-processing scheme to correctly account the monthly cross-correlation between spot prices and hydro generation. The candidate contract for which the WtS curve was assessed was chosen to be a five years flat energy contract similar to the standards contracts auctioned every year by Discos in order to re-contract the existing energy. Thus, an annual PLUF was specified according to section III assuming that 50 MMR$ is the necessary annual income revenue to achieve the operational breakeven, 80 MMR$ provides reserves needed to distribute dividends for shareholders and 100 MMR$ is the total annual income that makes the company to achieve its annual goal. Thus, Gencos segment weights were determined according to the proposed interest rate marginal utility rationale for the segments delimitated by the specified annual critical points. In Table I the first slope (revenues in (-∞,0]) was set to 100 in order to extremely penalize negative annual income results. The utility discount factor was set to J=10% per year in order to express the temporal preference.

Table I. Piecewise Linear Utility Function coefficients

Revenue segments (MMR$)	(-∞,0]	(0,50]	(50,80]	(80,100]	(100,+∞)
Utility slope (%)	100	20	15	12	10

The $CVaR_{95\%}$ based preference was selected to reflect a 50% $CVaR_{95\%}$ + 50% expected value in each year (λ=50%). Then, the WtS curve was assessed for three different risk profiles: (i) risk-neutral, (ii) risk-averse according to the Table I PLUF and (iii) the (λ=50%, α=95%)-CVaR based functional. Xpress MP 2008 was used to solve the respective deterministic LP equivalent problems. In Fig. 6 the WtS curves are shown for price range from 50 to 150 R$/MWh.

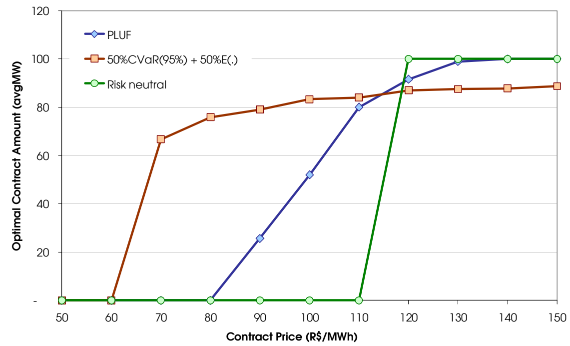

Fig. 6 – Willing to Supply Curve for a hydroelectric Genco (contract horizon from Jan/2011 to Dec/2015).

According to Fig. 6 the risk-neutral curve is a stepwise curve that shifts from 0 to 100% of the total FEC (100 avgMW) at a Spot Indifference Contract Price (SICP) of 120 R$/MWh, which is the contract price for which the total Expected Net Present Value (ENPV) of the candidate contract overtakes the ENPV of the spot sells. From this point on, the risk-neutral agent will be willing-to-supply as much energy as it possess thought such contract. This type of behavior is not verified in the risk-averse cases. In these cases, agents will search for a mix between spot and contract sales in order to mitigate the portfolio risk according to each risk-preference, even if the contract price is lower than the SISP. This behavior can be verified for both risk-averse agents. Furthermore, both risk-averse agents consider the over contracting risk (also kwon as quantity risk), by not contracting the 100% FEC amount, that hydro Gencos face in hydro based systems due to the negative correlation between spot prices and system production (see Fig. 1). In this sense, for some contract prices greater than the SICP both profiles shows to maintain a safe amount of the total FEC into the spot market to hedge against scenarios with deficit of production and high spot prices. Finally, as the CVaR based agent has a "relative" or a probability dependent utility representation, depending on the α-VaR quantile, the WtS curve of such agent persists on hedging in the spot market even for high contract prices, which do not occurs with the classical expected utility profile. The latter has a fixed utility that will exhibit a risk-neutral preference as the revenue scenarios are all on the same segment. Thus, from the expected utility maximizer point of view, for prices greater than 140 R$/MWh the quantity risk is not compensated in terms of marginal utility by the expected benefit of increasing the contract amount.

VI. Conclusions

This work has established a methodology to assess the WtS curve for the long-term forward trading for two types of risk-

[2] The risk aversion for a short cash flow stream is usually provided for the final net present value, but for a long-term contract, as many different cash flow patterns can result in the same net present value, Gencos should be averse to scenarios in which some periods exhibits very bad outcomes compensated in the total net present value by other periods.

aversive attitudes: expected utility maximizers and CVaR based maximizer agents. A PLUF form was adopted for the former and a breath discussion for the Genco preference build throughout the most relevant financial company's parameters was provided. A PDUF representation for the CVaR coherent risk measure, widely used in risk management applications, was established in order to compare and point out differences between both approaches in a single basis.

The WtS curve has shown to be quite convenient for the Brazilian Gencos as a bidding rule in the annual EE forward contract auctions. Thus, a case study with realistic data has illustrated the assessment of such curve for a Brazilian hydro based Genco. Future developments concern the implementation of a Brazilian EE contract auction scheme to obtain results for both Gencos and regulator. By means of the presented methodology, the effect on the final tariff, traded energy and on the market health can be monitored for different Gencos risk attitudes, market rules and auctions parameter values.

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VIII. Biographies

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