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5
Appendixes

5.1
Appendix of Chapter 1

5.1.1
The Model of Huang and Liu (2001)

In the model economy, the production of a final consumption good requires N stages of processing. The production of each good at stage 1 requires labor services only, with a constant-returns-to-scale technology given by $Y_1(j) = H_1(j)$, where $H_1(j)$ is the labor input and $Y_1(j)$ is the output. The production of each good at stage $n \in \{2, ..., N\}$ uses labor and all goods produced at the previous stage as inputs according to the following technology

$$Y_n(j) = \left[\int_0^1 Y_{n-1}(j,z)^{\frac{1}{\mu}} dz\right]^{r\mu} H_n(j)^{1-r},$$

where $Y_n(j)$ is the output of a stage-n firm of type j, $Y_{n-1}(j,z)$ is the input supplied to j by a stage-$(n-1)$ firm of type z, $\mu > 1$ is function of the elasticity of substitution between such goods, $H_n(j)$ is the labor input used by j, and $r \in (0,1)$ is the share of composite of stage-$(n-1)$ goods in j'production. Firms behave as imperfect competitors in their output markets and are price-takers in their input markets. Given the constant-returns-to-scale technologies, the unit cost is also the marginal cost and is firm-independent. The optimal price decision under flexible prices is just a mark-up over marginal costs

$$P_n(j) = \mu\left[\bar{r}(P_{n-1})^r (W)^{1-r}\right], \tag{5-1}$$

where $\bar{r} \equiv r^{-r}(1-r)^{-(1-r)}$, $P_{n-1} \equiv \left[\int_0^1 P_{n-1}(z)^{\frac{1}{1-\mu}} dz\right]^{1-\mu}$ is a price index for goods produced at stage $n-1$, and W is nominal wage per hour.

The representative household is infinitely lived and maximizes expected life utility. The following equation describes the labor supply decision of the household

$$\frac{W}{P_N} = C, \tag{5-2}$$

Table 5.1: Parameter Values

Parameter	Description	Interval
α	Private information precision	$[0.10, 50.0]$
β	Public or Semi-public information precision	$[0.10, 50.0]$
r	Share of goods produced at stage $(n-1)$	
	in the production of stage n	$[0.10, 0.90]$

where consumption C is a Dixit and Stiglitz (1977) composite of the final-stage goods

$$C = \left[\int_0^1 Y_N(z)^{\frac{1}{\mu}}\, dz\right]^{\mu} \equiv Y_N, \tag{5-3}$$

where $Y_N(z)$ is a type z good produced at stage N. We combine equations (5-1), (5-2), and (5-3) to obtain

$$\frac{P_n(j)}{P_N} = \mu\left(\frac{P_{n-1}}{P_N}\right)^r (Y_N)^{1-r},$$

which log-linearized yields $p_n^* \equiv rP_{n-1} + (1-r)\theta$, where $\theta \equiv Y_N + P_N$ is nominal aggregate demand.

5.1.2 Simulation

We simulate the model in order to understand the evolution of semi-public information precision $\hat{\beta}_n$. The model's structural parameters are α, β, and r. For ease of reference, we group all primitive parameter definitions and values in Table 5.1. We avoid extreme values of r even tough results don´t change dramatically in these cases. We comment about the extreme cases $r = 0$ and $r = 1$ bellow.

$\hat{\beta}^*$ limit

Substitute the lag of the expression for $\hat{\lambda}_n$ at (1-13) in (1-12) to obtain a recursive expression for $\hat{\beta}_n$

$$\hat{\beta}_n = \frac{\left[\beta\left(\alpha + \hat{\beta}_{n-1}\right)\right]^2 \hat{\beta}_{n-1}}{\beta\left(\alpha + \hat{\beta}_{n-1}\right)^2 \hat{\beta}_{n-1} + \left[(\beta - r\alpha)\hat{\beta}_{n-1} + r\alpha\beta\right]^2}. \tag{5-4}$$

If we set $\hat{\beta}_n = \hat{\beta}_{n-1} = \hat{\beta}^*$, we obtain

$$\hat{\beta}^* = \frac{\left[\beta\left(\alpha + \hat{\beta}^*\right)\right]^2 \hat{\beta}^*}{\beta\left(\alpha + \hat{\beta}^*\right)^2 \hat{\beta}^* + \left[\left(\beta - r\alpha\right)\hat{\beta}^* + r\alpha\beta\right]^2},$$

which is a root of the third degree equation

$$f(X) \equiv (X)^3 + c_1 (X)^2 + c_2 X + c_3, \tag{5-5}$$

where

$$\begin{aligned}
c_1 &\equiv \frac{\alpha}{\beta}\left[2\beta(1-r) + r^2\alpha\right] > 0,\\
c_2 &\equiv \alpha\left[\alpha\left(1 - 2r^2\right) - 2\beta(1-r)\right],\\
c_3 &\equiv -\alpha^2\beta\left(1 - r^2\right) < 0.
\end{aligned}$$

Considering the parameter values of Table 3.1, $\hat{\beta}_n$ decreases monotonically to $\hat{\beta}^*$, the unique positive root of (5-5) when stage n increases.

Extreme values of r

If $r = 1$, zero is necessarily one of the roots of $f(X)$ and thus we must define $\hat{\beta}^*$ as the unique strict positive root of (5-5). For values of r very close to zero, we need an extra condition regarding the exogenous precisions α, β, and r to guarantee monotonicity. If $r = 0$, for example, we need $\alpha \leq \frac{3}{4}\beta$. To see this, consider (5-5) when $r = 0$

$$f(X) \equiv (X)^3 + 2\alpha (X)^2 + \alpha(\alpha - 2\beta)X - \alpha^2\beta. \tag{5-6}$$

In order to $\hat{\beta}_n$ converges monotonically to $\hat{\beta}^*$, the first derivative of (5-4) must be non-negative on $\hat{\beta}^*$

$$\begin{aligned}
\left.\frac{\partial\hat{\beta}_n}{\partial\hat{\beta}_{n-1}}\right|_{\hat{\beta}_{n-1}=\hat{\beta}^*} &= \frac{\beta^2((\hat{\beta}^*)^2 - \alpha^2)}{\left[(\alpha + \hat{\beta}^*)^2 + \beta\hat{\beta}^*\right]^2} \geq 0\\
&\Rightarrow \hat{\beta}^* \geq \alpha.
\end{aligned}$$

