DEPARTAMENTO DE ENGENHARIA ELÉTRICA

Internal Research Reports

ISSN

Number 13 | December 2010

Contingency-Constrained Unit Commitment with n–K Security Criterion: A Robust Optimization Approach

Alexandre Street de Aguiar

Fabrício Oliveira

José M. Arroyo

PUC RIO

Internal Research Reports

Number 13 | December 2010

Contingency-Constrained Unit Commitment with n-K Security Criterion: A Robust Optimization Approach

Alexandre Street de Aguiar

Fabrício Oliveira

José M. Arroyo

CREDITS

Publisher:
MAXWELL / LAMBDA-DEE
Sistema Maxwell / Laboratório de Automação de Museus, Bibliotecas Digitais e Arquivos
http://www.maxwell.vrac.puc-rio.br/

Organizers:
Alexandre Street de Aguiar
Delberis Araújo Lima

Cover:
Ana Cristina Costa Ribeiro

Contingency-Constrained Unit Commitment with $n-K$ Security Criterion: A Robust Optimization Approach

Alexandre Street, *Member, IEEE*, Fabrício Oliveira, and José M. Arroyo, *Senior Member, IEEE*

***Abstract*—This paper presents a new approach for the contingency-constrained single-bus unit commitment problem. The proposed model explicitly incorporates an $n-K$ security criterion by which power balance is guaranteed under any contingency state comprising the simultaneous loss of up to K generation units. Instead of considering all possible contingency states, which would render the problem intractable, a novel method based on robust optimization is proposed. Using the notion of umbrella contingency, the robust counterpart of the original problem is formulated. The resulting model is a particular instance of bilevel programming which is solved by its transformation to an equivalent single-level mixed-integer programming problem. Unlike previously reported contingency-dependent approaches, the robust model does not depend on the size of the set of credible contingencies, thus providing a computationally efficient framework. Simulation results back up these conclusions.**

***Index Terms*—Bilevel Programming, Contingency-Constrained Unit Commitment, $n-K$ Security Criterion, Robust Optimization, Umbrella Contingency.**

I. Nomenclature

A. Functions

$C_{it}^{P}(\cdot)$ Cost function offered by generator i in period t to produce $p_i(t)$ in the pre-contingency state.

B. Constants

$A_i^k(t)$ Availability parameter that is equal to 0 if generator i is unavailable in period t under contingency state k, being 1 otherwise.

$C_i^f(t)$ Fixed production cost coefficient offered by generator i in period t.

$C_i^l(t)$ Linear production cost coefficient offered by generator i in period t.

$C_i^{NS}(t)$ Cost rate offered by generator i to provide non-spinning reserve in period t.

$C_i^S(t)$ Cost rate offered by generator i to provide up-spinning reserve in period t.

$D(t)$ System demand in period t.

K Number of unavailable generators (security criterion parameter).

n Number of generators.

n_H Number of periods.

$\overline{P}_i$ Capacity of generator i.

$\underline{P}_i$ Minimum power output of generator i.

$\overline{R}_i^{NS}$ Upper bound for the non-spinning reserve contribution of generator i.

$\overline{R}_i^{S}$ Upper bound for the up-spinning reserve contribution of generator i.

RD_i Ramp-down limit of generator i.

RU_i Ramp-up limit of generator i.

SD_i Shutdown ramp limit of generator i.

SU_i Startup ramp limit of generator i.

C. Decision Variables

$a_i(t)$ Binary variable that is equal to 0 if generator i is unavailable in the worst-contingency state of period t, being 1 otherwise.

$d^{wc*}(t)$ Maximum power that can be supplied under the worst-case contingency for a given schedule in period t.

$p_i(t)$ Power output of generator i in period t in the pre-contingency state.

$p_i^k(t)$ Power output of generator i in period t under contingency k.

$r_i^{NS}(t)$ Non-spinning reserve provided by generator i in period t.

$r_i^S(t)$ Up-spinning reserve provided by generator i in period t.

$v_i(t)$ Binary variable that is equal to 1 if generator i is scheduled in period t in the pre-contingency state, being 0 otherwise.

$v_i^k(t)$ Binary variable that is equal to 1 if generator i is scheduled in period t under contingency k, being 0 otherwise.

The work by J. M. Arroyo is partly supported by the Ministry of Science of Spain under CICYT Project ENE2009-07836, and by the Junta de Comunidades de Castilla-La Mancha under Project PAI08-0077-6243.

A. Street is with the Electrical Engineering Department, Pontifical Catholic University of Rio de Janeiro, Rio de Janeiro, RJ, Brazil (e-mail: street@ele.puc-rio.br).

F. Oliveira is with the Industrial Engineering Department, Pontifical Catholic University of Rio de Janeiro, Rio de Janeiro, RJ, Brazil (e-mail: fabricio.carlos@gmail.com).

J. M. Arroyo is with the Departamento de Ingeniería Eléctrica, Electrónica, Automática y Comunicaciones, E.T.S.I. Industriales, Universidad de Castilla-La Mancha, Ciudad Real, E-13071 Spain (e-mail: JoseManuel.Arroyo@uclm.es).

$v_i^{NS}(t)$ Binary variable that is equal to 1 if generator i provides non-spinning reserve in period t, being 0 otherwise.

$y(t)$ Dual variable of the $n-K$ security constraint at the lower-level optimization problem for period t.

$z_i(t)$ Dual variable of the upper bound for generator i availability at the lower-level optimization problem for period t.

D. Sets

$\mathcal{C}$ Set of contingency indexes.

H Set of period indexes.

N Set of generator indexes.

ϑ_i Minimum up and down time feasible set for the pre-contingency scheduling variables of generator i.

E. Vectors

$\boldsymbol{a}(t)$ Generator availability statuses in period t under the worst-case contingency.

$\boldsymbol{A}^k(t)$ Generator availability statuses in period t under contingency k.

$\boldsymbol{v_i}$ On/off statuses of generator i.

II. Introduction

CURRENT reliability policy and associated security standards in power systems worldwide mainly focus on events such as the random outage of a single or at most two transmission or generation assets. These standards materialize in the well-known $n-1$ and $n-2$ security criteria which in industry practice are implemented as deterministic approaches [1][2]. Hence, power systems are neither operated nor planned to withstand contingencies comprising multiple simultaneous outages. However, the adequacy of such reliability framework is questionable, as revealed by recent blackouts caused by the coincidence in time of several independent system component outages [3]. As a consequence, researchers have begun to analyze the impact of multiple contingencies, particularly in power system planning [4]-[8].

Within this context, this paper extends the consideration of multiple contingencies to the unit commitment problem. Unit commitment plays a central role in power system operation under both centralized and competitive frameworks [1][9] and, therefore, security is an issue of major concern. Security has been incorporated in the unit commitment through the definition of several types of reserves by which preventive and corrective actions can be implemented in order to handle outages [10][11].

From a deterministic viewpoint, reserves have been traditionally modeled in the generation scheduling by imposing different types of pre-specified requirements [12][13]. The main drawback of these approaches is their dependence on an a priori determination of system-wide or local reserve requirements that may lead to suboptimal or even infeasible solutions once contingencies occur.

In contrast to reserve-constrained unit commitment, contingency-constrained unit commitment (CCUC) [10][11][14]-[18] explicitly imposes power balance under both normal and contingency states. In [10], contingencies were accounted for by a set of credible generator and line outages, and a joint power and reserve scheduling model was presented in a one-period setting. In the same work, a reserve pricing scheme was proposed based on the Lagrange multipliers of the nodal power balance equations. Joint market models for energy and several types of reserves were also proposed in [14] and [11]. Reference [14] focused on the beneficial impact of demand-side bidding under $n-1$ and $n-2$ security criteria in a single-bus model. Reference [11] presented the concept of security price in a network-constrained system from both deterministic and stochastic viewpoints. In [15], uncertainty management in the unit commitment problem was reviewed and the tradeoff between system reliability and total cost was studied for a set of CCUC models with an $n-1$ security criterion. In [16], a Benders decomposition approach was proposed for a contingency-constrained model of a joint energy and ancillary services auction. Finally, the impact of transmission switching on contingency-dependent scheduling problems was analyzed in [17] for a single period, and in [18] for a multiperiod setting.

