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Pricing Flexible Natural Gas Supply Contracts Under Uncertainty in Hydrothermal Markets

Alexandre Street, Luiz Augusto Barroso, *Senior Member, IEEE*, Raphael Chabar, André T. S. Mendes, and Mario Veiga Pereira, *Member, IEEE*

***Abstract*—The worldwide development of the natural gas industry resulted in an integration process between electrical and gas sectors in several countries. In Brazil, this process has been taking place in a consistent manner, especially on account of the increase in gas consumption for industrial use and of the installation of thermoelectric plants. Due to the predominance of hydro plants in electric power generation, thermoelectric energy production is basically dependent on hydrology and, as a result, presents a wide annual variability. Consequently, the investment applied to gas production and transportation infrastructure may become under-utilized during a large part of the time; thus, it is important to find mechanisms apt to improve its utilization. In this respect, the present work investigates the creation of a flexible market for gas, where contracts for flexible gas supply would be offered to industrial users, who would receive the gas assigned to thermal power plants when the latter are not dispatched, and would resort to an alternate fuel when these plants are dispatched. The attractiveness of such a contract would depend, of course, on its price. The purpose of this work is to develop a stochastic model for pricing flexible gas supply contracts, taking into account the uncertainty associated to the supply—dependent on the dispatch of the thermal power plants, which have the priority of use of the gas—and the risk profile of potential consumers.**

***Index Terms*—Electricity-gas integration, natural gas, power system economics, risk management.**

I. Introduction

NATURAL gas (NG) is considered as one of the most promising sources to fulfill world energy demand, with a consumption expanding at a very accelerated pace. One of its major uses is as a source of industrial heat. The second largest use of NG is for electric power generation, which experienced a strong growth after the development of combined-cycle generation technology (CC-NG) in the 1980s. Besides being efficient, CC-NG is competitive in modules quite smaller than those of other technologies, such as that for coal. This has contributed to foster the implementation of power plants based on CC-NG in electricity markets worldwide, which has started the interdependency between the electricity and gas sectors.

In Brazil, the introduction of NG in the energy matrix took place in a more aggressive manner at the end of the 1990s, with the construction of the Bolivia–Brazil gas pipeline and the development of local gas production fields [1]. NG consumption for industrial and automotive use grew at quite significant rates and, in the electrical sector, the installation of gas-fired thermal generation also increased fast. These plants represent at present the second largest source in capacity (around 8000 MW installed) and the biggest potential market for the NG sector.

However, while the "nonpower" consumption of NG (for industrial, automotive, residential use, etc.) is practically constant (firm), gas consumption for thermal power plants is very variable. The reason for this is that the Brazilian electrical system is predominantly hydroelectric (85% of the generation capacity) and designed to fulfill the demand even if a severe drought occurs. This means that hydro plants are able, during most of the time, to produce energy above the "firm" level. The National Power System Operator, therefore, utilizes fully this additional hydro generation to reduce production of thermal power plants, which are then operated in the hydroelectric production complementation mode. This way, fossil fuels are saved[1] and the final consequence of such hydrothermal optimization is the cost reduction for the consumer.

An important consequence of this hydrothermal *coordination* is the great variability of thermoelectric energy produced each year, which may vary from zero—if the thermal plant is not operated—up to a base-load operation, in which the thermal plant operates at full load all year long. The electrical sector regulation requires that thermal power plants should have a gas supply and transportation capacity availability adequate for their dispatch at full load for the whole year (firm fuel supply agreement). This way, gas producers have to make an important investment in gas infrastructure, which may possibly remain idle during long periods. If, for instance, a gas thermal power plant is dispatched only during 40% of the time, gas infrastructure remains underutilized for 60% of the time. Thus, the attractiveness of operating flexibility of gas-fired plants conflicts with the need of investment recovery for the gas sector.

From the viewpoint of the gas producer, the natural way to stabilize the recovery of the investments made in production is the adoption of "take-or-pay" (ToP) clauses in gas supply contracts [2]. The ToP is an obligatory gas *commodity* purchase of x% of the gas supply agreement, which, for the Brazilian

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[1]In Brazil all generators are centrally dispatched by a National System Operator in order to minimize total costs.

thermal power plants using Bolivian gas, corresponds (at present) to 70%. In theory, the "prepaid" amount of gas from the ToP clause does not necessarily need to be immediately fired. Instead, since the ToP is a financial clause, its gas could be "virtually" stored for future use (make-up clause). In reality, however, what is seen is that gas-fired plants declare the ToP as a must-run (inflexible) generation. This forces the system operator to dispatch them in at least 70% of their capacities, using the "prepaid" gas. As a consequence, benefits from hydrothermal coordination are reduced. Furthermore, for gas transportation (pipelines) there is a similar fixed payment clause, known as "ship-or-pay" (SoP).[2] As a result, the ToP and SoP clauses, which are purely financial, guarantee a stable cash flow to the gas producer—to remunerate its investments in infrastructure—at the "cost" of deviating the hydrothermal coordination from its optimal generation scheduling (in the case of ToP clauses). Moreover, the fuel expenses related to unnecessary gas-fired plants inflexible dispatch will be reflected in the energy contracts prices paid by the distribution companies (Discos), who will, at the end, transfer them via electricity tariffs to the final consumers. In this sense, it is important to find solutions leading to a better utilization of gas production and transportation infrastructure, satisfying the needs of the gas producers and of the electricity sector.