If we substitute $\hat{\beta}^* = \alpha$ in the third degree equation (5-6), we obtain the result

$$f(\alpha) = 0 \Rightarrow \alpha = \frac{3}{4}\beta.$$

5.2 Appendix of Chapter 2

5.2.1 Proofs of Results

Proof 19 (Proof of Result 12: ERPT under Complete Information) *If information is complete, the external price of a given exporter is*

$$p_f^* = (1 - r^*)\,\theta^* + r^* P_F^*.$$

Because firms are identical, all set the same price. As a result,

$$P_F^* = (1 - r^*)\,\theta^* + r^* P_F^* \Rightarrow P_F^* = \theta^*,$$

and the price of an imported good is $p_f = P_F = e + \theta^*$. Now, consider the prices of domestic firms. Under complete information, all final goods' firms set the same price

$$p_h = (1 - r)\,\theta + r\,[e + \theta^*]\,.$$

Setting $r = 1$ establishes the result

$$p_h = P_F = e + \theta^*.$$

Proof 20 (Proof of Result 13: Incomplete ERPT (Morris and Shin (2002))) *Under dispersed information, the optimal response for an exporter is*

$$p_f^* = E\left[(1 - r^*)\,\theta^* + r^* P_F^* \mid \Im_f^*\right].$$

After iterating and writing $E\,[\theta^* \mid \Im^*]$ for the average expectation of θ^* across exporters, we have

$$p_f^* = (1 - r^*) \textstyle\sum_{k=0}^{\infty} (r^*)^k\, E\left[E^k\,[\theta^* \mid \Im^*] \mid \Im_f^*\right]. \qquad (5\text{-}7)$$

Given the signals, it is straightforward to compute the high-order beliefs. The average expectation of θ^* across exporters is

$$\begin{aligned} E\,[\theta^* \mid \Im^*] &\equiv \int_{f\in[0,1]} E\left[\theta^* \mid \Im_f^*\right] df \\ &= (1 - \lambda^*) \int_{f\in[0,1]} x_f^* df + \lambda^* \left[\int_{f\in[0,1]} y_f^* df - e\right] \\ &= (1 - \lambda^*)\,\theta^* + \lambda^*\,[\theta - e]\,. \end{aligned}$$

Now, the expected value of the average expectation for a given exporter is

$$
\begin{aligned}
E\left[E\left[\theta^* \mid \Im^*\right] \mid \Im_f^*\right] &= E\left[\left(1-\lambda^*\right)\theta^* + \lambda^*\left[\theta - e\right] \mid \Im_f^*\right] \\
&= \left(1-\lambda^*\right) E\left[\theta^* \mid \Im_f^*\right] + \lambda^*\left[E\left[\theta \mid \Im_f^*\right] - e\right] \\
&= \left[1 - \lambda^*\left(\frac{1-\left(1-\lambda^*-\eta^*\right)^2}{\lambda^*+\eta^*}\right)\right] x_f^* \\
&+ \lambda^*\left(\frac{1-\left(1-\lambda^*-\eta^*\right)^2}{\lambda^*+\eta^*}\right)\left[y_f^* - e\right].
\end{aligned}
$$

More generally, we have

$$
\begin{aligned}
E\left[E^k\left[\theta^* \mid \Im^*\right] \mid \Im_f^*\right] &= \left[1 - \lambda^*\left(\frac{1-\left(1-\lambda^*-\eta^*\right)^{k+1}}{\lambda^*+\eta^*}\right)\right] x_f^* \\
&+ \lambda^*\left(\frac{1-\left(1-\lambda^*-\eta^*\right)^{k+1}}{\lambda^*+\eta^*}\right)\left[y_f^* - e\right]. \qquad (5\text{-}8)
\end{aligned}
$$

If we replace (5-8) in (5-7), we obtain

$$
\begin{aligned}
p_f^* &= \left(1-r^*\right)\textstyle\sum_{k=0}^{\infty}\left(r^*\right)^k\left[1 - \lambda^*\left(\frac{1-\left(1-\lambda^*-\eta^*\right)^{k+1}}{\lambda^*+\eta^*}\right)\right] x_f^* \\
&+ \left(1-r^*\right)\textstyle\sum_{k=0}^{\infty}\left(r^*\right)^k\lambda^*\left(\frac{1-\left(1-\lambda^*-\eta^*\right)^{k+1}}{\lambda^*+\eta^*}\right)\left[y_f^* - e\right] \\
&= \left(1-\delta^*\right)x_f^* + \delta^*\left[y_f^* - e\right].
\end{aligned}
$$

where

$$
\begin{aligned}
\delta^* &\equiv \frac{\lambda^*}{1 - r^*\left(1-\lambda^*-\eta^*\right)} \\
&= \underbrace{\frac{\sigma_x^{2*}}{\sigma_{x^*}^2 + \sigma_{y^*}^2 + \left(1-r^*\right)\sigma_e^2}}_{:\ \textit{signal effect for exporters}} \in (0,1)
\end{aligned}
$$

As a result, import prices are

$$
\begin{aligned}
p_f &= e + p_f^* \\
&= e + \left(1-\delta^*\right)x_f^* + \delta^*\left[y_f^* - e\right] \\
&= \left(1-\delta^*\right)\left[x_f^* + e\right] + \delta^* y_f^*.
\end{aligned}
$$

Setting $r^* = 0$ establishes our result.

Proof 21 (Proof of Result 14: ERCP Puzzle) *Given Result 13, aggre-*

gate imports' prices are

$$\begin{aligned} P_F &\equiv \int_{f\in[0,1]} p_f df \\ &= (1-\delta^*)\left[\int_{f\in[0,1]} x_f^* df + e\right] + \delta^* \int_{f\in[0,1]} y_f^* df \\ &= (1-\delta^*)\left[\theta^* + e\right] + \delta^*\theta. \end{aligned}$$

As a result, domestic prices are

$$\begin{aligned} p_h &= E\left[(1-r)\theta + rP_F \mid \Im_h\right] \\ &= E\left[(1-r)\theta + r\left[(1-\delta^*)\left[\theta^*+e\right] + \delta^*\theta\right] \mid \Im_h\right] \\ &= \left[1 - r(1-\delta^*)\right] E\left[\theta \mid \Im_h\right] + r(1-\delta^*) E\left[\theta^* + e \mid \Im_h\right] \\ &= (1-\delta)x_h + \delta\left[y_h + e\right], \end{aligned}$$

where

$$\begin{aligned} \delta &\equiv \lambda + r(1-\delta^*)(1-\lambda-\eta) \\ &= 1 - \underbrace{\frac{\sigma_y^2 + \left[1 - r(1-\delta^*)\right]\sigma_e^2}{\sigma_x^2+\sigma_y^2+\sigma_e^2}} \\ &\quad : \textit{signal effect for domestic firms} \end{aligned}$$

Setting $r = 1$ establishes our result.

