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A
Appendix

Definition 42 *(Conditional Expectation). The conditional expectation of a nonnegative random variable ξ with respect to the σ-algebra $\mathcal{G}$ is a nonnegative extended random variable, denoted by $\mathbb{E}[\xi|\mathcal{G}]$ or $\mathbb{E}[\xi|\mathcal{G}](\omega)$, such that*

(a) $\mathbb{E}[\xi|\mathcal{G}]$ *is $\mathcal{G}$-measurable;*

(b) For every $A \in \mathcal{G}$,

$$\int_A \xi d\mathbb{P} = \int_A \mathbb{E}[\xi|\mathcal{G}]d\mathbb{P}. \tag{A-1}$$

Theorem 43 *Let $\mathcal{G}$, $\mathcal{H}$ be σ-algebras such that $\mathcal{G} \subset \mathcal{H}$. Then*

$$E[X|\mathcal{G}] = E[E[X|\mathcal{H}]|\mathcal{G}] \tag{A-2}$$

Proof: If $G \in \mathcal{G}$ then $G \in \mathcal{H}$ and therefore

$$\int_G E[X|\mathcal{H}]dP = \int_G XdP. \tag{A-3}$$

Hence

$$E[E[X|\mathcal{H}]|\mathcal{G}] = E[X|\mathcal{G}]. \tag{A-4}$$

□

Corollary 44 *(Iterated expectations)*

$$E[E[X|\mathcal{H}]] = E[X]. \tag{A-5}$$

Proof: In particular take $G = \Omega$ and we will have

$$\int_{\Omega} E[X|\mathcal{H}]dP = \int_{\Omega} XdP \tag{A-6}$$

□

Theorem 45 *(Baye's rule) Let μ and ν be two probability measures on a measurable space $(\Omega, \mathcal{G})$ such that*

$$d\nu(\omega) = f(\omega)d\mu(\omega)$$

for some $f \in L^1(\mu)$. Let X be a random variable on $(\Omega, \mathcal{G})$ such that

$$E_\nu[|X|] = \int_{\Omega} |X(\omega)|f(\omega)d\mu(\omega) < \infty \quad (X \textit{ is } \nu - \textit{integrable}) \tag{A-7}$$

Let $\mathcal{H}$ be a σ-algebra, $\mathcal{H} \subset \mathcal{G}$. Then,

$$E_\nu[X|\mathcal{H}].E_\mu[f|\mathcal{H}] = E_\mu[fX|\mathcal{H}] \quad a.s.$$

or

$$E_\nu[X|\mathcal{H}] = \frac{E_\mu[fX|\mathcal{H}]}{E_\mu[f|\mathcal{H}]} \quad a.s. \tag{A-8}$$

Proof: By the definition of conditional expectation we have that if $H \in \mathcal{H}$ then

$$\int_H E_\nu[X|\mathcal{H}]fd\mu = \int_H E_\nu[X|\mathcal{H}]d\nu = \int_H Xd\nu \tag{A-9}$$

$$= \int_H Xfd\mu = \int_H E_\mu[fX|\mathcal{H}]d\mu. \tag{A-10}$$

On the other hand, by iterated expectations we have

$$\int_H E_\nu[X|\mathcal{H}]fd\mu = E_\mu[E_\nu[X|\mathcal{H}]f\chi_H] \tag{A-11}$$

$$= E_\mu[E_\mu[E_\nu[X|\mathcal{H}]f\chi_H|\mathcal{H}]] \tag{A-12}$$

$$= E_\mu[\chi_H E_\nu[X|\mathcal{H}]E_\mu[f|\mathcal{H}]] \tag{A-13}$$

$$= \int_H E_\nu[X|\mathcal{H}]E_\mu[f|\mathcal{H}]d\mu \tag{A-14}$$

Combining (3.3) and (3.8) we get

$$\int_H E_\nu[X|\mathcal{H}]E_\mu[f|\mathcal{H}]d\mu = \int_H E_\mu[fX|\mathcal{H}]d\mu \tag{A-15}$$

Since this holds for all $H \in \mathcal{H}$,

$$E_\nu[X|\mathcal{H}]E_\mu[f|\mathcal{H}] = E_\mu[fX|\mathcal{H}] \quad a.s. \tag{A-16}$$

□