3
Optimal Informational Interest Rate Rule

3.1.
Introduction

Any public policy may be understood as a public signal of the current state of the economy as it informs the views of the governmental authority to all agents. This paper consider the case where the interest rate is informative about aggregate nominal demand - the fundamental of our economy - to set light to two main issues: (i) how this informational effect changes price setting and, by consequence, inflation dynamics and (ii) what would be an optimal informational interest rate rule.

We consider a model where the central bank has a noisy signal of aggregate nominal demand growth. Based on this information, it sets the interest rate, which affects the dynamics of the fundamental. As firms needs to infer aggregate nominal demand to set their prices and they do not observe the information central bank has, when they observe an interest rate rise, they consider two opposite movements: (i) aggregate nominal demand is expected to rise and (ii) aggregate nominal demand will depress by the influence of the policy instrument. We name the first effect as the *informational power of interest rate* as it reveals the views of the monetary authority.

To evaluate this effect on prices, and by consequence, on inflation, we study price setting in a model where firms have heterogeneous information and face strategic complementarity on their actions. In this environment, firms choose prices knowing that their payoffs depend not only on their own actions, but also on the prices the other firms choose. Firms try to predict one another's actions using the information they have. We assume that information is both sticky (as in Mankiw and Reis (2002), each period only a fraction of the firms updates their information sets) and dispersed (as in Morris and Shin (2002), firms receive private signals of the fundamental when they update). Furthermore, the policy instrument becomes a public signal since it is observable to all firms, including those that have not been selected to update the information set.

We isolate the informational power of interest rate by studying how (equilibrium) price setting changes when firms ignore the fact that the monetary authority reveals information when it chooses the policy instrument. This strategy allows us to measure the influence of informational power of interest rate on welfare, evaluated by three different measures - inflation variance, ex-ante total profit and cross-sectional dispersion. Besides, it is possible to find an *optimal informational interest rate rule*, which is characterized by the parameters of the policy instrument that maximizes welfare, considering that *central bank knows that firms will take information from its actions*.[22]

The main consequence to inflation dynamics regards the persistence of monetary shocks. Without the informational effect, firms would understand changes in the policy instrument as isolated movements, instead of a response to changes in the fundamental. Under this short sight, interest rate affects inflation only instantaneously. In contrast, when interest rate is understood as a public signal of the fundamental, the whole realization of interest rate affects inflation, making the monetary shock persistent.

We also show that there is an optimal informational interest rule that simultaneously maximizes all three welfare criteria. To implement this rule, central bank should avoid adding any monetary shock to the policy instrument, while should use the level of informational response that minimizes inflation variance, one of our welfare measures.

We introduce our basic model in the next section. In Section 3.3 we give the basic steps we use to compute the equilibrium, considering that its derivation is analogous to the one described in chapter 2. In Section 3.4, we define the informational effect and build a counterfactual in other to isolate it. We also present some analyses based on how price setting changes when the informational effect is considered. The core of the paper is Section 3.5, where we use three different welfare criteria to measure the informational effect and analyze its

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[22]Our terminology *optimal informational interest rate rule* should not be misunderstood. By this expression, we do not mean finding which variables the monetary authority should look to when it wants to minimize a welfare criterion. For this approach, see Woodford (2003).

quantitative implications. We provide concluding remarks in Section 3.6 and proofs omitted in the main text in the Appendix.

3.2. The Model

Pricing Decisions

There is a continuum of firms indexed by $z \in [0,1]$. Every period $t \in \{1,2,\dots\}$, each firm z chooses a price $p_t(z)$. We can derive from a model of monopolistic competition à la Blanchard and Kiyotaki (1987) that the (log-linear) price decision that solves a firm's profit maximization problem, p_t^*, is given by

$$p_t^* = rP_t + (1-r)\theta_t, \qquad (3.1)$$

where r is the degree of strategic complementarity, $P_t \equiv \int_0^1 p_t(z)dz$ is the aggregate price level of the economy, and $\theta_t \in \mathbb{R}$ is a relevant fundamental. In our framework, we can interpret θ_t as being the current state of aggregate nominal demand of our economy.

Fundamental Dynamics

In this paper, we consider the case where the dynamics of θ_t is partially driven by a policy instrument, the interest rate i_t, that respond to the noisy information central bank has about changes in the state, y_t. That is

$$\theta_t = \theta_{t-1} - \sigma i_t + \varepsilon_t, \qquad (3.2)$$

$$i_t = \phi y_t + v_t, \qquad (3.3)$$

$$y_t = (\theta_t - \theta_{t-1}) + \eta_t, \qquad (3.4)$$

where the errors are independent of one another - $\varepsilon_t \perp \eta_{t+k} \perp v_{t+m}, \forall(t,k,m)$ - and distributed according to $\varepsilon_t \sim N(0,\alpha^{-1})$, $\eta_t \sim N(0,\gamma^{-1})$, and $v_t \sim N(0,\mu^{-1})$. The noise term ε_t may represent demand shock affecting the current state of the economy, while the noise v_t is a policy disturbance as, for example, a monetary policy shock. Finally, the shock η_t reflects that central banks' information about the state is also imprecise.

Information

As in Mankiw and Reis (2002), information is *sticky*. Every period only a fraction $\lambda \in (0,1)$ of firms receives new information about the fundamental. The probability of being selected to adjust information is the same across firms and independent of history. However, as in chapters 1 and 2, we depart from this standard sticky-information structure, by allowing information to be also *dispersed.* As before, following Morris and Shin (2002) and Angeletos and Pavan (2007), we assume that, when a firm updates its information set, it receives not only information regarding the past states of the economy,

$$\Theta_t = \{\theta_{t-k}\}_{k=1}^{\infty},$$

but also a *private* signal about the current state,

$$x_t(z) = \theta_t + \xi_t(z).$$

As before, we assume that $\xi_t(z) \sim N(0, \beta^{-1})$ is independent of all other errors. Furthermore, for each firm z, $\xi(z)$ is a idiosyncratic shock. That is, $\xi_t(z_1) \perp \xi_{t+k}(z_2)$, $\forall (t, k, z_1, z_2)$.