Current computing capabilities may allow incorporating $n-1$ and $n-2$ security criteria in the CCUC models presented in [10][11][14]-[18] for practical power systems. However, the extension to tighter security levels would lead to intractability due to the huge number of contingency states that should be considered. As a consequence, CCUC models only examine a limited set of credible contingencies, which is determined based on experience and engineering judgment.

This paper presents an alternative approach that efficiently incorporates a deterministic $n-K$ security criterion into the CCUC problem. This model is hereinafter referred to as $n-K$ CCUC. Unlike previously reported CCUC models [10][11][14]-[18] relying on a reduced set of credible contingencies, we propose a joint energy and reserve dispatch model based on robust optimization that allows considering all combinations of at most K unit outages, i.e., $\sum_{i=1}^{K}\binom{n}{i}$, in a computationally efficient manner.

Based on the widely used definition of the unit commitment problem [1][12][13][19], we make use of a single-bus model, recognizing that the use of such a simplified model leads to results that may be optimistic and that a complete study of the $n-K$ CCUC should also consider the effects of branch contingencies, reactive power support, transmission network loading, and even protection schemes. This generalization would, however, render the problem essentially intractable through optimization and would have to be solved by repeated simulations. These modeling limitations notwithstanding, the solution of the $n-K$ CCUC based on a single-bus model provides the system operator with a valuable scheduling tool accounting for security.

Robust optimization is an appropriate framework to model optimization problems where the optimal solution must remain feasible for some parameter variations in a given user-defined set (also called "uncertainty set"). In this framework, the unknown parameters (uncertainty) are treated as worst-case deterministic functions of the decision variables, which are set to perform the worst "feasibility damage" in the model for each proposed solution. Robust optimization was first introduced in the early 1970's by Soyster [20] for linear programming problems. Such approach has been criticized due to the over-conservativism of the solutions provided. In the late 1990's and early 2000's, robust optimization was further developed by Ben-Tal and Nemirovski's works [21][22]. These works proposed an ellipsoidal uncertainty set that allowed the user to define a much more accurate description of the uncertainty factors through robust counterparts. However, even in the case of linear models, the associated robust counterparts were nonlinear.

Recent theoretical advances by Bertsimas and Sim [23] allow an easy control of the degree of conservativism with moderate computational effort by controlling the number of coefficients that may change in each constraint of the problem. The main advantage of this technique is that robust counterparts do not increase in complexity compared to their original formulation. Such work has paved the way for an enormous number of applications in the optimization field due to its intuitive interpretation, easy implementation, and independence of any subjective probability specification process (see [24] for many well-known problem reformulations and references therein). Some examples of successful application of robust optimization in power systems can be found in [25][26].

The $n-K$ CCUC model presented here belongs to the class of problems suitable for robust optimization, where the parameters allowed to vary represent the generation unit availability under the contingency states. The robust counterpart of the original $n-K$ CCUC problem is first constructed. This problem is modeled as a worst-case bilevel programming problem [27] wherein contingency states are characterized as decision variables. Using recent findings from robust optimization [23][24], the $n-K$ CCUC bilevel program is subsequently transformed into an equivalent single-level mixed-integer programming (MIP) problem. The main advantage of the proposed solution is that the dimension of the resulting MIP problem does not depend on the security level defined by parameter K, thereby allowing an efficient solution by off-the-shell software [28].

The main contributions of this paper are:

1. Within the framework of the conventional single-bus unit commitment problem, the system operator is provided with a generation scheduling tool that accounts for tighter security levels than the traditional $n-1$ and $n-2$ security criteria.
2. A novel $n-K$ CCUC model is formulated and implemented based on robust optimization. This methodology is effective in attaining globally optimal or near-optimal solutions with moderate computational effort.
3. The performance of the proposed approach is successfully validated with numerical simulations.

The remainder of this paper is organized as follows. In Section III, the formulation of the $n-K$ CCUC problem is presented. Section IV describes the robust optimization approach. Section V provides and discusses results from several case studies. Finally, some relevant conclusions are drawn in Section VI.

III. Problem Formulation

The contingency-constrained unit commitment problem determines the optimal generation schedule and reserve allocation so that the power demand is supplied under both normal and contingency states over a specific short-term time span. Here we propose the explicit consideration of an $n-K$ security criterion by which all combinations of up to K unit outages are modeled in each period. For unit consistency, it should be noted that hourly time periods are considered.

Based on the models presented in [10][14][19], the $n-K$ CCUC problem can be formulated as:

$$\underset{\substack{p_i(t),p_i^k(t),\\ r_i^{NS}(t),r_i^S(t),\\ v_i(t),v_i^k(t),v_i^{NS}(t)}}{\text{Minimize}} \sum_{t\in H}\sum_{i\in N}\Big[C_{it}^P\big(p_i(t),v_i(t)\big)+\ C_i^S(t)r_i^S(t) + C_i^{NS}(t)r_i^{NS}(t)\Big] \quad (1)$$

subject to:

$$\sum_{i\in N} p_i(t) = D(t);\quad \forall\, t\in H \quad (2)$$

$$\sum_{i\in N} p_i^k(t) = D(t);\quad \forall\, k\in\mathcal{C},\forall\, t\in H \quad (3)$$

$$\underline{P}_i v_i(t) \le p_i(t) \le \overline{P}_i v_i(t);\quad \forall\, i\in N,\forall\, t\in H \quad (4)$$

$$\underline{P}_i A_i^k(t) v_i^k(t) \le p_i^k(t) \le \overline{P}_i A_i^k(t) v_i^k(t);\quad \forall\, i\in N,\forall\, k\in\mathcal{C},\forall\, t\in H \quad (5)$$

$$p_i^k(t) \le A_i^k(t)[p_i(t)+r_i^S(t)+r_i^{NS}(t)];\quad \forall\, i\in N,\forall\, k\in\mathcal{C},\forall\, t\in H \quad (6)$$

$$p_i(t)+r_i^S(t) \le \overline{P}_i v_i(t);\quad \forall\, i\in N,\forall\, t\in H \quad (7)$$

$$0\le r_i^S(t) \le \overline{R}_i^S v_i(t);\quad \forall\, i\in N,\forall\, t\in H \quad (8)$$

$$0\le r_i^{NS}(t) \le \overline{R}_i^{NS} v_i^{NS}(t);\quad \forall\, i\in N,\forall\, t\in H \quad (9)$$

$$v_i(t)+v_i^{NS}(t)\le 1;\quad \forall\, i\in N,\forall\, t\in H \quad (10)$$

$$A_i^k(t)v_i(t) \le v_i^k(t) \le A_i^k(t)[v_i(t)+v_i^{NS}(t)];\quad \forall\, i\in N,\forall\, k\in\mathcal{C},\forall\, t\in H \quad (11)$$

$$p_i(t-1) \le p_i(t) + RD_i v_i(t) + SD_i[v_i(t-1)-v_i(t)] + \overline{P}_i[1-v_i(t-1)];\quad \forall i\in N,\forall\, t\in H \quad (12)$$

$$p_i(t) \le p_i(t-1) + RU_i v_i(t-1) + SU_i[v_i(t)-v_i(t-1)] + \overline{P}_i[1-v_i(t)];\quad \forall i\in N,\forall\, t\in H \quad (13)$$

$$p_i^k(t) \le A_i^k(t)\big\{p_i(t-1)+RU_i v_i(t-1) + SU_i\big[v_i^k(t)-v_i(t-1)\big] + \overline{P}_i\big[1-v_i^k(t)\big]\big\};\quad \forall i\in N,\forall\, k\in\mathcal{C},\forall\, t\in H \quad (14)$$

$$\boldsymbol{v_i} \in \{0,1\}^{n_H}\cap\vartheta_i;\quad \forall\, i\in N \quad (15)$$

$$v_i^{NS}(t) \in \{0,1\}; \quad \forall\, i \in N, \forall\, t \in H \tag{16}$$

$$v_i^k(t) \in \{0,1\}; \quad \forall\, i \in N, \forall\, k \in \mathcal{C}, \forall\, t \in H. \tag{17}$$