An initial approach towards this improvement would be the creation of a flexible market for gas, making possible the resale of "prepaid" gas (ToP) not consumed by thermal plants, in a flexible operating strategy. In this market, industrial consumers would receive an offer of a contract modality, by which the industrialist could receive the gas assigned to thermal plants if the latter are not dispatched; in the opposite case, the industrialist could use an alternate fuel. The attractiveness of this contract would depend, of course, on its price. If implemented, the infrastructure costs could decrease substantially, since both consumption groups (industry and thermal plants) would be sharing the same gas production infrastructure.

The creation of flexible contracts for NG, where the consumption by the industry is reduced when the gas availability decreases due to the dispatch of the gas-fired plants, brings opportunities and challenges for its (industrial) buyers. The major challenge is how to evaluate the attractiveness of flexible contracts and their pricing. The willingness-to-pay for a flexible contract depends on the frequency, severity and economic cost of gas supply interruptions, which are a function of 1) the thermoelectric dispatch (stochastic variable); 2) the price of the alternate fuel, used whenever the gas supply is interrupted; and 3) the risk profile of the contract buyer, who may be more or less averse to the risk of being interrupted and having to use a more expensive alternate fuel.

Thus, the objective of this work is to develop a methodology for pricing flexible NG supply contracts, so as to determine the "value" of this contract according to the risk profile of its buyer. The approach adopted in this work is to construct a "willingness-to-contract" (demand) curve, for each consumer, that indicates the desired volume to be purchased in the flexible gas contracts market for each contract price offer. Once the curves for all consumers are calculated, a single-product auction can be easily simulated. This work describes the experiments carried out by the Brazilian gas authorities and consumers in order to assess the impacts of the creation of a flexible gas market.

The concept of flexible (or interruptible) consumption (FC) is not new in the electricity markets. [3] was one of the first works to propose a market for interruptible contracts for electric power, in which the consumer grants the power supplier the right to interrupt a given load unit in return for a price discount. Pricing of electricity flexible contracts is discussed in [4]–[6]. Flexible contracts and distributed generation (DG) can also be used by Discos to improve their market response capabilities, and change accordingly their passive positions in the markets.

A static single-period Disco energy acquisition market model with DGs and FC is presented in [7] under a market structure based on Pool and bilateral contracts. In [8] this model is extended to a multi-period energy acquisition model with multiple options including DGs, FC, and wholesale market purchase. Discos with DGs and FC can also use these instruments to increase their hedging strategies options to avoid supply shortage, which is discussed in [9]. Finally, flexible electricity contracts can be also used for operation planning purposes, as discussed in [10] and [11].

While a lot of work has been done on flexible contracts for electricity markets, we cannot say the same about the field of flexible contracts for the gas market, to which this work contributes. We highlight the work of [12] and [13], which optimizes short-term gas supply portfolios for electricity companies. This work follows the similar type of problem studied in [13], but focusing on the specific case of a gas consumer.

The sequence of this work is the following: Section II discusses the flexible gas contracts; Section III proposes a methodology for pricing these contracts; Section IV illustrates the application of the proposed methodology through a case study; finally, Sections V and VI present, respectively, the conclusions and suggestions for future work.

II. Natural Gas Flexible Supply Contracts

As previously mentioned, the fact that gas-fired power plants typically have fuel supply contracts with clauses that oblige a prepurchase of gas, suggests that this prepaid gas (when not used) could be "resold" to consumers willing to use it in a flexible manner during a prespecified period (for example, in three-year contracts). The buyer should opt for purchasing the flexible gas at a lower price, so that the risk of not receiving the product is compensated by the gain obtained through the difference between prices in the regular market (firm gas contracts) and in the flexible market.[3] This is the trade-off concept of a flexible contract.

In a general way, the flexible gas supply contract studied in this work is quite simple: in the electrical sector, thermal plants operate in a flexible way, and their NG "surplus" with respect to the ToP is sold to new industrial consumers via contracts establishing that:

[2]In Brazil the SoP is usually 95% of the gas supply agreement for power plants using Bolivian gas.

[3]The absence of spot market for natural gas is assumed, i.e., all gas purchases/sales are done through fixed price contracts. This is the Brazilian case.

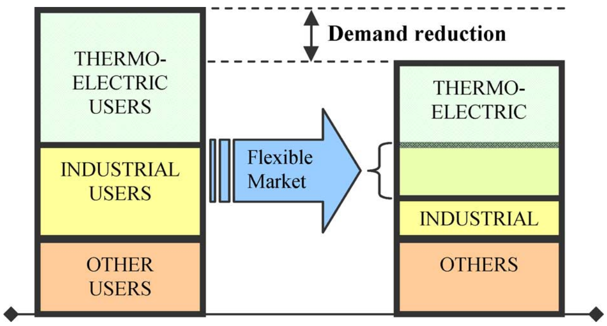

Fig. 1. Potential reduction of NG demand with the creation of the flexible market.