Proof 22 (Proof of Result 15: Macroeconomic Stability and ERPT) *Given Result 14, the first derivative of δ with respect to σ_e^2 is*

$$\frac{\partial\delta}{\partial\sigma_e^2} = \frac{r(1-\delta^*)\left(\sigma_x^2+\sigma_y^2\right) - r\frac{\partial\delta^*}{\partial\sigma_e^2}\sigma_e^2\left(\sigma_x^2+\sigma_y^2+\sigma_e^2\right) - \sigma_x^2}{\left(\sigma_x^2+\sigma_y^2+\sigma_e^2\right)^2},$$

where

$$\frac{\partial\delta^*}{\partial\sigma_e^2} = -\frac{(1-r^*)\sigma_{x^*}^2}{\left[\sigma_{x^*}^2+\sigma_{y^*}^2+(1-r^*)\sigma_e^2\right]^2}.$$

If we set $\sigma_x^2 = \sigma_{x^*}^2$, $\sigma_y^2 = \sigma_{y^*}^2$, we obtain

$$\frac{\partial\delta}{\partial\sigma_e^2} > 0 \Leftrightarrow r\left\{\left[\sigma_y^2 + (1-r^*)\sigma_e^2\right]\left(1+\frac{\sigma_y^2}{\sigma_x^2}\right) + \frac{(1-r^*)\sigma_e^2\left(\sigma_x^2+\sigma_y^2+\sigma_e^2\right)}{\left[\sigma_x^2+\sigma_y^2+(1-r^*)\sigma_e^2\right]}\right\} > 1.$$

If we also set $r = 1$, $r^* = 0$, we obtain

$$\frac{\partial\delta}{\partial\sigma_e^2} = \frac{\left(\sigma_y^2-\sigma_x^2+\sigma_e^2\right)\left(\sigma_x^2+\sigma_y^2\right)}{\left(\sigma_x^2+\sigma_y^2+\sigma_e^2\right)^3} > 0 \Leftrightarrow \sigma_e^2 > \sigma_x^2 - \sigma_y^2.$$

If instead we have $r = r^*$

$$\frac{\partial \delta}{\partial \sigma_e^2} = \frac{1}{(\Delta)^2 (\Lambda)^2} \left[2r(1-r)\sigma_x^2 \sigma_e^2 \left(\Delta - \frac{r}{2}\sigma_e^2 \right) + \left(r\sigma_y^2 - \sigma_x^2 \right)(\Lambda)^2 \right],$$

where

$$\Delta \equiv \sigma_x^2 + \sigma_y^2 + \sigma_e^2,$$
$$\Lambda \equiv \sigma_x^2 + \sigma_y^2 + (1-r)\sigma_e^2.$$

Note that if

$$r > \frac{\sigma_x^2}{\sigma_y^2} \in (0,1) \Rightarrow \frac{\partial \delta}{\partial \sigma_e^2} > 0.$$

Note also that

$$\frac{\partial^2 \delta}{\partial r \partial \sigma_e^2} = \frac{(1-\delta^*)\left(\sigma_x^2 + \sigma_y^2\right) - \frac{\partial \delta^*}{\partial \sigma_e^2}\sigma_e^2 \left(\sigma_x^2 + \sigma_y^2 + \sigma_e^2\right)}{\left(\sigma_x^2 + \sigma_y^2 + \sigma_e^2\right)^2} > 0,$$
$$\frac{\partial^2 \delta}{\partial r^* \partial \sigma_e^2} = \frac{r\sigma_{x^*}^2 \sigma_e^2}{\left[\sigma_{x^*}^2 + \sigma_{y^*}^2 + (1-r^*)\sigma_e^2\right]^2 \left(\sigma_x^2 + \sigma_y^2 + \sigma_e^2\right)^2}$$
$$\times \left[\frac{(1-r^*)\sigma_e^4 - \left[\sigma_{x^*}^2 + \sigma_{y^*}^2\right]\left(2\sigma_x^2 + 2\sigma_y^2 + \sigma_e^2\right)}{\left[\sigma_{x^*}^2 + \sigma_{y^*}^2 + (1-r^*)\sigma_e^2\right]} \right].$$

5.2.2
Production Chains

For $n = 1$, domestic prices are

$$p_{H,1} = E\left[\theta \mid \Im_1\right] = (1-\lambda)x_1 + \lambda\left[y_1 + e\right],$$
$$p_{F,1}^* = E\left[\theta^* \mid \Im_1^*\right] = (1-\lambda^*)x_1^* + \lambda^*\left[y_1^* - e\right].$$

Then, foreign prices are

$$p_{H,1}^* = p_{H,1} - e = (1-\lambda)\left[x_1 - e\right] + \lambda y_1,$$
$$p_{F,1} = e + p_{F,1}^* = (1-\lambda^*)\left[x_1^* + e\right] + \lambda^* y_1^*.$$

If we aggregate:

$$P_{H,1} \equiv \frac{1}{m}\int_0^m p_{H,1}(j)\,dj = (1-\lambda)\theta + \lambda\left[\theta^* + e\right],$$
$$P_{F,1}^* \equiv \frac{1}{1-m}\int_m^1 p_{F,1}^*(j)\,dj = (1-\lambda^*)\theta^* + \lambda^*\left[\theta - e\right],$$

and then

$$P_{H,1}^* \equiv P_{H,1} - e = (1-\lambda)[\theta - e] + \lambda\theta^*,$$
$$P_{F,1} \equiv e + P_{F,1}^* = (1-\lambda^*)[\theta^* + e] + \lambda^*\theta.$$

The countries price levels are

$$\begin{aligned} P_1 &\equiv (1-\chi) P_{H,1} + \chi P_{F,1} \\ &= (1-\chi)[(1-\lambda)\theta + \lambda[\theta^* + e]] + \chi[(1-\lambda^*)[\theta^* + e] + \lambda^*\theta] \\ &= (1-\delta_1)\theta + \delta_1[\theta^* + e], \\ P_1^* &\equiv (1-\chi^*) P_{F,1}^* + \chi^* P_{H,1}^* \\ &= (1-\chi^*)[(1-\lambda^*)\theta^* + \lambda^*[\theta - e]] + \chi^*[(1-\lambda)[\theta - e] + \lambda\theta^*] \\ &= (1-\delta_1^*)\theta^* + \delta_1^*[\theta - e], \end{aligned}$$

where

$$\delta_1 \equiv (1-\chi)\lambda + \chi(1-\lambda^*),$$
$$\delta_1^* \equiv (1-\chi^*)\lambda^* + \chi^*(1-\lambda).$$

Now, consider $n = 2$:

$$\begin{aligned} p_{H,2} &= E[(1-r)\theta + rP_1 \mid \Im_2] \\ &= E[(1-r)\theta + r[(1-\delta_1)\theta + \delta_1[\theta^* + e]] \mid \Im_2] \\ &= (1-r\delta_1) E[\theta \mid \Im_2] + r\delta_1 E[\theta^* + e \mid \Im_2] \\ &= (1-\lambda_2) x_2 + \lambda_2[y_2 + e], \\ p_{F,2}^* &= E[(1-r^*)\theta^* + r^* P_1^* \mid \Im_2^*] \\ &= E[(1-r^*)\theta^* + r^*[(1-\delta_1^*)\theta^* + \delta_1^*[\theta - e]] \mid \Im_2^*] \\ &= (1-r^*\delta_1^*) E[\theta^* \mid \Im_2^*] + r^*\delta_1^* E[\theta - e \mid \Im_2^*] \\ &= (1-\lambda_2^*) x_2^* + \lambda_2^*[y_2^* - e]. \end{aligned}$$