Parameters $A_i^k(t)$ are used to characterize contingency states. Thus, $A_i^k(t)$ is a constant equal to 1 if unit i is available in period t under contingency state k, being 0 otherwise. In each period, the $n-K$ security criterion is enforced by considering all contingency states such that

$$\sum_{i\in N} A_i^k(t) \geq n-K\,; \quad \forall\, k \in \mathcal{C}, \forall t \in H. \tag{18}$$

The objective function to be minimized (1) consists of the sum of the offered cost functions for generating power, including startup and shutdown costs, plus the cost of all up-spinning and non-spinning reserves offered by the generators. It should be noted that in a single-bus unit commitment the loss of any set of generation units does not require the reduction of the production of any remaining available unit to keep the power balance. Thus, only upward reserves such as up-spinning and non-spinning reserves are deployed and, consequently, down-spinning reserve needs not be included in (1)-(17). Moreover, as done in [10], the cost of the corrective actions in the event of a contingency, that is, the actual use of the reserves, is not included in the objective function.

Constraints (2) and (3) represent the power balance equations under the pre-contingency and contingency states, respectively. Constraints (4) and (5) set the generation limits for the pre-contingency and contingency states, respectively. On/off variables, $v_i^k(t)$, are used to allow those generators that are scheduled off in the pre-contingency state to participate in corrective actions during contingency states. Constraints (6) and (7) relate the reserve contributions to the power levels produced under the pre-contingency and contingency states. Constraints (8) and (9) provide the bounds for the up-spinning and non-spinning reserve contributions, respectively. Contraints (10) enforce that non-spinning reserve is only provided by generators that are not scheduled in the pre-contingency state. Constraints (11) set the logic of the scheduling variables. Constraints (12)-(14) represent the ramping limits for both the pre-contingency and contingency states. Finally, the binary nature of the scheduling variables $v_i(t)$, $v_i^{NS}(t)$, and $v_i^k(t)$ is expressed in (15)-(17). In (15), minimum up and down times for the pre-contingency state are formulated in a compact way by the feasibility set ϑ_i. A detailed description of such constraints can be found in [19].

Since the redispatch cost is not included in the objective function, problem (1)-(17) can be equivalently reformulated by dropping variables $p_i^k(t)$ and $v_i^k(t)$. As can be noted, by summing over i in (6),

$$\sum_{i\in N} p_i^k(t) \leq \sum_{i\in N} A_i^k(t)[p_i(t) + r_i^S(t) + r_i^{NS}(t)]\,; \quad \forall\, k \in \mathcal{C}, \forall\, t \in H, \tag{19}$$

and introducing (3) in (19) yields:

$$\sum_{i\in N} A_i^k(t)[p_i(t) + r_i^S(t) + r_i^{NS}(t)] \geq D(t); \quad \forall\, k \in \mathcal{C}, \forall\, t \in H. \tag{20}$$

Furthermore, in order to guarantee the minimum power output requirement in the case of a non-spinning reserve dispatch (5), expression (9) is modified as follows:

$$\underline{P}_i v_i^{NS}(t) \leq r_i^{NS}(t) \leq \overline{R}_i^{NS} v_i^{NS}(t); \quad \forall\, i \in N, \forall\, t \in H. \tag{21}$$

Likewise, constraints (14) can be replaced by the following reserve-based expressions:

$$\begin{aligned} p_i(t) + r_i^S(t) \leq{}& p_i(t-1) + RU_i v_i(t-1) \\ &+ SU_i[v_i(t) - v_i(t-1)] \\ &+ \overline{P}_i[1 - v_i(t)]; \ \forall i \in N, \forall\, t \in H \end{aligned} \tag{22}$$

$$\begin{aligned} r_i^{NS}(t) \leq{}& p_i(t-1) + RU_i v_i(t-1) \\ &+ \overline{R}_i^{NS}[1 - v_i(t-1)]; \\ &\forall i \in N, \forall\, t \in H \end{aligned} \tag{23}$$

$$\begin{aligned} r_i^{NS}(t) \leq{}& SU_i[v_i^{NS}(t) - v_i(t-1)] \\ &+ \overline{R}_i^{NS}\{1 - [v_i^{NS}(t) - v_i(t-1)]\}; \\ &\forall i \in N, \forall\, t \in H. \end{aligned} \tag{24}$$

Constraints (22) enforce ramp-up and startup limits for the maximum synchronized power output that can be supplied under any contingency state. Constraints (23) and (24) model the ramp-up and startup limits on the provision of non-spinning reserve.

Thus, by replacing constraints (3), (5), (6), (9), (11), (14), and (17) in the original model by expressions (21)-(24), post-contingency variables $p_i^k(t)$ and $v_i^k(t)$ can be dropped. For the sake of clarity, the equivalent model is stated below:

$$\begin{aligned} \underset{\substack{p_i(t), r_i^{NS}(t), r_i^S(t),\\ v_i(t), v_i^{NS}(t)}}{\text{Minimize}} \sum_{t\in H}\sum_{i\in N} \Big[& C_{it}^P\big(p_i(t), v_i(t)\big) \\ & + C_i^S(t) r_i^S(t) + C_i^{NS}(t) r_i^{NS}(t)\Big] \end{aligned} \tag{25}$$

subject to:

$$\sum_{i\in N} p_i(t) = D(t); \quad \forall\, t \in H \tag{26}$$

$$\sum_{i\in N} A_i^k(t)[p_i(t) + r_i^S(t) + r_i^{NS}(t)] \geq D(t); \quad \forall\, k \in \mathcal{C}, \forall\, t \in H \tag{27}$$

$$\underline{P}_i v_i(t) \leq p_i(t) \leq \overline{P}_i v_i(t); \quad \forall\, i \in N, \forall\, t \in H \tag{28}$$

$$p_i(t) + r_i^S(t) \leq \overline{P}_i v_i(t); \quad \forall\, i \in N, \forall\, t \in H \tag{29}$$

$$0 \leq r_i^S(t) \leq \overline{R}_i^S v_i(t); \quad \forall\, i \in N, \forall\, t \in H \tag{30}$$

$$\underline{P}_i v_i^{NS}(t) \leq r_i^{NS}(t) \leq \overline{R}_i^{NS} v_i^{NS}(t); \quad \forall\, i \in N, \forall\, t \in H \tag{31}$$

$$v_i(t) + v_i^{NS}(t) \leq 1; \quad \forall\, i \in N, \forall\, t \in H \tag{32}$$

$$\begin{aligned} p_i(t-1) \leq{}& p_i(t) + RD_i v_i(t) \\ &+ SD_i[v_i(t-1) - v_i(t)] \\ &+ \overline{P}_i[1 - v_i(t-1)]; \\ &\forall i \in N, \forall\, t \in H \end{aligned} \tag{33}$$

$$\begin{aligned} p_i(t) \leq{}& p_i(t-1) + RU_i v_i(t-1) \\ &+ SU_i[v_i(t) - v_i(t-1)] \\ &+ \overline{P}_i[1 - v_i(t)]; \quad \forall i \in N, \forall\, t \in H \end{aligned} \tag{34}$$

$$p_i(t) + r_i^S(t) \le p_i(t-1) + RU_i v_i(t-1) + SU_i[v_i(t) - v_i(t-1)] + \overline{P}_i[1 - v_i(t)];\ \ \forall i \in N, \forall\, t \in H \quad (35)$$

$$r_i^{NS}(t) \le p_i(t-1) + RU_i v_i(t-1) + \overline{R}_i^{NS}[1 - v_i(t-1)];\ \ \forall i \in N, \forall\, t \in H \quad (36)$$

$$r_i^{NS}(t) \le SU_i[v_i^{NS}(t) - v_i(t-1)] + \overline{R}_i^{NS}\{1 - [v_i^{NS}(t) - v_i(t-1)]\};\ \ \forall i \in N, \forall\, t \in H \quad (37)$$

$$\boldsymbol{v_i} \in \{0,1\}^{n_H} \cap \vartheta_i;\ \ \forall\, i \in N \quad (38)$$

$$v_i^{NS}(t) \in \{0,1\};\ \ \forall\, i \in N, \forall\, t \in H. \quad (39)$$

In spite of the absence of post-contingency variables, it is worth mentioning that the equivalent model implicitly guarantees a feasible post-contingency schedule for each period assuming no contingency has occurred in previous periods.