1) if the thermal plant is not dispatched, the gas supply for the flexible contract buyer occurs normally;
2) if the thermal plants are dispatched, the gas supply for the flexible contract is interrupted, and the buyer must rely on an alternate fuel at his own expense.

These contracts could be supported by the "prepaid" ToP of thermal power plants or by "additional" gas derived from seasonal variations in other sectors. Since industrial consumers usually have the capability of using alternate fuels—such as fuel oil, diesel oil or even wood—in their production processes, it would be possible to create a flexible market for NG consumption. In practical terms, the effect of industrial clients' migration to the flexible market is shown in Fig. 1, where an efficient way of sharing gas by two classes of users can be observed.

Obviously, other contract modalities could be established with additional flexibilities, such as contracts for liquefied natural gas, contracts where the consumer has the right to select the interruption timing and frequency, etc. This work deals with the simple flexible gas supply contract described before, but the methodology presented next may easily be adapted to other modalities.

A. Attractiveness of the Flexible Gas Contracts for Consumers

The attractiveness of the flexible gas supply contracts to its buyers depends on the probability of interruption of gas supply, on the price of the alternate fuel and on the risk profile of the consumer, who would be interested to receive a discount for the gas with an interruptible (flexible) supply when compared with the gas price of a firm contract. For instance, for a risk-adverse buyer, the value of a flexible contract that is interrupted 80% of the time is less than that of another contract with a lower probability of interruption, and both are cheaper than a firm contract.

Extrapolating this concept, gas could be sold at different reliability levels, following obviously an economic logic, and its allocation would be made in such a way as to benefit whoever pays more for it. The allocation process could take place through direct (bilateral) negotiation, or even through auctions, and the commercialization could be made by thermal power plants (this is not yet foreseen in the Brazilian regulations), by distribution companies or by gas producers.

Thus, the aim of the following sections is to develop a methodology for pricing NG supply flexible contracts, in order to determine the "value" of such contracts according to the consumer's risk profile. It is assumed that the flexible contracts allocation process takes place through a single-product (gas, at the same supply reliability level) auction involving different candidate buyers.

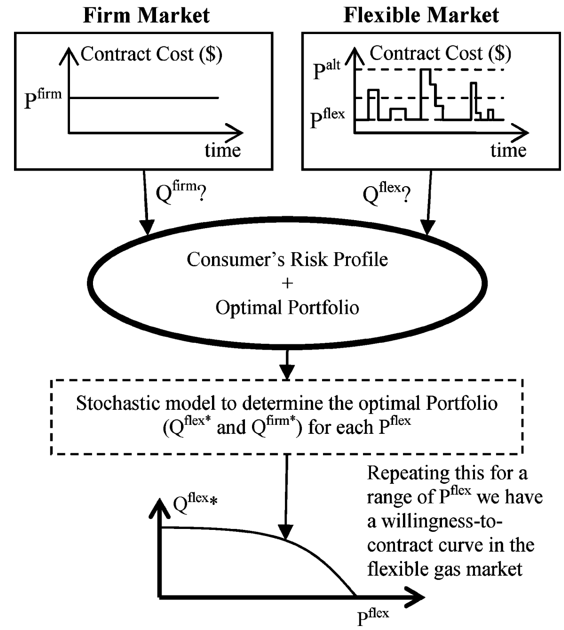

Fig. 2. Overview of the methodology for pricing flexible gas supply contracts.

III. Methodology and Stochastic Model for Pricing Flexible Gas Supply Contracts

The core of the methodology lies on a stochastic model which is able to compare gas contracts sold to consumers with different reliability levels. In other words, given the gas price for a firm contract (P^{firm}), the price for a flexible gas contract (P^{flex}) and an associated probability of interruption that results in the use of an alternate fuel with price (P^{alt}), the stochastic model determines the optimal gas volume ($Q^{\text{flex}*}, Q^{\text{firm}*}$) that should be bought of each contract type (firm – $Q^{\text{firm}*}$ – and flexible – $Q^{\text{flex}*}$) taking into account the consumers' risk profiles. By varying the price of the flexible contract, one can determine a curve that relates flexible gas price and volumes to be purchased, resulting in a "willingness-to-buy" curve.

Fig. 2 presents an outlook of the methodology developed in this work to price flexible gas supply contracts.

The details of the stochastic model will be described in the next subsections.

A. Calculation of Natural Gas Volume Available for the Flexible Market

As an initial step to apply the proposed methodology, it is necessary to estimate the volume of gas that would be available in the flexible market, as well as the probability and severity of the interruptions that occur when the thermal plants are dispatched. Since thermal plants dispatch is strongly influenced by hydrology, we use dispatch scenarios obtained from the simulation of the Brazilian long-term hydrothermal scheduling for a set of hydrological conditions (Monte Carlo simulation). Then, from the scenarios of thermal plants dispatch, the gas consumption for the power generation is calculated. By subtracting from the ToP volume the total gas actually consumed, we can obtain the gas availability for the flexible gas market for each hydrological scenario.