Foreign prices are

$$p_{H,2}^* = p_{H,2} - e = (1-\lambda_2)[x_2 - e] + \lambda_2 y_2,$$
$$p_{F,2} = e + p_{F,2}^* = (1-\lambda_2^*)[x_2^* + e] + \lambda_2^* y_2^*,$$

where

$$\lambda_2 \equiv (1 - r\delta_1)\,\lambda + r\delta_1\,(1 - \eta)\,,$$
$$\lambda_2^* \equiv (1 - r^*\delta_1^*)\,\lambda^* + r^*\delta_1^*\,(1 - \eta^*)\,.$$

If we aggregate

$$P_{H,2} \equiv \frac{1}{m}\int_0^m p_{H,2}\,(j)\,dj = (1 - \lambda_2)\,\theta + \lambda_2\,[\theta^* + e]\,,$$
$$P_{F,2}^* \equiv \frac{1}{1-m}\int_m^1 p_{F,2}^*\,(j)\,dj = (1 - \lambda_2^*)\,\theta^* + \lambda_2^*\,[\theta - e]\,,$$

and then

$$P_{H,2}^* \equiv P_{H,2} - e = (1 - \lambda_2)\,[\theta - e] + \lambda_2\theta^*,$$
$$P_{F,2} \equiv e + P_{F,2}^* = (1 - \lambda_2^*)\,[\theta^* + e] + \lambda_2^*\theta.$$

The countries price levels are

$$\begin{aligned}
P_2 &\equiv (1 - \chi)\,P_{H,2} + \chi P_{F,2} \\
&= (1 - \chi)\,[(1 - \lambda_2)\,\theta + \lambda_2\,[\theta^* + e]] + \chi\,[(1 - \lambda_2^*)\,[\theta^* + e] + \lambda_2^*\theta] \\
&= (1 - \delta_2)\,\theta + \delta_2\,[\theta^* + e]\,, \\
P_2^* &\equiv (1 - \chi^*)\,P_{F,2}^* + \chi^* P_{H,2}^* \\
&= (1 - \chi^*)\,[(1 - \lambda_2^*)\,\theta^* + \lambda_2^*\,[\theta - e]] + \chi^*\,[(1 - \lambda_2)\,[\theta - e] + \lambda_2\theta^*] \\
&= (1 - \delta_2^*)\,\theta^* + \delta_2^*\,[\theta - e]\,,
\end{aligned}$$

where

$$\delta_2 \equiv (1 - \chi)\,\lambda_2 + \chi\,(1 - \lambda_2^*)\,,$$
$$\delta_2^* \equiv (1 - \chi^*)\,\lambda_2^* + \chi^*\,(1 - \lambda_2)\,.$$

For a generic $n \geq 1$:

$$p_{H,n} = (1 - \lambda_n)\,x_n + \lambda_n\,[y_n + e]\,,$$
$$p_{F,n}^* = (1 - \lambda_n^*)\,x_n^* + \lambda_n^*\,[y_n^* - e]\,,$$
$$p_{H,n}^* = (1 - \lambda_n)\,[x_n - e] + \lambda_n y_n,$$
$$p_{F,n} = (1 - \lambda_n^*)\,[x_n^* + e] + \lambda_n^* y_n^*,$$

where

$$\lambda_n \equiv \lambda + r\left(1-\lambda-\eta\right)\delta_{n-1},\ \ \lambda_1 = \lambda,$$
$$\lambda_n^* \equiv \lambda^* + r^*\left(1-\lambda^*-\eta^*\right)\delta_{n-1}^*,\ \ \lambda_1^* = \lambda,$$

and

$$\delta_n \equiv \left(1-\chi\right)\lambda_n + \chi\left(1-\lambda_n^*\right),$$
$$\delta_n^* \equiv \left(1-\chi^*\right)\lambda_n^* + \chi^*\left(1-\lambda_n\right).$$

If we aggregate

$$P_{H,n} = \left(1-\lambda_n\right)\theta + \lambda_n\left[\theta^* + e\right],$$
$$P_{F,n}^* = \left(1-\lambda_n^*\right)\theta^* + \lambda_n^*\left[\theta - e\right],$$
$$P_{H,n}^* = \left(1-\lambda_n\right)\left[\theta - e\right] + \lambda_n\theta^*,$$
$$P_{F,n} = \left(1-\lambda_n^*\right)\left[\theta^* + e\right] + \lambda_n^*\theta,$$

which yields

$$P_n \equiv \left(1-\delta_n\right)\theta + \delta_n\left[\theta^* + e\right],$$
$$P_n^* \equiv \left(1-\delta_n^*\right)\theta^* + \delta_n^*\left[\theta - e\right].$$

We can obtain recursive expressions for (λ_n, λ_n^*)

$$\begin{aligned}\lambda_n &\equiv \lambda + r\left(1-\lambda-\eta\right)\delta_{n-1}\\ &= \lambda + r\left(1-\lambda-\eta\right)\left[\left(1-\chi\right)\lambda_{n-1} + \chi\left(1-\lambda_{n-1}^*\right)\right],\\ \lambda_n^* &\equiv \lambda^* + r^*\left(1-\lambda^*-\eta^*\right)\delta_{n-1}^*\\ &= \lambda^* + r^*\left(1-\lambda^*-\eta^*\right)\left[\left(1-\chi^*\right)\lambda_{n-1}^* + \chi^*\left(1-\lambda_{n-1}\right)\right].\end{aligned}$$

Write the weights in matrix format

$$\Delta_n = \Delta + A\Lambda_n,$$
$$\Lambda_n = \Lambda + B\Delta_{n-1},$$

where

$$\Delta_n \equiv \begin{bmatrix} \delta_n \\ \delta_n^* \end{bmatrix}, \quad \Lambda_n \equiv \begin{bmatrix} \lambda_n \\ \lambda_n^* \end{bmatrix},$$
$$\Delta \equiv \begin{bmatrix} \chi \\ \chi^* \end{bmatrix} \neq \Delta_1 \equiv \begin{bmatrix} \delta_1 \\ \delta_1^* \end{bmatrix}, \quad \Lambda \equiv \begin{bmatrix} \lambda \\ \lambda^* \end{bmatrix} = \Lambda_1,$$
$$A \equiv \begin{bmatrix} 1-\chi & -\chi \\ -\chi^* & 1-\chi^* \end{bmatrix}, ...B \equiv \begin{bmatrix} r\left(1-\lambda-\eta\right) & 0 \\ 0 & r^*\left(1-\lambda^*-\eta^*\right) \end{bmatrix}.$$

If we combine the two equations

$$\begin{aligned} \Delta_n &= \Delta + A\left[\Lambda + B\Delta_{n-1}\right] \\ &= \textstyle\sum_{j=0}^{n-1} C^j \Delta_1, \end{aligned}$$

where

$$\Delta_1 \equiv \Delta + A\Lambda.$$
$$C \equiv AB.$$

In order to solve the matricial difference equation, we must obtain the inverse of $I-C$ and calculate the powers of C. Under our simplifying assumptions $\left(r = r^*, \sigma_x^2 = \sigma_{x^*}^2, \sigma_y^2 = \sigma_{y^*}^2,\right)$, the weights on e are $\lambda = \lambda$ and $\eta = \eta^*$. As a result

$$C = \tilde{r}\begin{bmatrix} 1-\chi & -\chi \\ -\chi^* & 1-\chi^* \end{bmatrix},$$

where $\tilde{r} \equiv r\left(1-\lambda-\eta\right)$.