Problem (1)-(17) and its equivalent (25)-(39) both explicitly model the $n-K$ security criterion by considering all $\sum_{i=1}^{K}\binom{n}{i}$ combinations of unit outages in (3) or (27). For realistic power systems comprising hundreds of units, this number of contingency states may become prohibitive and render the problem essentially intractable even for low values of K. The approach presented next addresses such issue while keeping the modeling accuracy.

IV. Robust Optimization Approach

Problem (25)-(39) can be viewed as a particular instance of robust optimization [23][24] in which the parameters allowed to vary are parameters $A_i^k(t)$ representing the availability of generation units in each period under each contingency state. Based on this fact, we propose a robust optimization approach to solve the $n-K$ CCUC problem (25)-(39), where K is identified as the robustness parameter used to adjust the conservativism level. First, the contingency-dependent model (25)-(39) is equivalently reformulated as a robust bilevel counterpart. The resulting robust formulation embeds all contingencies associated with the $n-K$ security criterion but does not depend on the size of the contingency set $\mathcal{C}$. Using recent findings from robust optimization, the resulting bilevel program is subsequently transformed into an equivalent single-level MIP problem suitable for commercially available software.

A. Robust Bilevel Counterpart

The contingency dependence of problem (25)-(39) is introduced in (27), which can be referred to as the complicating constraints. This set of constraints requires that the sum of the pre-contingency power outputs and reserve contributions of all available units be greater than or equal to the system demand in each period for each contingency state. Since this requirement must hold for all contingencies $k \in \mathcal{C}$, it is sufficient to guarantee that it holds for the worst case, i.e., the contingency with the tightest left-hand side of (27). This contingency is also known as the umbrella contingency [29]. Therefore, constraints (27) can be expressed in a compact way as:

$$d^{wc*}(t) \ge D(t);\ \ \forall t \in H \quad (40)$$

$$d^{wc*}(t) = \min_{k\in\mathcal{C}}\left\{\sum_{i\in N} A_i^k(t)\big[p_i(t) + r_i^S(t) + r_i^{NS}(t)\big]\right\};\ \ \forall t \in H, \quad (41)$$

where $d^{wc*}(t)$ denotes the maximum power that can be supplied in period t under the worst-case contingency for given values of scheduled power output, spinning reserve, and non-spinning reserve.

The minimum function in (41) can be formulated as an optimization problem by defining a new vector of decision variables $\boldsymbol{a}(t) = [a_1(t), \dots, a_n(t)]^T$ associated with the worst-case contingency in each period. Hence, $a_i(t)$ is a binary variable which is equal to 0 if generator i is unavailable in period t in the worst-contingency state, being 1 otherwise.

Therefore, the $n-K$ CCUC problem can be restated as a bilevel programming problem [27]. As shown in Fig. 1, the upper-level agent (the system operator) determines the schedule of power and reserves so that the overall cost is minimized. This cost minimization problem is also subject to the worst-case contingency in each period, which is modeled by the lower-level optimization. Thus, the lower level determines the combination of out-of-service generators so that the available post-contingency power output in each period is minimized.

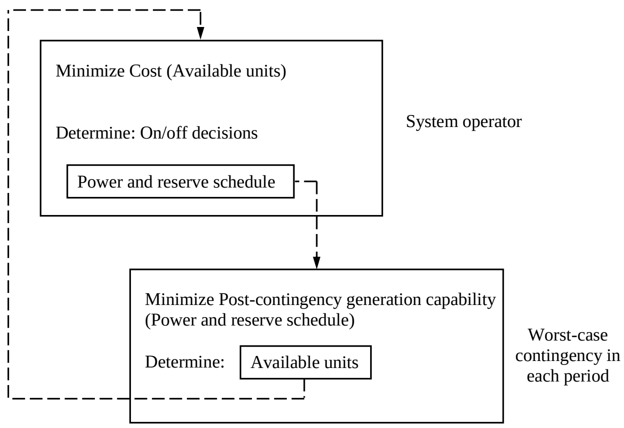

Fig. 1. Bilevel model.

The robust bilevel counterpart for the $n-K$ CCUC problem is formulated as follows:

$$\underset{\substack{p_i(t), r_i^{NS}(t), r_i^S(t),\\ v_i(t), v_i^{NS}(t)}}{\text{Minimize}} \sum_{t\in H}\sum_{i\in N}\big[C_{it}^P\big(p_i(t), v_i(t)\big) + C_i^S(t) r_i^S(t) + C_i^{NS}(t) r_i^{NS}(t)\big] \quad (42)$$

subject to:

$$\sum_{i \in N} p_i(t) = D(t); \quad \forall\, t \in H \tag{43}$$

$$\underline{P}_i v_i(t) \le p_i(t) \le \overline{P}_i v_i(t); \quad \forall\, i \in N, \forall\, t \in H \tag{44}$$

$$p_i(t) + r_i^S(t) \le \overline{P}_i v_i(t); \quad \forall\, i \in N, \forall\, t \in H \tag{45}$$

$$0 \le r_i^S(t) \le \overline{R}_i^S v_i(t); \quad \forall\, i \in N, \forall\, t \in H \tag{46}$$

$$\underline{P}_i v_i^{NS}(t) \le r_i^{NS}(t) \le \overline{R}_i^{NS} v_i^{NS}(t); \quad \forall\, i \in N, \forall\, t \in H \tag{47}$$

$$v_i(t) + v_i^{NS}(t) \le 1; \quad \forall\, i \in N, \forall\, t \in H \tag{48}$$

$$\begin{aligned} p_i(t-1) \le p_i(t) &+ RD_i v_i(t) \\ &+ SD_i[v_i(t-1) - v_i(t)] \\ &+ \overline{P}_i[1 - v_i(t-1)]; \\ &\qquad \forall i \in N, \forall\, t \in H \end{aligned} \tag{49}$$

$$\begin{aligned} p_i(t) \le p_i(t-1) &+ RU_i v_i(t-1) \\ &+ SU_i[v_i(t) - v_i(t-1)] \\ &+ \overline{P}_i[1 - v_i(t)]; \quad \forall i \in N, \forall\, t \in H \end{aligned} \tag{50}$$

$$\begin{aligned} p_i(t) + r_i^S(t) \le p_i(t-1) &+ RU_i v_i(t-1) \\ &+ SU_i[v_i(t) - v_i(t-1)] \\ &+ \overline{P}_i[1 - v_i(t)]; \ \forall i \in N, \forall\, t \in H \end{aligned} \tag{51}$$

$$\begin{aligned} r_i^{NS}(t) \le p_i(t-1) &+ RU_i v_i(t-1) \\ &+ \overline{R}_i^{NS}[1 - v_i(t-1)]; \\ &\qquad \forall i \in N, \forall\, t \in H \end{aligned} \tag{52}$$

$$\begin{aligned} r_i^{NS}(t) \le SU_i[v_i^{NS}(t) &- v_i(t-1)] \\ &+ \overline{R}_i^{NS}\{1 - [v_i^{NS}(t) - v_i(t-1)]\}; \\ &\qquad \forall i \in N, \forall\, t \in H \end{aligned} \tag{53}$$

$$\boldsymbol{v_i} \in \{0,1\}^{n_H} \cap \vartheta_i; \quad \forall\, i \in N \tag{54}$$

$$v_i^{NS}(t) \in \{0,1\}; \quad \forall\, i \in N, \forall\, t \in H \tag{55}$$

$$d^{wc*}(t) \ge D(t); \ \ \forall t \in H \tag{56}$$

$$d^{wc*}(t) = \min_{a_i(t)} \left\{ \sum_{i \in N} a_i(t)\left[p_i(t) + r_i^S(t) + r_i^{NS}(t)\right] \right. \tag{57}$$

subject to:

$$\sum_{i \in N} a_i(t) \ge n - K \ : y(t) \tag{58}$$

$$\left. 0 \le a_i(t) \le 1 \qquad : z_i(\mathrm{t}); \forall i \in N \right\}; \ \forall t \in H. \tag{59}$$

