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On Implementable Transfer Matrices

PEDRO M. G. FERREIRA

Abstract—**A straightforward proof is given of the sufficient condition for implementable transfer matrices. A similar result is obtained also when the controlled output is not accessible for feedback.**

I. Introduction

In [1] the interesting and useful definition of implementable transfer matrices was introduced and explored. The definition goes as follows.

Let $P \in M(R(s))$ be a proper transfer matrix, with input u and output y. Let r be an exogenous signal. Then the transfer matrix H_{yr} between y and r is implementable if there exists a configuration and compensators so that the resulting system implements H_{yr} under the following conditions.

1) All forward paths from r to y pass through the plant P.

2) The closed-loop transfer matrix of every possible input–output pair is well defined, proper, and stable.

3) All compensators have proper transfer matrices.

Remarks:

i) The definition is phrased in a slightly different, but equivalent way in [1].

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ii) The inputs referred to in condition 2) above are exogenous inputs entering the configuration, including r. The outputs are y and any signal between plant and compensators and between the compensators themselves.

II. Feedback of the Controlled Output

The following result, given in [1], is proved here in a straightforward way, using factorizations over proper and stable rational matrices.

Theorem 1: Assume that y is accessible for feedback. Then H_{yr} is implementable if and only if it is proper and stable and can be decomposed as $H_{yr} = PT$, where T is proper and stable.

Proof: As pointed out by [1], the necessity follows directly from conditions 1) and 2) above. Indeed, from condition 1) it follows that $H_{yr} = PT$, for some $T \in M(R(s))$. From condition 2) it follows that T is proper and stable because T is the transfer matrix between u and r.

(Sufficiency): Let S denote the set (a Euclidean domain) of proper and stable rational functions. Let $N, D \in M(S)$, right coprime and such that

$$P = ND^{-1}$$

$$\therefore H_{yr} = ND^{-1}T.$$

Now $H_{yr} \in M(S)$ implies, in view of the coprimeness of N and D (see [4]), $D^{-1}T \in M(S)$. Hence, $H_{yr} = NQ$, for some $Q \in M(S)$. Next choose the following compensator with two degrees of freedom [4, sect. 5.6]:

$$u = \tilde{D}_c^{-1}[Q \mid -\tilde{N}_c]\begin{bmatrix} r \\ y \end{bmatrix}$$

The lemma in [4, p. 114] guarantees the existence of $\tilde{D}_c^{-1}\tilde{N}_c$ which stabilizes the loop and which is proper.

Moreover, with such a compensator we obtain indeed [4, sect. 5.6] $H_{yr} = NQ$, with conditions 1)-3) satisfied. □

Some remarks are in order.

1) In [1] the theorem is stated formally only as a necessary condition without the assumption on the accessibility of y for feedback, an assumption which is not used, as a matter of fact, in the proof of the necessity of the theorem. Then [1] gives a proof of the sufficiency of the theorem without mentioning the above assumption but in fact using it.

2) "H_{yr} proper and stable" is obviously included in condition 2), so it could be omitted if the theorem were stated only as a necessary condition.

3) The proof would be the same if the linear systems were defined in the more general algebraic structure (g, h, i, j), which encompasses, as a special case, transfer matrices which are rational and proper: see, e.g., [5] and the references therein. The only additional assumption would be that the elements of P are in the Jacobson radical of g (see [5, lemma 1.14]). This assumption corresponds, for lumped-parameter systems, to strictly proper transfer matrices.

4) The proof of the sufficiency above cannot be carried out with polynomial factorizations. To see this, let N_0 and D_0 be right coprime polynomial matrices with D_0^{-1} proper and such that $P = N_0D_0^{-1}$. Then, using a reasoning which is slightly more involved than the one used above, it is easy to see that there exists Q_0, proper and stable and such that $H_{yr} = N_0Q_0$.

Let $\tilde{D}_q$ and $\tilde{N}_q$ be left coprime polynomial matrices such that $Q_0 = \tilde{D}_q^{-1}\tilde{N}_q$. Hence,

$$H_{yr} = N_0\tilde{D}_q^{-1}\tilde{N}_q. \tag{1}$$

Let C_0 be a compensator with two degrees of freedom, i.e., $C_0 = \tilde{D}_1^{-1}[\tilde{N}_1 \ \ \tilde{N}_2]$, a left coprime factorization over polynomial matrices. It is straightforward to see that $H_{yr} = N_0(\tilde{D}_1D_0 + \tilde{N}_2N_0)^{-1}\tilde{N}_1$. In view of (1), choose $\tilde{D}_1$, $\tilde{N}_1$, and $\tilde{N}_2$ such that $\tilde{D}_q = \tilde{D}_1D_0 + \tilde{N}_2N_0$ and $\tilde{N}_1 = \tilde{N}_q$. Are we done? No, we are not, because there is no guarantee that C_0 is proper. Moreover, let an exogenous signal w enter the loop at the plant's input. It is easy to see that H_{uw} and H_{yw} are $D_0(\tilde{D}_1D_0 + \tilde{N}_2N_0)^{-1}\tilde{D}_1$ and $N_0(\tilde{D}_1D_0 + \tilde{N}_2N_0)^{-1}\tilde{D}_1$, respectively. It is clear that there is no guarantee that these matrices are proper with the above choices of $\tilde{D}_1$ and $\tilde{N}_2$. Again we reach the same conclusion if we consider the transfer matrices with respect to an exogenous signal entering the loop at the plant's output.

5) In [1] Chen and Zhang proved the sufficiency of the theorem using polynomial factorizations. In order to take care of the properness problem shown above, their proof is much less straightforward. The bonus is that their constructive proof is more ready to be implemented as an algorithm for actual design.

III. Feedback of the Measured Output

In many practical situations the controlled output (y) is not accessible for feedback. Assume then that y is not accessible for feedback, but y_m is. Let $y_m = P_m u$. Define the right coprime factorization over S

$$\begin{bmatrix} P \\ P_m \end{bmatrix} = \begin{bmatrix} \bar{N} \\ N_m \end{bmatrix} D^{-1}.$$

Implement a two-parameter compensator

$$u = \tilde{D}_c^{-1}[Q \mid -\tilde{N}_c]\begin{bmatrix} r \\ y_m \end{bmatrix}$$

with $\tilde{D}c$, $\tilde{N}_c$, and $Q \in M(S)$ and $\tilde{D}_c^{-1}\tilde{N}_c$ a stabilizer of the loop. It is well known [5] that there exists such a loop stabilizer if and only if N_m and D are right coprime. If this is the case, it is easy to prove (see [5]) that $H_{yr} = \bar{N}Q$.

Now let $H_{yr} = PT = \bar{N}D^{-1}T, T \in M(S)$.

But $H_{yr} \in M(S)$ implies $D^{-1}T \in M(S)$ if $\bar{N}$ and D are right coprime. So we have the following.

Theorem 2: Let y_m be the accessible output for feedback. Assume that both (N_m, D) and $(\bar{N}, D)$ are right coprime pairs. Then H_{yr} is implementable if and only if it is proper and stable and can be decomposed as $H_{yr} = PT$, where T is proper and stable.

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