We can combine equations (3.3) and (3.4) to show that interest rate is a public signal of the fundamental's change, which is available to all firms, including those who have not been selected to update their information sets. As a result, the information set of a firm z that was selected to update its information j periods ago is

$$\Im_{t-j}(z) = \{x_{t-j}(z), \Theta_{t-j-1}, I_t\},$$

where $I_t = \{i_{t-k}\}_{k=0}^{\infty}$. The introduction of a public signal on a sticky-dispersed information model has already been studied in chapter 2. Nevertheless, now the public signal is a policy instrument that also interferes on the dynamics of the fundamental. This fact changes how firms compute the equilibrium.

3.3. Equilibrium

In equilibrium, a firm z that updated its information set at period $t - j$ chooses

$$p_t(z) = E[p_t^* \mid \Im_{t-j}(z)]. \tag{3.5}$$

From (3.1), it is clear that firm z will have to predict not only the current state θ_t, but also the aggregate price level P_t. As P_t encompasses the prices set by other firms, firm z must also predict the behavior of the other firms in the economy by forecasting other firms' forecasts about the state, forecasting the forecasts of other firms' forecasts about the state, and so on and so forth. This explanation highlights the importance of computing order beliefs to find the equilibrium. However, because of the linearity of the best-response condition (3.1) and the Gaussian specification of the information structure, the equilibrium prices are linear combinations of the observed signals. As a result, it is possible to obtain the unique linear equilibrium of this game using the much simpler approach of matching coefficient. In chapters 1 and 2, we use both methods.

Reduced Form

In order to derive the equilibrium exactly in the same way as we did in chapter 2, we rewrite this model as

$$\theta_t = \theta_{t-1} + u_t, \tag{3.6}$$

$$i_t = \phi\,(\theta_t - \theta_{t-1}) + \phi\, e_t, \tag{3.7}$$

where

$$e_t \equiv \eta_t + \phi^{-1} v_t \ \text{ and } \ u_t \equiv \frac{\varepsilon_t - \sigma\phi e_t}{1+\sigma\phi}. \tag{3.8}$$

The term u_t aggregates the policy shocks, while the noise e_t encompasses all the shocks affecting the state. As ε_t is independent of e_t, we obtain that $e_t \sim N\left(0, \omega^{-1}\right)$, and $u_t \sim N\left(0, \varphi^{-1}\right)$ where

$$\omega^{-1} = \gamma^{-1} + \left(\phi^2 \mu\right)^{-1} \ \text{ and } \ \varphi^{-1} \equiv \left(\frac{1}{1+\sigma\phi}\right)^2 \left[\alpha^{-1} + (\sigma\phi)^2 \omega^{-1}\right]. \tag{3.9}$$

Although rewriting our information structure as (3.6) and (3.7) makes our model similar to the one studied at chapter 2, there are important differences to consider. First, we can analyze the impact of the policy parameter ϕ, while in chapter 2 we fix $\phi = 1$. However, the main difference between them lies on the fact that u_t is not independent of e_t. Although this fact is not surprising, since it captures the endogeneity of the variables, it affects the way firms compute their beliefs about the fundamental and, consequently, alters the equilibrium.

Expectations

It is important to understand how a firm z that updated its information set at $t-j$ computes its beliefs about a fundamental θ_{t-m}. Since at the moment it adjusts its information set the firm observes all previous states, Θ_{t-j-1}, the firm will know for sure the value of θ_{t-m} when $m > j$. Therefore, $E[\theta_{t-m} \mid \Im_{t-j}(z)] = \theta_{t-m}$. For $m \leq j$, θ_{t-m} is not in the information set of firm z. However, it knows that

$$\theta_{t-m} = \theta_{t-j-1} + \sum_{i=m}^{j} u_{t-i}.$$

Since $\theta_{t-j-1} \in \Im_{t-j}(z)$, it computes $E[\theta_{t-m} \mid \Im_{t-j}(z)]$ as

$$E[\theta_{t-m} \mid \Im_{t-j}(z)] = \theta_{t-j-1} + \sum_{i=m}^{j} E[u_{t-i} \mid \Im_{t-j}(z)].$$

As the process is Markovian, past values of θ does not help to predict u_{t-i}. Therefore, the only signals of u_{t-i} the firm can build from $\Im_{t-j}(z)$ are

$$w_{t-k} \equiv \phi^{-1} i_{t-k} = u_{t-k} + e_{t-k} \text{ and}$$
$$t_{t-j}(z) \equiv x_{t-j}(z) - \theta_{t-j-1} = u_{t-j} + \xi_{t-j}(z).$$

But, u_{t-i} is independent of w_{t-k}, $\forall k \neq i$, and of $t_{t-j}(z)$, if $i \neq j$. Therefore,[23]

$$\begin{aligned} E[\theta_{t-m} \mid \Im_{t-j}(z)] &= \theta_{t-j-1} + E[u_{t-j} \mid w_{t-j}, t_{t-j}(z)] + \sum_{i=m}^{j-1} E[u_{t-i} \mid w_{t-i}] \\ &= \left(1-\hat{\delta}\right) x_{t-j} + \hat{\delta}\theta_{t-j-1} + \hat{\delta}\hat{\kappa} i_{t-j} + \hat{\kappa} \sum_{i=m}^{j-1} i_{t-i}, \end{aligned} \tag{3.10}$$

where

$$\hat{\delta} = \left(\frac{\alpha+\omega}{\alpha+\omega+\beta}\right) \quad \text{and} \quad \hat{\kappa} = \phi^{-1}\left(\frac{\omega - \alpha\sigma\phi}{\alpha+\omega}\right). \tag{3.11}$$

The first three components present in the expectation represent $E[\theta_{t-j} \mid \Im_{t-j}(z)]$ and can be expressed as a convex combination of private and public information. That is,

$$\theta_{t-j} \mid x_{t-j}(z), r_{t-j}, s_{t-j} \sim N\left((\alpha+\beta+\gamma)^{-1}[\beta x_{t-j}(z) + \omega r_{t-j} + \alpha s_{t-j}], (\alpha+\beta+\omega)^{-1}\right),$$

where $r_{t-j} \equiv \theta_{t-j-1} - \sigma i_{t-j} = \theta_{t-j} - \varepsilon_{t-j}$, and $s_{t-j} \equiv \phi^{-1} i_{t-j} + \theta_{t-1} = \theta_{t-j} + e_{t-j}$ are two signals of the fundamental θ_{t-j}. As the errors associated to each of these signals are independent of one another, the weights represent the relative precision