This process is divided into the following steps:

a) Construction of an electricity supply and demand scenario for the study horizon—in this step, a demand growth scenario is prepared, based on GDP growth hypotheses. Next, a generation expansion plan is adjusted to this scenario, taking into account the competitiveness of the available generation options (hydroelectric plants; gas, coal and other thermal power plants; international interconnections), the transmission limits and the characteristics of the Brazilian market.

b) Computational model for spot prices and generation forecast—following, a stochastic hydro-thermal dispatch model is used to calculate the optimal system operation policy for the preceding electric energy supply and demand scenario, taking into account the hydro-thermal system constraints and the inflows uncertainties [14]. The hydrothermal scheduling model is based on multistage stochastic Benders decomposition, whose methodology is described in [15].

c) Natural gas supply for the flexible market—the total amount of NG to be offered in the flexible market is calculated at each stage (month, trimester, semester, year etc) as being the total of all thermal plants volumes of NG from the ToP, i.e,

$$\mathrm{Supply}_t^{\mathrm{NG}} = \mathrm{ToP}_t^{\mathrm{total}} = \sum_{i \in UNG} \mathrm{ToP}_{i,t} \qquad (1)$$

where $\mathrm{ToP}_t^{\mathrm{Total}}$ is the total (potential) volume of NG to be offered in the flexible market (ToP of thermal plants in each stage); $\mathrm{ToP}_{i,t}$ represents the "take-or-pay" of gas contract of thermal plant i in stage t; and UNG is the group of NG-fired thermal plants (or units using NG).

In order to determine the percentage of NG actually available to the flexible market, for each stage t and hydrological scenario s, we have to subtract from the ToP volume of each NG power unit the amount of gas used on its dispatch. Because the actual gas consumption of a given unit can exceed its ToP amount, we truncate the negative values on zero, meaning that, whenever this event occurs, this unit will not contribute to the flexible market.

Thus, by summing the contribution of NG (volume) of each unit, the total amount of gas available for the flexible market consumption can be assessed. This is done in the following expression:

$$\mathrm{Av}_{t,s}{}^{\mathrm{NG}} = \sum_{i \in UNG} \max(0, \mathrm{ToP}_{i,t} - \mathrm{V}_{i,t,s}^{\mathrm{D}}). \qquad (2)$$

where $\mathrm{Av}_{t,s}^{\mathrm{NG}}$ is the NG availability at stage t and scenario s and $\mathrm{V}_{i,t,s}^{\mathrm{D}}$ is the volume of gas used on the dispatch of the NG thermal unit i dispatched at stage t and scenario s.

The gas availability for the flexible market ($\mathrm{X}_{t,s}$), expressed in percentage of the total ToP offered by thermal plants, is calculated dividing (2) by (1) at each stage and hydrological scenario

$$\mathrm{X}_{t,s} = \frac{\mathrm{Av}_{t,s}^{\mathrm{NG}}}{\mathrm{Supply}_t^{\mathrm{NG}}}. \qquad (3)$$

For example, if the total ToP ($\mathrm{Supply}_t^{\mathrm{NG}}$) for a one-month contract is 20 $\mathrm{MMm}^3/\mathrm{day}$ and the actual dispatch of thermal plants in the first hydrological scenario requires 8 $\mathrm{MMm}^3/\mathrm{day}$, the gas availability for the flexible contracts in this hydrological scenario ($\mathrm{Av}_{t,1}^{\mathrm{NG}}$) is $20 - 8 = 12$ $\mathrm{MMm}^3/\mathrm{day}$, or, expressing it equivalently in percentage terms, $\mathrm{X}_{t,1} = 12/20 = 60\%$ of the potential volume. Observe that, according to (3), we consider that every interruption is shared equally among all flexible contracts.

B. Expenditure With a Flexible Gas Supply Contract

Once defined the percentage of gas actually available for the flexible market, the gas consumer expenditure $\mathrm{E}_{t,s}$, relative to its flexible and firm contract portfolio in this stage t and hydrological scenario s, may be defined as

$$\mathrm{E}_{t,s} = (1-\mathrm{Q}^{\mathrm{flex}})\mathrm{P}^{\mathrm{firm}} + \mathrm{Q}^{\mathrm{flex}}[\mathrm{X}_{t,s}\mathrm{P}^{\mathrm{flex}} + (1-\mathrm{X}_{t,s})\mathrm{P}^{\mathrm{alt}}] \qquad (4)$$

where $\mathrm{Q}^{\mathrm{flex}}$ is the quantity contracted in the flexible market (in % of the consumer's demand); the portion $(1 - \mathrm{Q}^{\mathrm{flex}})$ defines the percentage contracted in the firm market (in % of the consumer's demand); $\mathrm{P}^{\mathrm{firm}}$ is the price of the firm gas contract, $\mathrm{P}^{\mathrm{flex}}$ is the price of the flexible gas contract and $\mathrm{P}^{\mathrm{alt}}$ is the price of the alternate fuel. All prices are defined in US$/MMBTU.

In other words, the expenditure is equal to the expense with the firm consumption plus that for the consumption of gas contracted under flexible conditions, which depends on the probability of dispatch of the thermal plants, and on the prices of the flexible gas contract and of the alternate fuel.