Problem (42)-(59) comprises an upper-level problem (42)-(56) and a set of lower-level problems (57)-(59), one for each period. The upper-level problem consists in the determination of the generation unit schedule, including pre-contingency power outputs as well as up-spinning and non-spinning reserve contributions, where the unavailability of generation units in the worst-case contingency results from the solution to the lower-level problems. Upper-level decision variables are $p_i(t)$, $r_i^{NS}(t)$, $r_i^S(t)$, $v_i(t)$, and $v_i^{NS}(t)$, whereas $a_i(t)$ are the lower-level decision variables. The dual variables associated with (58) and (59) are $y(t)$ and $z_i(t)$, respectively.

The upper-level objective function (42) is identical to (25). Analogously, upper-level constraints (43)-(55) are identical to (26), (28)-(39), respectively. Constraints (56) impose that the generation capability in each period under the worst-case contingency is greater than or equal to the system demand, i.e., power balance is guaranteed under all contingencies.

The lower-level objective function (57) represents the total post-contingency power output that can be supplied in each period by the generation units under any combination of available units. Such availability is modeled by the lower-level decision variables $a_i(t)$. Therefore, minimizing this objective function leads to the worst-case contingency. Constraint (58) enforces the $n - K$ security criterion in each period. Finally, constraints (59) set the upper and lower bounds for variables $a_i(t)$.

It should be noted that constraints (58) and (59) have a unimodular matrix structure (see proposition 3.2 in [30]), which guarantees that, for integer values of K, the lower-level problems (57)-(59) always provide integer (binary) optimal solutions for vector $\boldsymbol{a}(t)$. In other words, vectors composed of the generator availability parameters in each period t, $\boldsymbol{A}^k(t) = [A_1^k(t), \dots, A_n^k(t)]^T \ \forall\, k \in \mathcal{C}$, are the vertexes of the polyhedral set defined by constraints (58) and (59). In Fig. 2, such polytope and the set $\{\boldsymbol{A}^k(t)\}_{k \in \mathcal{C}}$ in a generic period t are both illustrated for the case of a three-generator system with $K = 1$. Circles represent the vertexes whereas the dashed volume, above the two-sum plane, is the so-called polyhedral uncertainty set, in robust optimization nomenclature. This set comprises all data variations in the generator availability space, $\boldsymbol{a}(t) \in [0,1]^n$, under which the system demand in a given period t is met even if a single generator happens to fail. Note that such uncertainty set also contains fractional (non-binary) availability vectors for which the system demand is also met (points inside the dashed volume). However, for any feasible upper-level scheduling decision the worst-case contingency can always be characterized by one of the vertexes of such polytope.

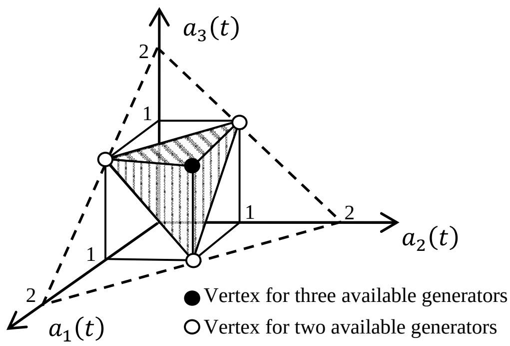

Fig. 2. Generator availability set for $n = 3$ and $K = 1$.

Besides its intrinsic complexity due to the two levels of optimization, problem (42)-(59) is mixed integer (containing both continuous and binary variables) and nonlinear due to the products $a_i(t)p_i(t)$, $a_i(t)r_i^S(t)$, and $a_i(t)r_i^{NS}(t)$ in (57).

B. Robust Single-Level Counterpart

Based on the robust optimization approach presented in [23], an efficient single-level equivalent formulation is provided for the bilevel problem (42)-(59).

Note that the only requirement on each lower-level optimization problem (57)-(59) imposed at the upper level is that its optimal objective function value, $d^{wc*}(t)$, be at least $D(t)$ in each period t. Thus, this requirement is also met by imposing that a lower bound for $d^{wc*}(t)$, given by the dual objective function of each lower-level problem, be greater than or equal to $D(t)$. Therefore, the procedure to derive the final single-level robust counterpart is summarized as follows:

1) replace $d^{wc*}(t)$ in (56) by the dual objective function of the lower-level optimization problem (57)-(59) for each period t;

2) replace each lower-level optimization problem (57)-(59) by its dual feasibility constraints.

Step 2 guarantees that the dual objective function of each lower-level problem at step 1 provides a lower bound for $d^{wc*}(t)$ in each period. The interested reader is referred to [23] for a detailed proof of the transformation. For the sake of completeness, the equivalent single-level MIP model for the robust bilevel $n-K$ CCUC problem (42)-(59) is as follows:

$$\underset{\substack{p_i(t), r_i^{NS}(t), r_i^S(t),\\ v_i(t), v_i^{NS}(t),\\ y(t), z_i(t)}}{\text{Minimize}} \sum_{t\in H}\sum_{i\in N}\Big[C_{it}^P\big(p_i(t), v_i(t)\big) + C_i^S(t) r_i^S(t) + C_i^{NS}(t) r_i^{NS}(t)\Big] \tag{60}$$

subject to:

$$\sum_{i\in N} p_i(t) = D(t); \quad \forall\, t\in H \tag{61}$$

$$\underline{P}_i v_i(t) \le p_i(t) \le \overline{P}_i v_i(t); \quad \forall\, i\in N, \forall\, t\in H \tag{62}$$

$$p_i(t) + r_i^S(t) \le \overline{P}_i v_i(t); \quad \forall\, i\in N, \forall\, t\in H \tag{63}$$

$$0 \le r_i^S(t) \le \overline{R}_i^S v_i(t); \quad \forall\, i\in N, \forall\, t\in H \tag{64}$$

$$\underline{P}_i v_i^{NS}(t) \le r_i^{NS}(t) \le \overline{R}_i^{NS} v_i^{NS}(t); \quad \forall\, i\in N, \forall\, t\in H \tag{65}$$

$$v_i(t) + v_i^{NS}(t) \le 1; \quad \forall\, i\in N, \forall\, t\in H \tag{66}$$

$$p_i(t-1) \le p_i(t) + RD_i v_i(t) + SD_i[v_i(t-1) - v_i(t)] + \overline{P}_i[1 - v_i(t-1)]; \quad \forall i\in N, \forall\, t\in H \tag{67}$$

$$p_i(t) \le p_i(t-1) + RU_i v_i(t-1) + SU_i[v_i(t) - v_i(t-1)] + \overline{P}_i[1 - v_i(t)]; \quad \forall i\in N, \forall\, t\in H \tag{68}$$

$$p_i(t) + r_i^S(t) \le p_i(t-1) + RU_i v_i(t-1) + SU_i[v_i(t) - v_i(t-1)] + \overline{P}_i[1 - v_i(t)]; \quad \forall i\in N, \forall\, t\in H \tag{69}$$