[23] See Appendix A for details.

associated to each of these signals. This standard result is present in the models of Morris and Shin (2002) and Angeletos and Pavan (2007). The last term shows how to build expectations for θ_{t-m}, when $m < j$. The weight κ captures the importance of u_{t-k} on the signal $w_{t-k} = u_{t-k} + e_{t-k}$. It is worth noting that κ is affected by the public and policy precisions, α and ω, as well as the policy and structural parameters, ϕ and σ. However, as $x_{t-j}(z)$ is not informative about u_{t-i}, when $i < j$, κ does not depend on the precision of private information, β. In summary, a firm z that last updated its information set j periods ago has the following forecasts about the state θ_{t-m} of the economy

$$E[\theta_{t-m} \mid \Im_{t-j}(z)] = \begin{cases} (1-\hat{\delta})x_{t-j}(z) + \hat{\delta}\theta_{t-j-1} + \hat{\delta}\hat{\kappa} i_{t-j} + \hat{\kappa}\sum_{i=m}^{j-1} i_{t-i} & : m \le j \\ \theta_{t-m} & : m > j, \end{cases} \tag{3.12}$$

which are used to compute the linear equilibrium of the model.

Linear Equilibrium

In chapter 2, we use an expression analogous to (3.12) to derive the unique linear equilibrium of the game. The expression for the equilibrium price index is $\hat{P}_t \equiv P_t(\hat{\delta}, \hat{\kappa})$ where

$$P_t(\delta,\kappa) = \sum_{k=0}^{\infty} c_k \theta_{t-k} + \sum_{k=0}^{\infty} d_k i_{t-k}. \tag{3.13}$$

and the coefficients (c_k, d_k) are functions of (δ, κ) given by[24]

$$\rho(\delta) = 1 - \lambda(1-\delta),$$

$$c_k(\rho(\delta)) = \begin{cases} \left(\frac{1-r}{r}\right)\left[\frac{1}{1-r(1-\rho)} - 1\right] & \text{if } k = 0 \\ \left(\frac{1-r}{r}\right)\left[\frac{1}{1-r\left[1-\rho(1-\lambda)^k\right]} - \frac{1}{1-r\left[1-\rho(1-\lambda)^{k-1}\right]}\right] & \text{if } k \ge 1, \end{cases} \tag{3.14}$$

[24]The terminology (c_j, d_j) is a notational abuse, since it does not refer to a pair of generic coefficients, but to a pair of sequences $\left(\{c_j\}_{j=0}^{\infty}, \{d_j\}_{j=0}^{\infty}\right)$. However, we use it throughout the text.

$$d_k(\rho(\delta),\kappa) = \kappa\left[\frac{\rho(1-\lambda)^k}{1-r+r\rho(1-\lambda)^k}\right]. \tag{3.15}$$

Equation (3.13) shows that the price index (and, by consequence, inflation) depends not only on the realizations of θ, but also on the realization of i. The presence of the policy instruments on prices is a not exactly new result. For instance, Ravenna and Walsh (2006) studied the cost channel. A cost channel is present when firms have to finance their productions, making marginal cost depending directly on the nominal rate of interest. Nevertheless, not only this result comes from a different source, the informational power of interest rate, but it also has a much more permanent effect, since the whole realization of i appears on (3.13). Form (3.15), when stickiness is very high (λ is small), this persistence becomes more relevant.

3.4. Informational Effect

In other to measure the informational effect, it is important to obtain the dynamics of P_t when firms *observe the interest rate but they do not see it as a public signal.*

3.4.1. Counterfactual

If firms ignored that interest rate is informative about the current state of the economy, the dynamics of P_t would change because firms would modify the way they compute $E\lfloor\theta_{t-m} \mid \Im_{t-j}(z)\rfloor$. For $m \leq j$, a firm z that updated its information set at period $t-j$ will obtain $E\lfloor\theta_{t-m} \mid \Im_{t-j}(z)\rfloor$ from

$$\theta_{t-m} = \theta_{t-j} - \sigma \sum\nolimits_{i=m}^{j-1} i_{t-i} + \sum\nolimits_{i=m}^{j-1} \varepsilon_{t-i}.$$

Firm z has two signals of θ_{t-j} : a private signal

$$x_{t-j}(z) = \theta_{t-j} + \xi_{t-j}(z),$$

and a public signal composed by past information about the fundamental and the policy instrument

$$q_t \equiv \theta_{t-j-1} - \sigma i_{t-j} = \theta_{t-j} - \varepsilon_{t-j}.$$

If firms did not consider i_{t-k} informative about ε_{t-k}, $\forall k < j$,

$$\begin{aligned} E[\theta_{t-m} \mid \Im_{t-j}(z)] &= E[\theta_{t-j} \mid \Im_{t-j}(z)] - \sigma \textstyle\sum_{i=m}^{j-1} i_{t-i} \\ &= \left(1-\tilde{\delta}\right) x_{t-j}(z) + \tilde{\delta} q_t - \sigma \textstyle\sum_{i=m}^{j-1} i_{t-i} \\ &= \left(1-\tilde{\delta}\right) x_{t-j}(z) + \tilde{\delta}\theta_{t-j-1} + \tilde{\delta}\tilde{\kappa} i_{t-j} + \tilde{\kappa} \textstyle\sum_{i=m}^{j-1} i_{t-i}, \end{aligned} \tag{3.16}$$

where

$$\tilde{\delta} = \frac{\alpha}{\alpha+\beta}, \text{ and } \tilde{\kappa} = -\sigma.$$

It is clear that expression (3.16) is the same of (3.10), when we have $\omega = 0$ in (3.11). This result has an easy interpretation: ignoring the informational power of interest rate is equivalent to saying that interest rate is not informative about the current state of the economy (i.e., the variance associated to the interest rate is infinity). From (3.16), we obtain the expression for (the equilibrium) price index,

$$\tilde{P}_t = P_t\left(\tilde{\delta}, \tilde{\kappa}\right), \tag{3.17}$$

where P_t is the function specified in (3.13).