C. Willingness-to-Contract Curve (WCC)

The main purpose of the methodology of this work is to define a model for determining the optimal price-dependent quantity of flexible NG to be contracted $\mathrm{Q}^{\mathrm{flex}*}(\mathrm{P}^{\mathrm{flex}})$ that minimizes the expected consumer expenditure (4) subject to a risk constraint. By calculating $\mathrm{Q}^{\mathrm{flex}*}$ for several discrete values of $\mathrm{P}^{\mathrm{flex}}$, we can obtain a demand curve that reflects the consumer's willingness-to-contract (WCC) the gas supply contract for the given price and reliability level. This is the main concept of this work.

D. Risk-Aversion

The consideration of the risk profile of each consumer, when building his WCC, is extremely important in order to reflect his behavior towards the presence of uncertain gas availability. For the purpose of our work, risk is defined as the probability of purchasing an alternate fuel at a higher price, which happens when the gas supply is interrupted. Risk-aversion is related to the behavior of consumers under this uncertainty.

Because the interruption is probabilistic, we assume that each consumer will define his own reliability level in calculating his willingness-to-contract. We translate this into a maximum financial expenditure considering a risk level.

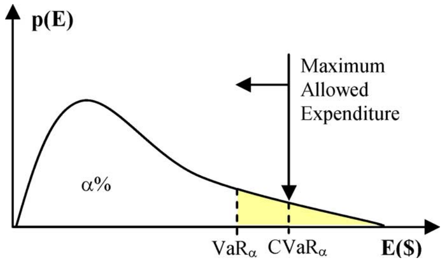

Fig. 3. Expenditure CVaR constraint.

For example, in order to minimize the total expected gas purchase cost a consumer may contract part of his gas demand on the flexible gas market. He can specify a contracting strategy to ensure that there is a probability greater than 80% to have a total gas purchase cost lower than 135% of the cost that he would have incurred if he were integrally in the firm market. In contrast, a more risk-averse consumer may define for the same supply reliability a more tight financial constraint, demanding the whole portfolio expenditure to be at most 130% of the firm market reference cost.

Obviously, for a risk-averse agent who minimizes expected costs, the willingness-to-contract a flexible gas supply will only make sense when the average cost in the flexible market[4] is lower than in the firm market—otherwise, the firm market option dominates the flexible supply option (no risk). The contracting strategy will be constructed in order to take advantage of the flexible market opportunity, but hedging against its volatility with some consumption in the firm market.

Several measures may be applied to model risk-aversion, such as the expected utility, downside risk, value-at-risk (VaR), etc. In this work, we have opted for the conditional-value-at-risk – CVaR [16]–[18] as a risk control measure. This metric has been widely used as a supporting tool for decisions in many risk management problems [19]–[21]. It is also easy to understand, since it represents the expected loss at the worst $\mathrm{S}(1-\alpha)$ expenditure scenarios (S is the number of scenarios and α is the reliability level); and to implement, once it can be written as an expected value optimization problem with linear constraints, which can be implemented and solved as a classical linear programming problem.

Fig. 3 shows a one-period expenditure α-CVaR constraint and its relation to α-VaR.

E. Risk-Constrained WCC for Flexible Gas Supply Contracts

In this way, the WCC of a flexible contract consumer may be obtained by solving the following stochastic linear programming problem, in which all decisions variables are highlighted in italic

$$\mathrm{Q}^{\mathrm{flex}*}(\mathrm{P}^{\mathrm{flex}}) = \arg\,\mathrm{Min}\left(\frac{1}{\mathrm{S}}\right)\sum_{\mathrm{t,s}} E_{\mathrm{t,s}}(1+\mathrm{J})^{-\mathrm{t}} \quad (5)$$

Subject to:

$$E_{\mathrm{t,s}} = (1-Q^{\mathrm{flex}})\mathrm{P}^{\mathrm{firm}} + Q^{\mathrm{flex}}[\mathrm{X}_{\mathrm{t,s}}\mathrm{P}^{\mathrm{flex}} + (1-\mathrm{X}_{\mathrm{t,s}})\mathrm{P}^{\mathrm{alt}}], \forall \mathrm{t,s} \quad (6)$$

$$\theta_{\mathrm{t,s}} \geq E_{\mathrm{t,s}} - \lambda_{\mathrm{t}}, \quad \forall \mathrm{t,s} \quad (7)$$

$$\theta_{\mathrm{t,s}} \geq 0, \quad \forall \mathrm{t,s} \quad (8)$$

$$\lambda_{\mathrm{t}} + \sum_{\mathrm{s}} \frac{\theta_{\mathrm{t,s}}}{[\mathrm{S}(1-\alpha)]} \leq (1+\delta)\mathrm{P}^{\mathrm{firm}}, \quad \forall \mathrm{t} \quad (9)$$

$$0 \leq Q^{\mathrm{flex}} \leq 1$$

$$\text{where } \mathrm{t}\in\{1,\ldots,\mathrm{T}\},\ \mathrm{s}\in\{1,\ldots,\mathrm{S}\}. \quad (10)$$

[4]Weighted average of the purchases in the flexible market, when it is available, and the price of the alternate fuel when it is not.