$$r_i^{NS}(t) \le p_i(t-1) + RU_i v_i(t-1) + \overline{R}_i^{NS}[1 - v_i(t-1)]; \quad \forall i\in N, \forall\, t\in H \tag{70}$$

$$r_i^{NS}(t) \le SU_i[v_i^{NS}(t) - v_i(t-1)] + \overline{R}_i^{NS}\{1 - [v_i^{NS}(t) - v_i(t-1)]\}; \quad \forall i\in N, \forall\, t\in H \tag{71}$$

$$\boldsymbol{v_i} \in \{0,1\}^{n_H} \cap \vartheta_i; \quad \forall\, i\in N \tag{72}$$

$$v_i^{NS}(t) \in \{0,1\}; \quad \forall\, i\in N, \forall\, t\in H \tag{73}$$

$$(n-K)y(t) - \sum_{i\in N} z_i(t) \ge D(t); \quad \forall t\in H \tag{74}$$

$$y(t) - z_i(t) \le p_i(t) + r_i^S(t) + r_i^{NS}(t); \quad \forall\, i\in N, \forall t\in H \tag{75}$$

$$z_i(t) \ge 0; \quad \forall\, i\in N, \forall t\in H \tag{76}$$

$$y(t) \ge 0; \quad \forall t\in H. \tag{77}$$

Note that expressions (60)-(73) are respectively identical to (42)-(55). Constraints (74) correspond to (56). Finally, constraints (75)-(77) are the dual feasibility constraints of the lower-level problems (57)-(59).

Table I compares the size of the three formulations for the $n-K$ CCUC problem addressed in this paper, namely both contingency-dependent models (1)-(17) and (25)-(39), and the proposed robust equivalent (60)-(77). Model sizes are expressed in terms of the number of constraints excluding variable bounds, the number of continuous variables, and the number of binary variables. As can be seen, even for the simplest case of $K=1$, i.e., n contingency states ($|\mathcal{C}| = n$), the number of constraints and continuous variables of the reformulated contingency-dependent model (25)-(39) and the robust equivalent model grows linearly with the number of generation units whereas this increase is quadratic for the original contingency-dependent model (1)-(17). For tighter security levels, the size growth of the robust equivalent model remains linear with respect to the number of generation units whereas for both contingency-dependent models the size increases at higher polynomial orders. This comparison reveals the theoretical superiority of the proposed robust approach over the contingency-dependent models. In the next section, such advantage is shown in terms of significant computational time savings when solving a set of realistic case studies.

TABLE I
SIZE COMPARISON

Model	# constraints	# continuous variables	# binary variables
(1)-(17)	$6n_H n\|\mathcal{C}\| + n_H\|\mathcal{C}\| + 10n_H n + n_H$	$n_H n\|\mathcal{C}\| + 3n_H n$	$n_H n\|\mathcal{C}\| + 2n_H n$
(25)-(39)	$n_H\|\mathcal{C}\| + 14n_H n + n_H$	$3n_H n$	$2n_H n$
(60)-(77)	$15n_H n + 2n_H$	$4n_H n + n_H$	$2n_H n$

*$|\mathcal{C}| = \sum_{i=1}^{K}\binom{n}{i}$.

V. Case Studies

Results from several case studies are presented in this Section. For didactical purposes, the robust formulation has been first applied to an illustrative example comprising three generation units and a single period. In order to assess the practical applicability of the proposed robust model and the influence of the problem size on its computational performance, a 10-unit system has been replicated to analyze systems including up to 100 generation units. For the sake of simplicity, generators offer linear cost functions of the form $C_{it}^{P}(p_i(t), v_i(t)) = C_i^{f}(t)v_i(t) + C_i^{l}(t)p_i(t)$. Therefore, the robust single-level counterpart is a mixed-integer linear programming problem. The model has been implemented on a Pentium Intel i7, 3.2 GHz processor with 16 GB of RAM using Xpress-MP 7.0 under MOSEL [28].

A. Three-Unit Example

Data for the generators are given in Table II. In this illustrative example, only up-spinning reserve is considered. The system demand is equal to 50 MW.

TABLE II
Generator Data for the 3-Unit System

Unit	$\underline{P}_i$ (MW)	$\overline{P}_i$ (MW)	$\overline{R}_i^S$ (MW)	C_i^f (\$)	C_i^l (\$/MW)	C_i^S (\$/MW)
1	10	100	100	300	10	1
2	10	100	100	200	20	2
3	10	100	100	150	30	3

For $K = 0$, i.e., without imposing any security criterion, the optimal value of the objective function is \$800. In the optimal solution, the cheapest generator 1 is the only generator scheduled, thereby supplying the whole demand, and no spinning reserve contribution is required.

Due to the small size of this problem (8 possible generation schedules), it can be solved by enumeration of the scheduling variables $\boldsymbol{v}$. Table III provides the results associated with the feasible schedules for $K = 1$ and $K = 2$. For $K > 2$ the problem is infeasible.

TABLE III
Results for the Feasible Schedules of the 3-Unit System

	$K = 1$				$K = 2$
$\boldsymbol{v}$	$[1\ 1\ 0]^T$	$[1\ 0\ 1]^T$	$[0\ 1\ 1]^T$	$[1\ 1\ 1]^T$	$[1\ 1\ 1]^T$
p_1/r_1^S (MW)	40/10	40/10	0/0	30/0	30/20
p_2/r_2^S (MW)	10/40	0/0	40/10	10/20	10/40
p_3/r_3^S (MW)	0/0	10/40	10/40	10/10	10/40
Cost (\$)	1190	1280	1590	1520	1670

For $K = 1$, there are only four feasible schedules comprising the commitment of at least two generation units. The optimal solution requires the commitment of the expensive generator 2. This generator operates at its minimum power output of 10 MW, also providing 40 MW of up-spinning reserve. In addition, preventive security requires that the power output of the cheapest generator 1 be reduced with respect to the unconstrained case. The optimal value of the objective function for this security-constrained case is \$1190, which is considerably higher than the \$800 that it costs to operate the system without security.

For $K = 2$, there is only one feasible solution. This optimal solution requires the commitment of all available generation units. Similar to the previous case, the most expensive generators (units 2 and 3) produce the minimum power output of 10 MW while providing 40 MW of up-spinning reserve. Generator 1 reduces its output down to 30 MW and increases its up-spinning reserve contribution up to 20 MW. As expected, the higher level of conservativism yields an increase in the optimal value of the objective function, which is equal to \$1670.

The robust optimization model (60)-(77) was applied to this illustrative example for $K = 1$ and $K = 2$. The optimal solutions were attained in 0.2 s.

B. Real-Size Case Studies

The proposed robust formulation has been applied to solve several real-size case studies built on a base test system comprising 10 generators. The data for the generators of the base test system can be found in [19]. Fixed and linear production cost coefficients respectively correspond to the fixed and linear coefficients of the quadratic cost functions reported in [19]. Maximum up-spinning and non-spinning reserve contributions, $\overline{R}_i^S$ and $\overline{R}_i^{NS}$, are both equal to each generator capacity. Additionally, spinning and non-spinning cost rates, $C_i^S(t)$ and $C_i^{NS}(t)$, are respectively set to 8% and 10% of the linear production cost coefficient $C_i^l(t)$ offered by each generator. All cost coefficients are assumed constant over the time span. The demand profile is 50% of that reported in [19].

Nine additional case studies have been generated by replicating the base test system and scaling the system demand accordingly.

In Xpress [28], an optimality parameter can be specified to decide whether to find the optimal solution or to quickly obtain a suboptimal solution, referred to as an ε-optimal solution. In these case studies, the execution of Xpress was stopped when the value of the objective function was within 0.5% of the optimal solution, which is a reasonable choice in terms of solution accuracy. In addition, a time limit of one hour (3600 s) was set.