Two main facts come straightforward from the comparison of $(\hat{\delta}, \hat{\kappa})$ with $(\tilde{\delta}, \tilde{\kappa})$: (i) $\hat{\delta} > \tilde{\delta}$ and (ii) $\tilde{\kappa}$ *is* negative, while $\hat{\kappa}$ *can be* negative. The first observation tells us that the private signal becomes less important when agents consider that interest rate is informative about the current state. That is, the informational power of interest rate makes public information more valuable (relative to private information) to agents. This result resembles the one obtained in Morris and Shin (2002).

Using (3.15), the second observation shows that interest rate have a positive impact on prices when $\hat{\kappa} > 0$ (or equivalently, $\omega > \alpha\sigma\phi$). Therefore, if interest rate is very informative about the current state of the economy (i.e., the precision of i_t, ω, is sufficiently high), an interest rate upturn is understood as a rise on the aggregate nominal demand, inducing firms to raise their prices. In this context, the informational power of interest rate is more important than its capability of reducing aggregate nominal demand. This is clear the case when σ is small, since the policy instrument will have a small impact on the aggregate demand.

The analysis for ϕ is not so obvious. We can used (3.8) to rewrite the condition $\hat{\kappa} > 0$ as $F(\phi) < 0$, where

$$F(\phi) = (\alpha\sigma\mu)\phi^2 - (\mu\gamma)\phi + \gamma\alpha\sigma. \tag{3.18}$$

Because $F(\phi)$ is convex, when there is no real root for $F(\phi)$ (condition $\mu\gamma < (2\alpha\sigma)^2$), for no value of ϕ interest rate will have a positive impact over prices. However, when $F(\phi)$ has two real roots ($\gamma\mu > (2\alpha\sigma)^2$), we have $\hat{\kappa} > 0$ when $\phi \in (r_1, r_2)$, where r_1, r_2 are the roots of $F(\phi)$. As both roots are positive, ϕ should be small enough ($\phi \in (0, r_1)$) or high enough ($\phi \in (r_2, \infty)$), if the monetary authority does no wants the informational power of interest rate to be dominant over its capability of reducing aggregate nominal demand. When ϕ is high, the policy instrument becomes strong enough to overcome the difficulty imposed by the informational power of interest rate. When ϕ is small, the policy instrument is just not informative about movements on aggregate nominal demand (at the limit case, $\phi = 0$, the policy instrument becomes a white noise, $i_t = v_t$). As a consequence, firms pay no attention on the public signal.

3.5. Inflation Dynamics

In order to analyze the impact of the informational power of interest rate on inflation, we use the expressions for $\hat{P}_t$ and $\tilde{P}_t$ to derive inflation dynamics and study how its response to shocks is sensitive to the parameters of the model.

Inflation

As $\hat{P}_t$ and $\tilde{P}_t$ differ only by the coefficients (c_k, d_k), we can express inflation as $\hat{\pi}_t = \pi_t(\hat{\delta}, \hat{\kappa})$ and $\tilde{\pi}_t = \pi_t(\tilde{\delta}, \tilde{\kappa})$, where π_t is written as a combination of independent shocks given by

$$\begin{aligned}\pi_t(\delta,\kappa) &= P_t - P_{t-1} = \textstyle\sum_{j=0}^{\infty} c_j(\theta_{t-j}-\theta_{t-j-1}) + \sum_{j=0}^{\infty} d_j(i_{t-j}-i_{t-j-1}) \\ &= \textstyle\sum_{j=0}^{\infty} c_j u_{t-j} + \phi\sum_{j=0}^{\infty} d_j(u_{t-j}+e_{t-j}-u_{t-j-1}-e_{t-j-1}) \\ &= \left(\frac{1}{1+\sigma\phi}\right)\left[\textstyle\sum_{j=0}^{\infty}(c_j+\phi l_j)\varepsilon_{t-j} + \phi\sum_{j=0}^{\infty}(l_j-\sigma c_j)e_{t-j}\right]\end{aligned} \tag{3.19}$$

and the coefficients (c_k, d_k) are given by (3.14) and (3.15), while

$$l_k(\delta,\kappa) = \begin{cases} d_0 & = \kappa(1-c_0) \quad \text{, if } k=0 \\ d_k - d_{k-1} & = -\kappa c_k \quad \text{, if } k \geq 1. \end{cases}$$

Using this expression, we can analyze how inflation dynamics changes by the informational power of the interest rate.

Calibration

The model's structural parameters are r, λ, α, β, γ, μ, ϕ, and σ. Following Mankiw and Reis (2002), we use $\lambda = 0.25$ and $r = 0.9$ as our baseline values (see Table 3.1).

Table 3.1: Baseline calibration

Parameter	Description	Range	Benchmark Value
r	Degree of strategic complementarity	$[0,1]$	0.90
λ	Degree of informational stickiness	$[0,1]$	0.25
α	Precision of the aggregate demand shock ε_t	$\mathbb{R}_+$	1.00
β	Precision of the private information	$\mathbb{R}_+$	1.00
γ	Precision of of the information available to central bank	$\mathbb{R}_+$	1.00
μ	Precision of monetary shock	$\mathbb{R}_+$	1.00
ϕ	Central bank's informational response coefficient	$\mathbb{R}$	1.00
σ	Elasticity of the fundamental with respect to interest rate	$\mathbb{R}$	0.67

The value $\lambda = 0.25$ implies that firms adjust their private information once a year, which is compatible with the most recent microeconomic evidence on price-setting.[25] For the remaining parameters, we set $\alpha = \beta = \gamma = \mu = 1$ as our

[25] See, for example, Klenow and Malin (2010).

benchmark value to keep the baseline calibration as neutral as possible regarding the relative importance of each type of information.

To highlight how the informational power of interest rate changes inflation dynamics, we vary four parameters of the model. More specifically, we chose the parameters associated to the precision ω (i.e., γ, μ, and ϕ) and the primitive parameter σ. The importance of ω to the informational effect is clear, since the parameters ($\tilde{\kappa}$, $\tilde{\delta}$) and ($\hat{\kappa}$, $\hat{\delta}$) differ solely by whether $\omega = 0$ or not. Besides, precision ω encompasses all policy dimensions, including the precision of policy information γ, policy instrument shocks μ, and the degree to which policy responds systematically to information ϕ. The importance of the primitive parameter σ comes from the fact that it represents the endogeneity of the variables. When $\sigma = 0$, the interest rate does not interfere on the dynamics of the fundamental, although it continues to be a public signal of the fundamental growth.