In the problem (5)–(10), J (in % per period) represents the consumer's capital cost and δ represents the percentage of maximum "additional cost" (with respect to the firm gas contract expenses) that the consumer is willing to pay for the average of the $\mathrm{S}(1-\alpha)$ worst scenarios, as illustrated in Section III-D. In other words, we represent that the consumer would be willing to pay up to $(1+\delta)100\%$ of the firm contract's cost at every stage for the average of the worst scenarios. The CVaR constraint ensures this limit assuming that the expectation over the $\mathrm{S}(1-\alpha)$ worst scenarios is a good proxy for the worst case expenditure. Auxiliary variables such as λ_{t} and $\theta_{\mathrm{t,s}}$ are used to implement the CVaR risk-criterion: they are auxiliary variables used to assess, for each period, the VaR in the optimal solution and a positive truncate function, both needed in the CVaR formulation (for more details see [16] and [21] for an energy application); $1-\alpha$ is the significance level of the CVaR (defined for each consumer) and represents the percentage of the worst scenarios of expenditures whose average must be lower than a given threshold $(1+\delta)\mathrm{P}^{\mathrm{firm}}$; S is the number of hydrological scenarios simulated, and T represents the duration of the contract being analyzed.

The objective function of this model (5) represents the minimization of the consumer's total gas expected expenditure, which is computed in (6). Equations (7) and (8) represent a two-segment piecewise linear function, which computes in $\theta_{\mathrm{t,s}}$ the cost violations for the scenarios whose expenditure exceed the λ_{t} threshold in every period. As shown in [16] for a convex optimization problem, this threshold will reach, in the optimum solution, the α-value-at-risk and thus, (9) will provide a bound for the conditioned expected VaR violations (which is the CVaR) based on a percentage over the firm cost. Finally, (10) represents the flexible quantity bounds, which limit the purchases in this market to be at most 100% of the consumer's gas needs. Problem (5)–(10) is a stochastic linear programming problem that can be solved by commercially available packages such as Xpress-MP or CPLEX.

IV. Case Study

In this section, the developed methodology will be applied to a realistic case study for the Brazilian electricity-gas system, where the objective is to investigate the attractiveness of a flexible gas market to industrial consumers. It has been assumed that the flexible contract has a three-year duration, and that it would be available starting in 2010.

A. System Characteristics

Since this market does not exist in the country, as a starting point, the profile of flexible gas potential buyers has been assumed. The potential size of this market is still unknown (recent studies carried out in Brazil [22] indicate that at least 15% of the gas consumed at present by the industrial sector could already

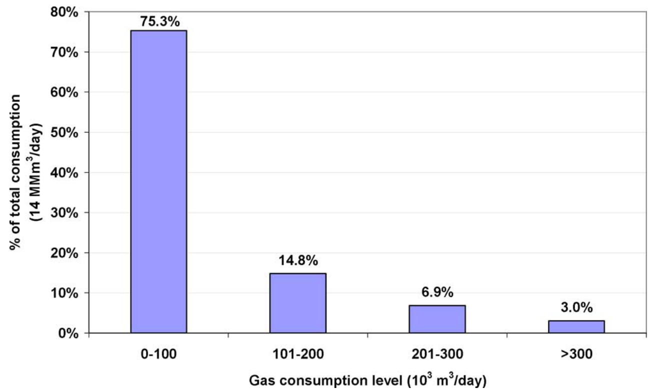

Fig. 4. Profile of flexible gas potential buyers.

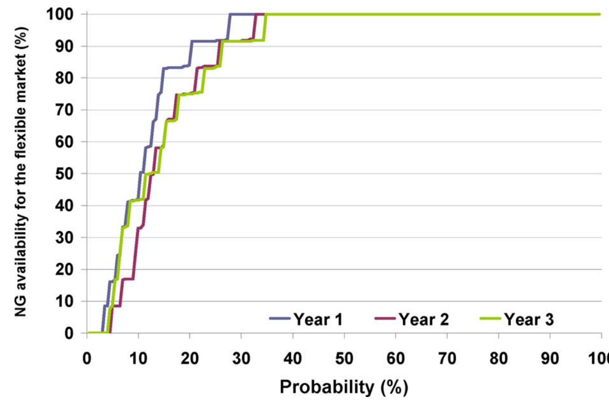

Fig. 5. Empirical inverse accumulated probability distribution function of NG availability (per year).

TABLE I
CONSUMER PROFILES

Cons.	Class Consumption range (10^3 m³/day)	Total Class Consumption (10^6 m³/day)	δ (%)	α (%)
1	0 – 100	10.66	30	80
2	101 – 200	2.09	35	80
3	201 – 300	0.97	40	80
4	> 300	0.43	45	80

have a flexible supply). Thus, in a simplified manner, we considered as probable buyers the industrial clients of the Southeast region using NG in their activities.

Even though the sets of data represent several types and profiles of industrial consumers (steel, glass, aluminum and other sectors), in order to simplify and to aggregate the profiles, these data have been interpreted as if they came from four distinct consumers, having their profiles defined according to the consumed volume, as shown in Fig. 4.

