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Comments on "System Zeros Determination from an Unreduced Matrix Fraction Description"[1]

Pedro M. G. Ferreira

Abstract—**The main result of the above paper[1] is a straightforward consequence of the general definition of system zeros by Rosenbrock [1],[2].**

The authors of the above paper[1] seem to be unaware of the general definition of system zeros given by Rosenbrock [1]. We show below that their main result follows in a straightforward way from that definition.

Let the relationship between the Laplace-transforms of the input and output of a linear time-invariant system be given by

$$G(s) = V(s)T^{-1}(s)U(s) + W(s) \tag{1}$$

where V, T, U, and W are polynomial matrices, V is $m \times n$, T is $n \times n$, U is $n \times p$. V and T are not necessarily right coprime, and T and U are not necessarily left coprime.

The system matrix [2] of (1) is

$$P(s) = \begin{bmatrix} T(s) & U(s) \\ -V(s) & W(s) \end{bmatrix}. \tag{2}$$

Let $n + k$ be the normal rank of $P(s)$, $k = \min(p, m)$.

Definition [1]: Consider a minor of $P(s)$ formed with the rows of order $1, 2, \cdots, n, n+i_1, n+i_2, \cdots, n+i_k$ and the columns of order $1, 2, \cdots, n, n+j_1, n+j_2, \cdots, n+j_k$. The system zeros of (1) are the zeros of the greatest common divisor of all these minors. □

Consider now the system in the form studied by the above paper, namely

$$G(s) = V(s)T^{-1}(s) \tag{3}$$

where it is assumed that $m \geq n (= p)$. It is clear that

$$P(s) = \begin{bmatrix} T(s) & I \\ -V(s) & 0 \end{bmatrix}.$$

Furthermore, the minors of this matrix as specified in the definition above are

$$\begin{vmatrix} T(s) & I \\ -V(s)_{i_1, i_2, \cdots, i_n} & 0 \end{vmatrix}$$

where $|\cdot|$ means determinant and $V(s)_{i_1, i_2, \cdots, i_n}$ is formed with the rows of order $i_1, i_2, \cdots, i_n$ of $V(s)$. Through elementary column operations on the above minor, it is easy to see that it is equal to (up to a change of sign) $|V(s)_{i_1, i_2, \cdots, i_n}|$. Hence the system zeros of (3) are the zeros of the greatest common divisor of all these minors. Or, in other words, the system zeros of (3) are the common roots of all the $p \times p (= n \times n)$ determinants contained in $V(s)$, which is the main result of the above paper.[1]

If $m \leq p$, we take a left factorization of the transfer matrix, namely, $G(s) = T^{-1}(s)U(s)$, and proceed as above; the system zeros will then be the common roots of all the $m \times m (= n \times n)$ determinants contained in $U(s)$.

Finally we note that even though the authors assume that the system is strictly proper, this assumption is not necessary either in our proof or in theirs.

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[2] H. H. Rosenbrock, *State-Space and Multivariable Theory.* London: Nelson, 1970.

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