We can show that matrix $I-C$ is invertible. Just note that

$$|I-C| = \begin{vmatrix} 1-\tilde{r}\left(1-\chi\right) & \tilde{r}\chi \\ \tilde{r}\chi^* & 1-\tilde{r}\left(1-\chi^*\right) \end{vmatrix} = 0$$

if and only if

$$\left(1-\tilde{r}\right)\left[1-\tilde{r}\left(1-\chi-\chi^*\right)\right] = 0.$$

Thus, matrix $I-C$ is invertible as long as $\tilde{r} \neq \left\{1, \left(1-\chi-\chi^*\right)^{-1}\right\}$. This condition is not restrictive because $\tilde{r} \in (0,1)$. In this case

$$\left(I-C\right)^{-1} = \frac{1}{\left(1-\tilde{r}\right)\left[1-\tilde{r}\left(1-\chi-\chi^*\right)\right]}\begin{bmatrix} 1-\tilde{r}\left(1-\chi^*\right) & -\tilde{r}\chi \\ -\tilde{r}\chi^* & 1-\tilde{r}\left(1-\chi\right) \end{bmatrix}.$$

We must also calculate the powers of C. The eigenvalue ϕ is such that

$$\begin{vmatrix} \tilde{r}\left(1-\chi\right)-\phi & -\tilde{r}\chi \\ -\tilde{r}\chi^{*} & \tilde{r}\left(1-\chi^{*}\right)-\phi \end{vmatrix} = 0,$$

or

$$\left[\tilde{r}\left(1-\chi-\chi^{*}\right)-\phi\right]\left[\tilde{r}-\phi\right]=0.$$

As a result, we obtain

$$\phi=\left\{\tilde{r},\tilde{r}\left(1-\chi-\chi^{*}\right)\right\}.$$

The eigenvectors $\{v\}$ are

$$\tilde{r}\begin{bmatrix} 1-\chi & -\chi \\ -\chi^{*} & 1-\chi^{*} \end{bmatrix}\begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix}=\tilde{r}\begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix},$$
$$\tilde{r}\begin{bmatrix} 1-\chi & -\chi \\ -\chi^{*} & 1-\chi^{*} \end{bmatrix}\begin{bmatrix} v_{12} \\ v_{22} \end{bmatrix}=\tilde{r}\left(1-\chi-\chi^{*}\right)\begin{bmatrix} v_{12} \\ v_{22} \end{bmatrix},$$

or

$$v_{21}=-v_{11},$$
$$v_{22}=\frac{\chi^{*}}{\chi}v_{12}.$$

If we set $v_{11}=v_{12}=1$, we can rewrite C as

$$C\equiv TDT^{-1},$$

where

$$T\equiv\begin{bmatrix} 1 & 1 \\ -1 & \frac{\chi^{*}}{\chi} \end{bmatrix},\ T^{-1}\equiv\frac{\chi}{\chi+\chi^{*}}\begin{bmatrix} \frac{\chi^{*}}{\chi} & -1 \\ 1 & 1 \end{bmatrix}$$
$$D\equiv\begin{bmatrix} \tilde{r} & 0 \\ 0 & \tilde{r}\left(1-\chi-\chi^{*}\right) \end{bmatrix},$$

and then

$$\begin{aligned} C^{n} &= TD^{n}T^{-1} \\ &= \tilde{r}^{n}\frac{\chi}{\chi+\chi^{*}}\begin{bmatrix} \frac{\chi^{*}}{\chi}+\left(1-\chi-\chi^{*}\right)^{n} & -\left[1-\left(1-\chi-\chi^{*}\right)^{n}\right] \\ -\frac{\chi^{*}}{\chi}\left[1-\left(1-\chi-\chi^{*}\right)^{n}\right] & 1+\frac{\chi^{*}}{\chi}\left(1-\chi-\chi^{*}\right)^{n} \end{bmatrix} \end{aligned}$$

Finally, we obtain

$$\Delta_{n}=\left(I-C^{n}\right)\left(I-C\right)^{-1}\Delta_{1}.$$

If we ignore home-bias ($\bar{\chi} = \bar{\chi}^* = 1$), then $\chi + \chi^* = 1$. We can rewrite $(I - C)^{-1}$ and C^n as

$$(I - C)^{-1} = \frac{1}{1 - \tilde{r}} \begin{bmatrix} 1 - \tilde{r}\chi & -\tilde{r}\chi \\ -\tilde{r}(1-\chi) & 1 - \tilde{r}(1-\chi) \end{bmatrix}$$

and

$$C^n = \tilde{r}^n \begin{bmatrix} (1-\chi) & -\chi \\ -(1-\chi) & \chi \end{bmatrix}.$$

After tedious calculation, we obtain

$$\begin{aligned}
\delta_n &= \chi + (\chi^* - \chi)\left(\frac{1 - \tilde{r}^n}{1 - \tilde{r}}\right)\lambda \\
&< \delta_{n-1} \Longleftrightarrow \chi^* < \chi, \\
\delta_n^* &= \chi^* - (\chi^* - \chi)\left(\frac{1 - \tilde{r}^n}{1 - \tilde{r}}\right)\lambda \\
&< \delta_{n-1}^* \Longleftrightarrow \chi^* > \chi, \\
\lambda_n &\equiv \lambda + \tilde{r}\delta_{n-1} \\
&< \lambda_{n-1} \Leftrightarrow \delta_{n-1} < \delta_{n-2} \Leftrightarrow \chi^* < \chi, \\
\lambda_n^* &\equiv \lambda + \tilde{r}\delta_{n-1}^* \\
&< \lambda_{n-1}^* \Leftrightarrow \delta_{n-1}^* < \delta_{n-2}^* \Leftrightarrow \chi^* > \chi.
\end{aligned}$$

5.3 Appendix of Chapter 3

Proof 23 (Proof of expression (3-21): Equilibrium Aggregate Price Level) *First, we replace (3-1) in (3-7) to obtain:*

$$\begin{aligned}
P_t &= \textstyle\sum_{j=0}^{\infty}\int_{\Lambda_{t-j}} E\left[rP_t + (1-r)\theta_t \mid \Im_{t-j}(z)\right] dz \\
&= r\textstyle\sum_{j=0}^{\infty}\int_{\Lambda_{t-j}} E\left[P_t \mid \Im_{t-j}(z)\right] dz + (1-r)\textstyle\sum_{j=0}^{\infty}\int_{\Lambda_{t-j}} E\left[\theta_t \mid \Im_{t-j}(z)\right] dz.
\end{aligned}$$