B. Consumers Risk Profiles

Based on a poll of consumers, Table I presents the consumer classes and their profiles. Consumers with a lower daily consumption usually have greater risk aversion than large consumers. For instance, type "1" consumers have $\delta = 30\%$, indicating that they accept to pay, in the worst case (represented by the conditional expected value of the $1 - \alpha\%$ worst scenarios), 30% more than if they were fully in the firm market. On the other hand, type "4" consumers, with higher gas consumption, accept to stay in the flexible market paying up to 45% more than they would do for the firm gas contract, for the reliability level they have chosen, discussed next.

For the sake of simplicity, in this case study the second risk aversion parameter (α) was fixed at 80% for all consumers, indicating that every agent will use the same reliability level to measure risk. Consumer's risk profiles will only differ in the level of extra cost they are willing to pay (δ parameter).

The price of gas in the firm market was assumed to be 4 US$/MMBTU and the price for the alternate fuel (used when gas supply is interrupted) was defined at 8 US$/MMBTU (price of fuel oil at the time of this study) for all consumers. As the firm market is the reference and our objective function is to minimize the expected value of the gas contracts costs, whenever the average cost in the flexible market (composed of the convex combination between the flexible and alternate prices) exceeds the cost of the firm market, the model will choose to be integrally in the latter (firm). Therefore, in the cases where the consumer starts to compose a contract portfolio between the firm and flexible markets, a reduction in the average expenditure will take place, in exchange for an increase in volatility. The reason is simple: the flexible gas market will have a lower average price than the firm market, but in the scenarios where consumption is interrupted, the use of a more costly fuel becomes necessary. Thus, the trade-off relies on the replacement of a constant expenditure by an uncertain one, with a lower average.

C. Procedure

Next, the procedure described in Section III was applied. An electricity supply x demand configuration was defined, and the hydrothermal dispatch of the system was simulated in a monthly basis for the period 2010–2012, thus encompassing the contract period (2010–2012). The official system configuration used for January 2007 power system's operations planning purposes was used and the hydrothermal scheduling was carried out for a sample of 200 hydrological scenarios. Following the procedure described in Section III-A, Fig. 5 shows the accumulated probability distribution curve of NG availability for the flexible gas market in each of the three years simulated.

It can be observed that the probability distribution curves are very skewed. For all years, in 65% of the scenarios, gas-fired plants are not dispatched at all, i.e., in 65% of the scenarios, 100% of the NG initially offered (Supply_t^{GN}) would be available for the flexible market consumers. In other words, in these scenarios, the flexible market consumers would have an integral NG supply without interruptions, so that it would not be necessary to use the alternate fuel. From the same chart, it may be also concluded that: the risk (or probability) of an interruption of more than 50% of the supply is lower than 15% for all years, which demonstrates a non-negligible opportunity for consumers in a possible future NG supply flexible market.

D. Results

Next, the scheme described in Fig. 2 and in Section III-C was applied to obtain the WCC of the consumption class for several

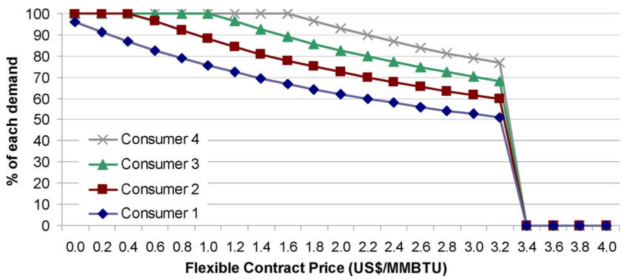

Fig. 6. Willingness-to-contract curve for each consumer class in the flexible market (the complement would be contracted in the firm market).

TABLE II
OPTIMAL STRATEGY FOR A PRICE OF 1.6 US$/MMBTU

Consumer	(1+δ)	α	Q^{flex} (%)	Q^{firm} (%)
1	130%	80%	67	33
2	135%	80%	78	22
3	140%	80%	89	11
4	145%	80%	100	0

NG flexible contracts price levels, i.e., the stochastic linear programming problem (5)–(10) was solved for 20 different flexible prices (P^{flex}) from 0 to $P^{firm} = 4$ US$/MMBTU. Xpress-MP v2006B solver, from Dash Optimization, has been applied to solve this problem.

The result of the individual WCC of each consumption class is shown in Fig. 6.

It may be noted that the consumers contracting strategy is sensitive to the risk profile, and the attractiveness of the flexible market varies according to the possible price of this product.

For instance, Fig. 6 shows that up to a price of US$ 1.6/MMBTU, consumer 4 is interested in buying 100% of his gas demand in the flexible market. For prices higher than this value, consumer 4 starts to have a portfolio of firm and flexible contracts. Type "1" consumer, the most risk averse, starts to hedge against the use of alternate fuel even for a flexible contract price of zero, maintaining 4% of its portfolio in the firm market (first point of consumer 1 in Fig. 6). This behavior is due to the risk of having to buy the alternate fuel. Finally, from US$ 3.4/MMBTU and up, there is no more interest for this market (the price is not low enough to compensate the interruptions, providing an expected price that is greater than the firm price).

Table II presents the quantities contracted in the flexible (Q^{flex}) and firm ($1 - Q^{flex}$) markets, at a price of 1.6 US$/MMBTU for the three-year flexible gas contract. It is verified that the consumers' contracting strategy varies according to their risk profile.