Impulse response

We study inflation's impulse responses to two types of shocks - ε_t and e_t. From (3.2), ε_t is a shock that has a direct impact on the aggregate nominal demand, while, from (3.7), e_t is a composite policy shock that affects the fundamental through i_t.

Figure (F1a) shows how inflation's responses to a demand shock ε_t changes with the parameters of the model - ϕ, σ, γ, and μ.

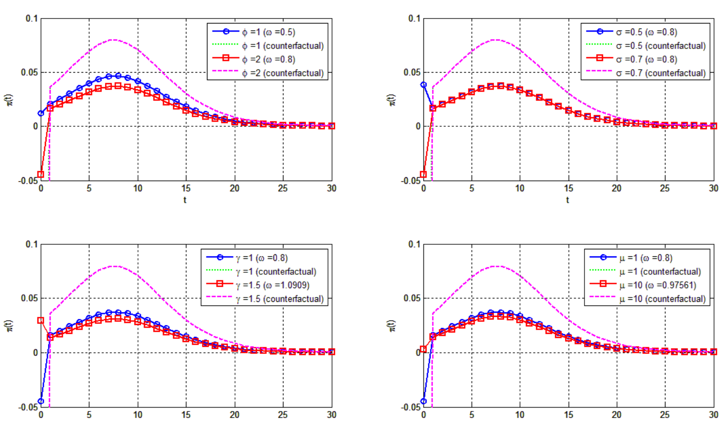


Figure 3.1: Inflation's impulse responses to a (unit) shock ε_t

Some patterns appear in the four graphs. For all parameters, when firms consider that interest rate is informative about the state of the economy, the response is attenuated. This result suggests that, when firms are better informed, price setting becomes more sensible, as firms estimate better the magnitude of the shock. Furthermore, all four graphs show the same pattern for the counterfactual: inflation drops hugely at $t=0$ and becomes positive afterwards. The rationale behind this observation is simple: if firms do not consider the interest rate informative about the current state, they infer that the interest rate rise they observe will make inflation drop. This behavior is stronger at $t=0$, when no firm has information about the state θ_t, but gradually vanishes over time as firms get informed about the state and correctly identify the occurrence of a positive shock on aggregate demand. When firms consider the interest rate informative about the current state, we do not observe a strong drop at $t=0$, since the informational power of interest rate makes firms correctly predict that the rise they observe at interest rate can come from a positive shock on aggregate nominal demand. For some set of parameters, inflation can be even positive at $t=0$.

It is also important to analyze the influence of each parameter separately. Considering that firms take information from interest rate, inflation increases less when: (i) central bank responds more aggressively to the fundamental growth (higher values of ϕ), (ii) the precision of the information central bank has (γ) is higher or (iii) the monetary shock has smaller variance. However, changes in the elasticity of the fundamental with respect to interest rate (σ) modify inflation's impulse response only at $t=0$. We can derive this result analytically: using the definition of l_j and the constant $\hat{\kappa}$ in equation (3.19), we see that both $c_j+\phi l_j$ and $l_j-\sigma c_j$ do not depend on σ, $\forall j \geq 1$.

When we analyze the counterfactual, we see that it does not move with any of these parameters when $t \geq 1$. Although not plotted in Figure (F1a), inflation drops more intensively at point $t=0$ when ϕ and σ increases, while it remains unchanged for variations of γ and μ. As firms ignore the informational power of interest rate, parameters strictly associated to the precision of interest rate, ω, do not move inflation on the counterfactual case. As parameters ϕ and σ are associated to the transmission of the policy instrument to inflation, they move inflation when no firm has information about the shock. Afterwards, however, firms do not consider that interest rate will continue to rise, since firms do not see interest rate as a response to the information central bank has about the fundamental growth.

Figure (F2a) shows how inflation's responses to a demand shock e_t changes with the parameters of the model - ϕ, σ, γ, and μ.

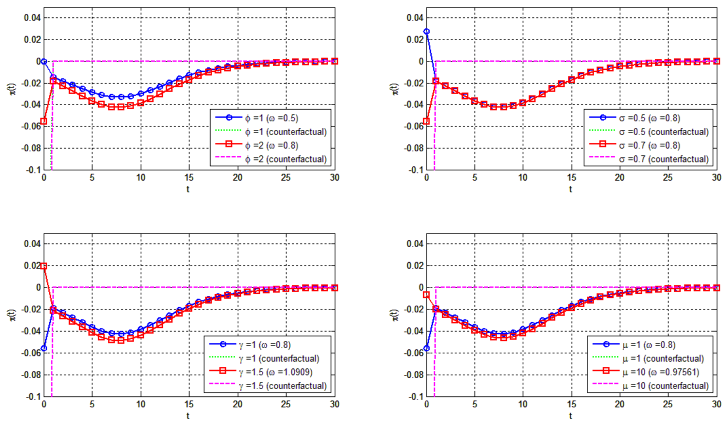


Figure 3.2: Inflation's impulse responses to a (unit) shock e_t.

The first important observation we take from Figure (F2a) is that shock e_t is not persistent when firms ignore the fact that interest rate is informative about the state of the economy. This observation is easy to understand analytically: defining $\tilde{l}_j \equiv l_j\left(\tilde{\delta}, \tilde{\kappa}\right)$ and $\tilde{c}_j \equiv l_j\left(\tilde{\delta}, \tilde{\kappa}\right)$, we have $\tilde{l}_j - \sigma\tilde{c}_j = 0$, $\forall j \geq 1$ in equation (3.19). As interest rate is not persistent by itself, persistence on interest rate comes from the fact that central bank is reacting to changes in the fundamental, which evolves according to a Markovian process. If firms did not see the interest rate as a public signal about the fundamental, they would never incorporate this inertial behavior in their forecasts. This short-sight behavior generates the same pattern in all four graphs when we analyze the counterfactual cases: inflation hugely drops at $t = 0$, and becomes null afterwards. It is important to stress that what creates persistence on inflation is not how central bank is reacting, but rather how firms change price setting when they understand that changes in interest rate are persistent.

