Pontifícia Universidade Católica
do Rio de Janeiro

Renato Alencar Adelino da Costa

Risk Neutral Option Pricing under some special GARCH models

Tese de Doutorado

Thesis presented to the Postgraduate Program in Engenharia Elétrica of the Departamento de Engenharia Elétrica, PUC–Rio as partial fulfillment of the requirements for the degree of Doutor em Engenharia Elétrica

Advisor : Prof. Álvaro de Lima Veiga Filho
Co–Advisor: Prof. Tak Kuen Siu

Rio de Janeiro
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Abstract

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Option pricing is a very important issue nowadays. The use of probabilistic methods is required for risk neutral pricing. Here we apply the method of Siu et al. for two classes of GARCHs, viz., the FC-GARCH and the Mixture of GARCHs.

In both models we derive the risk neutral version of the model which is essential for pricing contracts, in two different cases, when the noise is normal as well as when it is shifted gamma.

We also performed simulations with both models and compared to the benchmark Black Scholes model, checked for the smile effect and made some sensibility analysis in the parameters.

Keywords

Option Pricing. Change of measure. GARCH. FC-GARCH. Mixture of GARCHs. Conditional Esscher transform.

Resumo

Costa, Renato Alencar Adelino da; Veiga, Álvaro de Lima ; Siu, Tak Kuen. **Apreçamento neutro ao risco de opcões sob modelos GARCH especiais**. Rio de Janeiro, 2010. 103p. Tese de Doutorado — Departamento de Engenharia Elétrica, Pontifícia Universidade Católica do Rio de Janeiro.

O apreçamento de opções é um assunto muito importante nos dias de hoje. Métodos probabilisticos são necessários para fazer o apreçamento neutro ao risco. Usaremos o método de Siu et al. para duas classes de GARCHs, o FC-GARCH e a mistura de GARCHs

Em ambos os modelos nós encontramos a versão neutra ao risco do modelo que é necessária para a precificação de contratos, em dois diferentes casos, quando o ruído é normal e quando é shifted gamma.

Fizemos também simulações para ilustrar e comparamos os resultados com o valor de Black Scholes, verificamos a existência de smile e fizemos uma análise de sensibilidade nos parâmetros

Palavras–chave

Apreçamento de opções. Mudança de medida. GARCH. FC-GARCH. Mistura de GARCHs. Transformada de Esscher condicional.

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