3
The Bresnahan and Reiss Model

The objective of the model proposed by Bresnahan & Reiss (1991) is to capture cross-section variations in bank presence, resulted from a simultaneous entry game. In addition, this method allows for estimating the market size necessary to support different market structures, with the use of a limited quantity of data. The authors name the market size per firm estimated as entry-threshold per firm and, in order to compute it, a functional form assumption is necessary. Then it is possible to compute the market size that induces firms to have zero long-run profits[7].

Suppose a representative consumer initially and define demand per market as $Q = d(\boldsymbol{X}, P) \cdot S(\boldsymbol{Y})$, where $d(\cdot)$ is an individual demand function and $S(\cdot)$, a function for market size. Note that $d(\boldsymbol{X}, P)$ depends not only on $\boldsymbol{X}$, which indicates variables that shift individual demand, but also on P, the price. Note that $S(\boldsymbol{Y})$ can also vary with different factors, denoted by $\boldsymbol{Y}$. Observe that $\boldsymbol{Y}$ also affects aggregate demand. An important observation is that the price elasticity of demand is invariant to market size, which means that if we have a sample with different market sizes, the price elasticity will only vary if there is a competition regime change, given all other factors constant. This is an important characteristic, because since the objective is to make inference in market competition based on their size, having an aggregate demand invariant to size simplifies the entry threshold per firm calculations[8].

Next it is needed to specify the profit function for firm i in a market with N firms:

$$\prod\nolimits_{\mathrm{N}}^{\mathrm{i}}\left(S_N, P_N^i, F_N^i\right) = S_N \cdot d(\boldsymbol{X}, P_N^i\,)(P_N^i - AVC_N^i(\cdot)) - F_N^i \qquad (3)$$

---

[7] Note that the Bresnahan & Reiss (1991) approach assumes firms that typically offer a single and homogenous product. Since the banking sector offers a vast array of differentiated products, the use of this model ends up presenting itself as a limitation of this approach. An analysis on concentrated markets was not made as well.

[8] Bresnahan & Reiss (1991) find that this assumption does not alter the results.

PUC-Rio - Certificação Digital Nº 0812359/CA

where F_N^i is firm i's fixed costs and AVC_N^i its average variable costs. With the assumption of a representative firm, subscript i can be omitted. Moreover, note that it is still possible to include variables that represent costs of entry (fixed and variable), omitted to save notation.

Therefore, the zero profit long-run condition implies:

$$S_N = \frac{F_N}{V_N} \tag{4}$$

where V_N is the firm's per capita long-run variable profit, or $V_N = d(X, P_N) \cdot (P_N - AVC)$, when overall profit is equal to zero. As mentioned before, the per firm entry-threshold is given by $s_N = S_N / N$. It relates to the market competition variability test the following manner: as new firms enter the market, variable profits per consumer diminish and fixed costs rise, in which a larger market is needed to reach zero profits. If this value is the same in every competitive structure, then doubling market size translates to the double number of firms, keeping profits constant. This is equivalent to no market competition variability[9].

Note it is possible to build an interesting statistic from the entry-threshold per firm: the s_{N+1}/s_N ratio. It is capable of measuring precisely how competition varies as banks enter a market. If larger than unity, there will be competition intensification. Observe also that the non-linearity in competition increases can be explicitly analyzed by this variable. The more it oscillates, the more non-linear is this competitiveness intensification.

3.1. Econometric Specification

In order to estimate the model's parameters, rewrite expression (3) as $\prod_N^i(\mathrm{S}, \mathbf{X}) = \overline{\prod}_\mathrm{N}^\mathrm{i}(\mathrm{S}, \mathbf{X}) + \xi_N^i$, where $\overline{\prod}_\mathrm{N}(\mathrm{S}, \mathbf{X})$ represents all firm's i observed profits components in a market with N firms, while ξ^i represents the unobservable components.

Moreover, note that firms enter a market while economic profits are non-negative. This means, assuming profits are non-increasing in the number of firms, if two banks are observed in a market, duopoly profits are non-negative and in the perspective of a third bank entering, triopoly profits are negative.

[9] This equivalence, as mentioned in their paper, also signals evidences of cartelization in this specific market.