From the definition of the average 1^{st} order belief in (3-8):

$$P_t = r\bar{E}[P_t] + (1-r)\bar{E}[\theta_t].$$

If we iterate one time, we obtain:

$$\begin{aligned} P_t &= r\bar{E}\left[r\bar{E}\left[P_t\right] + (1-r)\,\bar{E}\left[\theta_t\right]\right] + (1-r)\,\bar{E}\left[\theta_t\right] \\ &= r^2\bar{E}\left[\bar{E}\left[P_t\right]\right] + r\,(1-r)\,\bar{E}\left[\bar{E}\left[\theta_t\right]\right] + (1-r)\,\bar{E}\left[\theta_t\right] \\ &= r^2\bar{E}^2\left[P_t\right] + r\,(1-r)\,\bar{E}^2\left[\theta_t\right] + (1-r)\,\bar{E}\left[\theta_t\right]. \end{aligned}$$

If we iterate N times:

$$P_t = r^N\bar{E}^N\left[P_t\right] + (1-r)\textstyle\sum_{k=1}^{N} r^{k-1}\bar{E}^k\left[\theta_t\right].$$

Taking the limit as $N \to \infty$, we obtain expression (3-9):

$$P_t = (1-r)\textstyle\sum_{k=1}^{\infty} r^{k-1}\bar{E}^k\left[\theta_t\right],$$

which proves the result.

Proof 24 (Obtaining the Expectations) $E\left[\theta_{t-m} \mid \Im_{t-j}(z)\right]$: *First, we calculate the distribution of the fundamental θ_{t-j} given that the firm updated its information set at period $t-j$. We can compute $f\left(\theta_{t-j} \mid \Theta_{t-j-1}, x_{t-j}\right)$ as*

$$\begin{aligned} f\left(\theta_{t-j} \mid \theta_{t-j-1}, x_{t-j}\right) &= \frac{f\left(\theta_{t-j}, \theta_{t-j-1}, x_{t-j}\right)}{\int_{-\infty}^{\infty} f\left(\theta_{t-j}, \theta_{t-j-1}, x_{t-j}\right) d\theta_{t-j}} \\ &= \frac{f\left(\theta_{t-j-1}, x_{t-j} \mid \theta_{t-j}\right) f\left(\theta_{t-j}\right)}{\int_{-\infty}^{\infty} f\left(\theta_{t-j}, \theta_{t-j-1}, x_{t-j}\right) d\theta_{t-j}} \\ &= \frac{f\left(\theta_{t-j-1} \mid \theta_{t-j}\right) f\left(x_{t-j} \mid \theta_{t-j}\right) f\left(\theta_{t-j}\right)}{\int_{-\infty}^{\infty} f\left(\theta_{t-j}, \theta_{t-j-1}, x_{t-j}\right) d\theta_{t-j}} \end{aligned}$$

where the last equality holds due to the independence of $\xi_t(z)$ and ϵ_{t-j}. As

$$\begin{aligned} x_{t-j}(z) &= \theta_{t-j} + \xi_{t-j}(z), \\ \theta_{t-j-1} &= \theta_{t-j} - \epsilon_{t-j}, \end{aligned}$$

where $\xi_t(z) \sim N\left(0, \beta^{-1}\right)$ and $\epsilon_{t-j} \sim N\left(0, \alpha^{-1}\right)$, we have that $f\left(x_{t-j} \mid \theta_{t-j}\right) = N\left(\theta_{t-j}, \beta^{-1}\right)$ and $f\left(\theta_{t-j-1} \mid \theta_{t-j}\right) = N\left(\theta_{t-j}, \alpha^{-1}\right)$. If the dynamics of θ_t was

$$\theta_{t-j-1} = \rho\theta_{t-j} - \epsilon_{t-j},$$

we would have

$$E[\theta_{t-j}] = E[\theta_t] = \frac{E[\epsilon_t]}{1-\rho} = 0,$$
$$Var[\theta_{t-j}] = Var[\theta_t] = \frac{Var[\epsilon_t]}{1-\rho^2} = \frac{\alpha^{-1}}{1-\rho^2}.$$

Therefore, the distribution of θ_{t-j} *would be given by* $f(\theta_{t-j}) = N(0, \Psi^{-1})$ *where* $\Psi = \alpha(1-\rho^2)$*. Thus, we would obtain*

$$\begin{aligned} f(\theta_{t-j}, \theta_{t-j-1}, x_{t-j}) &= c \times \exp\left\{ -\frac{1}{2}\left[\frac{(x_{t-j}(z) - \theta_{t-j})^2}{\beta^{-1}} + \frac{(\theta_{t-j-1} - \rho^{-1}\theta_{t-j})^2}{(\rho^2\alpha)^{-1}} + \frac{\theta_{t-j}^2}{\Psi^{-1}} \right] \right\} \\ &= c \times \exp\left\{ -\frac{1}{2}\left[(\beta + \alpha + \Psi)\theta_{t-j}^2 - 2(\beta x_{t-j}(z) + \alpha\rho\theta_{t-j-1})\theta_{t-j} \right] \right\} \\ &\times \exp\left\{ -\frac{1}{2}\left[\beta x_{t-j}^2(z) + \alpha\rho^2\theta_{t-j-1}^2 \right] \right\} \\ &= c \times d \times \frac{1}{\sqrt{2\pi}\sigma\Sigma} \times \exp\left\{ -\frac{1}{2}\frac{(\theta_{t-j} - \mu)^2}{\Sigma^2} \right\}, \end{aligned}$$

where

$$c = (2\pi)^{-3/2}(\beta\alpha\Psi)^{1/2}, \qquad d = \sqrt{2\pi}\sigma \exp\left\{ -\tfrac{1}{2}\left[-\mu^2\Sigma^{-2} + \beta x_{t-j}^2(z) + \alpha\rho^2\theta_{t-j-1}^2 \right] \right\},$$
$$\mu = [\Delta x_{t-j}(z) + (1-\Delta) z_{t-j-1}], \qquad \Delta = \beta(\beta+\alpha+\Psi)^{-1},$$
$$z_{t-j-1} = \Lambda\rho\theta_{t-j-1}, \qquad \Lambda = \alpha(\beta+\alpha)^{-1}.$$
$$\Sigma^2 = (\beta+\alpha+\Psi)^{-1},$$

As $\rho \to 1$*, we have* $\Psi \to 0$*,* $\Delta \to \delta$*, and* $\Sigma^2 \to (\beta+\alpha)^{-1}$*. Thus* $f(\theta_{t-j} \mid \theta_{t-j-1}, x_{t-j}) = N(\mu, \sigma^2)$ *where* $\mu = [\delta x_{t-j}(z) + (1-\delta)\theta_{t-j-1}]$*, and* $\sigma^2 = (\beta+\alpha)^{-1}$*.*