Table IV provides information on costs attained by the robust model for different values of the security parameter K ranging between 0 and 5. The second column lists the costs for the unconstrained case ($K = 0$). Columns 3-7 show the percent cost increase over the unconstrained case when security is accounted for. Note that the base case problem with 10 units is infeasible for $K = 3$, 4, and 5, whereas the case with 20 units is infeasible for $K = 5$. In this table, symbol "I" represents an infeasible problem.

It is remarkable that for each security level, the percent cost increase experiences a significant reduction as the system size grows. As an example, for $K = 2$, the cost increase due to security is reduced from 9.8% down to 1.2%. Furthermore, for each test system, higher values of the security parameter K yield higher cost increases, as expected. However, it is worth mentioning that this cost increase is moderate, reaching values between 9.5% and 2.8% for $K = 5$. Both results suggest that tighter security levels than currently used $n - 1$ and $n - 2$ criteria might be used in real-size systems without drastically increasing the total cost.

TABLE IV

IMPACT OF SECURITY ON COST FOR REAL-SIZE SYSTEMS

# units	Cost without security ($)	Cost increase due to security (%)				
		K				
		1	2	3	4	5
10	268978.5	5.5	9.8	I	I	I
20	528523.4	3.2	6.0	9.0	11.0	I
30	790214.2	1.5	3.5	5.6	7.9	9.5
40	1053379.3	1.4	2.7	4.5	5.9	7.5
50	1312876.8	1.3	2.4	3.8	5.0	6.0
60	1576606.0	0.9	1.9	2.9	3.9	5.4
70	1838217.9	0.7	1.6	2.4	3.5	4.2
80	2100310.8	0.5	1.4	2.1	2.8	3.6
90	2357599.8	0.5	1.4	1.9	2.5	3.4
100	2624203.3	0.4	1.2	1.8	2.3	2.8

The relationship between the total cost and the system size can be expressed through a linear regression for each security level K. Table V lists the angular and linear coefficients of such regression for values of K ranging between 0 and 5. The angular coefficient represents the increase in cost due to security (additional \$ per additional security level K). It should be noted that the angular coefficient varies slightly with K. This result means that, despite the increase in fixed cost due to the commitment of extra units, tighter security levels do not significantly change the rate with which the total cost varies with the system size. As a consequence, the unitary cost, defined as the total cost divided by the number of generation units, decreases with the system size for all security levels. This result is depicted in Fig. 3.

TABLE V

COEFFICIENTS OF THE LINEAR REGRESSION FOR COST VS. SYSTEM SIZE

	K					
	0	1	2	3	4	5
Angular ($/security)	26170.2	26125.2	26201.4	26149.1	26127.4	26139.4
Linear ($)	5615.1	21933.2	33597.1	53068.4	70402.4	85686.2

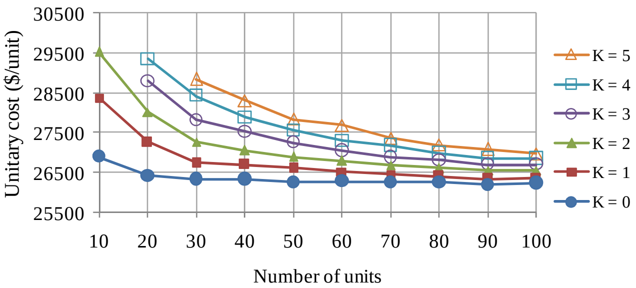

Fig. 3. Optimal unitary cost vs. system size.

Table VI presents the computing times required by the proposed robust model, denoted by R, for all test systems and values of K up to 5. Symbols "I" and "OM" represent "Infeasible" and "Out of memory", respectively. The computational performance of the robust approach (60)-(77) is assessed through the comparison with the reserve-based contingency-dependent formulation (25)-(39), referred to as CD. For the sake of tractability of problem (25)-(39), the set of contingencies in CD only includes those with exactly K units simultaneously out of service, i.e., the cardinality of $\mathcal{C}$ is equal to $\binom{n}{K}$ rather than $\sum_{i=1}^{K}\binom{n}{i}$. Note that this reduced contingency set covers all contingencies with fewer unavailable generators and hence optimality is not affected. As can be seen, the robust approach is able to find a solution satisfying the pre-specified optimality tolerance with little computational effort for all case studies. In contrast, the contingency-dependent model requires much larger computing times for most cases, particularly for large-scale systems. Moreover, for large numbers of units and more conservative security criteria, the contingency-dependent model is unable to find an ε-optimal solution within the time limit and even leads to intractable problems for which not enough memory is available. These results clearly back the superiority of the robust model over the contingency-dependent formulation from a computational viewpoint.

TABLE VI

COMPARISON OF COMPUTING TIMES FOR REAL-SIZE SYSTEMS (S)

# units	*K*									
	1		2		3		4		5	
	R	CD	R	CD	R	CD	R	CD	R	CD
10	0.8	1.0	1.1	7.4	I	I	I	I	I	I
20	2.9	6.1	10.0	24.6	10.8	288.0	0.0	2302.0	I	I
30	3.6	0.8	3.3	20.1	6.5	3299.8	25.0	3600.0	9.5	3600.0
40	7.2	17.7	9.8	2394.9	290.2	3600.0	12.6	3600.0	561.9	OM
50	14.5	13.1	12.1	3600.0	623.3	3600.0	15.7	OM	31.0	OM
60	11.8	16.8	11.8	161.8	18.7	3197.6	14.7	OM	2576.3	OM
70	10.9	15.9	1.8	257.8	13.3	3600.0	3520.5	OM	17.4	OM
80	14.4	20.8	14.9	384.9	15.4	OM	14.3	OM	11.8	OM
90	9.5	21.2	9.4	542.2	8.9	OM	10.9	OM	10.6	OM
100	18.4	33.6	21.0	836.5	15.0	OM	15.5	OM	18.8	OM

VI. CONCLUSIONS

This paper presents a robust optimization approach for the contingency-constrained single-bus unit commitment problem with an $n-K$ security criterion. As a major contribution of this paper, the model described allows system operators to schedule power and reserves while explicitly considering all combinations of up to K generation unit outages. The original contingency-dependent model is first formulated as a robust bilevel counterpart. The resulting bilevel program is subsequently transformed into an equivalent single-level mixed-integer program that is efficiently solved using available commercial software.

Numerical results show that the proposed robust model outperforms the contingency-dependent formulation since solutions within the optimality tolerance are achieved in moderate computing times. Moreover, the robust model can handle problems that are essentially intractable for the contingency-dependent model.

Research is currently underway to address a network-constrained model. This further work considers two aspects: (i) the impact of line flow capacities on the definition of the worst-case contingency, and (ii) the extension of the contingency set to include line outages. Numerical analyses of such network-constrained model will be discussed in our future publications. Finally, further research will also be devoted to pricing energy and reserves under the $n-K$ security criterion and to assessing the tradeoff between cost and security.

Acknowledgment

The authors would like to thank FICO (Xpress-MP developer) for the academic partnership program with the Electrical Engineering Department of the Pontifical Catholic University of Rio de Janeiro (PUC-Rio). The authors are also grateful to Alexandre Moreira da Silva (B.Sc. student of the Electrical Engineering Department of PUC-Rio) for helping in running the case studies.

References

[1] A. J. Wood and B. F. Wollenberg, *Power Generation, Operation, and Control*, 2nd ed. New York: Wiley, 1996.

[2] P. Kundur, J. Paserba, V. Ajjarapu, G. Andersson, A. Bose, C. Canizares, N. Hatziargyriou, D. Hill, A. Stankovic, C. Taylor, T. van Cutsem, and V. Vittal, "Definition and classification of power system stability," *IEEE Trans. Power Syst.*, vol. 19, no. 2, pp. 1387-1401, May 2004.