PUC-Rio - Certificação Digital Nº 0812359/CA

It is possible to find a general expression for the number of firms in a given market[10]:

$$N = \begin{cases} 0, & if \prod_1 < 0 \\ 1, & if \prod_2 < 0 \le \prod_1 \\ 2, & if \prod_3 < 0 \le \prod_2 \\ \vdots & \\ J-1, & if \prod_J < 0 \le \prod_{J-1} \end{cases} \qquad \text{and}$$

$$N \ge J, \qquad if\ 0 \le \prod_J$$

for a J defined afterwards.

Imposing now that the profits' unobservable components are equal and follow a standard normal distribution for every market structure, it is possible to estimate by maximum likelihood the following ordered probit:

$$p(N = 0|\mathrm{I}) = p\left(\overline{\prod}_1 < 0|\mathrm{I}\right) = \mathrm{p}\left(-\overline{\prod}_1 < \xi|\mathrm{I}\right) = 1 - \Phi(\overline{\prod}_1|\mathrm{I})$$

$$p(N = 1|\mathrm{I}) = p\left(\overline{\prod}_2 < 0 \le \overline{\prod}_1|\mathrm{I}\right) = \Phi\left(\overline{\prod}_1|\mathrm{I}\right) - \Phi\left(\overline{\prod}_2|\mathrm{I}\right)$$

$$\vdots$$

$$p(N = J|\mathrm{I}) = \Phi\left(\overline{\prod}_J|\mathrm{I}\right), \tag{5}$$

where I is defined as the explanatory variables.

It is needed to specify a functional form for the bank's observed components. Suppose bank homogeneity, so that we can omit the superscript i. For every market j,

$$\overline{\prod}_{N,j}\left(S_j, \boldsymbol{Q}_j\right) = S_j \cdot L_{N,j}\left(\boldsymbol{Q}_j\right) - F_{N,j}(\boldsymbol{W}_j) \tag{6}$$

in which $L_{N,j}$ represents the banks' per capita variable profits in market j with N banks[11], $\boldsymbol{Q}_j$ all factors that affect every bank's variable profit and $\boldsymbol{W}_j$, the ones that affect bank costs. Observe that, since it is hard to obtain price and costs data, we must assume an alternative specification for the firm's per capita profits (omitting j to save notation):

$$L_N(D, \boldsymbol{Q}) = \alpha_1 + \textstyle\sum_{n=2}^{N} D_n \cdot \alpha_n + \beta \cdot \boldsymbol{Q} \tag{7}$$

where D_n is a binary variable equal to 1 when there are n banks in the market in question, $\alpha_1 \ge 0$ and $\{\alpha_n\}_2^N \le 0$. These two hypotheses are made in order to obtain decreasing profits in the number of firms.

[10] Observe that working with the number of firms in a market and not differentiating between each firm's identity allows us to ignore regions of multiple equilibria.

[11] It would be identical to V_N^j,except it is not associated to long-run zero profits.

PUC-Rio - Certificação Digital Nº 0812359/CA

Analogously, we can specify a fixed costs function in a market with N firms as:

$$F_N = \sum_{n=1}^{N} \widetilde{D}_n \cdot \gamma_n \ + \eta \cdot \boldsymbol{W} \tag{8}$$

where $\widetilde{D}_n$ is similar to the binary variable described above and $\{\gamma_N\}_1^N \geq 0$, also necessary for non-increasing profits in the market's number of banks. Furthermore, this specification also includes the fact that banks entering in already existent structures incur higher fixed costs.

Finally, it is also possible to specify a functional form for market size:

$$S(\boldsymbol{Y}) = S_0 + \lambda \cdot \boldsymbol{Y}, \tag{9}$$

where $\boldsymbol{Y}$ are the factors that affect it.

3.1.1.
Entry-Threshold per firm Calculation

In light of equation (4), entry-threshold per firm is obtained from the following equation:

$$\hat{s}_N = \frac{1}{N} \cdot \frac{\hat{F}_N}{\hat{L}_N} = \frac{1}{N} \cdot \frac{\sum_{n=1}^{N} \widetilde{D}_n \cdot \hat{\gamma}_n \ + \hat{\eta} \cdot \overline{\boldsymbol{W}}}{\hat{\alpha}_1 + \sum_{n=2}^{N} \hat{\alpha}_n + \hat{\beta} \cdot \overline{\boldsymbol{Q}}}, \tag{10}$$

Observe that the mean values of $\boldsymbol{W}$ and $\boldsymbol{Q}$ will be calculated for every market structure, which means for a variable W_A, $\hat{s}_1$ will use the mean value of W_A conditional on municipalities that have only one bank.

PUC-Rio - Certificação Digital Nº 0812359/CA