When we analyze the influence of each parameter separately, we observe that inflation becomes less negative for smaller values of ϕ. The impulse response does not move with σ, when we have $t \geq 1$.

We define the *informational effect over a variable* as the difference that occurs on this variable when we replace (3.13) with (3.17). We are going to analyze three different criteria to measure the informational effect: (i) inflation variance; (ii) cross-sectional price dispersion, and (iii) ex ante aggregate profit of the firms.

3.6. Efficiency Criteria

The first criterion we analyze is inflation variance. Woodford (2003) derives a welfare based loss function as the second order approximation of the utility function of a representative household. On a standard sticky prices model à la Calvo (1983), this loss function is a weighted average of squared output gap and inflation. This means that any central bank that wants to minimize the expected value of this welfare based loss function should care about inflation variance. Although output gap is also present in this loss function, Woodford (2003) shows that, under standart parameters values, its weight is very small compared to inflation's. As a result, the variance of inflation is as a proxy of welfare for a vast literature of monetary models.

Nevertheless, as we do not consider the existence of price rigidity in our model, it would not be optimal to pursue the minimization of inflation variance as the policy objective. Ball et al. (2005) show that the welfare based policy objective when there is *informational* rigidity is to minimize a cross-sectional dispersion. Therefore we consider this our second efficiency criterion.

Finally, following the efficiency benchmark for dispersed information models proposed by Angeletos and Pavan (2007), we consider *ex-ante* aggregate profits as our third optimal criterion. Under this criterion, welfare is evaluated from the perspective of firms.

To highlight the informational effect, we show how these three criteria evolve with the four parameters ϕ, σ, γ, and μ. We have already discussed the importance of these parameters for the model.

We also obtain the parameters of the interest rule that minimizes those criteria. We assume that central bank cannot change γ, the precision of the policy information y_t. Nevertheless, we consider that the central bank can choose not only ϕ, the central bank's informational response coefficient, but also μ, the precision of the monetary shock.

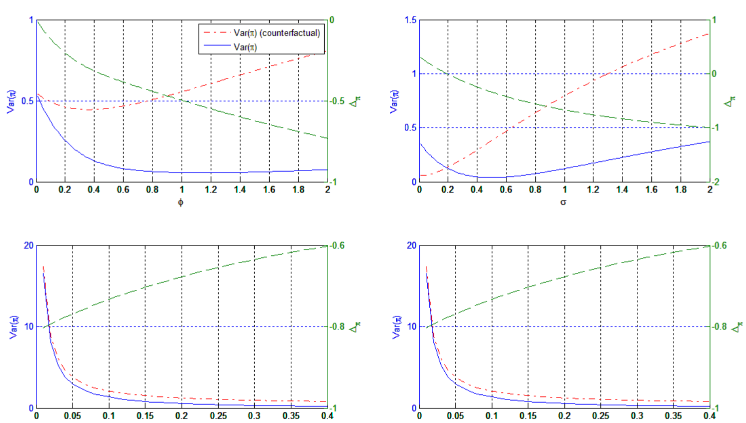
3.6.1. Inflation Variance

Considering expression (3.19), the variance of inflation can be written as

$$var(\pi_t(c_j, d_j)) = \left(\frac{1}{1+\sigma\phi}\right)^2 \left[\sum_{j=0}^{\infty}(c_j+\phi l_j)^2 E\left[\varepsilon_{t-j}^2\right] + \phi^2 \sum_{j=0}^{\infty}(l_j - \sigma c_j)^2 E\left[e_{t-j}^2\right]\right]$$
$$= \left(\frac{1}{1+\sigma\phi}\right)^2 \left[\alpha^{-1}\sum_{j=0}^{\infty}(c_j+\phi l_j)^2 + \phi^2\omega^{-1}\sum_{j=0}^{\infty}(l_j-\sigma c_j)^2\right],$$

where (c_j, d_j) is either $(\hat{c}_j, \hat{d}_j)$ or $(\tilde{c}_j, \tilde{d}_j)$. This expression shows that inflation variance depends on ω, even when firms ignore the fact that the interest rate is informative about the current state of the economy. The rationale behind this observation is: firms may discard part of the information they have, but this behavior does not change the way central bank reacts to information on aggregate nominal demand growth. Therefore, we continue to have $i_t = \phi u_t + \phi e_t$. As the interest rate interferes on the dynamics of the fundamental, and by extension on prices, the variance of e continue to influence the variance of inflation. Under this framework, we define the informational effect over the inflation variance as

$$\Delta_\pi = var(\hat{\pi}) - var(\tilde{\pi}).$$



Fugure 3.3: Evolution of Δ_π for different parameters.

Figure (F3a) shows how the informational effect over variance evolves when we modify the policy parameters ϕ, γ, and μ and the primitive parameter σ regarding two situations: (i) firms consider the policy instrument informative, $\text{var}(\hat{\pi})$, and (ii) they do not, $\text{var}(\tilde{\pi})$.

Equation (3.7) decomposes interest rate in two parts: the response to the fundamental ($\phi(\theta_t - \theta_{t-1})$) and a noise ($\phi e_t$). As the variance of the noisy part diminishes (γ or μ grows), interest rate becomes more systematic. As these parameters are exclusively associated to the quality of public information, the informational effect over the inflation variance grows with them. Whether Δ_π becomes positive when γ or μ grows, it depends on the calibration. For small values of σ, we can obtain $\text{var}(\hat{\pi}) > \text{var}(\tilde{\pi})$, if γ and μ are sufficiently high. When σ is small, the policy instrument does not interfere much on the fundamental dynamics, being just a public signal. As we have seen in chapter 2, the precision of the public signal increases inflation variance. The explanation is identical to the one presented in Angeletos and Pavan (2007): as the public signal increases coordination in price setting, the variance of inflation increases. However, for higher values of σ, as shown in figure (F3a), inflation variance diminishes when γ or μ grows, even when we consider that firms take information from interest rate. Therefore, in order to decrease inflation variance, central bank should pursue a continuous improvement on quality of the information used to take policy decisions, as well as not adding any monetary shock.