[3] G. Andersson, P. Donalek, R. Farmer, N. Hatziargyriou, I. Kamwa, P. Kundur, N. Martins, J. Paserba, P. Pourbeik, J. Sanchez-Gasca, R. Schulz, A. Stankovic, C. Taylor, and V. Vittal, "Causes of the 2003 major grid blackouts in North America and Europe, and recommended means to improve system dynamic performance," *IEEE Trans. Power Syst.*, vol. 20, no. 4, pp. 1922-1928, Nov. 2005.

[4] G. C. Oliveira, S. Binato, and M. V. F. Pereira, "Value-based transmission expansion planning of hydrothermal systems under uncertainty," *IEEE Trans. Power Syst.*, vol. 22, no. 4, pp. 1429-1435, Nov. 2007.

[5] M. Carrión, J. M. Arroyo, and N. Alguacil, "Vulnerability-constrained transmission expansion planning: A stochastic programming approach," *IEEE Trans. Power Syst.*, vol. 22, no. 4, pp. 1436-1445, Nov. 2007.

[6] J. Choi, T. D. Mount, and R. J. Thomas, "Transmission expansion planning using contingency criteria," *IEEE Trans. Power Syst.*, vol. 22, no. 4, pp. 2249-2261, Nov. 2007.

[7] N. Alguacil, M. Carrión, and J. M. Arroyo, "Transmission network expansion planning under deliberate outages," *Int. J. Electr. Power Energy Syst.*, vol. 31, no. 9, pp. 553-561, Oct. 2009.

[8] J. M. Arroyo, "Bilevel programming applied to power system vulnerability analysis under multiple contingencies," *IET Gener. Transm. Distrib.*, vol. 4, no. 2, pp. 178-190, Feb. 2010.

[9] M. Shahidehpour, H. Yamin, and Z. Li, *Market Operations in Electric Power Systems*. New York: Wiley, 2002.

[10] J. M. Arroyo and F. D. Galiana, "Energy and reserve pricing in security and network-constrained electricity markets," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 634-643, May 2005.

[11] F. D. Galiana, F. Bouffard, J. M. Arroyo, and J. F. Restrepo, "Scheduling and pricing of coupled energy and primary, secondary, and tertiary reserves," *Proc. IEEE*, vol. 93, no. 11, pp. 1970-1983, Nov. 2005.

[12] N. S. Rau, "Optimal dispatch of a system based on offers and bids — A mixed integer LP formulation," *IEEE Trans. Power Syst*., vol. 14, no. 1, pp. 274-279, Feb. 1999.

[13] M. Shahidehpour, W. F. Tinney, and Y. Fu, "Impact of security on power systems operation," *Proc. IEEE*, vol. 93, no. 11, pp. 2013-2025, Nov. 2005.

[14] J. Wang, N. E. Redondo, and F. D. Galiana, "Demand-side reserve offers in joint energy/reserve electricity markets," *IEEE Trans. Power Syst.*, vol. 18, no. 4, pp. 1300-1306, Nov. 2003.

[15] P. A. Ruiz, C. R. Philbrick, E. Zak, K. W. Cheung, and P. W. Sauer, "Uncertainty management in the unit commitment problem," *IEEE Trans. Power Syst.*, vol. 24, no. 2, pp. 642-651, May 2009.

[16] J. Wang, M. Shahidehpour, and Z. Li, "Contingency-constrained reserve requirements in joint energy and ancillary services auction," *IEEE Trans. Power Syst.*, vol. 24, no. 3, pp. 1457-1468, Aug. 2009.

[17] K. W. Hedman, R. P. O'Neill, E. B. Fisher, and S. S. Oren, "Optimal transmission switching with contingency analysis," *IEEE Trans. Power Syst.*, vol. 24, no. 3, pp. 1577-1586, Aug. 2009.

[18] K. W. Hedman, M. C. Ferris, R. P. O'Neill, E. B. Fisher, and S. S. Oren, "Co-optimization of generation unit commitment and transmission switching with n-1 reliability," *IEEE Trans. Power Syst.*, vol. 25, no. 2, pp. 1052-1063, May 2010.

[19] M. Carrión and J. M. Arroyo, "A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem," *IEEE Trans. Power Syst*., vol. 21, no. 3, pp. 1371-1378, Aug. 2006.

[20] A. L. Soyster, "Convex programming with set-inclusive constraints and applications to inexact linear programming," *Oper. Res.*, vol. 21, no. 5, pp. 1154-1157, Sep.-Oct. 1973.

[21] A. Ben-Tal and A. Nemirovski, "Robust convex optimization," *Math. Oper. Res.*, vol. 23, no. 4, pp. 769-805, Nov. 1998.

[22] A. Ben-Tal and A. Nemirovski, "Robust optimization – Methodology and applications," *Math. Program.*, vol. 92, no. 3, pp. 453-480, May 2002.

[23] D. Bertsimas and M. Sim, "The price of robustness," *Oper. Res.*, vol. 52, no. 1, pp. 35-53, Jan.-Feb. 2004.

[24] D. Bertsimas and M. Sim, "Robust discrete optimization and network flows," *Math. Program.*, vol. 98, no. 1-3, pp. 49-71, Sep. 2003.

[25] S. Takriti and S. Ahmed, "On robust optimization of two-stage systems," *Math. Program.*, vol. 99, no. 1, pp. 109-126, Jan. 2004.

[26] A. T. Sariç and A. M. Stankoviç, "A robust algorithm for volt/var control," in Power Systems Conference and Exposition, PSCE'09, pp. 1-8, Seattle, USA, Mar. 2009.

[27] S. Dempe, *Foundations of Bilevel Programming*. Norwell: Kluwer Academic Publishers, 2002.

[28] Xpress Optimization Suite. [Online]. Available: http://www.fico.com/.

[29] F. Bouffard, F. D. Galiana, and J. M. Arroyo, "Umbrella contingencies in security-constrained optimal power flow," in 15th Power Systems Computation Conference, PSCC'05, Liège, Belgium, Aug. 2005.

[30] L. Wolsey, *Integer Programming*. New York: Wiley Inter-Science, 1998.

Alexandre Street (S'06–M'10) holds an M.Sc. and a D.Sc. in Electrical Engineering (Operations Research) from the Pontifical Catholic University of Rio de Janeiro (PUC-Rio), Rio de Janeiro, Brazil.

From 2003 to 2007 he participated in several projects related to strategic bidding in the Brazilian energy auctions and market regulation at the Power System Research Consulting (PSR), Rio de Janeiro, Brazil. From August 2006 to March 2007 he was a visiting researcher at the Universidad de Castilla-La Mancha, Ciudad Real, Spain. In the beginning of 2008 he joined PUC-Rio to teach Optimization in the Electrical Engineering Department as an Assistant Professor. His research interests include: strategic bidding in energy contract auctions, energy markets regulation, optimization methods, and decision making under uncertainty. He is also involved with gas-energy integrated portfolios studies and renewable energy trading strategies.

Fabrício Oliveira is a second-year Ph.D. student at the Industrial Engineering Department of the Pontifical Catholic University of Rio de Janeiro (PUC-Rio), Rio de Janeiro, Brazil. His degree in Industrial Engineering was obtained from the same university in 2008.

Since 2007, he works at the Center of Excellence in Optimization Solutions (NExO/PUC-Rio) where he has participated in several research and consulting projects. His main research interests are in optimization methods for decision making under uncertainty, especially in the oil and gas industries and power systems.

José M. Arroyo (S'96–M'01–SM'06) received the Ingeniero Industrial degree from the Universidad de Málaga, Málaga, Spain, in 1995, and the Ph.D. degree in power systems operations planning from the Universidad de Castilla-La Mancha, Ciudad Real, Spain, in 2000.

From June 2003 through July 2004 he held a Richard H. Tomlinson Postdoctoral Fellowship at the Department of Electrical and Computer Engineering of McGill University, Montreal, QC, Canada. He is currently a Full Professor of electrical engineering at the Universidad de Castilla-La Mancha. His research interests include operations, planning, and economics of power systems, as well as optimization and parallel computation.