Proof 25 (Proof of Lemma 17: Higher Order Beliefs) *In this Appendix we derive the general formula of the k-th order average expectation*

$$\bar{E}^k[\theta_t] = \lambda \textstyle\sum_{m=0}^{\infty} (1-\lambda)^m [\kappa_{m,k}\theta_{t-m} + \delta_{m,k}\theta_{t-m-1}]$$

with the weights $(\kappa_{m,k}, \delta_{m,k})$ *recursively defined for* $k \geq 1$

$$\begin{bmatrix} \kappa_{m,k+1} \\ \delta_{m,k+1} \end{bmatrix} = \begin{bmatrix} (1-\delta) \\ \delta \end{bmatrix} [1 - (1-\lambda)^m]^k + A_m \begin{bmatrix} \kappa_{m,k} \\ \delta_{m,k} \end{bmatrix},$$

where the matrix A_m *is given by*

$$A_m \equiv \begin{bmatrix} [(1-\delta)\left[1-(1-\lambda)^{m+1}\right] + \delta[1-(1-\lambda)^m]] & 0 \\ \delta\left[\left[1-(1-\lambda)^{m+1}\right] - [1-(1-\lambda)^m]\right] & \left[1-(1-\lambda)^{m+1}\right] \end{bmatrix},$$

and the initial weights are $(\kappa_{1,k}, \delta_{1,k}) \equiv (1-\delta, \delta)$.
We start by computing $\bar{E}^1[\theta_t]$ *as*

$$\begin{aligned}\bar{E}^1[\theta_t] &= \textstyle\sum_{j=0}^{\infty}\int_{\Lambda_j} E\left[\bar{E}^0[\theta_t] \mid \Im_{t-j}(z)\right] dz \\ &= \textstyle\sum_{j=0}^{\infty}\int_{\Lambda_j} E\left[\theta_t \mid \Im_{t-j}(z)\right] dz \\ &= \textstyle\sum_{j=0}^{\infty}\int_{\Lambda_j} \left[(1-\delta) x_{t-j}(z) + \delta\theta_{t-j-1}\right] dz \\ &= \lambda\textstyle\sum_{j=0}^{\infty}(1-\lambda)^j\left[(1-\delta)\theta_{t-j} + \delta\theta_{t-j-1}\right].\end{aligned}$$

We can use this result to obtain $\bar{E}^2[\theta_t]$ *as*

$$\begin{aligned}\bar{E}^2[\theta_t] &= \textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m} E\left[\bar{E}^1[\theta_t] \mid \Im_{t-m}(z)\right] dz \\ &= \lambda\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=0}^{\infty}(1-\lambda)^j E\left[(1-\delta)\theta_{t-j} + \delta\theta_{t-j-1} \mid \Im_{t-m}(z)\right] dz.\end{aligned}$$

We know that

$$E\left[\theta_{t-j} \mid \Im_{t-m}(z)\right] = \begin{cases} (1-\delta) x_{t-m}(z) + \delta\theta_{t-m-1} & : m \geq j, \\ \theta_{t-j} & : m < j. \end{cases}$$

Thereafter,

$$\begin{aligned}\bar{E}^2[\theta_t] &= \lambda\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=0}^{m-1}(1-\lambda)^j\left\{(1-\delta)E\left[\theta_{t-j} \mid \Im_{t-m}(z)\right] + \delta E\left[\theta_{t-j-1} \mid \Im_{t-m}(z)\right]\right\} dz \\ &+ \lambda\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}(1-\lambda)^m\left\{(1-\delta)E\left[\theta_{t-m} \mid \Im_{t-m}(z)\right] + \delta\theta_{t-m-1}\right\} dz \\ &+ \lambda\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=m+1}^{\infty}(1-\lambda)^j\left[(1-\delta)\theta_{t-j} + \delta\theta_{t-j-1}\right] dz \\ &= \lambda\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=0}^{m-1}(1-\lambda)^j\left[(1-\delta)x_{t-m}(z) + \delta\theta_{t-m-1}\right] dz \\ &+ \lambda\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}(1-\lambda)^m\left[(1-\delta)\left[(1-\delta)x_{t-m}(z) + \delta\theta_{t-m-1}\right] + \delta\theta_{t-m-1}\right] dz \\ &+ \lambda\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=m+1}^{\infty}(1-\lambda)^j\left[(1-\delta)\theta_{t-j} + \delta\theta_{t-j-1}\right] dz \\ &= \lambda^2\textstyle\sum_{m=0}^{\infty}(1-\lambda)^m\left[(1-\delta)\theta_{t-m} + \delta\theta_{t-m-1}\right]\sum_{j=0}^{m-1}(1-\lambda)^j \\ &+ \lambda^2\textstyle\sum_{m=0}^{\infty}(1-\lambda)^{2m}\left[(1-\delta)^2\theta_{t-m} + \left[1-(1-\delta)^2\right]\theta_{t-m-1}\right] \\ &+ \lambda^2\textstyle\sum_{j=1}^{\infty}(1-\lambda)^j\left[(1-\delta)\theta_{t-j} + \delta\theta_{t-j-1}\right]\sum_{m=0}^{j-1}(1-\lambda)^m \\ &= \lambda\textstyle\sum_{m=0}^{\infty}(1-\lambda)^m\left[(1-\delta)\theta_{t-m} + \delta\theta_{t-m-1}\right]\left[1-(1-\lambda)^m\right] \\ &+ \lambda^2\textstyle\sum_{m=0}^{\infty}(1-\lambda)^{2m}\left[(1-\delta)^2\theta_{t-m} + \left[1-(1-\delta)^2\right]\theta_{t-m-1}\right] \\ &+ \lambda\textstyle\sum_{j=1}^{\infty}(1-\lambda)^j\left[(1-\delta)\theta_{t-j} + \delta\theta_{t-j-1}\right]\left[1-(1-\lambda)^j\right] \\ &= \lambda\textstyle\sum_{m=0}^{\infty}(1-\lambda)^m\, 2\left[1-(1-\lambda)^m\right]\left[(1-\delta)\theta_{t-m} + \delta\theta_{t-m-1}\right] \\ &+ \lambda^2\textstyle\sum_{m=0}^{\infty}(1-\lambda)^{2m}\left[(1-\delta)^2\theta_{t-m} + \left[1-(1-\delta)^2\right]\theta_{t-m-1}\right].\end{aligned}$$

We can write this expression as

$$\bar{E}^2[\theta_t] = \lambda\textstyle\sum_{j=0}^{\infty}(1-\lambda)^j\left[\kappa_{j,2}\theta_{t-j} + \delta_{j,2}\theta_{t-j-1}\right],$$

where

$$\kappa_{j,2} = \left(1-\delta^2\right)\left[1-(1-\lambda)^j\right] + (1-\delta)^2\left[1-(1-\lambda)^{j+1}\right]$$
$$= \left[1-(1-\lambda)^{j+1}\right]\kappa_{j,1}^2 + \left[1-(1-\lambda)^j\right]\left(1-\delta_{j,1}^2\right),$$
$$\delta_{j,2} = \delta^2\left[1-(1-\lambda)^j\right] + \left[1-(1-\delta)^2\right]\left[1-(1-\lambda)^{j+1}\right]$$
$$= \left[1-(1-\lambda)^{j+1}\right]\left(1-\kappa_{j,1}^2\right) + \left[1-(1-\lambda)^j\right]\delta_{j,1}^2.$$