When we analyze the influence of σ, we have to consider two effects: (i) the influence of the policy instrument on the fundamental dynamics increases and (ii) the shocks incorporated in the policy instrument are amplified. The first effect helps to lower inflation variance, as firms will change the way they compute expectations when they perceive the policy instrument as an effective mean of driving the fundamental (i.e., central bank will need smaller variations on the policy instrument to move the fundamental). The second effect increases inflation variance as the fundamental becomes more volatile. Firms do not compute the first effect when they disregard the fact that the policy instrument is a response to changes in the fundamental. Therefore, we observe increasing inflation volatility when we analyze the counterfactual case. When firms take information from the interest rate, both effects are considered. For small values of σ, the first effect is dominant, while the second effect pushes inflation variance up when σ increases. The net effect, measured by Δ_π, is a decreasing function of σ that is positive for small values of σ.

The most complex analysis is related to the central bank's informational response coefficient, ϕ. Depending on the calibration, it can either produce the smooth behavior shown in Figure (F3a) or it can produce an overshooting on $\mathrm{var}(\hat{\pi})$, and by consequence on Δ_π, for small values of ϕ. This overshooting shows that there is a region where inflation variance increases with ϕ when firms consider the policy instrument informative about the state. Although not proven analytically, we believe that this overshooting is produced when we have two positive real roots in (3.18), in which case there is a region where interest rate increases prices. Besides the existence of an overshooting, we observe the same two effects we studied for the case of σ. As before the first effect is dominant for small values of ϕ, while the second effect makes inflation variance increases. Again, Δ_π is a decreasing function of ϕ, when this parameter is sufficiently high. It is not a coincidence that σ and ϕ have similar influence on the inflation variance, once the factor $\sigma\phi$ measures how intensively central bank's information hits the fundamental dynamics.

Considering our baseline calibration, minimization of inflation variance recommends that the central bank should not add any noise to its policy rule $(\mu \to \infty)$, making the interest rate as much informative about the fundamental as possible. Besides, the central bank should positively react to the information received $(\phi \approx 1.5)$.

3.6.2.
Ex ante Total Profit

As in chapter 2, we use a modified version of the efficiency criterion proposed by Angeletos and Pavan (2007) that represents ex-ante total profits.

$$E\Pi = -\lambda \int_{(\Theta_t, I_t)} \left[\sum_{j=0}^{\infty} (1-\lambda)^j \int_{x_{t-j}} n(x_{t-j}, \Theta_{t-j-1}, I_t)^2 dF(x_{t-j} \mid \Theta_t, I_t) \right] dF(\Theta_t, I_t)$$
$$+ \int_{(\Theta_t, I_t)} \eta(\Theta_t, I_t) h(\Theta_t, I_t) dF(\Theta_t, I_t),$$

where $n(x_{t-j}, \Theta_{t-j-1}, I_t) \equiv p_t(x_{t-j}, \Theta_{t-j-1}, I_t) - [(1-r)\theta_t + rP_t(\Theta_t, I_t)]$ is the objective function that guarantees profit maximization and $\eta(\Theta_t, I_t)$ is the Lagrange multiplier associated to the constraint

$$h(\Theta_t, I_t) \equiv P_t(\Theta_t, I_t) - \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \int_{x_{t-j}} p_t(x_{t-j}, \Theta_{t-j-1}, I_t) dF(x_{t-j} \mid \Theta_t, I_t).$$

Using (3.13), the generic expression for the equilibrium aggregate price index, we can write $n(x_{t-j}, \Theta_{t-j-1}, I_t)$ as a function of the parameters (κ, δ) and of the independent shocks

$$n(x_{t-j}, \Theta_{t-j-1}, I_t) = \Omega_j \left\{ \left(\frac{\phi\kappa - 1}{1 + \sigma\phi} \right) \delta\varepsilon_{t-j} + (1-\delta)\xi_{t-j}(z) + \left(\frac{\phi(\sigma+\kappa)}{1+\sigma\phi} \right) \delta e_{t-j} \right\}$$
$$+ \left(\frac{\phi\kappa - 1}{1+\sigma\phi} \right) \sum_{k=0}^{j-1} \Omega_k \varepsilon_{t-k} + \left(\frac{\phi(\sigma+\kappa)}{1+\sigma\phi} \right) \sum_{k=0}^{j-1} \Omega_k e_{t-k},$$

where

$$\Omega_j(\rho) = \left[\frac{1-r}{1 - r\left[1 - \rho(1-\lambda)^j\right]} \right]. \tag{3.20}$$

Using this expression, we obtain *ex ante* total profit as a function of (κ, δ) and of the variances of the shocks. However, we can come with a much simpler expression when we use (3.11) to write $E\Pi$ as a function of ω,

$$E\Pi(\omega) = -\left(\frac{\rho}{\alpha + \omega} \right) \sum_{j=0}^{\infty} (1-\lambda)^j \Omega_j^2. \tag{3.21}$$

For the case where firms ignore the informational power of interest rate (the counterfactual), we have to evaluate this expression considering $\omega = 0$. It is important to highlight that Ω_j is also a function of ω, since it depends on ρ. Under this framework, we define the informational effect over *ex ante* total profit as

$$\Delta_{E\Pi} = E\Pi(\omega) - E\Pi(0).$$

Figure (F4a) shows how $\Delta_{E\Pi}$ changes with some parameters of the model.

For all cases, we have that $\Delta_{E\Pi}$ is positive, meaning that welfare is higher when firms use interest rate to take information about the state. We have shown that $E\Pi$ is a function of the precision of interest rate, ω. Therefore, the counterfactual does not change with any parameter, since we consider $\omega = 0$ when firms do not take information from interest rate. As the elasticity of the fundamental with respect to interest rate, σ, does not affect ω, $E\Pi$, and by extension $\Delta_{E\Pi}$, does not vary with σ. Furthermore, the first derivative of $E\Pi$ with respect to ω is always positive, meaning that $E\Pi$ is an increasing function of the parameters ϕ, γ, and μ.