In summary, the example shows how a consumer can price these contracts considering their uncertainty of the interruption and the consumer's individual risk profile.

E. Simulation of an Auction Mechanism to Allocate the Flexible Contracts Among Consumers

Once established the methodology to determine each consumers' willingness-to-contract for each price level, it is now

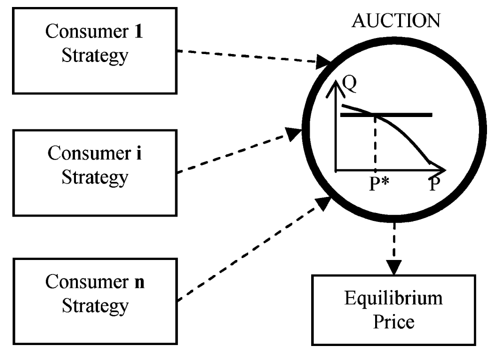

Fig. 7. Single product uniform auction.

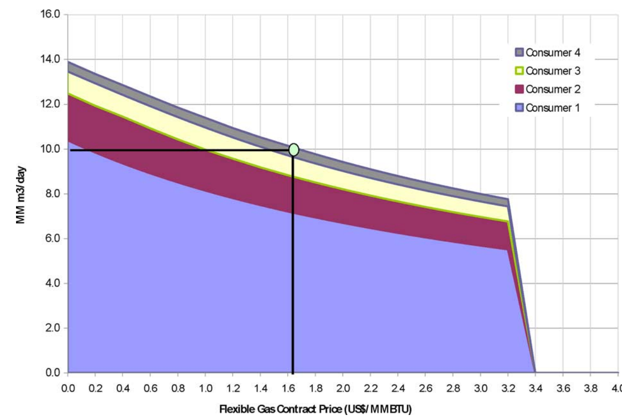

Fig. 8. Accumulated demand curve.

possible to simulate an allocation mechanism for the flexible gas contracts. A usual mechanism is an auction [23].

For the case of iterative auctions, such as the descending price clock auctions, it suffices to implement the auction rules and procedures and, for each auction round, allocate the flexible gas contracts to consumers that pay more for it.

Alternatively, for the single-product case, the auction result may be obtained by the simple aggregate composition of the individual WCC curves and the determination of the single price, such that supply and demand are matched, as described in Fig. 7.

Fig. 8 shows the aggregated WCC (for all consumers) for each possible price at the auction. This curve is obtained by multiplying the data in Fig. 6 by each consumer's demand.

Finally, it is sufficient to find the price that makes the supply equal to the demand. For example, if the total to be offered at the auction is 10 $\mathrm{MMm^3/day}$, the auction balance price would be US$ 1.6/MMBTU (applying the supply to the aggregated WCC), as illustrated in Fig. 8.

V. CONCLUSIONS

Differently from many countries, the NG sector in Brazil is newborn and several challenges are arising due to its creation and to the complexity of integrating gas-fired plants in a hydroelectric system.

In this sense, this work investigates the creation of a flexible gas market in Brazil, in which flexible contracts for gas supply would be offered to new industrial users, who could receive the gas allocated to thermal power plants if the latter were not dispatched, and could use an alternate fuel or even have their supply interrupted in the opposite case. The creation of this market would lead to a better utilization of the gas production and transportation infrastructure, preventing or postponing investments on gas supply enlargement and on systems interconnections. This would be attractive for the flexible dispatch of thermal power plants, leading to a better use of the country's hydroelectric resources.

As it has been shown, the attractiveness of such a contract would depend, of course, on its price, and the purpose of this work was to develop a methodology for pricing gas flexible contracts, taking into account the uncertainty associated to the supply (which depends on the priority of the use of gas by thermal plants) and the risk profile of these consumers. The proposed methodology allows the evaluation of the attractiveness of the contracts in bilateral negotiations or through auctions, and was illustrated by actual examples.

Overall, the main contribution of this work is the development of a stochastic pricing model for flexible gas contracts with risk constraints. The framework presented here has been used by gas consumers in Brazil to verify the attractiveness to implement such a market.

VI. Future Work

Several important subjects have not been discussed in this work, as for instance: the migration of existing consumers in case they would have access to these contracts; who would perform the sale of the contracts; the property of the income from the contracts' sale, and others. With regard to the model, other subjects for analysis would be: 1) representation of gas transportation constraints (how to make feasible the creation of a NG flexible market, with distribution companies or sellers located in different regions and not physically interconnected, meaning that NG is not always available for delivery at the site of the consumer disposed to consume it); 2) evaluation of the model behavior when submitted to other risk aversion measurements. Another interesting simulation would be to associate the cost of the alternate fuel to the size or consumption profile of the consumers; it may be assumed that a larger-sized consumer is able to negotiate a back-up contract under better conditions than those for a smaller-sized consumer, and, on the other hand, for small consumers (the major type—Fig. 4) it would be interesting to model the alternate fuel uncertainty for the case in which consumers are exposed to alternate fuel spot prices. Finally, a major extension would be to estimate the potential economical benefits for implementing this flexible gas supply market, considering all the costs incurred in this process, such as cost of building additional pipeline and to transport the NG to potential costumers, and the expenses for organizing an auction market.

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