Note that

$$\kappa_{j,2}+\delta_{j,2} = \textstyle\sum_{n=0}^{1}\left[1-(1-\lambda)^j\right]^n\left[1-(1-\lambda)^{j+1}\right]^{1-n}.$$

We use induction to obtain the general case. Suppose that (3-18) holds for $k-1$*. Then*

$$\bar{E}^{k-1}[\theta_t] = \lambda\textstyle\sum_{m=0}^{\infty}(1-\lambda)^m\left[\kappa_{m,k-1}\theta_{t-m}+\delta_{m,k-1}\theta_{t-m-1}\right],$$

where

$$\textstyle\sum_{j=0}^{m-1}(1-\lambda)^j\left(\kappa_{j,k-1}+\delta_{j,k-1}\right) = \frac{1}{\lambda}\left[1-(1-\lambda)^m\right]^{k-1}.$$

As a result,

$$\bar{E}^k[\theta_t] = \textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}E\left[\bar{E}^{k-1}[\theta_t]\mid\Im_{t-m}(z)\right]dz$$
$$= \textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}E\left[\lambda\sum_{j=0}^{\infty}(1-\lambda)^j\left[\kappa_{j,k-1}\theta_{t-j}+\delta_{j,k-1}\theta_{t-j-1}\right]\mid\Im_{t-m}(z)\right]dz$$
$$= \lambda\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=0}^{m-1}(1-\lambda)^j\left\{\kappa_{j,k-1}E\left[\theta_{t-j}\mid\Im_{t-m}(z)\right]+\delta_{j,k-1}E\left[\theta_{t-j-1}\mid\Im_{t-m}(z)\right]\right\}dz$$
$$+\lambda\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}(1-\lambda)^m\left\{\kappa_{m,k-1}E\left[\theta_{t-m}\mid\Im_{t-m}(z)\right]+\delta_{m,k-1}\theta_{t-m-1}\right\}dz$$
$$+\lambda\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=m+1}^{\infty}(1-\lambda)^j\left[\kappa_{j,k-1}\theta_{t-j}+\delta_{j,k-1}\theta_{t-j-1}\right]dz$$
$$= \lambda\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=0}^{m-1}(1-\lambda)^j\left(\kappa_{j,k-1}+\delta_{j,k-1}\right)\left[(1-\delta)x_{t-m}(z)+\delta\theta_{t-m-1}\right]dz$$
$$+\lambda\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}(1-\lambda)^m\left[\kappa_{m,k-1}\left[(1-\delta)x_{t-m}(z)+\delta\theta_{t-m-1}\right]+\delta_{m,k-1}\theta_{t-m-1}\right]dz$$
$$+\lambda\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=m+1}^{\infty}(1-\lambda)^j\left[\kappa_{j,k-1}\theta_{t-j}+\delta_{j,k-1}\theta_{t-j-1}\right]dz$$
$$= \lambda^2\textstyle\sum_{m=0}^{\infty}(1-\lambda)^m\left[(1-\delta)\theta_{t-m}+\delta\theta_{t-m-1}\right]\sum_{j=0}^{m-1}(1-\lambda)^j\left(\kappa_{j,k-1}+\delta_{j,k-1}\right)$$
$$+\lambda^2\textstyle\sum_{m=0}^{\infty}(1-\lambda)^{2m}\left[\kappa_{m,k-1}(1-\delta)\theta_{t-m}+\left[\kappa_{m,k-1}\delta+\delta_{m,k-1}\right]\theta_{t-m-1}\right]$$
$$+\lambda^2\textstyle\sum_{j=1}^{\infty}(1-\lambda)^j\left[\kappa_{j,k-1}\theta_{t-j}+\delta_{j,k-1}\theta_{t-j-1}\right]\sum_{m=0}^{j-1}(1-\lambda)^m$$
$$= \lambda\textstyle\sum_{m=0}^{\infty}(1-\lambda)^m\left[1-(1-\lambda)^m\right]^{k-1}\left[(1-\delta)\theta_{t-m}+\delta\theta_{t-m-1}\right]$$
$$+\lambda^2\textstyle\sum_{m=0}^{\infty}(1-\lambda)^{2m}\left[\kappa_{m,k-1}(1-\delta)\theta_{t-m}+\left[\kappa_{m,k-1}\delta+\delta_{m,k-1}\right]\theta_{t-m-1}\right]$$
$$+\lambda\textstyle\sum_{m=0}^{\infty}(1-\lambda)^m\left[1-(1-\lambda)^m\right]\left[\kappa_{m,k-1}\theta_{t-m}+\delta_{m,k-1}\theta_{t-m-1}\right].$$

We can rewrite the last three lines above as

$$\bar{E}^k[\theta_t] = \lambda \textstyle\sum_{m=0}^{\infty} (1-\lambda)^m [\kappa_{m,k}\theta_{t-m} + \delta_{m,k}\theta_{t-m-1}],$$

where

$$\begin{aligned}
\kappa_{m,k} &\equiv (1-\delta)[1-(1-\lambda)^m]^{k-1} + [(1-\delta)\lambda(1-\lambda)^m + [1-(1-\lambda)^m]]\,\kappa_{m,k-1} \\
&= (1-\delta)[1-(1-\lambda)^m]^{k-1} \\
&\quad + \left[(1-\delta)\left[1-(1-\lambda)^{m+1}\right] + \delta[1-(1-\lambda)^m]\right]\kappa_{m,k-1}, \\
\delta_{m,k} &\equiv \delta[1-(1-\lambda)^m]^{k-1} + \delta\lambda(1-\lambda)^m\,\kappa_{m,k-1} + [\lambda(1-\lambda)^m + [1-(1-\lambda)^m]]\,\delta_{m,k-1} \\
&= \delta[1-(1-\lambda)^m]^{k-1} \\
&\quad + \delta\left[\left[1-(1-\lambda)^{m+1}\right] - [1-(1-\lambda)^m]\right]\kappa_{m,k-1} + \left[1-(1-\lambda)^{m+1}\right]\delta_{m,k-1},
\end{aligned}$$

since

$$\lambda(1-\lambda)^m = \left[1-(1-\lambda)^{m+1}\right] - [1-(1-\lambda)^m].$$

Rewriting these weights in matrix format, we obtain

$$\begin{bmatrix}\kappa_{m,k+1}\\ \delta_{m,k+1}\end{bmatrix} = \begin{bmatrix}(1-\delta)\\ \delta\end{bmatrix}[1-(1-\lambda)^m]^k + A_m\begin{bmatrix}\kappa_{m,k}\\ \delta_{m,k}\end{bmatrix},$$

where the matrix A_m is given by

$$A_m \equiv \begin{bmatrix}\left[(1-\delta)\left[1-(1-\lambda)^{m+1}\right] + \delta[1-(1-\lambda)^m]\right] & 0 \\ \delta\left[\left[1-(1-\lambda)^{m+1}\right] - [1-(1-\lambda)^m]\right] & \left[1-(1-\lambda)^{m+1}\right]\end{bmatrix},$$

which is exactly our result.