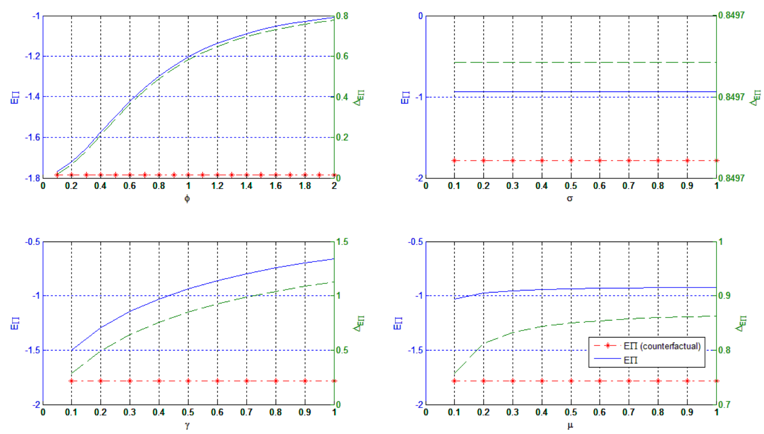


Figure 3.4: Evolution of $\Delta_{E\Pi}$ for different parameters

The first derivative of $E\Pi$ with respect to ω is always positive. Therefore, if the central bank is interested in this criterion, it should increase ω as much as possible. One way of attaining this objective is to increase the precision of the policy instrument, $\mu \to \infty$. This result tells us again that policy shocks, like monetary shocks, should be avoided.

3.6.3. Cross-Sectional Dispersion

Following similar steps to Woodford (2002), Ball et al. (2005) showed that the second order approximation of agents' utility function in a model with sticky information is a weighted average of output gap variance and cross-sectional dispersion plus terms that are independent of policy. As we focus on the cross-sectional dispersion, our criterion is a proxy of the criterion proposed in Ball et al.

(2005),

$$EV \equiv -E[Var_z(p_t(z) - P_t)],$$

where Var_z is given by

$$Var_z(p_t(z) - P_t) = \int (p_t(z) - P_t)^2 dz - \left[\int (p_t(z) - P_t) dz\right]^2.$$

Writing this criterion in a manner similar to Angeletos and Pavan (2007) we obtain[26]

$$EV = E\Pi + E\left[(p_t^* - P_t)^2\right].$$

This expression shows that the expected cross-section dispersion is related to the *ex-ante* total profit. As before we write $(p_t^* - P_t)$ as a function of independent shocks,

$$\left(\frac{1}{1+\sigma\phi}\right)\left[\frac{1-\kappa\phi}{\kappa}\sum_{k=0}^{\infty} d_k \varepsilon_{t-m} - \frac{\phi(\sigma+\kappa)}{\kappa}\sum_{k=0}^{\infty} d_k e_{t-m}\right].$$

Taking the expected value of this expression, we obtain a function of ω,

$$EV_1(\omega) \equiv E\left[(p_t^* - P_t)^2\right] = \left(\frac{\rho}{\alpha+\omega}\right)^2 \sum_{k=0}^{\infty} (1-\lambda)^{2j}\Omega_k^2, \qquad (3.22)$$

where Ω_k is defined in (3.20). As in the former criterion, we have to consider $\omega = 0$ to analyze the counterfactual. With the expression for EV_1, we can write the informational effect over cross-sectional dispersion as

$$\Delta_{EV} = E\Pi(\omega) + EV_1(\omega) - E\Pi(0) - EV_1(0).$$

As in the former criterion, we have that: (i) for all parameters, Δ_{EV} is always positive, (ii) EV is a growing function of ω, (iii) the counterfactual does not vary with ϕ, σ, γ, and μ, (iv) EV does not change with σ. Observations (ii) and (iii) leads to the result: Δ_{EV} is a increasing function of ϕ, γ, and μ. Therefore, we conclude that welfare, evaluated either from firms' point of view or as the second order approximation of agents' utility function, presents the same characteristics.

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[26]See Appendix G for details.

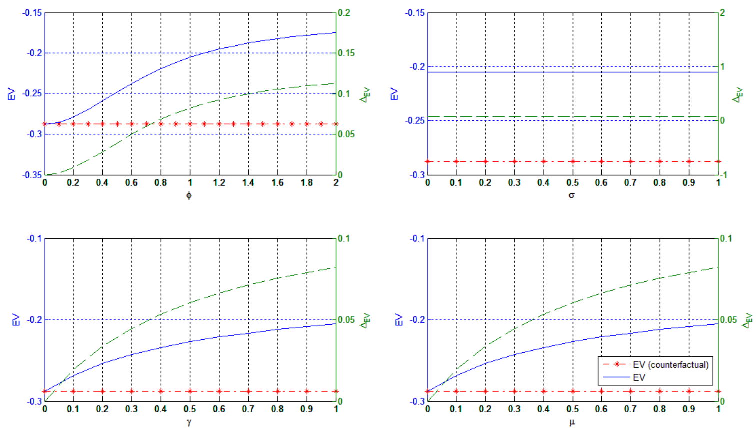

Figure 3.5: Evolution of Δ_{EV} for different parameters

3.7. Conclusions

We use a sticky-dispersed information model to show how the informational power of interest rate, defined as the information firms take from the policy instrument, changes price setting, and by consequence, inflation dynamics. Pricing decisions modifies due to the fact that firms alter the way they compute expectation on the current state of the economy.

The main consequence on inflation dynamics regards persistence of the monetary shock. As interest rate is not inertial, persistence on interest rate comes from central bank's reaction to information on changes in the fundamental, which evolves according to a Markovian process. If firms did not see interest rate as central bank's reaction function, any change in the policy instrument will be understood as an isolated movement that affects inflation only instantaneously. In contrast, when interest rate is understood as a public signal of the fundamental, the whole realization of interest rate affects inflation. It is important to stress that what creates persistence on inflation is not how central bank is reacting, but rather how firms change price setting when they understand that changes in interest rate are persistent.

We also analyze how the informational power of interest rate affects three different welfare criteria: (i) inflation variance, (ii) cross-sectional dispersion, and (iii) ex-ante total profit. We showed that the last two criteria depends exclusively on the precision of the policy instrument, ω. Therefore, to implement the optimal informational interest rule central bank should either increase as much as possible the informational response coefficient, ϕ, or the precision of the monetary shock, μ. When we focus on the inflation variance, we observe that, as before, the precision of the monetary shock should increase as much as possible. However, ϕ should assume a finite and positive value depending on the baseline calibration of the model. Following this recommendation, central bank maximizes all three criteria at the same time. The lessons we take from this optimal informational interest rule are: (i) central bank should avoid adding monetary shocks to the interest rate and (ii) there is always an optimal informational response coefficient, ϕ.